One Symmetry to Rule Them All

Arjun Bagchi

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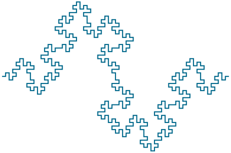

"Higher Spins, Strings and Duality", Galileo Galilei Institute, Florence.
May 9, 2013

OUTLINE

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- Conformal Symmetry: Primary tool in Theoretical Physics.
  - Central to understanding QFTs through RG fixed points.
  - Study of critical phenomena.

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  - *Space-time symmetry*: Extensively used in holographic studies as the symmetry of the field theory dual to AdS_3 (and dS_3).
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  - *Gauge symmetry*: On the world-sheet of String Theory. Residual symmetry after fixing conformal gauge in the closed bosonic string.
- Enormous success in both these avenues.

A DIFFERENT SYMMETRY

Another symmetry governed by the Galilean Conformal Algebra (GCA) has arisen in very different contexts recently.

- Constructed as a limit of the symmetries of a CFT.
- Infinite dimensional in all spacetime dimensions.

Today we will confine ourselves to 2 dimensions.

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Focus of the talk today: [Reference: A Bagchi 1303.0291]

- 2d GCA can be realised as a *Gauge symmetry*.
- On the world-sheet of String Theory in the *Tensionless Limit*.
- Residual symmetry after fixing analogue of the conformal gauge in the closed tensionless bosonic string.

TENSIONLESS STRINGS: WHY BOTHER?

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Lacking: An organising principle (like 2d CFT for string theory). We aim to rectify this.

OUTLINE

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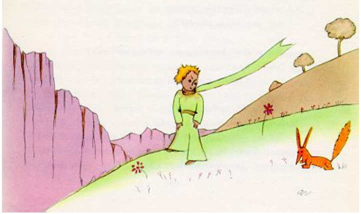
HOW TO TAKE LIMITS OR INÖNÜ-WIGNER CONTRACTIONS

A simple example.

$SO(3)$ maps the surface of the sphere (S^2) embedded in R_3 to itself.

- Equation for S^2: $x_1^2 + x_2^2 + x_3^2 = R^2$.
- Infinitesimal generators: $X_{ij} = x_i \partial_j - x_j \partial_i$
- Algebra: $[X_{ij}, X_{rs}] = X_{is}\delta_{jr} + X_{jr}\delta_{is} - X_{ir}\delta_{js} - X_{js}\delta_{ir}$

HOW TO TAKE LIMITS OR INÖNÜ-WIGNER CONTRACTIONS...



Take the limit $R \to \infty$. Look at the north pole: $x_{1,2} = 0$ and $x_3 = R$.

$$Y_{12} = \lim_{R\to\infty} X_{12} = x_1\partial_2 - x_2\partial_1, \quad P_i = \lim_{R\to\infty} \frac{1}{R} X_{i,3} = \lim_{R\to\infty} \frac{1}{R}(x_i\partial_3 - x_3\partial_i) \to -\partial_i$$

Redefined algebra: $\boxed{[Y_{12}, P_i] = P_1\delta_{2i} - P_2\delta_{1i}, \quad [P_1, P_2] = 0} \to$ ISO(2).

Will use this extensively to explain the limits we take.

GALILEAN CONFORMAL SYMMETRY

- Galilean algebra: symmetry algebra for Galilean invariant systems. Can be obtained as a limit of the Poincare algebra.
- Galilean Conformal Algebra (GCA): conformal generalisation of the Galilean algebra. Symmetry of non-relativistic conformal systems. Can be constructed as a limit of the relativistic Conformal algebra. [AB, Gopakumar 2009.]
- GCA is *infinite dimensional* in all spacetime dimensions. (Finite part obtained as limit and then given infinite lift.)
- In 2d, relativistic conformal symmetry is infinite dimensional.
- The infinite GCA_2 can be shown to emerge as a limit of 2d CFT symmetry. [AB, Gopakumar, Mandal, Miwa 2009.]
- The algebra looks like:

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} + C_1 m(m^2-1)\delta_{m+n,0}, \quad [M_m, M_n] = 0. \\ [L_m, M_n] &= (m-n)M_{m+n} + C_2 m(m^2-1)\delta_{m+n,0}. \end{aligned} \tag{1}$$

2D GCA AS SPACETIME SYMMETRY - I

NON-RELATIVISTIC LIMIT OF AdS_3/CFT_2

[AB, Gopakumar 2009; AB, Gopakumar, Mandal, Miwa 2009.]

Boundary Theory:

- Non-relativistic theory with 2d GCA symmetry (GCFT).
- Can compute e.g. correlation functions and Ward identities.
- Quantities obtained as a limit of 2d CFTs or directly from 2d GCFT.
- Realised e.g. in non-relativistic hydrodynamical systems.

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Bulk Theory:

- Bulk theory is a non-relativistic version of AdS_3.
- This has a structure of $AdS_2 \otimes R$ (non-trivially fibred).
- Generalisation of usual Newton-Cartan spacetimes.
- 2d-GCA is recovered as asymptotic symmetries.

2d GCA as Spacetime Symmetry - II

Flat Space Holography

- Asymptotic symmetries of 3d flat space $\Rightarrow$ BMS_3 algebra. [Barnich, Compere 2006.]
- This is isomorphic to the 2d GCA [AB 2010].

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Successes: (1) Entropy of 3d flat cosmological horizons. [AB, Detournay, Fareghbal, Simon 2012; Barnich 2012]

- Flat limit of non-extremal BTZ.
- Outer radius $\rightarrow \infty$. Spacetime ⇒ inside of BTZ outer horizon.
- Radial and time directions interchanged. Cosmology.
- Inner radius survives limit. Cosmological Horizon.
- Entropy reproduced by a Cardy-like analysis of dual field theory.

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- Many other recent advances. Contracted symmetries are central to this.

[AB, Detournay, Grumiller 2012; Barnich, Gomberoff, Gonzalez 2012; Barnich, Gonzalez 2013.]

OUTLINE

Classical Closed Strings

Isberg, Lindstrom, Sundborg, Theodoridis 1993

- Start with Nambu-Goto action

$$S = -T \int d^2\xi \sqrt{-\det \gamma_{\alpha\beta}} \tag{2}$$

- To take the tensionless limit, first switch to Hamiltonian framework.
- Generalised momenta: $P_m = T\sqrt{-\gamma}\gamma^{0\alpha}\partial_\alpha X_m$.
- Constraints: $P^2 + T^2\gamma\gamma^{00} = 0,\ \ P_m\partial_\alpha X^m = 0$.
- Hamiltonian of the system: $\mathcal{H} = \lambda(P^2 + T^2\gamma\gamma^{00}) + \rho^\alpha P_m\partial_\alpha X^m$.
- Action after integrating out momenta:

$$S = \frac{1}{2}\int d^2\xi \, \frac{1}{2\lambda}\left[\dot{X}^2 - 2\rho^\alpha \dot{X}^m\partial_\alpha X_m + \rho^a\rho^b\partial_b X^m\partial_a X_m - 4\lambda^2 T^2\gamma\gamma^{00}\right] \tag{3}$$

- Identifying $g^{\alpha\beta} = \begin{pmatrix} -1 & \rho \\ \rho & -\rho^2 + 4\lambda^2 T^2 \end{pmatrix}$

  Action takes the familiar Weyl-invariant form

$$S = -\frac{T}{2}\int d^2\xi \sqrt{-g} g^{\alpha\beta}\partial_\alpha X^m \partial_\beta X^n \eta_{mn}. \tag{4}$$

CLASSICAL TENSIONLESS CLOSED STRINGS

Isberg, Lindstrom, Sundborg, Theodoridis 1993

- Tensionless limit can be taken at various steps.
- Metric density $T\sqrt{-g}g^{\alpha\beta}$ degenerates and is replaced by a rank-1 matrix $V^\alpha V^\beta$ where V^α is a vector density

$$V^\alpha \equiv \frac{1}{\sqrt{2}\lambda}(1, \rho^a) \quad (5)$$

- Action in $T \to 0$ limit

$$S = \int d^2\xi \; V^\alpha V^\beta \partial_\alpha X^m \partial_\beta X^n \eta_{mn}. \quad (6)$$

- Tensionless action is invariant under world-sheet diffeomorphisms.
- Fixing gauge: "Conformal" gauge: $V^\alpha = (v, 0)$ (v: constant).

- Tensile: Residual symmetry after fixing conformal gauge = Vir $\otimes$ Vir. Central to understanding string theory.
- Tensionless: Similar residual symmetry left over after gauge fixing.

Tensionless Closed Strings: Symmetries

Isberg, Lindstrom, Sundborg, Theodoridis 1993

- Tensionless residual symmetries:

$$\delta\xi^\alpha = \lambda^\alpha,\ \lambda^\alpha = (f'(\sigma)\tau + g(\sigma), f(\sigma)) \qquad \text{where} \quad f, g = f(\sigma), g(\sigma)$$

- Define: $L(f) = f'(\sigma)\tau\partial_\tau + f(\sigma)\partial_\sigma, \quad M(g) = g(\sigma)\partial_\tau.$
- Expand: $f = \sum a_n e^{in\sigma}$, $g = \sum b_n e^{in\sigma}$

$$\begin{aligned} L(f) &= \sum_n a_n e^{in\sigma}(\partial_\sigma + in\tau\partial_\tau) = -i\sum_n a_n L_n \\ M(g) &= \sum_n b_n e^{in\sigma}\partial_\tau = -i\sum_n b_n M_n. \end{aligned}$$

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- 2d GCA!! (Central terms: Isberg et al find $C_1 = C_2 = 0$).

TENSIONLESS STRINGS: SYMMETRIES AS A LIMIT

A Bagchi 2013

- Tensile string: Residual symmetry in conformal gauge $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$:

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \quad [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}m(m^2-1)\delta_{m+n,0} \quad (8)$$

- World-sheet is a cylinder. Symmetry best expressed as 2d conformal generators on the cylinder.

$$\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}} \quad (9)$$

where $\omega, \bar{\omega} = \tau \pm \sigma$. Vector fields generate centre-less Virasoros.

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- Ends of closed string identified $\Rightarrow$ limit best viewed as $(\sigma \to \sigma, \tau \to \epsilon\tau, \epsilon \to 0)$.
- Define

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}). \tag{10}$$

- New vector fields (L_n, M_n) well-defined in limit and given by:

$$L_n = ie^{in\sigma}(\partial_\sigma + in\tau\partial_\tau), \quad M_n = ie^{in\sigma}\partial_\tau. \tag{11}$$

- These are *exactly the generators defined previously* . Close to form the 2d GCA.

Central Terms and Critical Dimensions

A Bagchi 2013

- A lot of debate about critical dimensions for tensionless strings.
- From the point of view of the contraction, the answer is simple.
- To generate non-zero C_1, the parent Virasoro should have $c \neq \bar{c}$.
- To generate non-zero $C_2 \Rightarrow c, \bar{c} \sim \epsilon^{-1}$.

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- Parent has no diffeomorphism anomaly $\Rightarrow c = \bar{c} \Rightarrow C_1 = 0$.
- Parent has finite number of world-sheet fields $\Rightarrow c, \bar{c} \not\sim \epsilon^{-1} \Rightarrow C_2 = 0$.

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   - Need a calculation of the analogue of the Weyl anomaly in 2d GCA.

- More interested in Option A $\Rightarrow$ limit does not generate additional constraints.
- Tensionless strings (as a limit of usual strings) consistent in *any dimensions*.
- Good feature. Don't want limit of a theory to be consistent in a dimension different from the original theory!

GCA EM-TENSOR

A Bagchi 2013

- Generators of Virasoro $\Rightarrow$ modes of E-M tensor.

GCA EM-TENSOR

A Bagchi 2013

- Generators of Virasoro ⇒ modes of E-M tensor.
- Spectrum of tensile string theory (in conformal gauge in flat space)
  - Quantise world sheet theory as a theory free scalar fields.
  - Constraint: vanishing of EOM of metric (which is fixed to be flat).
  - Op form: Physical states vanish under action of modes of E-M tensor.
  - Forms basis of decoupling negative norm states from Hilbert space.
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GCA EM-TENSOR

A Bagchi 2013

- Generators of Virasoro $\Rightarrow$ modes of E-M tensor.
- Spectrum of tensile string theory (in conformal gauge in flat space)
  - Quantise world sheet theory as a theory free scalar fields.
  - Constraint: vanishing of EOM of metric (which is fixed to be flat).
  - Op form: Physical states vanish under action of modes of E-M tensor.
  - Forms basis of decoupling negative norm states from Hilbert space.
- Important to construct EM-tensor of GCA and expand in modes.
- EM tensor for 2d CFT on cylinder:

$$T_{cyl} = z^2 T_{plane} - \frac{c}{24} = \sum_n \mathcal{L}_n e^{-in\omega} - \frac{c}{24}; \quad \bar{T}_{cyl} = \sum_n \bar{\mathcal{L}}_n e^{-in\bar{\omega}} - \frac{\bar{c}}{24} \tag{12}$$

- GCA EM tensor

$$T_{(1)} = \lim_{\epsilon\to 0} \left(T_{cyl} - \bar{T}_{cyl}\right) = \sum_n (L_n - in\tau M_n) e^{-in\sigma} - \frac{C_1}{2} \tag{13}$$

$$T_{(2)} = \lim_{\epsilon\to 0} \epsilon\left(T_{cyl} + \bar{T}_{cyl}\right) = \sum_n M_n e^{-in\sigma} - \frac{C_2}{2} \tag{14}$$

GCA EM-TENSOR ..

A Bagchi 2013

- Spectrum of tensionless strings: physical spectrum restricted by constraint

$$\langle \text{phys}|T_{(1)}|\text{phys}'\rangle = 0, \quad \langle \text{phys}|T_{(2)}|\text{phys}'\rangle = 0. \tag{15}$$

- Equivalently,

$$L_n|\text{phys}\rangle = 0, \quad M_n|\text{phys}\rangle = 0 \quad \text{for } n > 0 \tag{16}$$

- Important step in building tensionless spectrum from GCA methods.

OUTLINE

The Other Contraction

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- Euclidean worldsheet $\Rightarrow \sigma, \tau$ are on the same footing.
- So contraction in $\tau \equiv$ contraction in σ.
- $(\sigma, \tau) \to (\epsilon\sigma, \tau)$ should yield same symmetry algebra as $(\sigma, \tau) \to (\sigma, \epsilon\tau)$.
- We have seen $(\sigma, \tau) \to (\sigma, \epsilon\tau) \Rightarrow \text{Vir} \otimes \text{Vir} \to$ 2d-GCA.

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- Now define $\tilde{L}_n = \mathcal{L}_n + \bar{\mathcal{L}}_n, \quad \tilde{M}_n = \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n)$
- The generators take the form

$$\tilde{L}_n = ie^{in\tau}(\partial_\tau + in\sigma\partial_\sigma), \quad \tilde{M}_n = ie^{in\tau}\partial_\sigma. \tag{17}$$

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- Tensionless string behaves like a collection of massless point particles: as expected from the "dual" contraction.

THEORY ON A TORUS.

A Bagchi 2013

- As a further check: look at the theory on a torus.
- Modular invariance in 2d CFT $\rightarrow$ density of states via the Cardy formula.

- Demand $\Rightarrow$ 2d GCA derives a modular invariance in the limit.
- Use this to compute a Cardy-like formula.
- Can do this in both the limits discussed.
- States: $L_0|h_L, h_M\rangle = h_L|h_L, h_M\rangle, \quad M_0|h_L, h_M\rangle = h_M|h_L, h_M\rangle$

Cardy-like formula:
$$S = 2\pi\left(h_L\sqrt{\frac{C_2}{2h_M}} + C_1\sqrt{\frac{h_M}{2C_2}}\right). \tag{19}$$

- Final answers are identical and don't depend on details of the limit.
- Further evidence that the two contractions are equivalent.

SUMMARY.

Principle observations:

- 2d GCA governs field theories dual to non-relativistic AdS_3 and 3d flatspace.
- Surprisingly, 2d GCA also appears as the residual gauge symmetry after "conformal" gauge-fixing in tensionless strings.
- Role of 2d GCA in tensionless strings ⇔ Role of 2d CFTs in tensile strings.
- Can use techniques developed in other contexts to understand tensionless strings.
- Much of this can be developed in direct analogy to 2d CFTs. Answers can also be arrived at by a limiting procedure.

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Numerous open questions and possible directions.

OUTLINE

QUESTIONS AND FUTURE DIRECTIONS

1. Open Strings

- Existing body of literature about tensionless open strings. [Sagnotti, Tsulaia 2003]
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  1. Both correct $\Rightarrow$ tensionless open and closed strings behave very differently!
  2. One of them incorrect.
- Closed strings are more fundamental. Can have a theory of just closed strings.
- But open strings would always form closed strings in one-loop order.
- If this still holds for tensionless strings, need to re-examine open string analysis carefully from 2d-GCA point of view.

QUESTIONS AND FUTURE DIRECTIONS

2. Connections to Flat Holography

3. Connections to Higher-Spin Holography

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- Tensile strings on $AdS_3 \otimes X^7$. CFT_2 on world-sheet and CFT_2 in spacetime.
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3. Connections to Higher-Spin Holography

- Tensionless strings in AdS_3 ala Giveon-Kutasov-Seiberg.
- Appropriate limit of their construction?
- Other avenue: Strings on group manifolds $\rightarrow$ WZW construction.
- Tensionless limit: critical tuning of level of affine algebra [Lindstrom, Zabzine 2004].
- But they continue to use Virasoro constructions even at the critical point.
- Should use contracted algebra and redo things like Sugawara constructions.
- Revisit using GCA techniques.

QUESTIONS AND FUTURE DIRECTIONS

4. Others

- Construction of tensionless spectrum via GCA methods and comparing with existing literature.
- A inherent GCA way of calculating analogue of Weyl anomaly to determine critical dimension of tensionless string on its own.
- Supersymmetric versions.
- Many more possible avenues..
- Perhaps could write a tensionless version of Green-Schwarz-Witten. :-)

Thank you!