Cosmological $B-L$ Breaking: (Dark) Matter & Gravitational Waves

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I. $B-L$ breaking, inflation & dark matter

- Light neutrino masses can be explained by mixing with Majorana neutrinos with GUT scale masses from $B-L$ breaking (seesaw mechanism)

- Decays of heavy Majorana neutrinos natural source of baryon asymmetry (leptogenesis; thermal (Fukugita, Yanagida '86) or nonthermal (Lazarides, Shafi '91))

- In supersymmetric models with spontaneous $B-L$ breaking, natural connection with inflation (Copeland et al '94; Dvali, Shafi, Schaefer '94; ...)

- LSP (gravitino, higgsino,...) natural candidate for dark matter

- Consistent picture of inflation, baryogenesis and dark matter?

- Possible direct test: gravitational waves
Leptogenesis and gravitinos: for thermal leptogenesis and typical superparticle masses, thermal production yields observed amount of DM,

$$\Omega_{\tilde{G}} h^2 = C \left( \frac{T_R}{10^{10} \text{GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left( \frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2 , \quad C \sim 0.5 ;$$

$$\Omega_{\text{DM}} h^2 \sim 0.1$$ is natural value; but why $T_R \sim T_L$?

Starting point simple observation: heavy neutrino decay width

$$\Gamma_{N_1}^0 = \frac{\tilde{m}_1}{8\pi} \left( \frac{M_1}{v_{\text{EW}}} \right)^2 \sim 10^3 \text{ GeV} , \quad \tilde{m}_1 \sim 0.01 \text{ eV} , \quad M_1 \sim 10^{10} \text{ GeV} .$$

yields reheating temperature (for decaying gas of heavy neutrinos)

$$T_R \sim 0.2 \cdot \sqrt{\Gamma_{N_1}^0 M_P} \sim 10^{10} \text{ GeV} ,$$

wanted for gravitino DM. Intriguing hint or misleading coincidence?
II. Spontaneous $B-L$ breaking and false vacuum decay

Supersymmetric SM with right-handed neutrinos,

$$W_M = h_{ij}^u 10_i 10_j H_u + h_{ij}^d 5^*_i 10_j H_d + h_{ij}^\nu 5^*_i n^c_j H_u + h_{ij}^n n^c_i n^c_j S_1,$$

in $SU(5)$ notation: $10 = (q, u^c, e^c), 5^* = (d^c, l)$; electroweak symmetry breaking, $\langle H_{u,d} \rangle \propto v_{EW}$, and $B-L$ breaking,

$$W_{B-L} = \frac{\sqrt{\lambda}}{2} \Phi \left( v_{B-L}^2 - 2S_1 S_2 \right),$$

$\langle S_{1,2} \rangle = v_{B-L}/\sqrt{2}$ yields heavy neutrino masses.

Lagrangian is determined by low energy physics: quark, lepton, neutrino masses etc, but it contains all ingredients wanted in cosmology: inflation, leptogenesis, dark matter,..., all related!
Parameters of $B-L$ breaking sector: $\bar{m}_\nu = \sqrt{m_2 m_3} = 3 \times 10^{-2}$ eV, $M_1 \ll M_{2, 3} \simeq m_S$, $\tilde{m}_1 = (m_D^\dagger m_D)_{11}/M_1$, $v_{B-L}$.

**Spontaneous symmetry breaking:** consider Abelian Higgs model in unitary gauge (→ massive vector multiplet, no Wess-Zumino gauge!),

$$S_{1, 2} = \frac{1}{\sqrt{2}} S' \exp(\pm iT), \quad V = Z + \frac{i}{2g} (T - T^*) .$$

Inflaton field $\Phi$: slow motion (quantum corrections), changes mass of ‘waterfall’ field $S'$, rapid change after critical point where $m_S = 0$; basic mechanism of hybrid inflation.

Shift around time-dependent background, $s' = \frac{1}{\sqrt{2}} (\sigma' + i \tau)$, $\sigma' \to \sqrt{2} v(t) + \sigma$ with $v(t) = \frac{1}{\sqrt{2}} \langle \sigma'^2(t, \vec{x}) \rangle^{1/2}$; masses of fluctuations:
\[ m_\sigma^2 = \frac{1}{2} \lambda (3v^2(t) - v_{B-L}^2) , \quad m_\tau^2 = \frac{1}{2} \lambda (v_{B-L}^2 + v^2(t)) , \quad m_\phi^2 = \lambda v^2(t) , \]
\[ m_\psi^2 = \lambda v^2(t) , \quad m_Z^2 = 8g^2 v^2(t) , \quad M_i^2 = (h_i^n)^2 v^2(t) ; \]

time-dependent masses of \( B-L \) Higgs, inflaton, vector boson, heavy neutrinos, all supermultiplets!
Constraints from cosmic strings and inflation: upper bound on string tension
(Planck Collaboration '13)

\[ G\mu < 3.2 \times 10^{-7} , \quad \mu = 2\pi B(\beta)v_{B-L}^2 , \]

with \( \beta = \lambda/(8 g^2) \) and \( B(\beta) = 2.4 [\ln(2/\beta)]^{-1} \) for \( \beta < 10^{-2} \); further constraint from CMB (cf. Nakayama et al '10), yields

\[ 3 \times 10^{15} \text{ GeV} \lesssim v_{B-L} \lesssim 7 \times 10^{15} \text{ GeV} , \]
\[ 10^{-4} \lesssim \sqrt{\lambda} \lesssim 10^{-1} . \]

Final choice for range of parameters (analysis within FN flavour model):

\[ v_{B-L} = 5 \times 10^{15} \text{ GeV} , \quad 10^{-5} \text{ eV} \leq \tilde{m}_1 \leq 1 \text{ eV} , \]
\[ 10^9 \text{ GeV} \leq M_1 \leq 3 \times 10^{12} \text{ GeV} . \]

(range of \( \tilde{m}_1 \): uncertainty of \( \mathcal{O}(1) \) parameters)
**Tachyonic Preheating**

Hybrid inflation ends at critical value $\Phi_C$ of inflaton field $\Phi$ by rapid growth of fluctuations of $B-L$ Higgs field $S'$ (‘spinodal decomposition’):

\[
\langle \phi^2(t) \rangle^{1/2} / v, \quad n_B(t) \quad (\text{Tanh})
\]

in addition, particles which couple to $S'$ are produced by rapid increase of ‘waterfall field’ (Garcia-Bellido, Morales ’02); no coherent oscillations!
Decay of false vacuum produces long wave-length $\sigma$-modes, true vacuum reached at time $t_{PH}$ (even faster decay with inflaton dynamics),

$$\langle \sigma'^2 \rangle \bigg|_{t=t_{PH}} = 2v_{B-L}^2, \quad t_{PH} \simeq \frac{1}{2m_\sigma} \ln \left( \frac{32\pi^2}{\lambda} \right).$$

Initial state: nonrelativistic gas of $\sigma$-bosons, $N_{2,3}$, $\tilde{N}_{2,3}$, $A$, $\tilde{A}$, $C$ (contained in superfield $Z$), ...; energy fractions ($\alpha = m_X/m_S$, $\rho_0 = \lambda v_{B-L}^4/4$):

$$\rho_B/\rho_0 \simeq 2 \times 10^{-3} \, g_s \, \lambda \, f(\alpha, 1.3), \quad \rho_F/\rho_0 \simeq 1.5 \times 10^{-3} \, g_s \, \lambda \, f(\alpha, 0.8).$$

Time evolution: rapid $N_{2,3}$, $\tilde{N}_{2,3}$, $A$, $\tilde{A}$, $C$ decays, yields initial radiation, thermal $N_1$'s and gravitinos; $\sigma$ decays produce nonthermal $N_1$'s; $N_1$ decays produce most of radiation and baryon asymmetry; details of evolution described by Boltzmann equations.
Reheating Process

Major work: solve network of Boltzmann equations for all (super)particles; treat nonthermal and thermal contributions differently, varying equation of state; result: detailed time resolved description of reheating process, prediction of baryon asymmetry and gravitino density (possibly dark matter).

Illustrative example for parameter choice

\[ M_1 = 5.4 \times 10^{10} \text{GeV}, \quad \tilde{m}_1 = 4.0 \times 10^{-2} \text{eV}, \]
\[ m_{\tilde{G}} = 100 \text{GeV}, \quad m_{\tilde{g}} = 1 \text{TeV}; \quad G\mu = 2.0 \times 10^{-7} \]

fixes (within FN flavour model) all other masses, CP asymmetries etc. Note: emergence of temperature plateau at intermediate times; final result:

\[ \eta_B \simeq 3.7 \times 10^{-9} \simeq \eta^\text{nt}_B, \quad \Omega_{\tilde{G}} h^2 \simeq 0.11, \]

i.e., dynamical realization of original conjecture.
Thermal and nonthermal number densities

Comoving number densities of thermal and nonthermal $N_1$'s,..., $B-L$, gravitinos and radiation as functions of scale factor $a$.  

Inverse temperature $M_1 / T$
Gravitino abundance can be understood from ‘standard formula’ and effective ‘reheating temperature’ (determined by neutrino masses).
III. Gravitinos & Dark Matter

Thermal production of gravitinos is origin of DM; depending on pattern of SUSY breaking, gravitino DM or higgsino/wino DM. Mass spectrum of superparticles motivated ‘large’ Higgs mass measured at the LHC,

\[ m_{\text{LSP}} \ll m_{\text{squark,slepton}} \ll m_{\tilde{G}} . \]

LSP is typically ‘pure’ wino or higgsino (bino disfavoured, overproduction in thermal freeze-out), almost mass degenerate with chargino. Thermal abundance of wino (\( \tilde{w} \)) or higgsino (\( \tilde{h} \)) LSP significant for masses above 1 TeV, well approximated by (Arkani-Hamed et al ’06; Hisano et al ’07, Cirelli et al ’07)

\[ \Omega_{\tilde{w},\tilde{h}}^{th} h^2 = c_{\tilde{w},\tilde{h}} \left( \frac{m_{\tilde{w},\tilde{h}}}{1 \text{ TeV}} \right)^2 , \quad c_{\tilde{w}} = 0.014 , \quad c_{\tilde{h}} = 0.10 , \]

Heavy gravitinos (10 TeV \ldots 10^3 TeV) consistent with BBN, \( \tau_{\tilde{G}} \approx 24 \times \)
(10 TeV/$m_{\tilde{g}}$)$^3$ sec. Total higgsino/wino abundance

$$\Omega_{\tilde{w},\tilde{h}} h^2 = \Omega_{\tilde{w},\tilde{h}} h^2 + \Omega_{\tilde{w},\tilde{h}} h^2 ,$$

$$\Omega_{\tilde{G}, h} h^2 = \frac{m_{LSP}}{m_{\tilde{G}}} \Omega_{\tilde{G}, h} h^2 \approx 2.7 \times 10^{-2} \left( \frac{m_{LSP}}{100 \text{ GeV}} \right) \left( \frac{T_{RH}(M_1, \tilde{m}_1)}{10^{10} \text{ GeV}} \right) ,$$

with ‘reheating temperature’ determined by neutrino masses (takes reheating process into account),

$$T_{RH} \approx 1.3 \times 10^{10} \text{ GeV} \left( \frac{\tilde{m}_1}{0.04 \text{ eV}} \right)^{1/4} \left( \frac{M_1}{10^{11} \text{ GeV}} \right)^{5/4} .$$

Requirement of LSP dark matter, i.e. $\Omega_{\text{LSP}} h^2 = \Omega_{\text{DM}} h^2 \approx 0.11$, yields upper bound on the reheating temperature, $T_{RH} < 4.2 \times 10^{10}$ GeV; lower bound on $T_{RH}$ from successful leptogenesis (depends on $\tilde{m}_1$).
For each ‘reheating temperature’, i.e. pair \((M_1, \tilde{m}_1)\), lower bound on gravitino mass (taken from Kawasaki et al ’08) (left panel). Requirement of higgsino/wino dark matter puts upper bound on LSP mass, dependent on \(\tilde{m}_1\), ‘reheating temperature’ (right panel); more stringent for higgsino mass, since freeze-out contribution larger. E.g., \(m_1 = 0.05 \text{ eV}\) implies \(m_{\tilde{h}} \lesssim 900 \text{ GeV}, m_{\tilde{G}} \gtrsim 10 \text{ TeV}\).
IV. Gravitational Waves

Relic gravitational waves are window to very early universe; contributions from inflation, preheating and cosmic strings (Rubakov et al ’82; Garcia-Bellido and Figueroa ’07; Vilenkin ’81; Hindmarsh et al ’12); cosmological $B-L$ breaking: prediction of GW spectrum with all contributions!

Perturbations in flat FRW background,

$$ds^2 = a^2(\tau)(\eta_{\mu \nu} + h_{\mu \nu})dx^\mu dx^\nu , \quad \bar{h}_{\mu \nu} = h_{\mu \nu} - \frac{1}{2}\eta_{\mu \nu}h^\rho_\rho ,$$

determined by linearized Einstein equations,

$$\ddot{\bar{h}}_{\mu \nu}(x, \tau) + 2\frac{a'}{a}\dot{\bar{h}}_{\mu \nu}(x, \tau) - \nabla^2_x \bar{h}_{\mu \nu}(x, \tau) = 16\pi G T_{\mu \nu}(x, \tau) .$$

Spectrum of GW background,

$$\Omega_{GW}(k, \tau) = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} ,$$
\[
\int_{-\infty}^{\infty} d\ln k \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(\mathbf{x}, \tau) \dot{h}^{ij}(\mathbf{x}, \tau) \right\rangle ;
\]

use initial conditions for super-horizon modes from inflation, calculate correlation function of stress energy tensor.

Contribution from inflation (primordial spectrum, transfer function; cf. Nakayama et al '08):

\[
\Omega_{GW}(k, \tau) = \frac{\Delta_{t}^{2}}{12} \frac{k^{2}}{a_{0}^{2}H_{0}^{2}} T_{k}^{2}(\tau)
\]

\[
= \frac{\Delta_{t}^{2}}{12} \Omega_{r} \frac{k_{*}^{k}}{g_{*}^{0}} \left( \frac{g_{*,s}^{0}}{g_{*,s}^{k}} \right)^{4/3} \begin{cases} 
\frac{1}{2} \left( k_{\text{eq}}/k \right)^{2} , & k_{0} \ll k \ll k_{\text{eq}} \\
1 , & k_{\text{eq}} \ll k \ll k_{\text{RH}} \\
\frac{1}{2} C_{RH}^{3} \left( k_{\text{RH}}/k \right)^{2} , & k_{\text{RH}} \ll k \ll k_{\text{PH}}
\end{cases}
\]
with boundary frequencies \( f = k / (2\pi a_0) \),

\[
\begin{align*}
  f_0 &= 3.58 \times 10^{-19} \text{ Hz} \left( \frac{h}{0.70} \right), \\
  f_{eq} &= 1.57 \times 10^{-17} \text{ Hz} \left( \frac{\Omega_m h^2}{0.14} \right), \\
  f_{RH} &= 4.25 \times 10^{-1} \text{ Hz} \left( \frac{T_{RH}}{10^7 \text{ GeV}} \right), \\
  f_{PH} &= 1.99 \times 10^4 \text{ Hz} \left( \frac{\lambda}{10^{-4}} \right)^{1/6} \left( \frac{v_{B-L}}{5 \times 10^{15} \text{ GeV}} \right)^{2/3} \left( \frac{T_{RH}}{10^7 \text{ GeV}} \right)^{1/3}.
\end{align*}
\]

**Contribution from preheating (cf. Dufaux et al ’07):**

\[
\Omega_{GW}(k_{PH}) h^2 \approx c_{PH} \left( R_{PH} H_{PH} \right)^2 \frac{\alpha_{PH}}{a_{RH}} \Omega_T h^2 \frac{g_{*RH}}{g_*} \left( \frac{g^0_{*s}}{g^0_{*s}} \right)^{4/3},
\]
with characteristic scalar and vector scales
\[
\left( R_{\phi H}^{(s)} \right)^{-1} = (\lambda v_{B-L} |\dot{\phi}_c|)^{1/3}, \quad \left( R_{\phi H}^{(v)} \right)^{-1} \sim m_Z = 2\sqrt{2} g v_{B-L}.
\]

Contribution from cosmic strings (Abelian Higgs):
\[
\Omega_{GW}(k) \simeq \frac{1}{6\pi^2} F^r \left( \frac{v_{B-L}}{M_{Pl}} \right)^4 \Omega_r h^2 \left\{ \begin{array}{ll}
(k_{eq}/k)^2, & k_0 \ll k \ll k_{eq} \\
1, & k_{eq} \ll k \ll k_{RH} \\
(k_{RH}/k)^2, & k_{RH} \ll k
\end{array} \right.
\]

constant $F^r$ recently determined in numerical simulation (cf. Figueroa, Hindmarsh, Urrestilla '12). Result similar to contribution from inflation, but very different normalization!
Are macroscopically long cosmic strings Nambu-Goto strings? Energy loss of string network by ‘massive radiation’ or gravitational waves? GW radiation from NG strings, radiated by loops of length

\[ l(t, t_i) = \alpha t_i - \Gamma G\mu(t - t_i) ; \]

rate for amplitude \( h \) and frequency \( f \) (cf. Kuroyanagi et al ’12),

\[
\frac{d^2 R}{dzdh}(f, h, z) \simeq \frac{3}{4(1 + z)(\alpha + \Gamma G\mu)h\alpha\gamma^2t^4(z)} \cdot \frac{1}{dz} \frac{dV(z)}{dz} \Theta(1 - \theta_m) ;
\]

can be approximately integrated analytically over \( z \) and \( h \); results differs qualitatively from Abelian-Higgs prediction; five orders of magnitude difference in normalization! Truth somewhere inbetween?
Abelian-Higgs Strings vs. Nambu-Goto Strings

\[ a = 10^{-6} \]

\[ a = 10^{-12} \]

\[ \alpha = 10^{-6} \]

\[ \alpha = 10^{-12} \]

\[ \Omega_{GW} h^2 \]

\[ k[Mpc^{-1}] \]

\[ f[Hz] \]
Observational Prospects

\[ \Omega_{GW} h^2 \]

\[ f [\text{Hz}] \]

\[ k [\text{Mpc}^{-1}] \]
Summary and Outlook

• Decay of false vacuum of unbroken B-L symmetry leads to consistent picture of inflation, baryogenesis and dark matter (everything from heavy neutrino decays)

• Prediction: relations between neutrino and superparticle masses for gravitino or higgsino/wino dark matter

• Possible direct test: detection of relic gravitational wave background, may provide determination of reheating temperature
(Non)thermal leptogenesis in $M_1 - \tilde{m}_1$ plane

Upper bound on $M_1$ from inflation; lower bound from baryogenesis
Gravitino Dark Matter vs. leptogenesis

Gravitino mass range: \(10 \text{ GeV} \lesssim m_{\tilde{G}} \lesssim 700 \text{ GeV}\); heavy neutrino mass range: \(2 \times 10^{10} \text{ GeV} \lesssim M_1 \lesssim 2 \times 10^{11} \text{ GeV}\) (more stringent than inflation)