Tensorial superspace approach to higher spin theories

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Based on the papers with J. Lukierski, D. Sorokin, M. Tonin,
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(time ordering - from 1998 till present- is used)

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Outline

Introduction

- The interacting theory of higher spin fields was constructed by Misha Vasiliev in late 80th. [Fradkin & Vasiliev 87, Vasiliev 88-89]
- Misha's interacting massless h-spin theory is formulated with the use of noncommutative star product and has quite a complicated structure.
- Not so many exact solutions of this theory are known. The known action principle [P. Sundell, N. Boulanger, N. Colombo] is quite unusual. Some properties are to be clarified.
- This stimulates not only its extensive study, but also a search for alternative frameworks to reformulate it/to construct interacting higher spin theories.
- One of such frameworks is provided by 'tensorial superspace'.

Introduction

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- Not so many exact solutions of this theory are known. The known action principle [P. Sundell, N. Boulanger, N. Colombo] is quite unusual. Some properties are to be clarified.
- This stimulates not only its extensive study, but also a search for alternative frameworks to reformulate it/to construct interacting higher spin theories.
- One of such frameworks is provided by 'tensorial superspace'.
- Its brief review will be the subject of the present talk.

Outline

$\Sigma^{(10|4)}$

- Fronsdal [1985]: tensorial space

$$\Sigma^{(10|0)} = \{x^m, y^{mn}\}\ , \quad y^{mn} = -y^{nm} \quad m,n = 0,1,2,3$$

  is the natural space for the 4D massless (=)conformal higher spin theories.

- The reason is clearer if we notice that

$$\Sigma^{(10|0)} = \{X^{\alpha\beta}\}, \qquad X^{\alpha\beta} = X^{\beta\alpha}, \qquad \alpha,\beta = 1,..,4$$

$$X^{\alpha\beta} = X^{\beta\alpha} \Rightarrow \quad X^{\alpha\beta} = x^m \gamma_m^{\alpha\beta} + \frac{1}{2} y^{mn} \gamma_{mn}^{\alpha\beta}\ .$$

- The first dynamical model in the superspace generalization of $\Sigma^{(10|0)}$,

$$\Sigma^{(10|4)} = \{x^m, y^{mn}, \theta^\alpha\} = \{X^{\alpha\beta}, \theta^\alpha\}, \qquad \alpha,\beta = 1,..,4$$

  was constructed in 1998 [I.B. + J. Lukierski MPLA 1999].

- Its quantization [I.B. + J. Lukierski + D. Sorokin 1999] gave the tower of conformal massless higher spin fields in D=4.

- Actually this 'generalized superparticle model' [I.B.+ J. Lukierski 1999] was formulated in

$$\Sigma^{(\frac{n(n+1)}{2}|n)} = \{X^{\alpha\beta}, \theta^{\alpha}\}, \qquad \alpha, \beta = 1, .., n$$

  where n is dim. of a min. spinor representation in D dimensions.
- It is D dimensional as far as $x^m =\propto \Gamma^m_{\alpha\beta} X^{\alpha\beta}$, $m = 0, 1, ..., (D-1)$.
- The additional tensorial coordinates $y^{m_1 \dots m_p} =\propto \Gamma^{m_1 \dots m_p}_{\alpha\beta} X^{\alpha\beta}$
- correspond to tensorial central charges of most general D-dim SUSY algebra, $\{Q_\alpha, Q_\beta\} = \mathcal{P}_{\alpha\beta} = \Gamma^m_{\alpha\beta} P_m + \Gamma^{m_1 \dots m_p}_{(\alpha\beta)} Z_{m_1 \dots m_p}$.
- Only $Z_{m_1 \dots m_p}$ with p, D obeying $\Gamma^{m_1 \dots m_p}_{\alpha\beta} = \Gamma^{m_1 \dots m_p}_{(\alpha\beta)}$ are present. Hence

D	n	$\frac{n(n+1)}{2} = \frac{\#\ of\ central}{charges}$	$Z_{m_1 \dots m_p}$	$y^{m_1 \dots m_p}$
4	4	$10 = 4 + 6$	Z_{mn}	y^{mn}
6	8	$36 = 6 + 30$	$Z^{I(=1,2,3)}_{mnp}$	$y^{mnp}_I,$
10	16	$136 = 10 + 126$	$Z_{m_1 \dots m_5}$	$y^{m_1 \dots m_5}$
11	32	$528 = 11 + 517$	$Z_{mn}, Z_{m_1 \dots m_5}$	$y^{mn}, y^{m_1 \dots m_5}$

- The action of [I.B.+ J.L. 1999]: $S = \int d\tau \lambda_\alpha \lambda_\beta (\dot{X}^{\alpha\beta}(\tau) - i\dot{\theta}^{(\alpha}\theta^{\beta)})$
- contains a huge amount of additional coordinate functions in $X^{\alpha\beta}(\tau)$.

Preonic superparticle action

- In addition to coordinate functions $X^{\alpha\beta} = X^{\alpha\beta}(\tau)$, $\theta^\alpha = \theta^\alpha(\tau)$,

$$S = \int d\tau \mathcal{L} \quad = \quad \int d\tau \lambda_\alpha \lambda_\beta (\dot{X}^{\alpha\beta} - i\dot{\theta}^{(\alpha}\theta^{\beta)}) = \int \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta}$$

$$\Pi^{\alpha\beta} = dX^{\alpha\beta} - id\theta^{(\alpha}\,\theta^{\beta)}\,, \quad \Pi^{\alpha\beta}(\tau) := d\tau \Pi^{\alpha\beta}_\tau$$

  contains auxiliary bosonic spinor $\lambda_\alpha = \lambda_\alpha(\tau)$.
- The canonical momentum $\mathcal{P}_{\alpha\beta} := \frac{\partial \mathcal{L}}{\partial \dot{X}^{\alpha\beta}}$ is expressed through λ_α,

$$\boxed{\mathcal{P}_{\alpha\beta} = \lambda_\alpha \lambda_\beta}$$

- $\Leftarrow$ 'twistorial dimensional reduction': momentum d.o.f.s: $\frac{n(n+1)}{2} \mapsto n$.
- $4D: \;\; 10 \mapsto 4$, $6D: \;\; 36 \mapsto 8$, $10D: \;\; 136 \mapsto 16$, $11D: \;\; 528 \mapsto 32$,
- In D=4,6,10 (but not in D=11) we have also another two effects
- $p_m \propto \mathcal{P}_{\alpha\beta}\Gamma_m^{\alpha\beta} = \lambda\Gamma_m\lambda$ is light–like, $p_m p^m = 0$. $\Leftarrow$ famous $\Gamma_m^{\alpha(\beta}\Gamma^{\gamma\delta)\,a} = 0$.
- $p_m p^m = 0$ suggests that the spectrum of the quantum states of the model consists of masseless particles.
- But to this end one has to prove the spectrum is discreet.

Spectrum of D=4,6,10 preonic superparticle

- In D=4,6,10, this is the case due to 'twistorial compactification':
- the spaces $\{\lambda\}/\{p^m\}_{p_np^n=0} = \mathbb{S}^{2D-5}/\mathbb{S}^{D-2}$ is isomorphic to $\mathbb{S}^{D-3} = (\mathbb{S}^1, \mathbb{S}^3, \mathbb{S}^7)$ spheres (Hopf fibrations): $\boxed{\{\lambda\}/\{p^m\}_{p_np^n=0} = \mathbb{S}^{D-3}}$
- In $D = 11$ $p_mp^m = 0$ is nonvanishing (arbitrary!) nor S^{31}/S^{11} (nor S^{31}/S^9) is known to be a sphere (or a compact space).
- The interest to this case was due to an M-theoretical perspective ('BPS preons' [I.B., J. de Azcárraga, J. Izquierdo, J. Lukierski, 2000]).
- In $D = 4, 6, 10$ the space of additional momentum variables is compact, $\mathbb{S}^{D-3} = (\mathbb{S}^1, \mathbb{S}^3, \mathbb{S}^7)$, which implies that the spectrum of corresponding coordinate variables is discreet.
- These are helicity in $D = 4$ and its generalizations in $D = 6$ and 10.
- This implies that quantum state spectrum of the D=4,6,10 'tensorial' superparticle $S = \int d\tau\lambda_\alpha\lambda_\beta(\dot{X}^{\alpha\beta}(\tau) - i\dot{\theta}^{(\alpha}\theta^{\beta)})$ is given by the complete tower of massless higher spin fields. [I.B, J.Lukierski and D.Sorokin 99].
- Also the equations of motion for higher spin fields in $\Sigma^{\frac{n(n+1)}{2}|n)}$ can be obtained by quantizing $S = \int d\tau\lambda_\alpha\lambda_\beta(\dot{X}^{\alpha\beta} - i\dot{\theta}^{(\alpha}\theta^{\beta)})$.
- But to lighten the representation, we will go another way.

Outline

Higher spin equations in tensorial superspace

- Free equations of motion for all the field strengths of all bosonic and fermionic higher spin fields can be collected in [M. Vasiliev 2001]

$$\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0 \qquad \Leftrightarrow \qquad (\partial_{\alpha\beta}\partial_{\gamma\delta} - \partial_{\alpha\gamma}\partial_{\beta\delta})\, b(X) = 0\,,$$
$$\partial_{\alpha[\beta} f_{\gamma]}(X) = 0 \qquad \Leftrightarrow \qquad (\partial_{\alpha\beta} f_\gamma(X) - \partial_{\alpha\gamma} f_\beta(X)) = 0\,.$$

- where $\partial_{\alpha\beta} := \frac{1}{2}\frac{\partial}{\partial X^{\alpha\beta}}$ and $f_\beta(X) = f_\beta(X^{\alpha\beta})$ is fermionic.
- In D=4 $\{X^{\alpha\beta}\} = \{x^m, y^{mn}\}$

$$b(x,\, y) = \phi(x) + y^{m_1 n_1} F_{m_1 n_1}(x) + y^{m_1 n_1}\, y^{m_2 n_2}\, \hat{R}_{m_1 n_1, m_2 n_2}(x) + \\ + \sum_{s=3}^{\infty} y^{m_1 n_1} \dots y^{m_s n_s}\, \hat{R}_{m_1 n_1, \cdots, m_s n_s}(x)\,,$$
$$f^\alpha(x,y) = \psi^\alpha(x) + y^{m_1 n_1}\, \hat{\mathcal{R}}^\alpha_{m_1 n_1}(x) + \\ + \sum_{s=\frac{5}{2}}^{\infty} y^{m_1 n_1} \dots y^{m_{s-\frac{1}{2}} n_{s-\frac{1}{2}}}\, \hat{\mathcal{R}}^\alpha_{m_1 n_1, \cdots, m_{s-\frac{1}{2}} n_{s-\frac{1}{2}}}(x)\,.$$

- $\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0 \;\Rightarrow\; \begin{cases} \partial_{[m} F_{nk]} = 0\,,\; \partial_{[m_3} R_{m_1 m_2] n_1 n_2} = 0\,, \dots \\ \Box\phi(x) = 0\,, \partial^m F_{mn} = 0\,,\; \partial^{m_1} R_{m_1 m_2 n_1 n_2} = 0 \dots \end{cases},$
  $\partial_{\alpha[\beta} f_{\gamma]}(X) = 0 \;\Rightarrow\; \{\partial\!\!\!/\psi = 0, \dots$

Higher spin equations in 4D tensorial superspace

- In a more schematic notation $^{[2]} :=^{[mn]} \equiv\, _{-}{}^{[nm]};\ ^{[2]_1} :=^{[m_1 n_1]}$

$$b(x,\, y) = \phi(x) + y^{[2]} F_{[2]}(x) + y^{[2]_1}\, y^{[2]_2}\, \hat{R}_{[2]_1 [2]_2}(x) +$$
$$+ \sum_{s=3}^{\infty} y^{[2]_1} \cdots y^{[2]_s}\, \hat{R}_{[2]_1 \cdots [2]_s}(x)\,,$$
$$f^{\alpha}(x,y) = \psi^{\alpha}(x) + y^{[2]}\, \hat{\mathcal{R}}^{\alpha}_{[2]}(x) + \sum_{s=\frac{5}{2}}^{\infty} y^{[2]_1} \cdots y^{[2]_{s-1/2}}\, \hat{\mathcal{R}}^{\alpha}_{[2]_1 \cdots [2]_{s-1/2}}(x).$$

- and eqs. for higher spin curvatures are (with $D=4$)

$$\partial_{[m_1} R_{m_2\, n_1]n_2\ [2]_3 \cdots [2]_s} = 0\,, \qquad \partial^{m_1} R_{m_1 m_2\,,\,[2]_2 \cdots [2]_s} = 0\,.$$
$$R_{[m_1 m_2\ n_1]n_2\ [2]_3 \cdots [2]_s} = 0 \quad \Leftrightarrow \quad R = \underbrace{\begin{matrix}\square \ldots \square \\ \square \ldots \square\end{matrix}}_{s}$$

- $\Leftrightarrow\ R_{m_1 n_1, \cdots, m_s n_s} = \sigma_{m_1\ n_1}^{A_1 A_{s+1}} \cdots \sigma_{m_s\ n_s}^{A_s A_{2s}}\, C_{A_1 \cdots A_s A_{s+1} \cdots A_{2s}} +\ c.c.$
- where symmetric spin-tensor $C_{A_1 \cdots A_s A_{s+1} \cdots A_{2s}}$ and its c.c. obey the Bargmann–Wigner equations

$$\partial^{B\dot{B}} C_{BA_1 \ldots A_{2s-1}}(x) = 0\,, \quad \partial^{B\dot{B}} C_{\dot{B}\dot{A}_1 \ldots \dot{A}_{2s-1}}(x) = 0\,.$$

Higher spin equations in 10D tensorial superspace

- In D=10 we denote $^{[5]} :=^{[mnklp]}$; $^{[5]_1} :=^{[m_1 n_1 k_1 l_1 p_1]}$

$$b(x,\, y) = \phi(x) + y^{[5]} F_{[5]}(x) + y^{[5]_1}\, y^{[5]_2}\, \hat{R}_{[5]_1 [5]_2}(x) +$$
$$+ \sum_{s=3}^{\infty} y^{[5]_1} \cdots y^{[5]_s}\, \hat{R}_{[5]_1 \cdots [5]_s}(x)\,,$$
$$f_\alpha(x, y) = \psi_\alpha(x) + y^{[2]}\, \hat{\mathcal{R}}_{\alpha\,[5]}(x) + \sum_{s=\frac{5}{2}}^{\infty} y^{[5]_1} \cdots y^{[5]_{s-1/2}}\, \hat{\mathcal{R}}_{\alpha\,[5]_1 \cdots [5]_{s-1/2}}(x)\,.$$

- and eqs. for higher spin curvatures are (with $D = 10$)

$$\partial_{[m_6} R_{m_1 \cdots m_5],\, [\frac{D}{2}]_2 \cdots [\frac{D}{2}]_s} = 0\,, \qquad \partial^n R_{n[4]_1\,,\,[5]_2 \cdots [5]_s} = 0\,.$$

$$R_{[6]_1\, [4]_2\, [5]_3 \cdots [\frac{D}{2}]_s} \equiv R_{[m_1 \cdots m_5\ n_1] n_2 \cdots n_5\ [5]_3 \cdots [\frac{D}{2}]_s} = 0 \quad \Leftrightarrow \quad R = \underbrace{\begin{array}{l} \square\square\ldots\ldots\square \\ \square\square\ldots\ldots\square \\ \square\square\ldots\ldots\square \\ \square\square\ldots\ldots\square \\ \square\square\ldots\ldots\square \end{array}}_{s}$$

- $\Leftarrow$ $\boxed{\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0}$; fermionic counterparts $\Leftarrow$ $\boxed{\partial_{\alpha[\beta} f_{\gamma]}(X) = 0}$.

On relation with preonic superparticle

- How to see the relation with preonic (tensorial superparticle)?

$$S = \int d\tau \mathcal{L} \quad = \quad \int d\tau \lambda_\alpha \lambda_\beta (\dot{X}^{\alpha\beta} - i\dot{\theta}^{(\alpha}\theta^{\beta)}) = \int \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta}$$

- This produces the generalization of the Cartan-Penrose relation:

$$\boxed{\mathcal{P}_{\alpha\beta} = \lambda_\alpha \lambda_\beta}$$

- $\boxed{\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0}$ in the momentum representation reads $p_{\alpha[\beta}p_{\gamma]\delta}\, b(p) = 0$
- $\Rightarrow b(p) \neq 0$ when $rank(p_{\alpha\beta}) = 1 \Leftrightarrow p_{\alpha\beta} = \lambda_\alpha \lambda_\beta$ for some λ_α.
- $\Rightarrow \partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0$ is solved by $b(X) = \int d^n\lambda \Phi(X, \lambda)$, where

$$\boxed{(\partial_{\alpha\beta} - i\lambda_\alpha\lambda_\beta)\Phi(X, \lambda) = 0}$$

- Fermionic $\partial_{\alpha[\beta} f_{\gamma]}(X) = 0$ is solved by $f_\alpha(X) = \int d^n\lambda\, \lambda_\alpha \Phi(X, \lambda)$.
- "Preonic wave function" $\Phi(X, \lambda) = 0$ is not exactly wavefunction: it depends on both coordinates and momenta variables ($p_{\alpha\beta} = \lambda_\alpha\lambda_\beta$).

4D. Preonic equation and unfolding equations.

- In D=4 $\lambda_\alpha = (\lambda_A, \bar\lambda^{\dot A})$, $y^{mn} =\propto (\sigma^{[m}\tilde\sigma^{n]})_{AB} y^{AB} + c.c.$ and

$$\boxed{(\partial_{\alpha\beta} - i\lambda_\alpha\lambda_\beta)\Phi(X,\lambda) = 0} \quad \Leftrightarrow \quad \begin{cases} \left(\frac{\partial}{\partial x^{A\dot B}} - i\lambda_A\bar\lambda_{\dot B}\right)\Phi(x,y;\lambda,\bar\lambda) = 0\,, \\ \left(\frac{\partial}{\partial y^{AB}} - i\lambda_A\lambda_B\right)\Phi(x,y;\lambda,\bar\lambda) = 0\,, \\ \left(\frac{\partial}{\partial y^{\dot A\dot B}} - i\bar\lambda_{\dot A}\bar\lambda_{\dot B}\right)\Phi(x,y;\lambda,\bar\lambda) = 0\,, \end{cases}$$

- Misha Vasiliev prefers to work with a Fourier transfrom $C(X,y^\alpha) = \int d^n\lambda\, e^{i\lambda_\alpha y^\alpha}\Phi(X,\lambda)$ which obeys the unfolded eqs.

$$(\partial_{\alpha\beta} + i\frac{\partial}{\partial Y^\alpha}\frac{\partial}{\partial Y^\beta})C(X,y) = 0 \quad \Leftrightarrow \quad \begin{cases} \left(\frac{\partial}{\partial x^{A\dot B}} + i\frac{\partial}{\partial Y^A}\frac{\partial}{\partial \bar Y^{\dot B}}\right)C(x,y;Y,\bar Y) = 0\,, \\ \left(\frac{\partial}{\partial y^{AB}} + i\frac{\partial}{\partial Y^A}\frac{\partial}{\partial Y^B}\right)C(x,y;Y,\bar Y) = 0\,, \\ \left(\frac{\partial}{\partial y^{\dot A\dot B}} + i\frac{\partial}{\partial \bar Y^{\dot A}}\frac{\partial}{\partial \bar Y^{\dot B}}\right)C(x,y;Y,\bar Y)) = 0\,, \end{cases}$$

- One can show [Vasiliev 2001] that in $C(X,y) = b(X) + f_\alpha(X)\,y^\alpha + \sum_{n=2}^\infty C_{\alpha_1\cdots\alpha_n}(X)\,y^{\alpha_1}\cdots y^{\alpha_n}$ the only dynamical fields are scalar $b(X)$ and spinor (or 'svector') $f_\alpha(X)$ which satisfy $\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0$ and $\partial_{\alpha[\beta} f_{\gamma]}(X) = 0$.

Superfield generalization

- Let us introduce covariant Grassmann derivative in $\Sigma^{(\frac{n(n+1)}{2}|n)}$

$$D_\alpha = \partial/\partial\theta^\alpha + i\theta^\beta\partial_{\beta\alpha} \; , \qquad \{D_\alpha, D_\beta\} = 2i\partial_{\alpha\beta} \; .$$

- The (manifestly) $GL(n)$ covariant eq. [I.B., Pasti, Sorokin, Tonin 2004]

$$D_{[\alpha}D_{\beta]}\Phi(X,\theta) = 0$$

- $\Rightarrow$ in $\Phi(X^{\alpha\beta},\theta^\gamma) = b(X) + f_\alpha(X)\,\theta^\alpha + \sum_{i=2}^n \phi_{\alpha_1\cdots\alpha_i}(X)\,\theta^{\alpha_1}\cdots\theta^{\alpha_i}$ the only dynamical fields are scalar $b(X)$ and spinor (or 'svector') $f_\alpha(X)$ which satisfy $\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0$ and $\partial_{\alpha[\beta}f_{\gamma]}(X) = 0$.
- Actually this equation possesses $OSp(1|2n)$ invariance ($OSp(1|8)$ for D=4), like $S = \int \lambda_\alpha\lambda_\beta(dX^{\alpha\beta} - id\theta^{(\alpha}\,\theta^{\beta)})$ does [I.B., Lukierski 98].

Superfield generalization

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- $\Rightarrow$ in $\Phi(X^{\alpha\beta}, \theta^\gamma) = b(X) + f_\alpha(X)\,\theta^\alpha$,
- $\partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0$ and $\partial_{\alpha[\beta} f_{\gamma]}(X) = 0$.
- Actually this equation possesses $OSp(1|2n)$ invariance ($OSp(1|8)$ for D=4), like $S = \int \lambda_\alpha\lambda_\beta(dX^{\alpha\beta} - id\theta^{(\alpha}\,\theta^{\beta)})$ does [I.B., Lukierski 98].
- Its quantization [I.B., Lukierski, Sorokin 1999]: wave function is a Clifford superfield ($\chi\chi = 1$, $\chi\theta = -\theta\chi$)

$$\Upsilon(X,\theta,\lambda,\chi) = g_0(X,\theta,\lambda) + i\chi\, g_1(X,\theta,\lambda) = \Upsilon(X,\theta,-\lambda,-\chi))$$

obeying $\boxed{(D_\alpha - \chi\,\lambda_\alpha)\Upsilon(X,\theta,\lambda,\chi) = 0.}$

- $\Rightarrow \ (D_\alpha D_\beta + \lambda_\alpha\lambda_\beta)\, g_0(X,\theta,\lambda) = 0 \Rightarrow \ D_{[\alpha}D_{\beta]}\, g_0(X,\theta,\lambda) = 0.$
- $\Phi(X,\theta) = \int \ d^n\lambda\, g_0(X,\theta,\lambda) = b(X) + f_\alpha(X)\,\theta^\alpha$,
  $b(X) = \int \ d^n\lambda\, g_0(X,0,\lambda)$,
  $f_\alpha(X) = \int \ d^n\lambda\, D_\alpha\, g_0(X,\theta,\lambda)|_{\theta=0}$,

Outline

AdS higher spin equations. Superfield form

- Thus all the free massless conformal higher spin eqs. in D=4,6,10 can be collected in one scalar eq. in $\Sigma^{(\frac{n(n+1)}{2}|n)}$ with $n=4,8,16$:

$$\boxed{D_{[\alpha}D_{\beta]}\Phi(X,\theta)=0} \Rightarrow \begin{cases} \Phi(X^{\alpha\beta},\theta^{\gamma}) = b(X) + f_{\alpha}(X)\,\theta^{\alpha}\ , \\ \partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0\,, \partial_{\alpha[\beta} f_{\gamma]}(X) = 0 \end{cases}$$

- Can we do this with (massless conformal) *AdS* higher spin equations?
- 1) What is the AdS generalization of the tensorial superspace $\Sigma^{(\frac{n(n+1)}{2}|n)}$?
- [I.B., Lukierski, Preitschopf, Sorokin 2000]: $\boxed{AdS^{(\frac{n(n+1)}{2}|n)} = OSp(1|n)}$
- In particular, $\boxed{AdS^{(10|4)} = OSp(1|4)}$
- Indeed, it is natural as far as $AdS_4 = Sp(4)/SO(1,3)$.
- $N=1$ AdS superspace is $AdS^{(4|4)} = OSp(4)/SO(1,3)$.
- Abelian algebra of Z_{mn} can be considered as a contraction of $so(1,3)$ algebra.

AdS higher spin equations. Superfield form

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$$\boxed{D_{[\alpha}D_{\beta]}\Phi(X,\theta) = 0} \Rightarrow \begin{cases} \Phi(X^{\alpha\beta},\theta^\gamma) = b(X) + f_\alpha(X)\,\theta^\alpha \;, \\ \partial_{\alpha[\beta}\partial_{\gamma]\delta}\, b(X) = 0\,, \partial_{\alpha[\beta} f_{\gamma]}(X) = 0 \end{cases}$$

- Can we do this with (massless conformal) *AdS* higher spin equations?
- 1) [I.B., Lukierski, Preitschopf, Sorokin 2000]: $\boxed{AdS^{(\frac{n(n+1)}{2}|n)} = OSp(1|n)}$
- 2) Free conformal AdS higher spin equations can be collected in

$$\left(\nabla_{[\alpha}\nabla_{\beta]} + i\frac{\varsigma}{4}C_{\alpha\beta}\right)\Phi(X,\theta) = 0\;.$$

where $\varsigma \propto \frac{1}{R_{AdS}}$, $C_{\alpha\beta} = -C_{\beta\alpha}$ is the Sp 'metric' and the $OSp(1|n)$ covariant derivatives $\nabla_\alpha, \nabla_{\alpha\beta}$ obey the $osp(1|n)$ superalgebra

$$\{\nabla_\alpha, \nabla_\beta\} = 2i\nabla_{\alpha\beta}\;, \qquad [\nabla_{\alpha\alpha'}, \nabla_\beta] = \varsigma\, C_{\beta(\alpha}\nabla_{\alpha')},$$
$$[\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] = \varsigma C_{\alpha(\gamma}\nabla_{\delta)\beta} + \varsigma C_{\beta(\gamma}\nabla_{\delta)\alpha}\;.$$

AdS higher spin equations. Component form on $Sp(n)$ space

- $\boxed{\left(\nabla_{[\alpha}\nabla_{\beta]} + i\frac{\varsigma}{4}C_{\alpha\beta}\right)\Phi(X,\theta) = 0}$ ⇒ 'component equations on $Sp(n)$ [Sorokin, Plyushchay, Tsulaia 2003]

$$\nabla_{\alpha[\beta}\nabla_{\gamma]\delta}b(X) = \frac{\varsigma}{4}\left(C_{\alpha[\beta}\nabla_{\gamma]\delta} + C_{\delta[\beta}\nabla_{\gamma]\alpha} - C_{\beta\gamma}\nabla_{\alpha\delta}\right)b(X) + \\ + \frac{\varsigma^2}{16}\left(C_{\alpha\delta}C_{\beta\gamma} - C_{\alpha[\beta}C_{\gamma]\delta}\right)b(X),$$

$$\nabla_{\alpha[\beta}f_{\gamma]}(X) = -\frac{\varsigma}{4}\left(C_{\alpha[\gamma}f_{\beta]}(X) + C_{\beta\gamma}f_{\alpha}(X)\right) .$$

where $[\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] = \varsigma C_{\alpha(\gamma}\nabla_{\delta)\beta} + \varsigma C_{\beta(\gamma}\nabla_{\delta)\alpha}$.

- The counterpart of the Clifford superfield eq. $(D_\alpha - \chi\,\lambda_\alpha)\Upsilon = 0$,

$$(\nabla_\alpha - \chi\, Y_\alpha)\Upsilon(X,\theta,\lambda,\chi) = 0\,, \qquad Y_\alpha = \lambda_\alpha - \frac{i\varsigma}{4}\,C_{\alpha\beta}\,\frac{\partial}{\partial\lambda_\beta}\,.$$

was studied in [Didenko, Valsiliev 2003].

- To be more precise, they studied its Fourier transform

$$(\nabla_\alpha - \chi\, Y_\alpha)\Upsilon(X,\theta,y^\beta,\chi) = 0\,, \quad Y_\alpha \equiv i\frac{\partial}{\partial y^\alpha} + C_{\alpha\beta}\,\frac{\varsigma}{4}y^\beta \ .$$

AdS higher spin equations. Component form on $Sp(n)$ space

$$(\nabla_\alpha - \chi Y_\alpha)\Upsilon(X,\theta,\lambda,\chi) = 0\,, \qquad Y_\alpha = \lambda_\alpha - \frac{i\varsigma}{4} C_{\alpha\beta}\frac{\partial}{\partial\lambda_\beta}\,.$$

- It results in an 'AdS preonic' equation ($Y = g_0 + \chi g_1$)

$$\left[\nabla_{\alpha\beta} - iY_{(\alpha}Y_{\beta)}\right] g_0(X,\theta,\lambda) = 0\,, \quad Y_\alpha \equiv \lambda_\alpha - \frac{i\varsigma}{4}\frac{\partial}{\partial\lambda_\alpha},$$

- and in a more general

$$(\nabla_\alpha\nabla_\beta + Y_\beta Y_\alpha)\, g_0(X,\theta,\lambda) = 0\,, \qquad Y_\alpha = \lambda_\alpha - \frac{i\varsigma}{4} C_{\alpha\beta}\frac{\partial}{\partial\lambda_\beta}\,.$$

  which also includes $\left(\nabla_{[\alpha}\nabla_{\beta]} + i\frac{\varsigma}{4}C_{\alpha\beta}\right) g_0(X,\theta,\lambda) = 0$

- Then, $\Phi(X,\theta) = \int d^n\lambda\, g_0(X,\theta,\lambda)$ obeys the superfield version of the AdS higher spin equation

$$\boxed{\left(\nabla_{[\alpha}\nabla_{\beta]} + i\frac{\varsigma}{4}C_{\alpha\beta}\right)\Phi(X,\theta) = 0}\,.$$

Preonic superparticle on $OSp(1|n)$

- $(\nabla_\alpha - \chi Y_\alpha)\Upsilon(X,\theta,\lambda,\chi) = 0$ with $Y_\alpha = \lambda_\alpha - \frac{i\varsigma}{4} C_{\alpha\beta}\frac{\partial}{\partial\lambda_\beta}(= \lambda_\alpha * \ldots)$ can be obtained by quantization of $OSp(1|n)$ superparticle

$$S = \int_{W^1} \lambda_\alpha\lambda_\beta\hat{\mathcal{E}}^{\alpha\beta} = \int d\tau\lambda_\alpha\lambda_\beta\partial_\tau\hat{Z}^M\mathcal{E}_M^{\alpha\beta}(\hat{Z}(\tau)) \; ,$$

- where $\mathcal{E}^{\alpha\beta} = dZ^M\mathcal{E}_M^{\alpha\beta}(Z)$ and $\mathcal{E}^{\alpha} = dZ^M\mathcal{E}_M^{\alpha}(Z)$ obey

$$\begin{aligned} d\mathcal{E}^{\alpha\beta} &= -i\mathcal{E}^{\alpha}\wedge\mathcal{E}^{\beta} - \zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta\beta}C_{\gamma\delta} \; , \\ d\mathcal{E}^{\alpha} &= -\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta}C_{\gamma\delta} \; , \end{aligned}$$

- $Z^M = (X^{\check{\alpha}\check{\beta}}, \theta^{\check{\alpha}})$ are local coordinates of $OSp(1|n)$ supergroup manifold
- and $Z^M = \hat{Z}^M(\tau)$ defines the embedding of W^1 in $OSp(1|n)$.
- This action possesses rigid $OSp(1|2n)$ symmetry (generalized conformal symm.) and also $(n-1)$ local fermionic κ–symmetries (3 in D=4):

$$\delta_\kappa\hat{Z}^M\mathcal{E}_M^{\alpha\beta}(\hat{Z}) = 0 \; , \qquad \delta_\kappa\hat{Z}^M\mathcal{E}_M^{\alpha}(\hat{Z})\lambda_\alpha = 0 \; .$$

Outline

Preonic superparticle on $OSp(1|n)$ and on $\Sigma^{(n(n+1)/2|n)}$

- These symmetries survive - and become simpler- in flat SSP limit

$$OSp(1|n)\underset{\zeta\mapsto 0}{\longmapsto}\Sigma^{(\frac{n(n+1)}{2}|n)}\;, \quad \mathcal{E}^{\alpha\beta}\underset{\zeta\mapsto 0}{\longmapsto}\Pi^{\alpha\beta}=dX^{\alpha\beta}-id\theta^{(\alpha}\,\theta^{\beta)},\quad \mathcal{E}^{\alpha}\underset{\zeta\mapsto 0}{\longmapsto}d\theta^{(\alpha},$$

$$\left.\begin{array}{rcl} d\mathcal{E}^{\alpha\beta} &=& -i\mathcal{E}^{\alpha}\wedge\mathcal{E}^{\beta}-\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta\beta}C_{\gamma\delta}\;, \\ d\mathcal{E}^{\alpha} &=& -\zeta\mathcal{E}^{\alpha\gamma}\wedge\mathcal{E}^{\delta}C_{\gamma\delta}\;, \end{array}\right\}\underset{\zeta\mapsto 0}{\longmapsto}\left\{\begin{array}{l} d\Pi^{\alpha\beta}=-id\theta^{\alpha}\wedge d\theta^{\beta}\;, \\ dd\theta^{\alpha}\equiv 0\;, \end{array}\right.$$

- The κ–symmetry of the $\Sigma^{(n(n+1)/2|n)}$ superparticle action $S=\int_{W^1}\lambda_\alpha\lambda_\beta\hat{\Pi}^{\alpha\beta}=\int d\tau\lambda_\alpha\lambda_\beta(\partial_\tau X^{\alpha\beta}-i\partial_\tau\theta^{(\alpha}\,\theta^{\beta)})$ reads

$$\delta_\kappa X^{\alpha\beta}=i\delta_\kappa\theta^{(\alpha}\,\theta^{\beta)}\;,\qquad \delta_\kappa\theta^{\alpha}\,\lambda_\alpha=0\;.$$

- $\delta_\kappa\theta^{\alpha}\,\lambda_\alpha=0$ can be solved in terms of $(n-1)$ bosonic spinors 'orthogonal' to λ_α: $\delta_\kappa\theta^{\alpha}=\kappa^I u_I^{\alpha}$, $u_I^{\alpha}\lambda_\alpha=0$, $I=1,...,15$.
- This makes clear that we can gauge away all but one component of $\theta^{\alpha}(\tau)$: $\boxed{\eta=\theta^{\alpha}(\tau)\lambda_\alpha}$ which is κ-invariant.
- This is related to global $OSp(1|2n)$ symmetry of the system
- but also shows that its ground state preserves all but one SUSY
- $\equiv$ is a BPS preon [I.B., de Azcárraga, Izquierdo, Lukierski, 2001].

κ symmetry and preserved SUSY, or Why tensorial superparticle is preonic?

- $S = \int_{W^1} \lambda_\alpha \lambda_\beta \hat{\Pi}^{\alpha\beta} = \int d\tau \lambda_\alpha \lambda_\beta (\partial_\tau X^{\alpha\beta} - i\partial_\tau \theta^{(\alpha}\, \theta^{\beta)})$ is invariant
- under κ–symmetry $\delta_\kappa X^{\alpha\beta} = i\delta_\kappa \theta^{(\alpha}\, \theta^{\beta)}$, $\delta_\kappa \theta^\alpha = \kappa^\alpha$, $\kappa^\alpha \lambda_\alpha = 0$
- and under rigid SUSY $\delta_\epsilon X^{\alpha\beta} = -i\delta_\epsilon \theta^{(\alpha}\, \theta^{\beta)}$, $\delta_\epsilon \theta^\alpha = \epsilon^\alpha$
- Thus $\delta\theta^\alpha = \delta_\kappa \theta^\alpha + \delta_\epsilon \theta^\alpha = \kappa^\alpha + \epsilon^\alpha$, $\kappa^\alpha \lambda_\alpha = 0$
- In the ground state of the system fermions are equal to zero: $\theta^\alpha = 0$
- so that it can be preserved by symmetries which preserve $\theta^\alpha = 0$, i.e.
- which obey $0 = \delta\theta^\alpha = \kappa^\alpha + \epsilon^\alpha$, $\kappa^\alpha \lambda_\alpha = 0$.
- This identifies all but one SUSY parameters with nontrivial parameters of κ-symmetry, $\epsilon^\alpha = -\kappa^\alpha$ and set to zero only one linear combination of the components of ϵ^α: $\boxed{\epsilon^\alpha \lambda_\alpha = 0}$.
- Thus all but one target space supersymmetry are preserved by the ground state of the tensorial superparticle.
- This ground state is $\frac{n-1}{n}$BPS, i.e. $\frac{3}{4}$BPS in D=4, $\frac{15}{16}$BPS in D=10 and $\frac{31}{32}$BPS in D=11;
- this is to say it is a BPS preon [I.B., de Azcárraga, Izquierdo, Lukierski, 2001].

$OSp(1|2n)$ symmetry of $\Sigma^{(\frac{n(n+1)}{2}|n)}$ superparticle

- $S = \int\limits_{W^1} \lambda_\alpha \lambda_\beta \hat{\Pi}^{\alpha\beta} = \int \lambda_\alpha \lambda_\beta (d\hat{X}^{\alpha\beta} - id\hat{\theta}^{(\alpha}\, \hat{\theta}^{\beta)})$ can be rewritten as

$$S = \int_{W^1} (\lambda_\alpha d\mu^\alpha - \mu^\alpha d\lambda_\alpha - id\chi\chi)\, , \qquad \begin{cases} \mu^\alpha = \hat{X}^{\alpha\beta}\lambda_\beta - \frac{i}{2}\hat{\theta}^\alpha \chi\, , \\ \chi = \hat{\theta}^\alpha \lambda_\alpha \end{cases} ,$$

[I.B.+Lukierski 98] or equivalently as

$$S = \int_{W^1} d\Upsilon^\Sigma \Xi_{\Sigma\Omega} \Upsilon^\Omega\, , \quad \Upsilon^\Sigma = \begin{pmatrix} \mu^\alpha \\ \lambda_\alpha \\ \chi \end{pmatrix} , \quad \Xi_{\Sigma\Omega} = \begin{pmatrix} 0 & \delta_\alpha{}^\beta & 0 \\ -\delta^\alpha{}_\beta & 0 & 0 \\ 0 & 0 & -i \end{pmatrix} ,$$

[I.B.+Lukierski 98]. Here $\Xi_{\Sigma\Omega}$ is the $OSp(1|2n)$ 'metric' Υ^Σ is orthosymplectic supertwistor
- carrying the index of the fundamental representation of $OSp(1|2n)$.
- Thus $S = \int\limits_{W^1} \lambda_\alpha \lambda_\beta \hat{\Pi}^{\alpha\beta} = \int_{W^1} d\Upsilon^\Sigma \Xi_{\Sigma\Omega} \Upsilon^\Omega$ is manifestly $OSp(1|2n)$ invariant.

$OSp(1|2n)$ symmetry of $OSp(1|n)$ superparticle from GL flatness of $OSp(1|n)$ supergroup manifold

- The simplest way to show the $OSp(1|2n)$ symmetry of $OSp(1|n)$ superparticle $S = \int_{W^1} \lambda_\alpha \lambda_\beta \hat{\mathcal{E}}^{\alpha\beta}$ is to use the GL(n) flatness of $OSp(1|n)$
- [Plyushchay, Sorokin, Tsulaia 2003]:

$$\boxed{\mathcal{E}^{\alpha\beta} = \Pi^{\alpha\beta}\, \mathcal{G}_\gamma{}^\alpha(X,\theta)\, \mathcal{G}_\delta{}^\beta(X,\theta)}\,, \qquad \mathcal{E}^\alpha = e^{\rho(x,\theta)}(\mathcal{D}\theta^\alpha - \theta^\alpha \mathcal{D}\rho)$$

$$\mathcal{G}_\beta{}^\alpha(x,\theta) = G_\beta{}^\alpha(x) - \tfrac{i\varsigma}{4}\left(\Theta_\beta - 2G_\beta{}^\gamma(x)\Theta_\gamma\right)\Theta^\alpha,$$

$$G^{-1}_\beta{}^\alpha(x) = \delta_\beta{}^\alpha + \tfrac{\varsigma}{2} x_\beta{}^\alpha,$$

$$\theta^\alpha = \Theta^\beta\, G^{-1}_\beta{}^\alpha(x) e^{-\rho(\Theta)}\,, \qquad e^{\rho(\Theta)} = \sqrt{1 + \tfrac{i\varsigma}{4}\Theta^\beta\Theta_\beta},$$

$$\mathcal{D}\theta^\alpha = d\theta^\alpha - \varsigma\theta^\gamma C_{\gamma\beta}\mathcal{E}^{\beta\alpha}(X,0) = d\theta^\alpha - \varsigma\theta^\gamma C_{\gamma\beta}(G^T X G)^{\beta\alpha}\,.$$

- Hence $S = \int_{W^1} \lambda_\alpha \lambda_\beta \hat{\mathcal{E}}^{\alpha\beta} = \int_{W^1} \tilde{\lambda}_\alpha \tilde{\lambda}_\beta \hat{\Pi}^{\alpha\beta}$ with $\tilde{\lambda}_\alpha = \mathcal{G}_\alpha{}^\beta(X,\theta)\lambda_\beta$
- and $OSp(1|2n)$ superconformal invariance of the $OSp(1|n)$ superparticle follows from the $OSp(1|2n)$ superconformal invariance of the $\Sigma^{(\frac{n(n+1)}{2}|n)}$ superparticle.

Outline

Geometry of curved tensorial superspace

- Tensorial supergravity, a theory dynamical curved tensorial superspace $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$ is a natural candidate for interacting higher spin theory.
- $Z^M = (X^{\check\alpha\check\beta}, \theta^{\check\gamma})$= are local coordinates of $\mathcal{M}^{(\frac{n(n+1)}{2}|n)}$. We construct the theory from the objects which are invariant under superdiffeomorphisms, $Z'^M = f^M(Z^N)$: $(\mathrm{sdet}(\partial f^M/\partial Z^N) \neq 0)$:
- Supervielbein one forms $E^{\mathcal{A}} = (E^{\alpha\beta}, E^{\alpha}) = dZ^M E_M^{\mathcal{A}}(Z)$

$$E^{\alpha\beta}(Z) = E^{\beta\alpha}(Z) = dZ^M E_M{}^{\alpha\beta}(Z)\,, \qquad E^{\alpha}(Z) = dZ^M E_M{}^{\alpha}(Z)$$

- And $GL(n)$ connection $\Omega_\beta{}^\alpha := dZ^M \Omega_{M\beta}{}^\alpha \equiv E^{\mathcal{A}}\Omega_{\mathcal{A}\beta}{}^\alpha$,
- The torsion and $GL(n)$ curvature 2–forms

$$\begin{aligned}
T^{\alpha\beta} &:= \mathcal{D}E^{\alpha\beta} \equiv dE^{\alpha\beta} - 2E^{(\alpha|\gamma} \wedge \Omega_\gamma{}^{|\beta)} =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}}\, T_{\mathcal{A}\mathcal{B}}{}^{\alpha\beta}\,,\\
T^{\alpha} &:= \mathcal{D}E^{\alpha} \equiv dE^{\alpha} - E^{\beta} \wedge \Omega_\beta{}^\alpha =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}}\, T_{\mathcal{A}\mathcal{B}}{}^{\alpha}\,,\\
\mathcal{R}_\beta{}^\alpha &:= d\,\Omega_\beta{}^\alpha - \Omega_\beta{}^\gamma \wedge \Omega_\gamma{}^\alpha =: \frac{1}{2}E^{\mathcal{B}} \wedge E^{\mathcal{A}}\, \mathcal{R}_{\mathcal{A}\mathcal{B}\beta}{}^\alpha\,.
\end{aligned}$$

Superparticle in curved tensorial superspace

- As in the case of SUGRA in usual SSP we need to restrict the supervielbein and connection by superspace constraints.
- Their essential part can be obtained by the condition of preservation of the κ–symmetry of the preonic superparticle in curved tensorial SSP [I.B., Pasti, Sorokin, Tonin JHEP 2004]
- Its action $S = \int_{W^1} \lambda_\alpha \lambda_\beta \hat{E}^{\alpha\beta}$ possesses the κ–symmetry

$$\delta_\kappa \hat{Z}^M E_M^{\alpha\alpha'}(\hat{Z}) = 0 \, , \quad \delta_\kappa \lambda_\alpha = 0 \, , \qquad \delta_\kappa Z^M E_M^{\alpha}(\hat{Z}) \lambda_\alpha = 0 \, , \quad \Leftrightarrow$$
$$\Leftrightarrow \qquad \delta_\kappa \hat{Z}^M E_M^{\alpha}(\hat{Z}) = \mu^{\alpha I} \kappa_I(\tau) \, , \quad I = 1, .., 15, \quad \mu^{\alpha I} \lambda_\alpha = 0$$

  provided supervielbein is restricted by torsion constraints

$$T^{\alpha\beta} \quad = -iE^\alpha \wedge E^\beta - 2E^{(\alpha} \wedge E^{\beta)\gamma} t_\gamma(Z) + 2E^{\gamma(\alpha} \wedge E^{\beta)\delta} R_{\gamma\delta}(Z) \, ,$$

  with some fermionic $t_\gamma(Z)$ and bosonic $R_{\gamma\delta}(Z) = -R_{\delta\gamma}(Z)$.
- As usually, the theory is still reducible and we need to impose also a number of conventional constraints, counterparts of $T_{cb}{}^a = 0$ in General Relativity. One of this can be $t_\gamma(Z) = 0$, but there are a number of others.

SUGRA constraints and their consequences

- After imposing the essential and conventional constraints and studying their consistency conditions given by Bianchi identities

$$\begin{aligned} \mathcal{D}T^{\alpha\beta} + E^{\alpha\gamma}\wedge \mathcal{R}_\gamma{}^\beta + E^{\beta\gamma}\wedge\mathcal{R}_\gamma{}^\alpha &\equiv 0\,, \\ \mathcal{D}T^\alpha + E^\beta \wedge R_\beta{}^\alpha &\equiv 0\,, \\ \mathcal{D}\mathcal{R}_\beta{}^\alpha &\equiv 0 \end{aligned}$$

- the torsion and curvature 2-forms are expressed by

$$\begin{aligned} T^{\alpha\beta} &= -iE^\alpha\wedge E^\beta + 2E^{\gamma(\alpha}\wedge E^{\beta)\delta}R_{\gamma\delta}(Z)\,, \\ T^\alpha &= 2E^{\alpha\beta}\wedge E^\gamma R_{\beta\gamma} + E^{\alpha\beta}\wedge E^{\gamma\delta}U_{\beta\gamma\delta}\,, \\ \mathcal{R}_\beta{}^\alpha &= iE^{\gamma\delta}\wedge E^\alpha U_{\beta\gamma\delta} - E^{\alpha\gamma}\wedge E^\delta(F_{\delta\beta\gamma} + \mathcal{D}_\delta R_{\beta\gamma}) - \\ &- E^{\alpha\gamma}\wedge E^{\delta\epsilon}(\mathcal{D}_{(\beta}U_{\gamma)\delta\epsilon} + \mathcal{D}_{\delta\epsilon}R_{\beta\gamma})\,. \end{aligned}$$

- in terms of 'main superfields' $R_{\beta\alpha} = -R_{\alpha\beta}$, $U_{\beta\gamma\delta}U_{\beta(\gamma\delta)}$ and $F_{\alpha\beta\gamma} = 2iU_{(\beta\gamma)\alpha} - iU_{\alpha\beta\gamma} - 2\mathcal{D}_{(\beta}R_{\gamma)\alpha}$ which obey a number of relations

$$\mathcal{D}_{[\alpha}U_{\beta]\gamma\delta} = -\mathcal{D}_{\gamma\delta}R_{\alpha\beta}\,, \quad \mathcal{D}_{(\alpha}U_{\beta)\,\gamma\delta} = -i\mathcal{D}_{(\gamma}F_{\delta)\,\alpha\beta}$$

$$\mathcal{D}_{\alpha\beta}U_{\gamma\delta\sigma} - \mathcal{D}_{\delta\sigma}U_{\gamma\alpha\beta} + 2U_{\gamma\alpha(\sigma}R_{\delta)\beta} + 2U_{\gamma\beta(\sigma}R_{\delta)\alpha} = 0\,,$$

SUGRA constraints and their sulutions

- The constraints $T^{\alpha\beta} = -iE^\alpha \wedge E^\beta + 2E^{\gamma(\alpha} \wedge E^{\beta)\delta} R_{\gamma\delta}(Z)$,

$$\begin{aligned} T^\alpha &= 2E^{\alpha\beta} \wedge E^\gamma R_{\beta\gamma} + E^{\alpha\beta} \wedge E^{\gamma\delta} U_{\beta\gamma\delta} \ , \\ \mathcal{R}_\beta{}^\alpha &= iE^{\gamma\delta} \wedge E^\alpha U_{\beta\gamma\delta} - E^{\alpha\gamma} \wedge E^\delta (F_{\delta\beta\gamma} + \mathcal{D}_\delta R_{\beta\gamma}) - \\ &- E^{\alpha\gamma} \wedge E^{\delta\epsilon} (\mathcal{D}_{(\beta} U_{\gamma)\delta\epsilon} + \mathcal{D}_{\delta\epsilon} R_{\beta\gamma}) \ . \end{aligned}$$

- have $\Sigma^{(\frac{n(n+1)}{2}|n)}$ solution: $R_{\gamma\delta} = 0$ and $U_{\gamma\delta\epsilon} = 0$ ($\} \Rightarrow \ F_{\gamma\,\delta\epsilon} = 0$).
- Setting $R_{\alpha\beta} = -\frac{\varsigma}{2} C_{\alpha\beta}$ and $U_{\alpha\beta\gamma}(Z) = 0$ we find $\mathcal{R}_\alpha^{\ \beta} = 0 \Rightarrow$ we can gauge away $GL(n)$ connection ($\Omega_\alpha^{\ \beta} = 0$) and arrive at

$$\begin{aligned} d\mathcal{E}^{\alpha\beta} &= -i\mathcal{E}^\alpha \wedge \mathcal{E}^\beta - \zeta \mathcal{E}^{\alpha\gamma} \wedge \mathcal{E}^{\delta\beta} C_{\gamma\delta} \ , \\ d\mathcal{E}^\alpha &= -\zeta \mathcal{E}^{\alpha\gamma} \wedge \mathcal{E}^\delta C_{\gamma\delta} \ . \end{aligned}$$

  which are the Maurer–Cartan eqs. for $OSp(1|n)$.
- Thus $OSp(1|n)$ supergroup manifold is also a solution of the TSSP SUGRA constraints.

Scalar superfield eq. in SUGRA background and reduction of the holonomy group to SL(n)

- A natural generalization of the free superfield equations for higher spin fields to curved TSSP is $\boxed{\mathcal{D}_{[\alpha}\mathcal{D}_{\beta]}\,\Phi = \frac{i}{2}\,R_{\alpha\beta}\,\Phi}$.
- Its integrability condition results in a quite complicated equation
- the known solution of which reduces the holonomy group from $GL(n,\mathbb{R})$ to $SL(n,\mathbb{R})$ i.e. implies $\mathcal{R}_{\alpha}{}^{\alpha} = 0$.
- Such a reduction simplifies a bit equations for main superfields,
- but also makes possible to prove [I.B., Pasti, Sorokin, Tonin 2004]:
- the general solution of supergravity constraints is superconformally equivalent either to $OSp(1|n)$ or to the flat $\Sigma^{(\frac{n(n+1)}{2}|n))}$.
- Namely, they can be obtained by

$$E'^{\alpha\beta} = E^{\alpha\beta}, \qquad E'^{\alpha} = E^{\alpha} + E^{\alpha\beta} W_{\beta}$$
$$\Omega'_{\beta}{}^{\alpha} = \Omega_{\beta}{}^{\alpha} - iE^{\alpha} W_{\beta} - E^{\alpha\gamma}(\mathcal{D}_{\gamma} W_{\beta} + iW_{\gamma} W_{\beta}),$$

with $W_{\beta} = -i\mathcal{D}_{\beta} W$ (and $GL(n)$ gauge transformations, $e^{kW}\delta_{\alpha}{}^{\beta}$, if convenient) from flat or OSp supervielbein and (trivial) GL(n) connection.

Solution of SL(n) SUGRA in TSSP are superconformally OSp or superconformally flat

- The general solution for the main superfields

$$R_{\alpha\beta} = i\, e^{-\frac{2W}{n}} \left[i\tfrac{\varsigma}{2} C_{\alpha\beta} + \nabla_{[\alpha}\nabla_{\beta]} W + \tfrac{1}{2}\nabla_\alpha W \nabla_\beta W \right] ,$$
$$U_{\beta\gamma\delta} = e^{-\frac{3W}{n}} \left[-i\nabla_{\gamma\delta}\nabla_\beta W + \nabla_{(\gamma} W \nabla_{\delta)}\nabla_\beta W \right] .$$

- Note: the original OSp solution, $R_{\alpha\beta} = -\frac{\varsigma}{2} C_{\alpha\beta}$ and $U_{\alpha\beta\gamma}(Z) = 0$ is preserved by superWeyl (supplemented by certain $GL(n)$ gauge) transformations if the superfield parameter W obeys

$$\nabla_{[\alpha}\nabla_{\beta]} W + \tfrac{1}{2}\nabla_\alpha W \nabla_\beta W = -\tfrac{i\varsigma}{2} C_{\alpha\beta} \left(1 - e^{-\frac{W}{2}}\right)$$

- which is an equivalent form of the free eqs, for the free higher spin fields in AdS: $W = 2\ln\left(\frac{\Phi + a}{a}\right)$ where $a = const > 0$ and $\Phi(X,\theta)$ obeys

$$\left(\nabla_{[\alpha}\nabla_{\beta]} + i\frac{\varsigma}{4} C_{\alpha\beta}\right) \Phi(X,\theta) = 0 .$$

- However, the conclusion is that **supergravity in tensorial superspace is super-Weyl trivial**: It does not describe dynamical potentials of higher spin supergravity and describes, at most, the free higher spin eqs.

super-Weyl triviality of SUGRA in TSSP and possible ways out

- **Supergravity in tensorial superspace**, as it has been formulated, **is super-Weyl trivial** $\Rightarrow$ It does not describe dynamical potentials of higher spin SUGRA and describes, at most, the free higher spin eqs.
- Some deformation of the theory or introduction of new elements are needed to continue the search for interacting HSpin theory on this basis.
- Ex.: current project with Dima Sorokin and Per Sundel. To start form an a-deformed 4D tensorial superparticle [I.B., J. Lukierski and D. Sorokin 1999]. SUGRA constraints from preservation of its 2 (not 3) κ–symmetries.
- Other basis to construct interacting higher spin theories in tensorial SSP. Tensorial SYM?
- YM field appears in the multiplets of extended SUSY. $\Rightarrow$ some interest superfield theories in extended TSSP. These were studied in [I.B., J. de Azcárraga, C. Meliveo, 2011].

Outline

HSpin equations in $\mathcal{N}$–extended tensorial superspaces

- Higher spin equations in extended tensorial superspaces

$$\Sigma^{(\frac{n(n+1)}{2}|\mathcal{N}n)} = \{Z^M\} = \{(X^{\alpha\beta}, \theta^{\alpha I})\}\, , \qquad \left\{ \begin{matrix} \alpha, \beta = 1, ..., n, \\ I = 1, ..., \mathcal{N} \end{matrix} \right\}$$

for even $\mathcal{N}$: [I.B., J. de Azcárraga, C. Meliveo, 2011]

- is convenient to write in terms of complex fermionic coordinates

$$\Theta^{\alpha q} = \tfrac{1}{2}(\theta^{\alpha q} - i\theta^{\alpha(q+\mathcal{N}/2)}) = (\bar{\Theta}_q^{\alpha})^* \, , \qquad q = 1, \ldots, \mathcal{N}/2 \, ,$$

$$\partial_{\alpha q} := \frac{\partial}{\partial\Theta^{\alpha q}} = \frac{\partial}{\partial\theta^{\alpha q}} + i\frac{\partial}{\partial\theta^{\alpha(q+\mathcal{N}/2)}} \, ,$$

and complex fermionic covariant derivatives,

$$\mathcal{D}_{\alpha q} = \partial_{\alpha q} + 2i\partial_{\alpha\beta}\bar{\Theta}^{\beta}_q \, , \quad \bar{\mathcal{D}}_{\alpha}{}^{q} = \bar{\partial}_{\alpha}{}^{q} + 2i\partial_{\alpha\beta}\Theta^{\beta q} \, , \quad \partial_{\alpha q} := \frac{\partial}{\partial\Theta^{\alpha q}} \, ,$$

$$\boxed{\{\mathcal{D}_{\alpha q}, \bar{\mathcal{D}}^{p}_{\beta}\} = 4i\partial_{\alpha\beta}\delta^{p}_{q}} \, , \quad \{\mathcal{D}_{\alpha q}, \mathcal{D}^{p}_{\beta}\} = 0 = \{\bar{\mathcal{D}}_{\alpha q}, \bar{\mathcal{D}}^{p}_{\beta}\} \, .$$

- The free higher spin equations with extended SUSY read

$$\bar{\mathcal{D}}^{q}_{\alpha}\Phi(X, \Theta^{q'}, \bar{\Theta}_{p'}) = 0 \, , \qquad \mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X, \Theta^{q'}, \bar{\Theta}_{p'}) = 0 \, .$$

HSpin equations with $\mathcal{N}=2$–extended supersymmetry

- The superfield equations

$$\bar{\mathcal{D}}^q_\alpha \Phi(X,\Theta,\bar{\Theta}) = 0 \; , \qquad \mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X,\Theta,\bar{\Theta}) = 0 \; .$$

- For $\mathcal{N}=2$: $\Rightarrow$

$$\Phi(X,\Theta,\bar{\Theta}) = \phi(X_L) + i\Theta^\alpha \psi_\alpha(X_L)$$

with $X_L^{\alpha\beta} = X^{\alpha\beta} + 2i\Theta^{(\alpha}\bar{\Theta}^{\beta)}$ (notice that n=4 for D=4) and

$$\partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = 0 \; , \qquad \partial_{\alpha[\beta}\psi_{\gamma]}(X) = 0 \; .$$

HSpin equations with $\mathcal{N} = 4$–extended supersymmetry

- The superfield equations

$$\bar{\mathcal{D}}^q_\alpha \Phi(X, \Theta, \bar{\Theta}) = 0 \,, \qquad \mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X, \Theta, \bar{\Theta}) = 0 \,.$$

- For $\mathcal{N} = 4$: $\Rightarrow$
$\Phi(X, \Theta^q, \bar{\Theta}_q) = \phi(X_L) + i\Theta^{\alpha q}\psi_{\alpha q}(X_L) + \epsilon_{pq}\Theta^{\alpha q}\Theta^{\beta p}\mathcal{F}_{\alpha\beta}(X_L)$

$$\partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = 0 \,, \qquad \partial_{\alpha[\beta}\psi_{\gamma]q}(X) = 0 \,,$$

and $\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\beta\alpha}$ obeying $\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta}(X) = 0$

- It might seem that we have found a tensorial superspace counterparts of the usual Maxwell equations and Bianchi identities

$$\partial_{\dot{A}[B}F_{C]D} = 0 \,, \qquad \partial_{A[\dot{B}}F_{\dot{C}]\dot{D}} = 0 \,. \qquad (1)$$

- However, the general solution of this tensorial space equation is

$$\mathcal{F}_{\alpha\beta} = \partial_{\alpha\beta}\tilde{\phi}(X) \,, \qquad \partial_{\alpha[\gamma}\partial_{\delta]\beta}\tilde{\phi}(X) = 0 \,.$$

- Peccei-Quinn-like symmetry acting on the second scalar field $\tilde{\phi}(X)$:
$\tilde{\phi}(X) \mapsto \tilde{\phi}(X) + const$.

$\mathcal{N} > 4$. HSpin equations with $\mathcal{N} = 8$–extended supersymmetry

- In the generic case $\bar{\mathcal{D}}^q_\alpha \Phi(X, \Theta, \bar{\Theta}) = 0$, $\mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X, \Theta, \bar{\Theta}) = 0$ } $\Rightarrow$

$$\Phi(X, \Theta, \bar{\Theta}) = \phi(X_L) + i\Theta^{\alpha q}\psi_{\alpha q} + \sum_{k=2}^{\mathcal{N}/2} \frac{1}{k!}\Theta^{\alpha_k q_k} \ldots \Theta^{\alpha_1 q_1} \mathcal{F}_{\alpha_1 \ldots \alpha_k \, q_1 \ldots q_k}) \; ,$$

$$\mathcal{F}_{\alpha_1 \ldots \alpha_k \, q_1 \ldots q_k}(X_L) = \mathcal{F}_{(\alpha_1 \ldots \alpha_k) \, [q_1 \ldots q_k]}(X_L) \; , \; X_L^{\alpha\beta} = X^{\alpha\beta} + 2i\Theta^{q(\alpha}\bar{\Theta}^{\beta)}_q \; ,$$

the higher components satisfy $\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta_2 \ldots \beta_k \, q_1 \ldots q_k}(X_L) = 0$ which is solved in terms of derivatives of new scalar and spinor fields defined up to Peccei-Quinn-like symmetries.

$\mathcal{N} > 4$. HSpin equations with $\mathcal{N} = 8$–extended supersymmetry

- $\bar{\mathcal{D}}^q_\alpha \Phi(X, \Theta, \bar{\Theta}) = 0, \qquad \mathcal{D}_{q[\beta}\mathcal{D}_{\gamma]p}\Phi(X, \Theta, \bar{\Theta}) = 0$ for $\mathcal{N} = 8$ is solved by
$\Phi(X, \Theta, \bar{\Theta}) = \phi(X_L) + i\Theta^{\alpha q}\psi_{\alpha q} + \sum_{k=2}^{4} \frac{1}{k!}\Theta^{\alpha_k q_k} \ldots \Theta^{\alpha_1 q_1}\mathcal{F}_{\alpha_1\ldots\alpha_k\, q_1\ldots q_k}(X_L)$
the higher components $\mathcal{F}_{\alpha_1\ldots\alpha_k\, q_1\ldots q_k}(X_L) = \mathcal{F}_{(\alpha_1\ldots\alpha_k)\,[q_1\ldots q_k]}$ satisfy

$$\partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta\, q_1 q_2}(X) = 0\ , \quad \partial_{\alpha[\gamma}\psi_{\delta]\beta_2\beta_3}{}^q(X) = 0\, , \quad \partial_{\alpha[\gamma}\mathcal{F}_{\delta]\beta_2\beta_3\beta_4}(X) = 0\ .$$

- which implies $\mathcal{F}_{\alpha\beta\, q_1 q_2}(X) = \partial_{\alpha\beta}\phi_{q_1 q_2}(X)$,

$$\psi^q_{\alpha_1\alpha_2\alpha_3}(X) = \partial_{(\alpha_1\alpha_2}\tilde{\psi}^q_{\alpha_3)}(X)\ , \quad \mathcal{F}_{\alpha_1\ldots\alpha_4}(X) = \partial_{(\alpha_1\alpha_2}\partial_{\alpha_3\alpha_4)}\tilde{\phi}(X)\ .$$

- so that the $\mathcal{N} = 8$ multiplet contains only scalar and s-vector fields

$$\partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi(X) = 0\ , \qquad \partial_{\alpha[\beta}\psi_{\gamma]}(X) = 0\ ,$$
$$\partial_{\alpha[\gamma}\partial_{\delta]\beta}\phi_{qp}(X) = 0\ , \qquad \partial_{\alpha[\beta}\tilde{\psi}_{\gamma]}{}^q(X) = 0\ , \qquad \partial_{\alpha[\gamma}\partial_{\delta]\beta}\tilde{\phi}(X) = 0\ .$$

- and defined up to a more complicated P-Q like symmetries:

$$\phi_{qp}(X) \mapsto \phi_{qp}(X) + a_{qp}\ , \qquad \tilde{\psi}_\alpha{}^q(X) \mapsto \tilde{\psi}_\alpha{}^q(X) + \beta_\alpha{}^q\ ,$$
$$\tilde{\phi}(X) \mapsto \tilde{\phi}(X) + a + X^{\alpha\beta}a_{\alpha\beta}\ ,$$

Outline

$\mathcal{N} > 4$. HSpin equations with $\mathcal{N} = 8$–extended supersymmetry

- Tensorial superspace provides a beautiful basis to describe free conformal higher spin fields in $D = 4, 6, 10$.
- AdS Flat $\leftrightarrow OSp(1|n)$, flat $\leftrightarrow \Sigma^{(\frac{n(n+1)}{2}|n)}$. $(n = 4, 8, 16)$ $D_{[\alpha}D_{\beta]}\Phi = 0$.
- (and also exotic BPS states in M-theory- BPS preons (n=32)).
- $OSp(1|2n)$ superconformal symmetry ($OSp(1|8)$ in D=4, $OSp(1|32)$ in D=10, $OSp(1|64)$ in D=11).
- The attempts to describe the HSpin interactions have not succeed (yet?)
- SUGRA in tensorial superspace, formulated with preservation of manifest GL(n) symmetry (SL(n) holonomy) was shown to be superconformally equivalent to either $\Sigma^{(\frac{n(n+1)}{2}|n)}$ or $OSp(1|n)$.
- A possible way is to search for a deformation which breaks (deforms) the $GL(n)$ and $OSp(1|2n)$ symmetry.
- Probably the suggestion will come from studies [Vasiliev, Gelfond, 10, 13] of the currents in $\Sigma^{(\frac{n(n+1)}{2}|n)}$ through the hypothetical
- tensorial AdS/CFT = $\Sigma^{(\frac{n(n+1)}{2}|n} \leftrightarrow OSp(1|2n)$ correspondence.
- $\Leftarrow$ Flat = $\Sigma^{(\frac{n(n+1)}{2}|n}$. Its superconf. group = $OSp(1|2n) = AdS^{(n(2n+1)|2n)}$.