Cosmological $B-L$ Breaking: (Dark) Matter & Gravitational Waves

Wilfried Buchmüller

DESY, Hamburg

with Valerie Domcke, Kai Schmitz & Kohei Kamada
1202.6679; 1203.0285, 1210.4105, 1305.3392

GGI, Florence, June 2013

I. $B-L$ breaking, inflation & dark matter

- Light neutrino masses can be explained by mixing with Majorana neutrinos with GUT scale masses from $B-L$ breaking (seesaw mechanism)

- Decays of heavy Majorana neutrinos natural source of baryon asymmetry (leptogenesis; thermal (Fukugita, Yanagida '86) or nonthermal (Lazarides, Shafi '91))

- In supersymmetric models with spontaneous $B-L$ breaking, natural connection with inflation (Copeland et al '94; Dvali, Shafi, Schaefer '94; ...)

- LSP (gravitino, higgsino,...) natural candidate for dark matter

- Consistent picture of inflation, baryogenesis and dark matter?

- Possible direct test: gravitational waves

Leptogenesis and gravitinos: for thermal leptogenesis and typical superparticle masses, thermal production yields observed amount of DM,

$$\Omega_{\tilde{G}} h^2 = C \left(\frac{T_R}{10^{10}\,\mathrm{GeV}} \right) \left(\frac{100\,\mathrm{GeV}}{m_{\tilde{G}}} \right) \left(\frac{m_{\tilde{g}}}{1\,\mathrm{TeV}} \right)^2 , \quad C \sim 0.5 \; ;$$

$\Omega_{\mathrm{DM}} h^2 \sim 0.1$ is natural value; but why $T_R \sim T_L$?

Starting point simple observation: heavy neutrino decay width

$$\Gamma^0_{N_1} = \frac{\tilde{m}_1}{8\pi} \left(\frac{M_1}{v_{\mathrm{EW}}} \right)^2 \sim 10^3 \ \mathrm{GeV} \ , \quad \widetilde{m}_1 \sim 0.01 \ \mathrm{eV} \ , \quad M_1 \sim 10^{10} \ \mathrm{GeV} \ .$$

yields reheating temperature (for decaying gas of heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma^0_{N_1} M_P} \sim 10^{10} \ \mathrm{GeV} \ ,$$

wanted for gravitino DM. Intriguing hint or misleading coincidence?

II. Spontaneous $B{-}L$ breaking and false vacuum decay

Supersymmetric SM with right-handed neutrinos,

$$W_M = h^u_{ij}\mathbf{10}_i\mathbf{10}_j H_u + h^d_{ij}\mathbf{5}^*_i\mathbf{10}_j H_d + h^\nu_{ij}\mathbf{5}^*_i n^c_j H_u + h^n_i n^c_i n^c_i S_1 \ ,$$

in $SU(5)$ notation: $\mathbf{10} = (q, u^c, e^c)$, $\mathbf{5}^* = (d^c, l)$; electroweak symmetry breaking, $\langle H_{u,d}\rangle \propto v_{EW}$, and $B{-}L$ breaking,

$$W_{B-L} = \frac{\sqrt{\lambda}}{2}\Phi\left(v^2_{B-L} - 2S_1S_2\right) \ ,$$

$\langle S_{1,2}\rangle = v_{B-L}/\sqrt{2}$ yields heavy neutrino masses.

Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it contains all ingredients wanted in cosmology: inflation, leptogenesis, dark matter,..., all related!

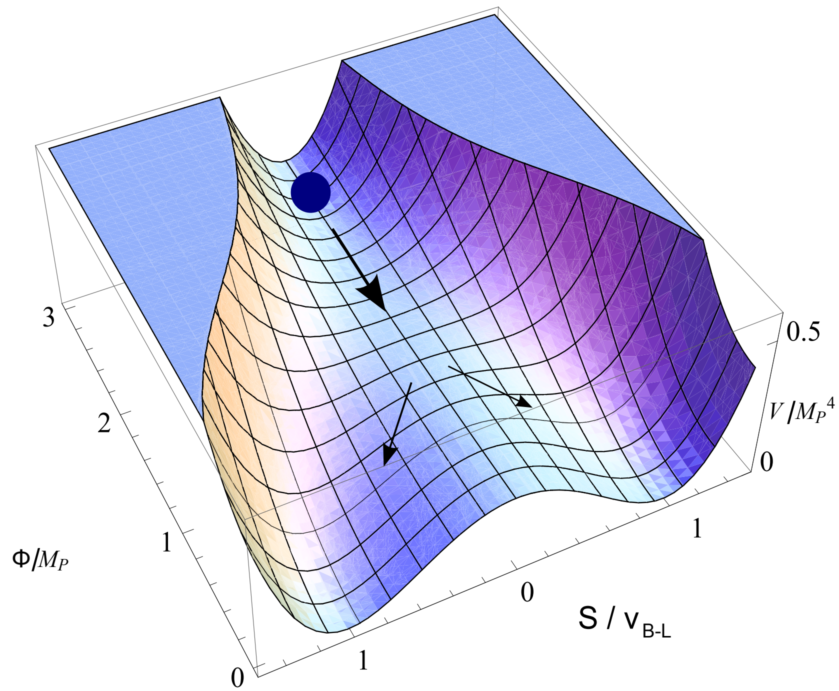
Parameters of $B-L$ breaking sector: $\overline{m}_\nu = \sqrt{m_2 m_3} = 3 \times 10^{-2}$ eV, $M_1 \ll M_{2,3} \simeq m_S$, $\widetilde{m}_1 = (m_D^\dagger m_D)_{11}/M_1$, v_{B-L}.

Spontaneous symmetry breaking: consider Abelian Higgs model in unitary gauge ($\rightarrow$ massive vector multiplet, no Wess-Zumino gauge!),

$$S_{1,2} = \frac{1}{\sqrt{2}} S' \exp(\pm iT) \ , \qquad V = Z + \frac{i}{2g}(T - T^*) \ .$$

Inflaton field Φ: slow motion (quantum corrections), changes mass of 'waterfall' field S, rapid change after critical point where $m_S = 0$; basic mechanism of hybrid inflation.

Shift around time-dependent background, $s' = \frac{1}{\sqrt{2}}(\sigma' + i\tau)$, $\sigma' \rightarrow \sqrt{2}v(t) + \sigma$ with $v(t) = \frac{1}{\sqrt{2}}\langle \sigma'^2(t, \vec{x})\rangle_{\vec{x}}^{1/2}$; masses of fluctuations:



$$m_\sigma^2 = \frac{1}{2}\lambda(3v^2(t) - v_{B-L}^2)\ , \quad m_\tau^2 = \frac{1}{2}\lambda(v_{B-L}^2 + v^2(t))\ , \quad m_\phi^2 = \lambda v^2(t)\ ,$$

$$m_\psi^2 = \lambda v^2(t)\ , \quad m_Z^2 = 8g^2v^2(t)\ , \quad M_i^2 = (h_i^n)^2 v^2(t)\ ;$$

time-dependent masses of $B-L$ Higgs, inflaton, vector boson, heavy neutrinos, all supermultiplets!

Constraints from cosmic strings and inflation: upper bound on string tension (Planck Collaboration '13)

$$G\mu < 3.2 \times 10^{-7} \; , \quad \mu = 2\pi B(\beta) v_{B-L}^2 \; ,$$

with $\beta = \lambda/(8\,g^2)$ and $B(\beta) = 2.4\,[\ln(2/\beta)]^{-1}$ for $\beta < 10^{-2}$; further constraint from CMB (cf. Nakayama et al '10), yields

$$3 \times 10^{15} \text{ GeV} \lesssim v_{B-L} \lesssim 7 \times 10^{15} \text{ GeV} \; ,$$
$$10^{-4} \lesssim \sqrt{\lambda} \lesssim 10^{-1} \; .$$

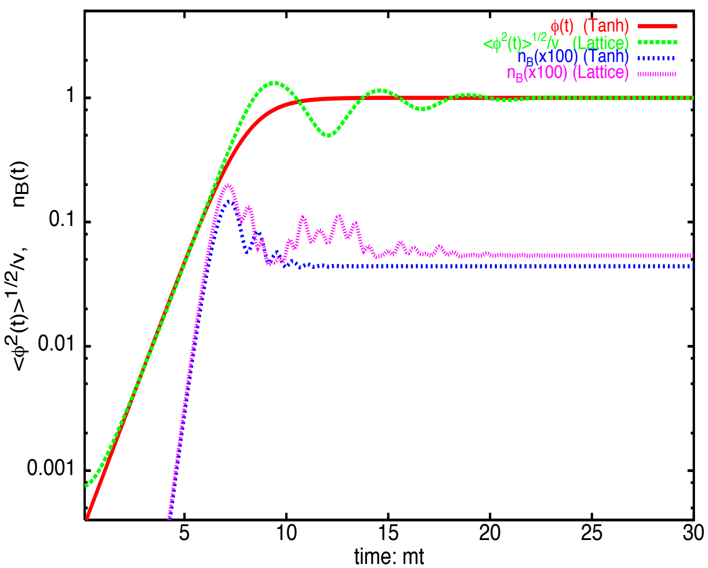
Final choice for range of parameters (analysis within FN flavour model):

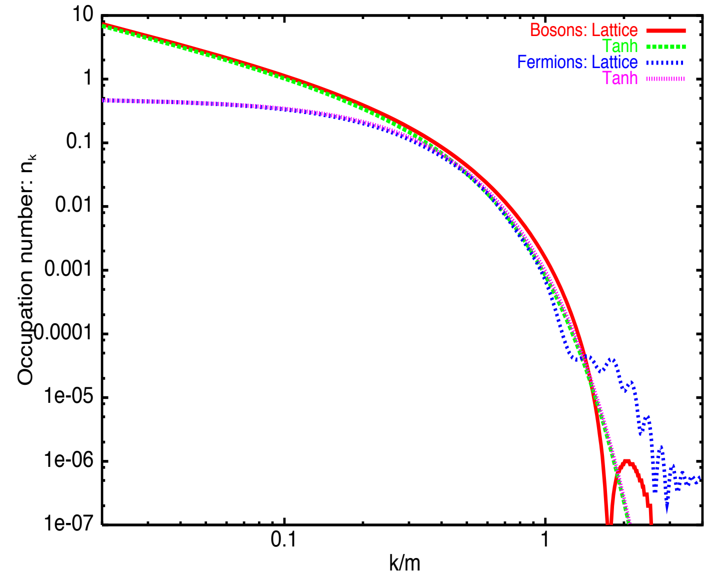
$$v_{B-L} = 5 \times 10^{15} \text{ GeV} \; , \quad 10^{-5} \text{ eV} \leq \widetilde{m}_1 \leq 1 \text{ eV} \; ,$$
$$10^{9} \text{ GeV} \leq M_1 \leq 3 \times 10^{12} \text{ GeV} \; .$$

(range of $\widetilde{m}_1$: uncertainty of $\mathcal{O}(1)$ parameters)

Tachyonic Preheating

Hybrid inflation ends at critical value Φ_c of inflaton field Φ by rapid growth of fluctuations of $B-L$ Higgs field S' (‘spinodal decomposition’):





in addition, particles which couple to S' are produced by rapid increase of ‘waterfall field’ (Garcia-Bellido, Morales ’02); no coherent oscillations!

Decay of false vacuum produces long wave-length σ-modes, true vacuum reached at time $t_{\rm PH}$ (even faster decay with inflaton dynamics),

$$\langle\sigma'^2\rangle\Big|_{t=t_{\rm PH}} = 2v_{B-L}^2\ , \qquad t_{\rm PH} \simeq \frac{1}{2m_\sigma}\ln\left(\frac{32\pi^2}{\lambda}\right)\ .$$

Initial state: nonrelativistic gas of σ-bosons, $N_{2,3}$, $\tilde{N}_{2,3}$, A, $\tilde{A}$, C (contained in superfield Z), ... ; energy fractions ($\alpha = m_X/m_S$, $\rho_0 = \lambda v_{B-L}^4/4$):

$$\rho_B/\rho_0 \simeq 2\times 10^{-3}\, g_s\, \lambda\, f(\alpha, 1.3)\ , \quad \rho_F/\rho_0 \simeq 1.5\times 10^{-3}\, g_s\, \lambda\, f(\alpha, 0.8)\ .$$

Time evolution: rapid $N_{2,3}$, $\tilde{N}_{2,3}$, A, $\tilde{A}$, C decays, yields initial radiation, thermal N_1's and gravitinos; σ decays produce nonthermal N_1's; N_1 decays produce most of radiation and baryon asymmetry; details of evolution described by Boltzmann equations.

Reheating Process

Major work: solve network of Boltzmann equations for all (super)particles; treat nonthermal and thermal contributions differently, varying equation of state; result: detailed time resolved description of reheating process, prediction of baryon asymmetry and gravitino density (possibly dark matter).

Illustrative example for parameter choice

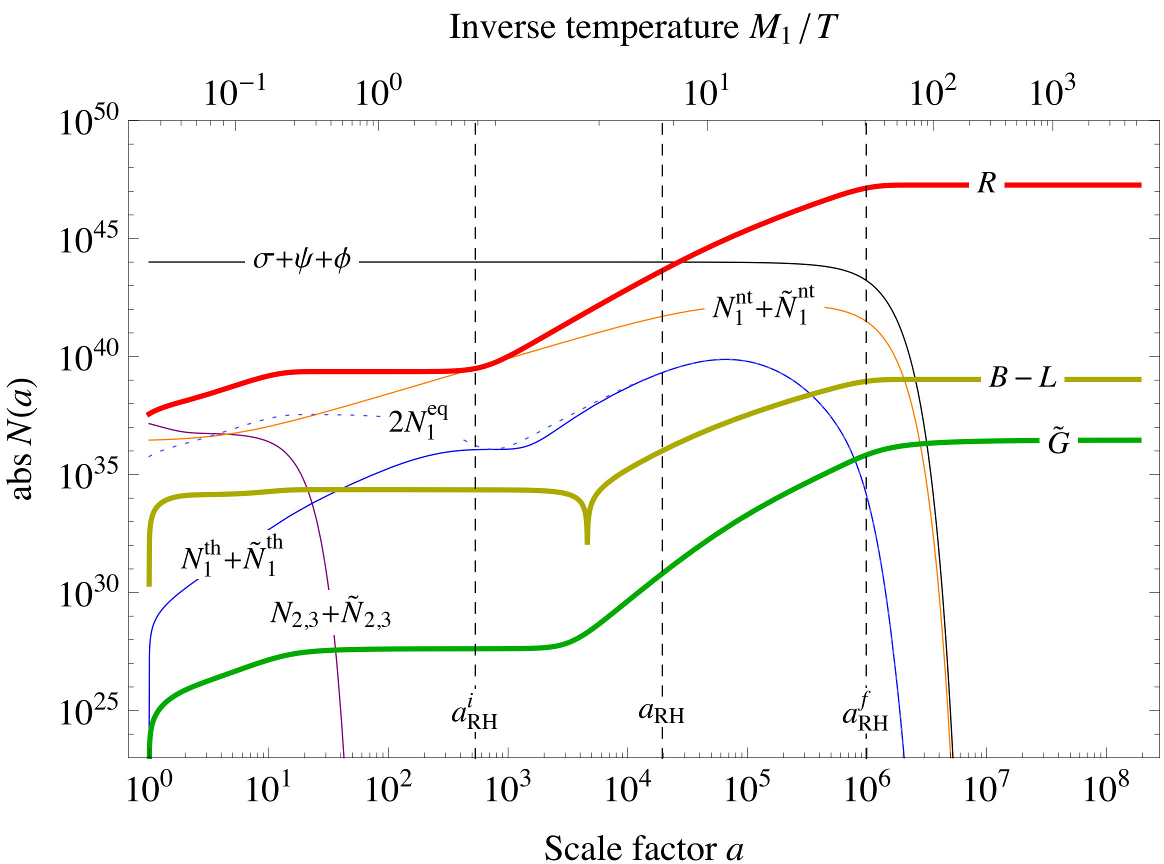
$$M_1 = 5.4 \times 10^{10}\,\mathrm{GeV}\ , \quad \widetilde{m}_1 = 4.0 \times 10^{-2}\,\mathrm{eV}\ ,$$
$$m_{\widetilde{G}} = 100\,\mathrm{GeV}\ , \quad m_{\tilde{g}} = 1\,\mathrm{TeV}\ ; \quad G\mu = 2.0 \times 10^{-7}$$

fixes (within FN flavour model) all other masses, CP asymmetries etc. Note: emergence of temperature plateau at intermediate times; final result:

$$\eta_B \simeq 3.7 \times 10^{-9} \ \simeq\ \eta_B^{\mathrm{nt}}\ , \quad \Omega_{\widetilde{G}} h^2 \simeq 0.11\ ,$$

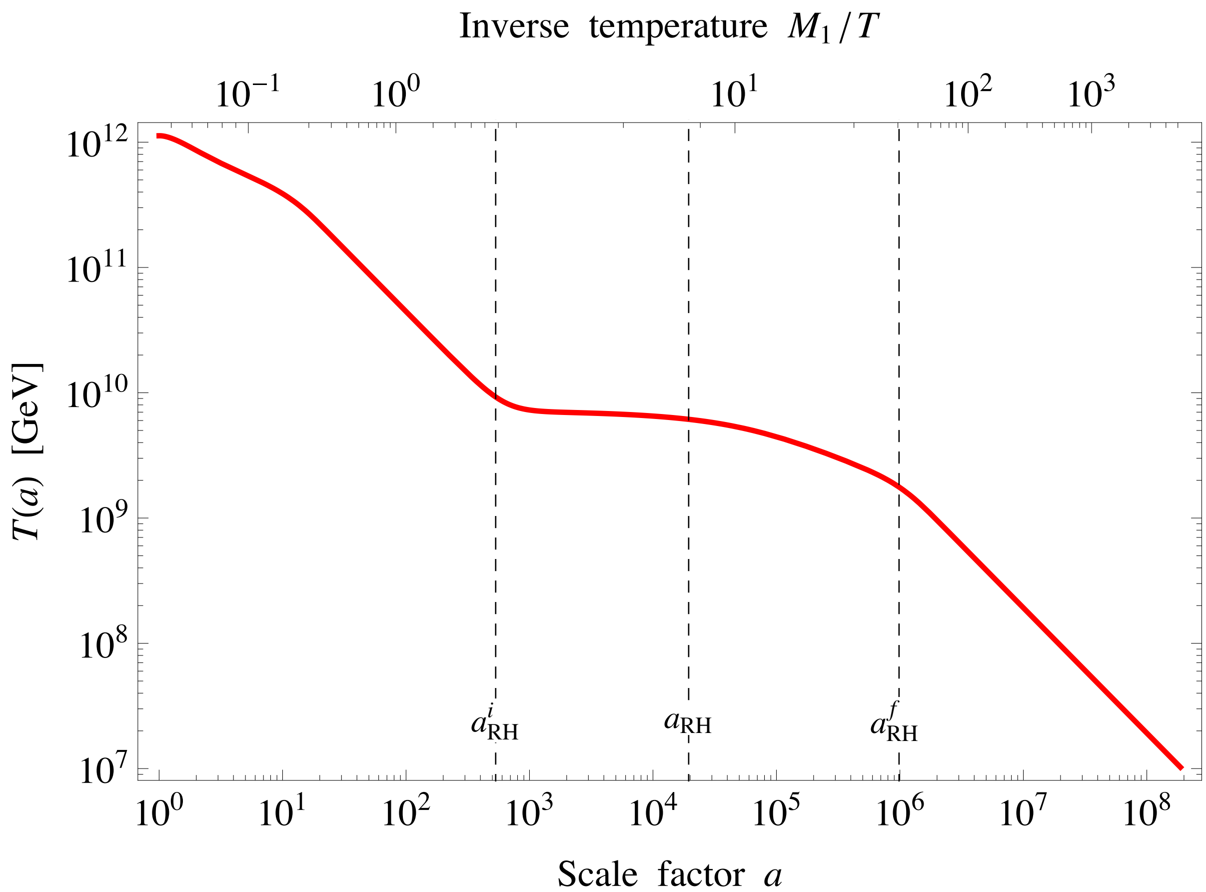
i.e., dynamical realization of original conjecture.

Thermal and nonthermal number densities



Comoving number densites of thermal and nonthermla N_1's,..., $B-L$, gravitinos and radiation as functions of scale factor a.

Time evolution of temperature: intermediate plateau



Gravitino abundance can be understood from 'standard formula' and effective 'reheating temperature' (determined by neutrino masses).

III. Gravitinos & Dark Matter

Thermal production of gravitinos is origin of DM; depending on pattern of SUSY breaking, gravitino DM or higgsino/wino DM. Mass spectrum of superparticles motivated 'large' Higgs mass measured at the LHC,

$$m_{\mathrm{LSP}} \ll m_{\mathrm{squark,slepton}} \ll m_{\widetilde{G}}\ .$$

LSP is typically 'pure' wino or higgsino (bino disfavoured, overproduction in thermal freeze-out), almost mass degenerate with chargino. Thermal abundance of wino ($\widetilde{w}$) or higgsino ($\widetilde{h}$) LSP significant for masses above $1\ \mathrm{TeV}$, well approximated by (Arkani-Hamed et al '06; Hisano et al '07, Cirelli et al '07)

$$\Omega^{\mathrm{th}}_{\widetilde{w},\widetilde{h}} h^2 = c_{\widetilde{w},\widetilde{h}} \left(\frac{m_{\widetilde{w},\widetilde{h}}}{1\ \mathrm{TeV}}\right)^2\ , \quad c_{\widetilde{w}} = 0.014\ , \quad c_{\widetilde{h}} = 0.10\ ,$$

Heavy gravitinos ($10\ \mathrm{TeV} \ldots 10^3\ \mathrm{TeV}$) consistent with BBN, $\tau_{\widetilde{G}} \simeq 24\ \times$

$(10\ \mathrm{TeV}/m_{\widetilde{G}})^3\mathrm{sec}$. Total higgsino/wino abundance

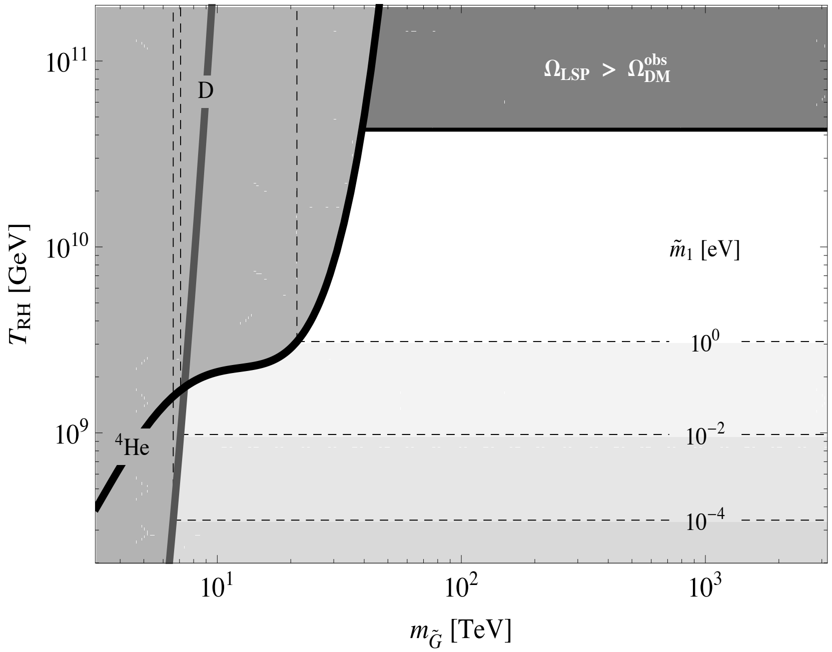
$$\Omega_{\widetilde{w},\widetilde{h}}h^2 = \Omega^{\widetilde{G}}_{\widetilde{w},\widetilde{h}}h^2 + \Omega^{\mathrm{th}}_{\widetilde{w},\widetilde{h}}h^2\ ,$$

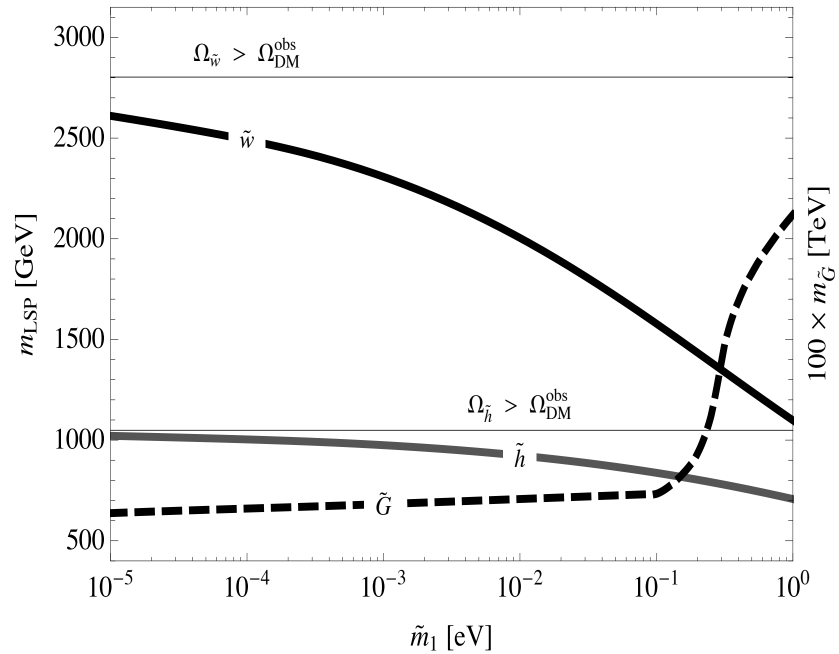
$$\Omega^{\widetilde{G}}_{\mathrm{LSP}}h^2 = \frac{m_{\mathrm{LSP}}}{m_{\widetilde{G}}}\Omega_{\widetilde{G}}h^2 \simeq 2.7\times 10^{-2}\left(\frac{m_{\mathrm{LSP}}}{100\ \mathrm{GeV}}\right)\left(\frac{T_{\mathrm{RH}}(M_1,\widetilde{m}_1)}{10^{10}\ \mathrm{GeV}}\right)\ ,$$

with 'reheating temperature' determined by neutino masses (takes reheating process into account),

$$T_{\mathrm{RH}} \simeq 1.3\times 10^{10}\,\mathrm{GeV}\left(\frac{\widetilde{m}_1}{0.04\,\mathrm{eV}}\right)^{1/4}\left(\frac{M_1}{10^{11}\,\mathrm{GeV}}\right)^{5/4}\ .$$

Requirement of LSP dark matter, i.e. $\Omega_{\mathrm{LSP}}h^2 = \Omega_{\mathrm{DM}}h^2 \simeq 0.11$, yields upper bound on the reheating temperature, $T_{\mathrm{RH}} < 4.2\times 10^{10}$ GeV; lower bound on T_{RH} from successfull leptogenesis (depends on $\widetilde{m}_1$).





For each 'reheating temperature', i.e. pair $(M_1, \widetilde{m}_1)$, lower bound on gravitino mass (taken from Kawasaki et al '08) (left panel). Requirement of higgsino/wino dark matter puts upper bound on LSP mass, dependent on $\widetilde{m}_1$, 'reheating temperature' (right panel); more stringent for higgsino mass, since freeze-out contribution larger. E.g., $m_1 = 0.05$ eV implies $m_{\widetilde{h}} \lesssim 900$ GeV, $m_{\widetilde{G}} \gtrsim 10$ TeV.

IV. Gravitational Waves

Relic gravitational waves are window to very early universe; contributions from inflation, preheating and cosmic strings (Rubakov et al '82; Garcia-Bellido and Figueroa '07; Vilenkin '81; Hindmarsh et al '12); cosmological $B-L$ breaking: prediction of GW spectrum with all contributions!

Perturbations in flat FRW background,

$$ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \ , \quad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\rho_\rho \ ,$$

determined by linearized Einstein equations,

$$\bar{h}''_{\mu\nu}(\mathbf{x},\tau) + 2\frac{a'}{a}\bar{h}'_{\mu\nu}(\mathbf{x},\tau) - \nabla^2_{\mathbf{x}}\bar{h}_{\mu\nu}(\mathbf{x},\tau) = 16\pi G T_{\mu\nu}(\mathbf{x},\tau) \ .$$

Spectrum of GW background,

$$\Omega_{GW}(k,\tau) = \frac{1}{\rho_c}\frac{\partial\rho_{GW}(k,\tau)}{\partial \ln k} \ ,$$

$$\int_{-\infty}^{\infty} d\ln k \frac{\partial \rho_{GW}(k,\tau)}{\partial \ln k} = \frac{1}{32\pi G}\left\langle \dot{h}_{ij}\left(\mathbf{x},\tau\right)\dot{h}^{ij}\left(\mathbf{x},\tau\right)\right\rangle \; ;$$

use initial conditions for super-horizon modes from inflation, calculate correlation function of stress energy tensor.

Contribution from inflation (primordial spectrum, transfer function; cf. Nakayama et al '08):

$$\Omega_{\mathrm{GW}}(k,\tau) = \frac{\Delta_t^2}{12}\frac{k^2}{a_0^2 H_0^2} T_k^2(\tau)$$

$$= \frac{\Delta_t^2}{12}\Omega_r \frac{g_*^k}{g_*^0}\left(\frac{g_{*,s}^0}{g_{*,s}^k}\right)^{4/3} \begin{cases} \frac{1}{2}\left(k_{\mathrm{eq}}/k\right)^2 , & k_0 \ll k \ll k_{\mathrm{eq}} \\ 1 , & k_{\mathrm{eq}} \ll k \ll k_{\mathrm{RH}} \\ \frac{1}{2} C_{\mathrm{RH}}^3 \left(k_{\mathrm{RH}}/k\right)^2 & k_{\mathrm{RH}} \ll k \ll k_{\mathrm{PH}} \end{cases}$$

with boundary frequences ($f = k/(2\pi a_0)$),

$$f_0 = 3.58 \times 10^{-19}\,\mathrm{Hz}\left(\frac{h}{0.70}\right) ,$$

$$f_{\rm eq} = 1.57 \times 10^{-17}\,\mathrm{Hz}\left(\frac{\Omega_m h^2}{0.14}\right) ,$$

$$f_{\rm RH} = 4.25 \times 10^{-1}\,\mathrm{Hz}\left(\frac{T_{\rm RH}}{10^7\,\mathrm{GeV}}\right) ,$$

$$f_{\rm PH} = 1.99 \times 10^{4}\,\mathrm{Hz}\left(\frac{\lambda}{10^{-4}}\right)^{1/6}\left(\frac{v_{B-L}}{5\times 10^{15}\,\mathrm{GeV}}\right)^{2/3}\left(\frac{T_{\rm RH}}{10^7\,\mathrm{GeV}}\right)^{1/3} .$$

Contribution from preheating (cf. Dufaux et al '07):

$$\Omega_{GW}(k_{\mathsf{PH}})\ h^2 \ \simeq\ c_{\mathsf{PH}}\,(R_{\mathsf{PH}}H_{\mathsf{PH}})^2\,\frac{a_{\mathsf{PH}}}{a_{\mathsf{RH}}}\,\Omega_r h^2\,\frac{g_*^{\mathsf{RH}}}{g_*^0}\left(\frac{g_{*,s}^0}{g_{*,s}^{\mathsf{RH}}}\right)^{4/3} ,$$

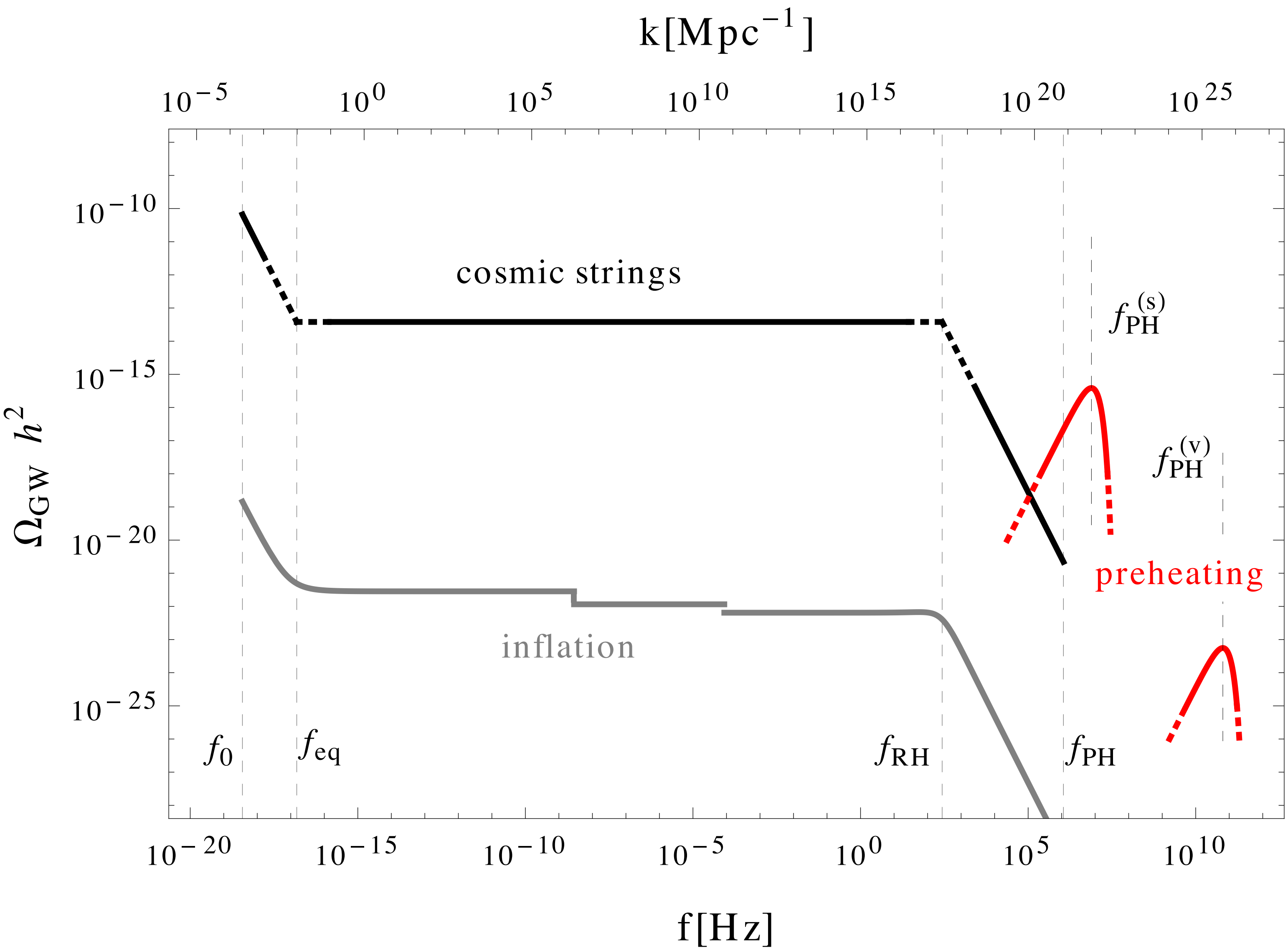
with characteristic scalar and vector scales

$$\left(R_{\mathsf{PH}}^{(s)}\right)^{-1} = (\lambda\, v_{B-L}\, |\dot{\phi}_c|)^{1/3}\ , \quad \left(R_{\mathsf{PH}}^{(v)}\right)^{-1} \sim m_Z = 2\sqrt{2}\, g\, v_{B-L}\, .$$

Contribution from cosmic strings (Abelian Higgs):

$$\Omega_{GW}(k) \simeq \frac{1}{6\pi^2} F^r \left(\frac{v_{B-L}}{M_{\rm Pl}}\right)^4 \Omega_r h^2 \begin{cases} (k_{\rm eq}/k)^2, & k_0 \ll k \ll k_{\rm eq} \\ 1, & k_{\rm eq} \ll k \ll k_{\rm RH} \\ (k_{\rm RH}/k)^2, & k_{\rm RH} \ll k \end{cases}$$

constant F^r recently determined in numerical simulation (cf. Figueroa, Hindmarsh, Urrestilla '12). Result similar to contribution from inflation, but very different normalization!


k[Mpc^{-1}]
10^{-5}
10^{0}
10^{5}
10^{10}
10^{15}
10^{20}
10^{25}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
$\Omega_{GW}\ h^2$
cosmic strings
$f_{PH}^{(s)}$
$f_{PH}^{(v)}$
preheating
inflation
f_0
f_{eq}
f_{RH}
f_{PH}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
10^{0}
10^{5}
10^{10}
f[Hz]

Are macroscopically long cosmic strings Nambu-Goto strings? Energy loss of string network by 'massive radiation' or gravitational waves? GW radiation from NG strings, radiated by loops of length

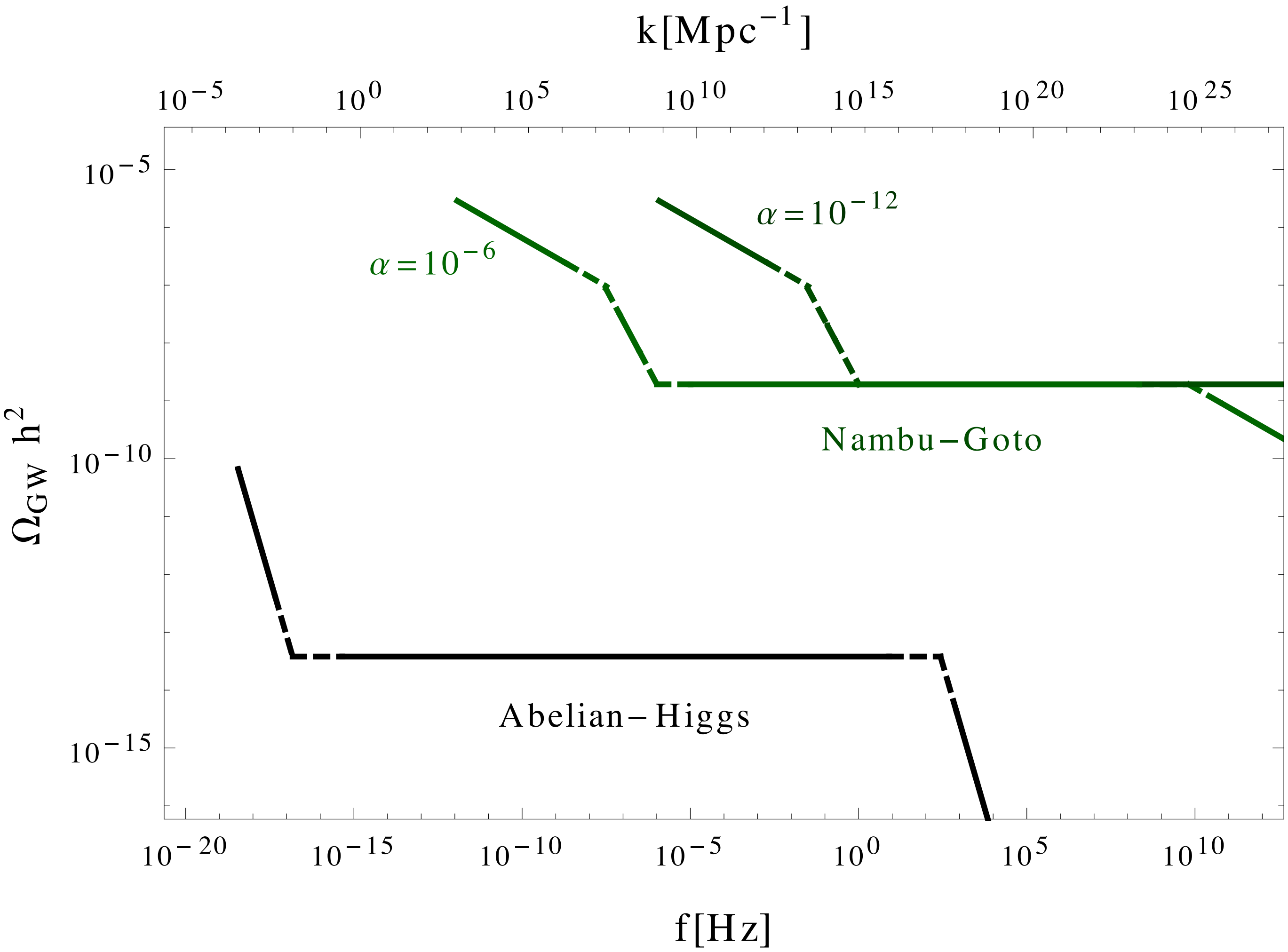
$$l(t, t_i) = \alpha t_i - \Gamma G\mu(t - t_i) \; ;$$

rate for amplitude h and frequency f (cf. Kuroyanagi et al '12),

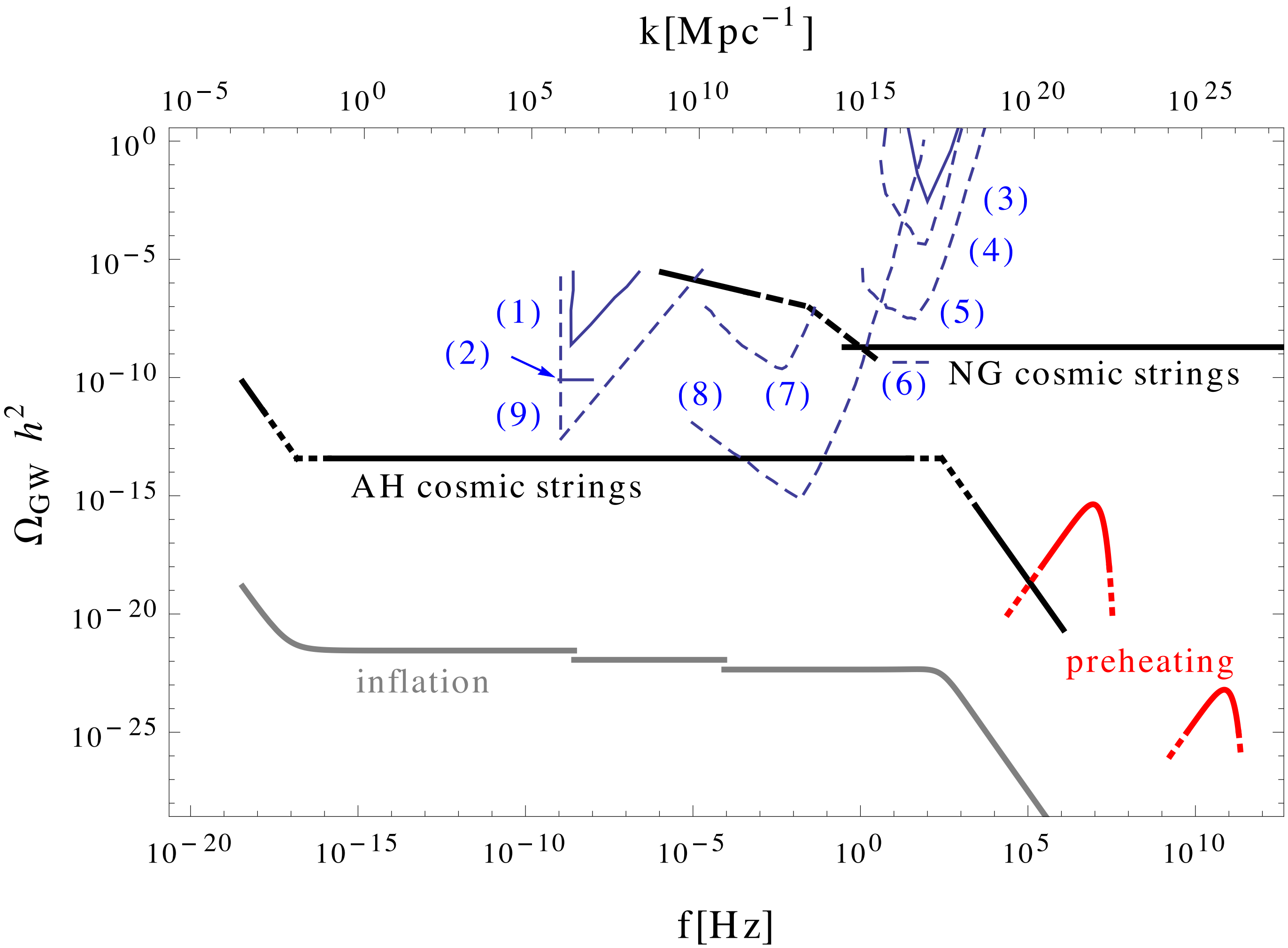
$$\frac{d^2R}{dzdh}(f, h, z) \simeq \frac{3}{4}\frac{\theta_m^2}{(1+z)(\alpha + \Gamma G\mu)h}\frac{1}{\alpha\gamma^2 t^4(z)}\frac{dV(z)}{dz}\Theta(1 - \theta_m) \; ;$$

can be approximately integrated analytically over z and h; results differs qualitatively from Abelian-Higgs prediction; five orders of magnitude difference in normalization! Truth somewhere inbetween?

Abelian-Higgs Strings vs. Nambu-Goto Strings



Observational Prospects



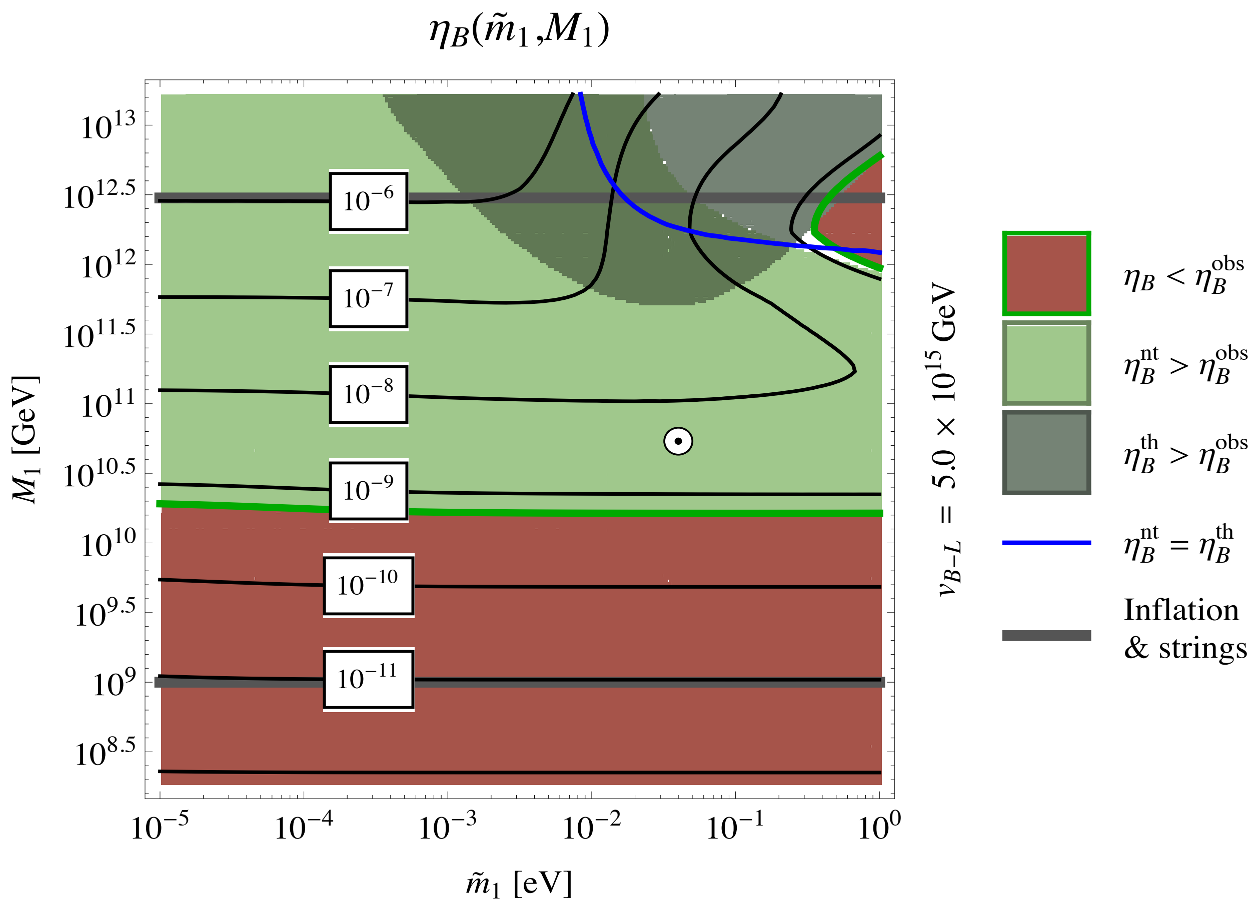
Summary and Outlook

- Decay of false vacuum of unbroken B-L symmetry leads to consistent picture of inflation, baryogenesis and dark matter (everything from heavy neutrino decays)

- Prediction: relations between neutrino and superparticle masses for gravitino or higgsino/wino dark matter

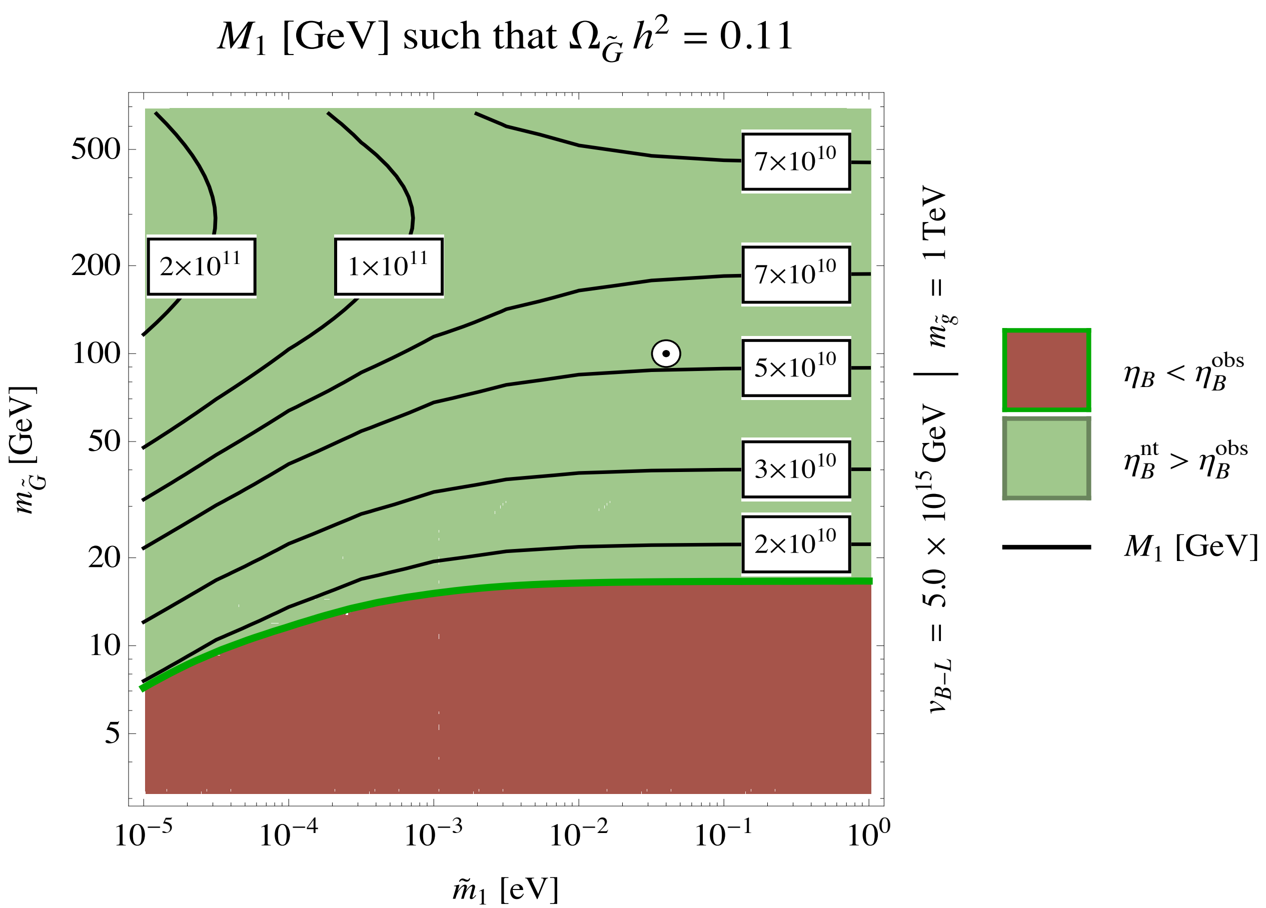
- Possible direct test: detection of relic gravitational wave background, may provide determination of reheating temperature

(Non)thermal leptogenesis in $M_1 - \tilde{m}_1$ plane



Upper bound on M_1 from inflation; lower bound from baryogenesis

Gravitino Dark Matter vs. leptogenesis



Gravitino mass range: $10\ \mathrm{GeV} \lesssim m_{\widetilde{G}} \lesssim 700\ \mathrm{GeV}$; heavy neutrino mass range: $2 \times 10^{10}\ \mathrm{GeV} \lesssim M_1 \lesssim 2 \times 10^{11}\ \mathrm{GeV}$ (more stringent than inflation)