The Conformal Higgs

Brando Bellazzini
University of Padova, SISSA, & INFN

based on a work in progress
with R. Franceschini, L. Martucci and R. Torre

GGI, Florence, July 10th 2013

FUTURO IN RICERCA
A new strong force, the origin of masses, and the LHC

The Conformal Higgs

or the role of the Dilaton in Composite Higgs models

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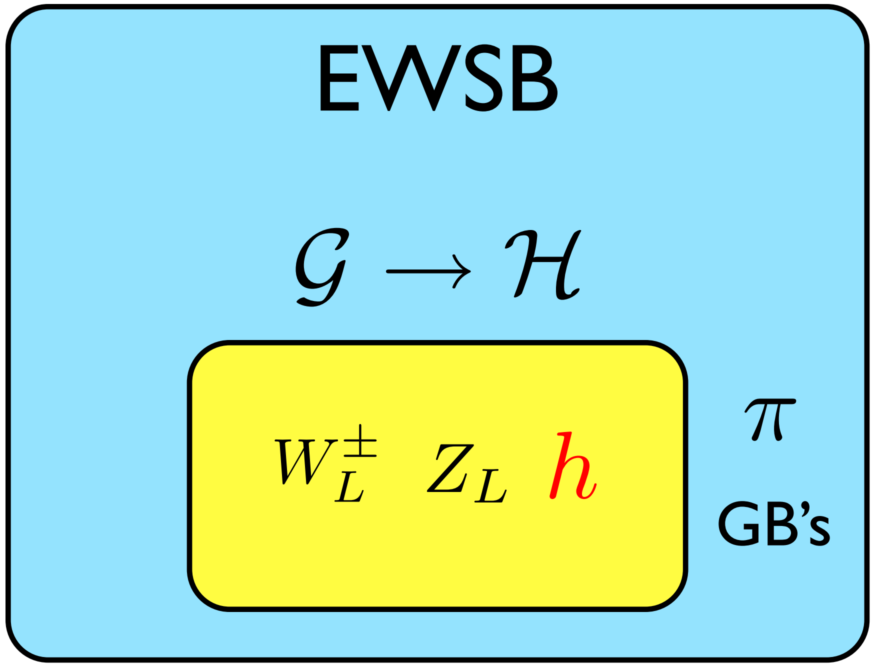
COMPOSITE HIGGS

strong sector

Georgi & Kaplan 1984



EWSB

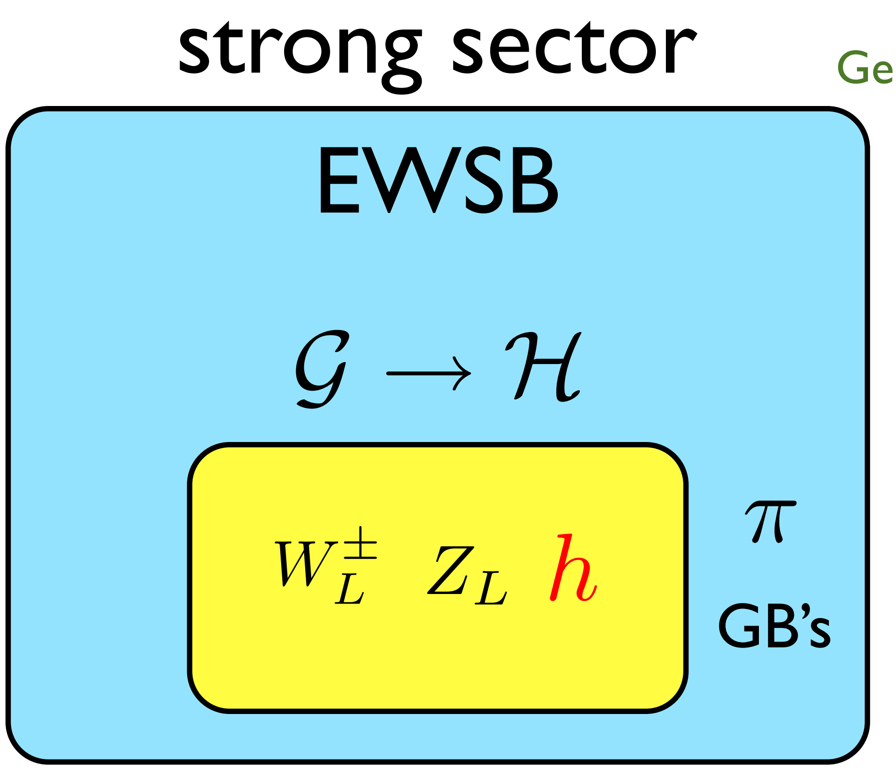
$\mathcal{G} \rightarrow \mathcal{H}$

$W_L^{\pm} \quad Z_L \quad h$

π

GB's

COMPOSITE HIGGS



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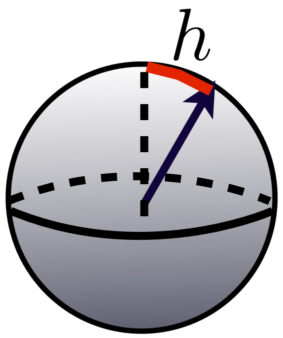
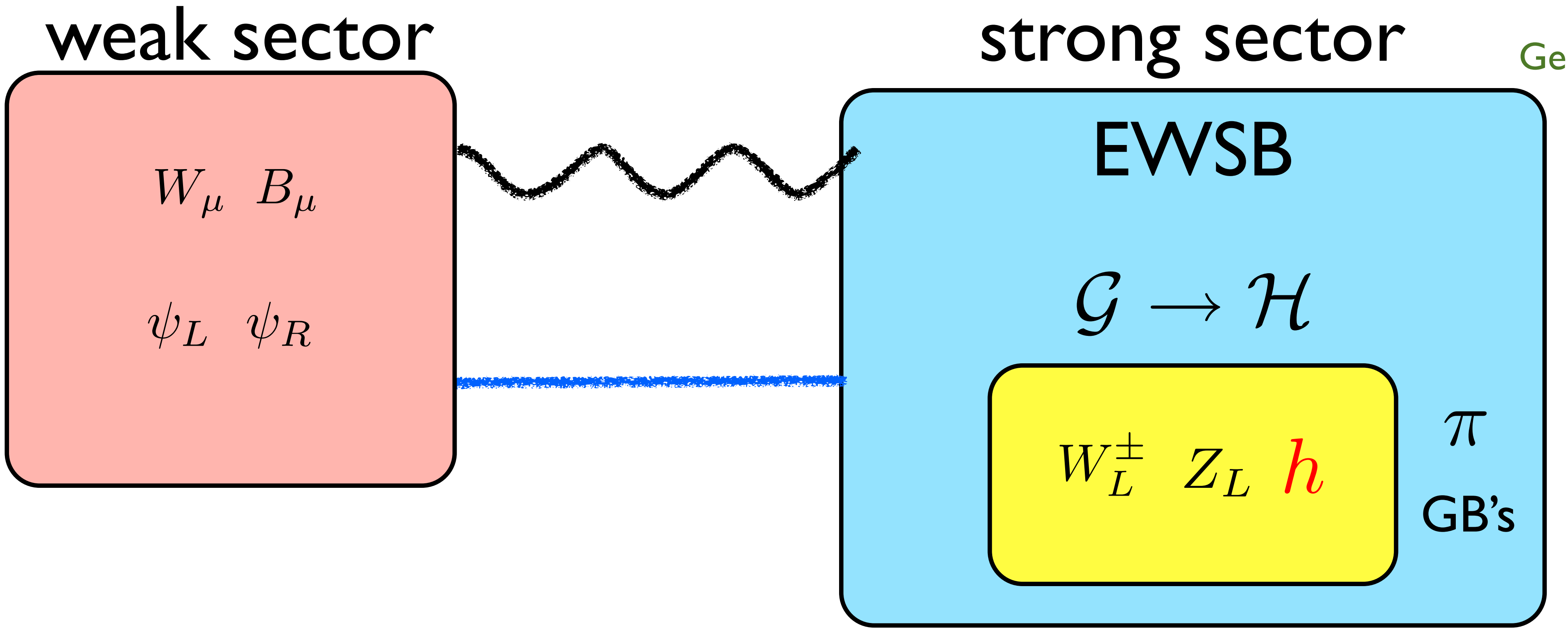
Minimal model: SO(5)/SO(4)= S_4

Agashe, Contino & Pomarol he-ph/0412089



COMPOSITE HIGGS



weak sector

W_μ B_μ

ψ_L ψ_R

strong sector

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$W_L^\pm$ Z_L h

π

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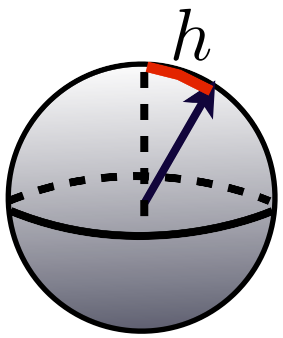
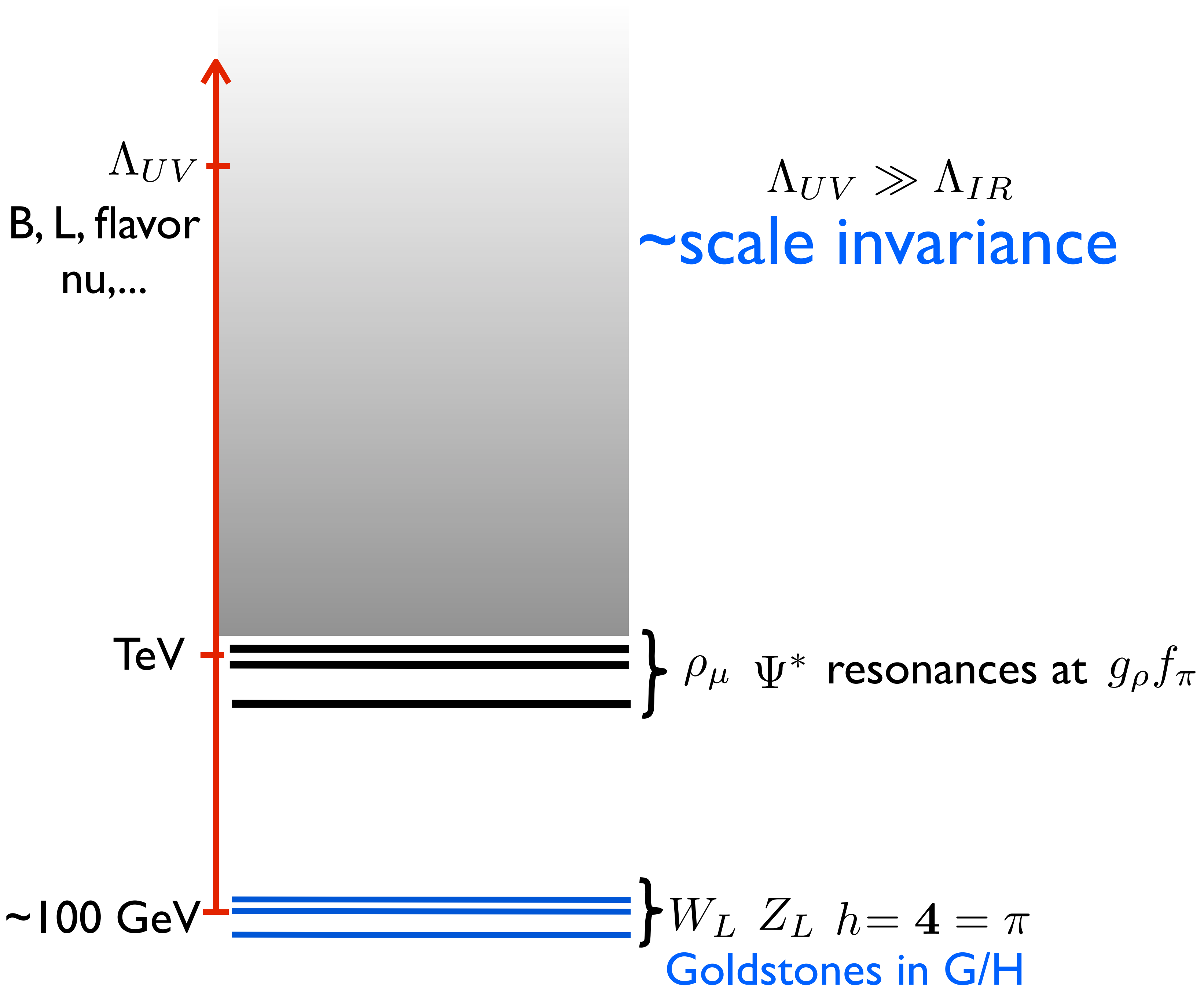


$$v = f_\pi \sin h$$

generically $\dfrac{v^2}{f_\pi^2} \sim \mathcal{O}(1)$

unless F.T.~few%

The spectrum



Λ_{UV}

B, L, flavor nu,...

TeV

~100 GeV

$\Lambda_{UV} \gg \Lambda_{IR}$

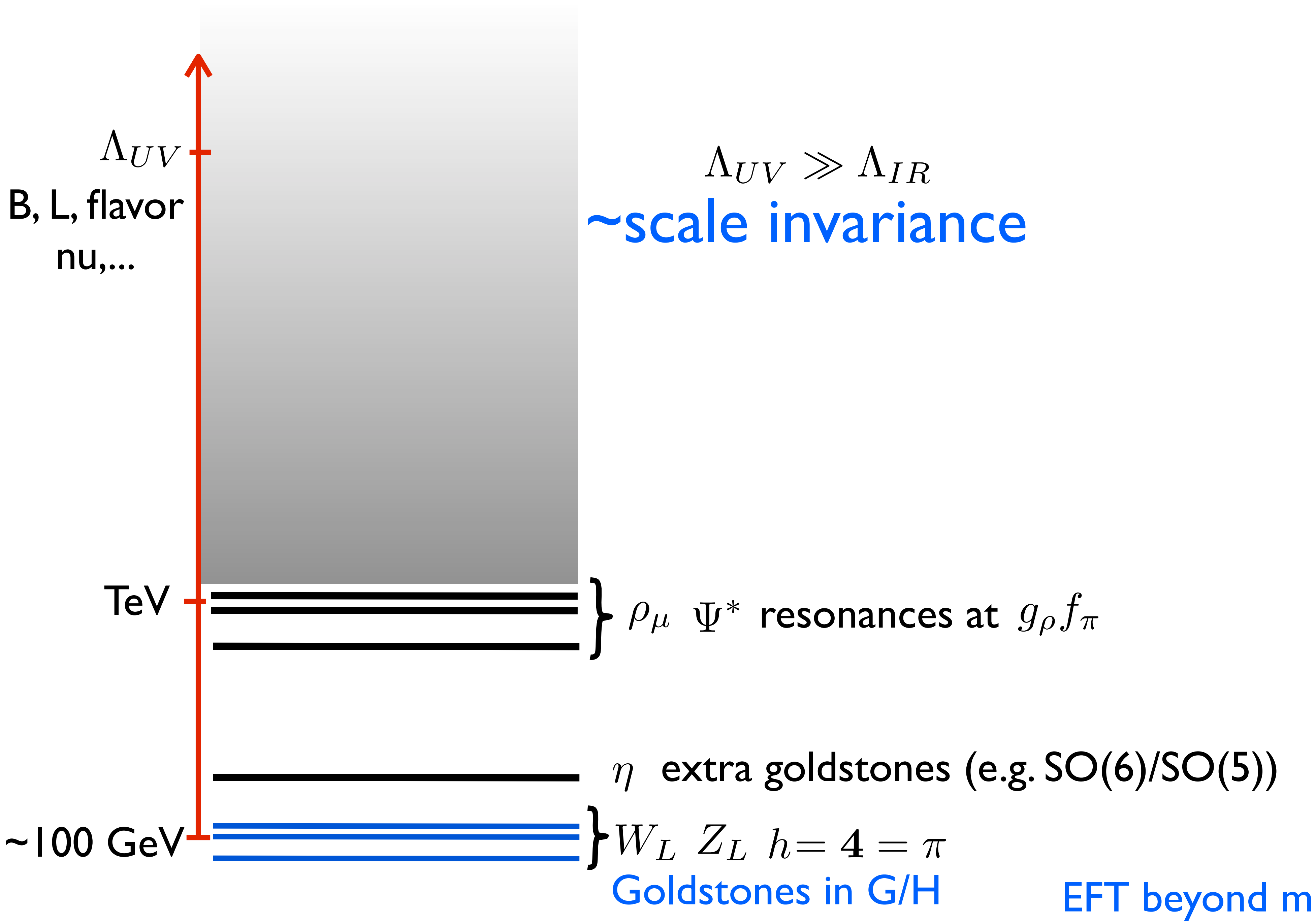
~scale invariance

$\rho_\mu \;\; \Psi^*$ resonances at $g_\rho f_\pi$

$W_L \;\; Z_L \;\; h = \mathbf{4} = \pi$

Goldstones in G/H

THE SPECTRUM

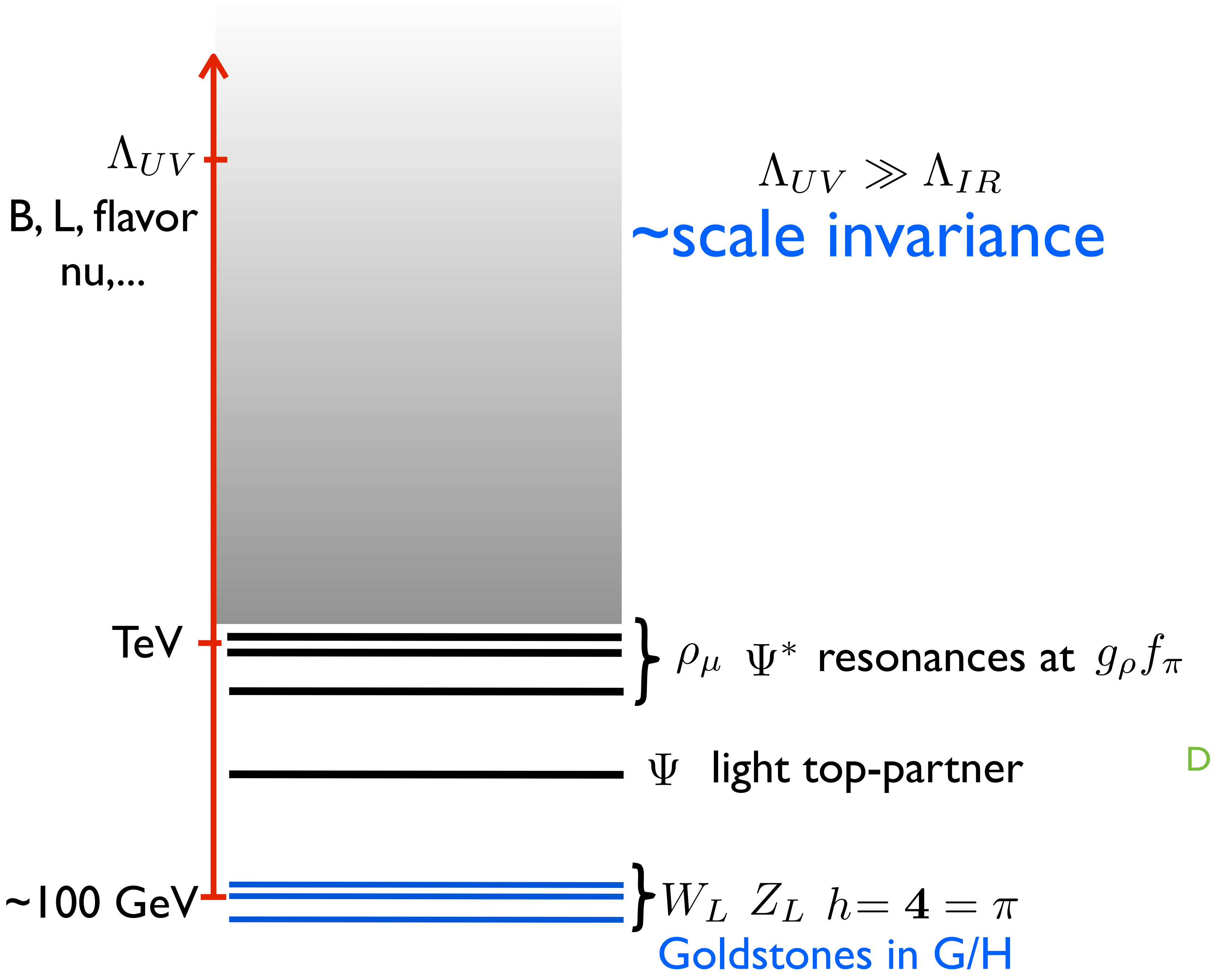


EFT beyond minimal GB sector

0902.1483 [hep-ph] Gripaios, Pomarol, Riva & Serra

1105.5403 [hep-ph] Mrazek, Pomarol, Rattazzi, Redi, & Serra

THE SPECTRUM



Λ_{UV}

B, L, flavor
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~scale invariance

TeV

ρ_μ Ψ^* resonances at $g_\rho f_\pi$

Ψ light top-partner

~100 GeV

W_L Z_L $h = \mathbf{4} = \pi$

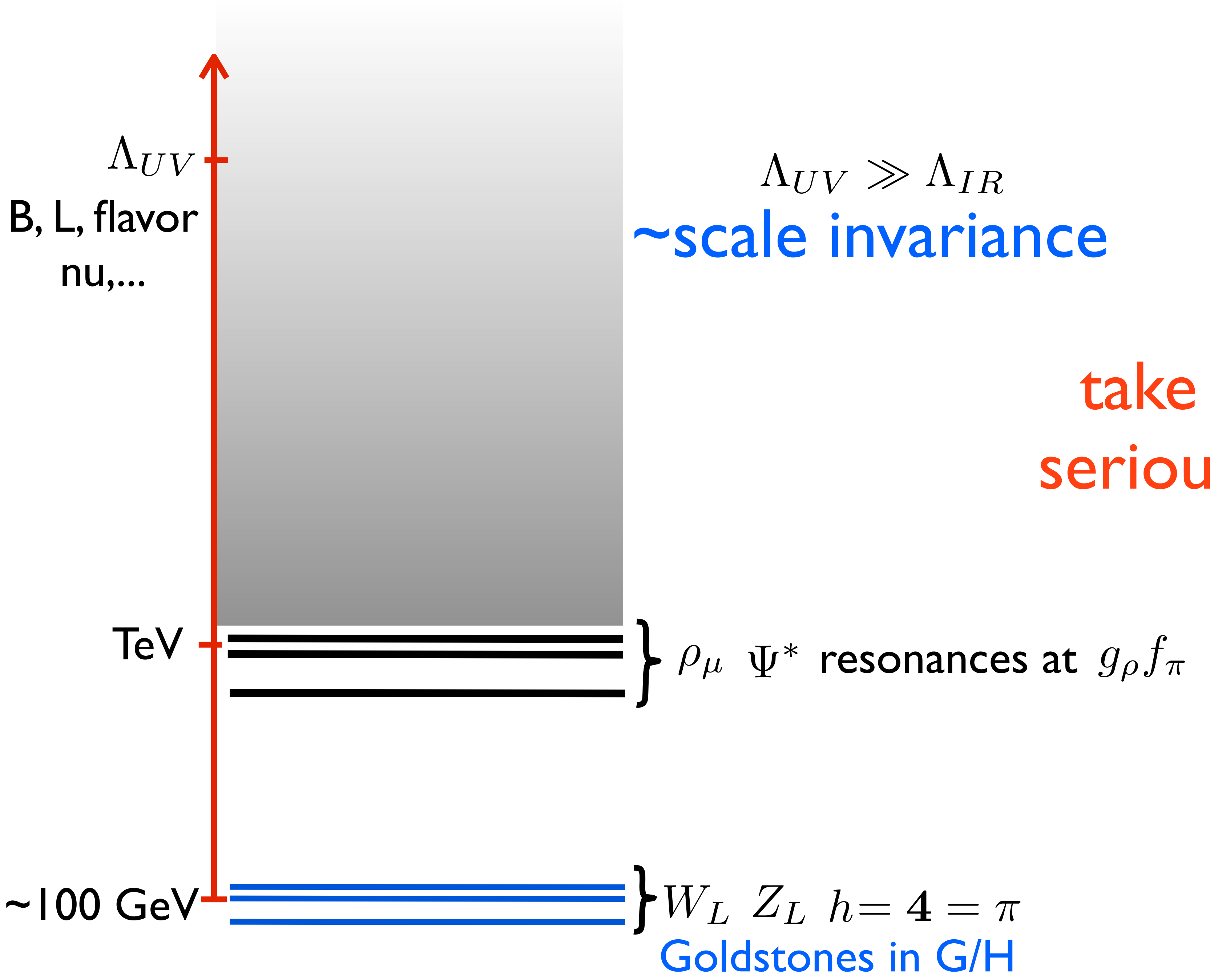
Goldstones in G/H

EFT for top-partners

see e.g. 1211.5663 [hep-ph]

De Simone, Matsedonskyi, Rattazzi & Wulzer

THE SPECTRUM



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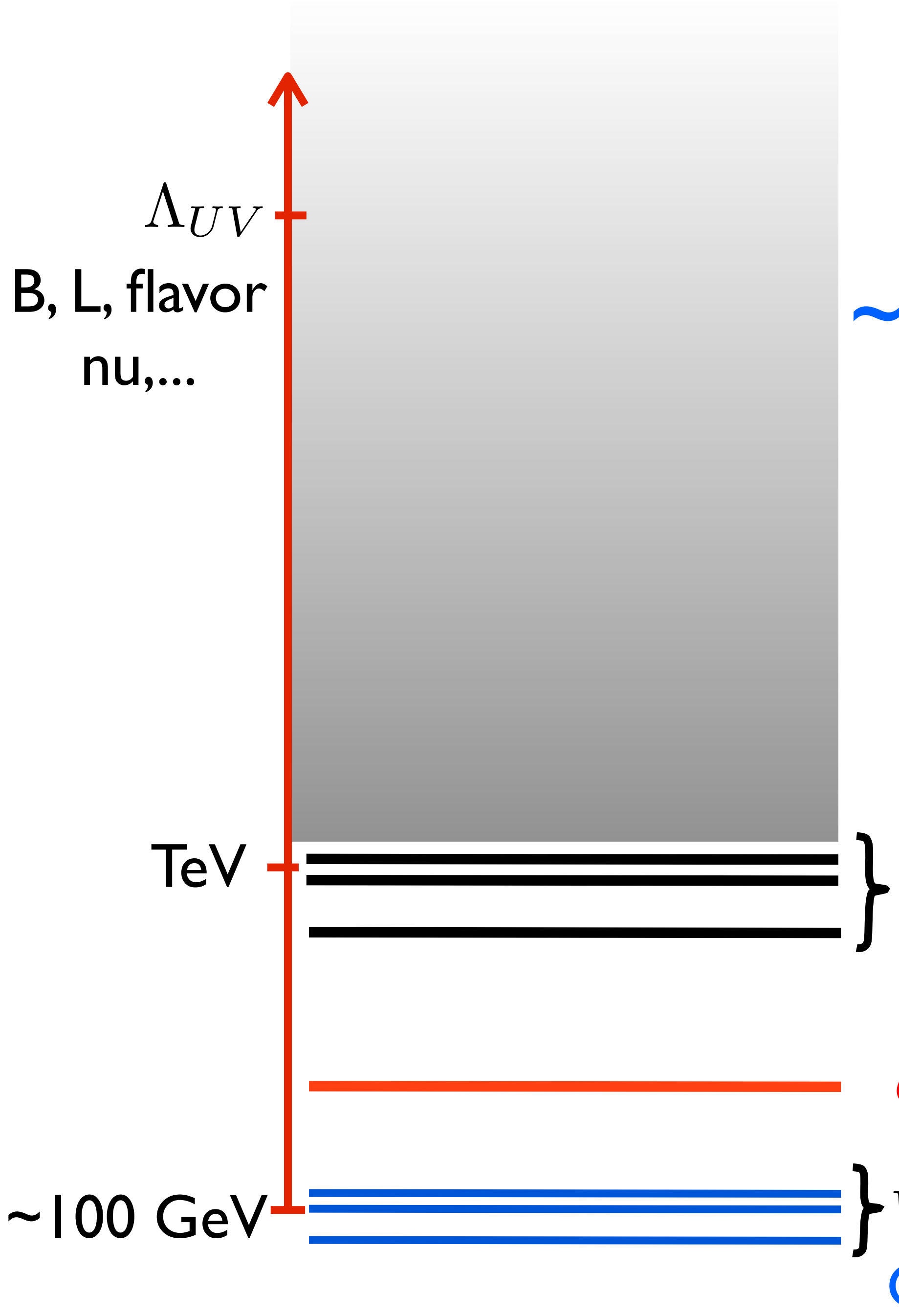
Goldstones in G/H

take scale invariance
seriously even in the IR

THE SPECTRUM



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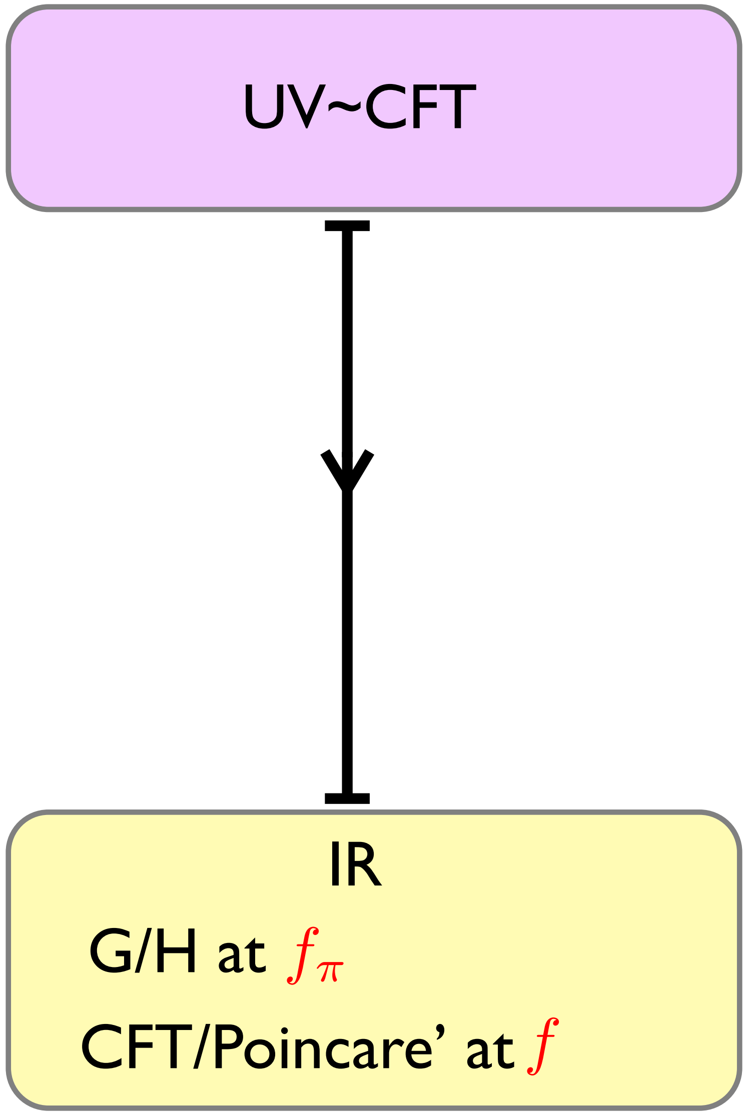
σ dilaton

W_L Z_L $h = \mathbf{4} = \pi$

Goldstones in G/H

EFT for Dilaton+Higgs



UV~CFT

IR

G/H at f_π

CFT/Poincare’ at f

Effective theory for

(G+CFT)/(H+Poincare’)?

global sym

spacetime sym

σ dilaton $m_\sigma \ll m_\rho$

π GBs $m_\pi \ll m_\rho$

for the EFT-Dilaton alone: see e.g. BB, Csaki, Hubisz, Serra &Terning 1305.3919 and1209.3299; Chacko & Mishra 1209.3022; Goldberger, Grinstein & Skiba 0708.1463; Hubisz, Csaki & Lee 0705.3844

DILATON BASICS

$$\text{CFT} \xrightarrow{\langle \mathcal{O}(x) \rangle = f^{\Delta}} \text{Poincare'}$$

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GOLDSTONE=DILATON

$$\sigma(x) \overset{\text{dilations}}{\longrightarrow} \sigma'(x') = \sigma(\underbrace{e^{-\alpha}}_{\text{spacetime tr.}} x') - \overbrace{\alpha}^{\text{shift-symmetry}}$$

$$\sigma(x) \overset{\text{special conform.}}{\longrightarrow} \sigma'(x') = \sigma(x(x')) - \frac{1}{4} \log J(x(x'))$$

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spacetime tr.

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scale=1

linear notation: $\chi(x) \equiv e^{\sigma(x)} \begin{cases} \to \chi'(x') = e^{-\alpha} \cdot \chi(e^{-\alpha} x') \\ \to \chi'(x') = J(x(x'))^{-\frac{1}{4}} \cdot \chi(x(x')) \end{cases}$

DILATON BASICS

DILATON BASICS

dilaton restores conformality

$$\mathcal{L}_{IR} \supset \mathcal{O}(x) \longrightarrow \mathcal{O}(x) \times \chi^{4-\Delta_{\mathcal{O}}}$$

dilaton couples to non-marginality

$$\mathcal{L}_{IR} \supset \mathcal{O}(x) \left[1 + (4-\Delta)\frac{\sigma}{f} + \ldots\right] = \mathcal{O}(x) + \frac{\sigma}{f}\partial_\mu D^\mu + \ldots$$

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like the Higgs: it couples to the mass

$$\frac{1}{f}\sigma T^\mu_\mu = \frac{v}{f}\sigma\left[2m_W^2 W^2 + m_\psi \psi\psi \dots\right]$$

overall rescaling

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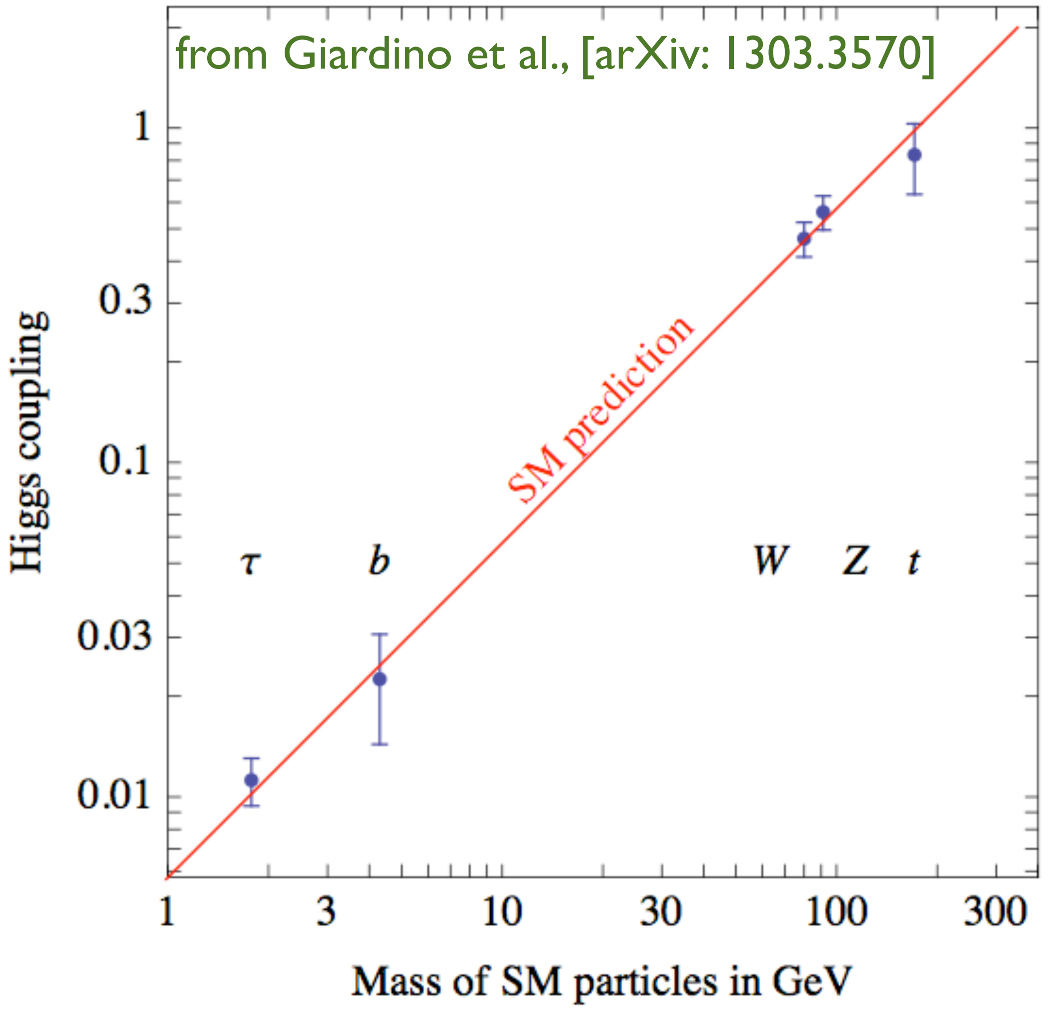
overall rescaling

Higgs-like Dilaton?

SO(4)/SO(3)+ dilaton with $v \sim f$ within 10%

BB, Csaki, Hubisz, Serra, Terning 1209.3299;
Chacko, Franceschini, Mishra 1209.3259

Fit to Higgs couplings



EFT: Goldstones+Dilaton

dilaton restores the CFT

covariant conformal derivatives

$$\mathcal{L}_{IR}(\phi, \partial_\mu \phi) \longrightarrow \mathcal{L}_{CFT} = \chi^4 \mathcal{L}_{IR}\left(\frac{\phi}{\chi^\Delta}, \frac{\nabla_\mu \phi}{\chi^{\Delta+1}}\right) + \frac{f^2}{2}(\partial_\mu \chi)^2 + \underbrace{\ldots}_{\text{O(p\textasciicircum 4)}}$$

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IR contains the Goldstones of G/H including the Higgs

$$\pi^{\hat{a}} = (\underset{(Z_L, W_L^\pm)}{\pi^{i=1,2,3}}, h, \ldots) \qquad \mathcal{L}_{IR}^{(2)} = \frac{1}{2} f_\pi^2 \partial_\mu \pi^{\hat{a}} \partial_\mu \pi^{\hat{b}} g_{\hat{a}\hat{b}}(\pi)$$

Pi's restore G

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scale dim. can be suitably chosen: $\Delta_\pi = 0$ “angles”

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$$\boxed{\mathcal{L}_{CFT+G}^{(2)} = \frac{1}{2}\chi^2 f_\pi^2\left[\partial_\mu\pi^{\hat{a}}\partial_\mu\pi^{\hat{b}}g_{\hat{a}\hat{b}}(\pi)\right] + \frac{1}{2}f^2(\partial\chi)^2}$$

all GB's restore G+CFT

THE CONE MANIFOLD

$$\mathcal{L}^{(2)}_{CFT+G} = \frac{1}{2}\left[\left(\frac{f_\pi^2}{f^2}\right)\chi^2\left(\partial_\mu\pi^{\hat{a}}\partial_\mu\pi^{\hat{b}}g_{\hat{a}\hat{b}}(\pi)\right) + \frac{1}{2}(\partial\chi)^2\right]$$

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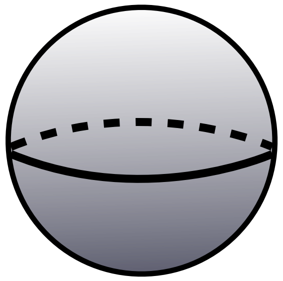

$\mathcal{G}/\mathcal{H}$

e.g. S_n

$$f_\pi^2 d\Pi^2 = f_\pi^2 g_{\hat{a}\hat{b}}(\pi)d\pi^{\hat{a}}d\pi^{\hat{b}}$$

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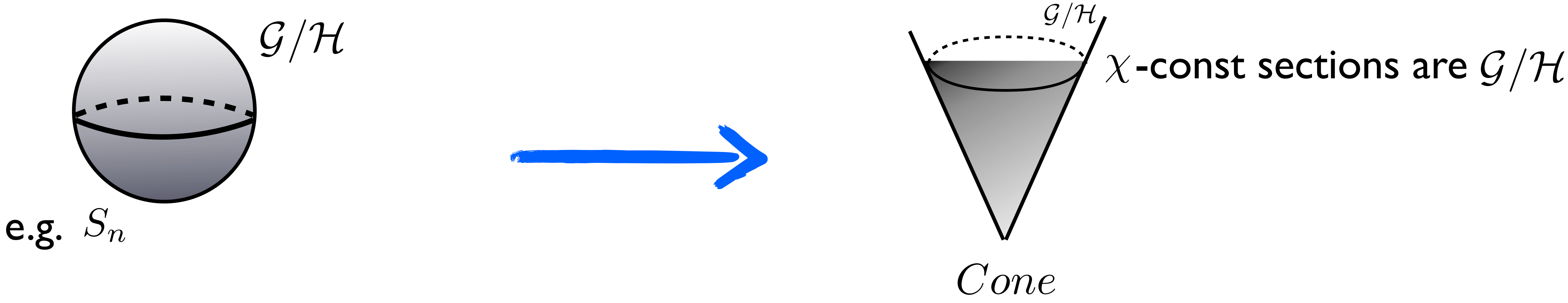

$$f_\pi^2 d\Pi^2 = f_\pi^2 g_{\hat{a}\hat{b}}(\pi)d\pi^{\hat{a}}d\pi^{\hat{b}}$$

$$d\Omega^2 = \left(\frac{f_\pi}{f}\right)^2\chi^2 d\Pi^2 + d\chi^2$$

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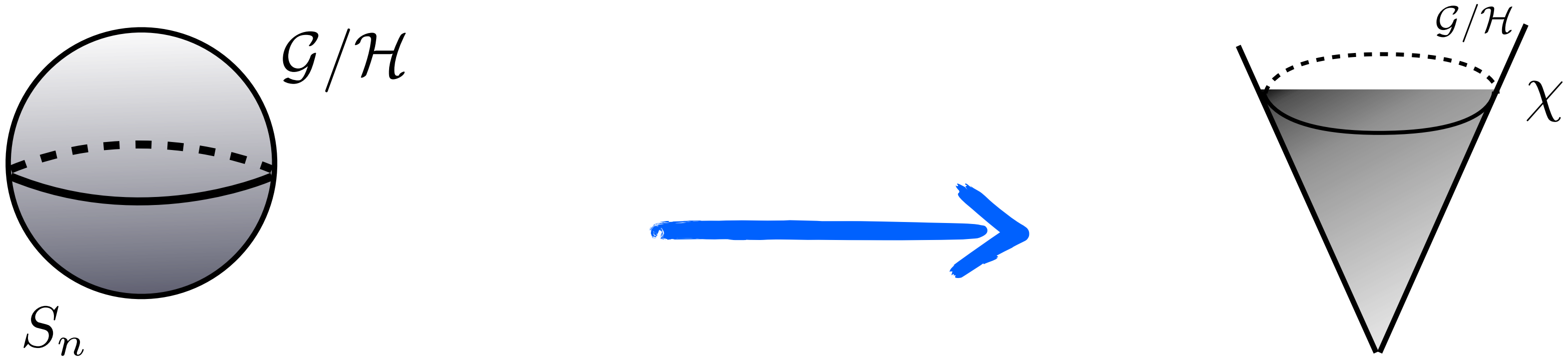

χ-const sections are $\mathcal{G}/\mathcal{H}$

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e.g. $|\Phi|^2 = f_\pi^2$ “radius”

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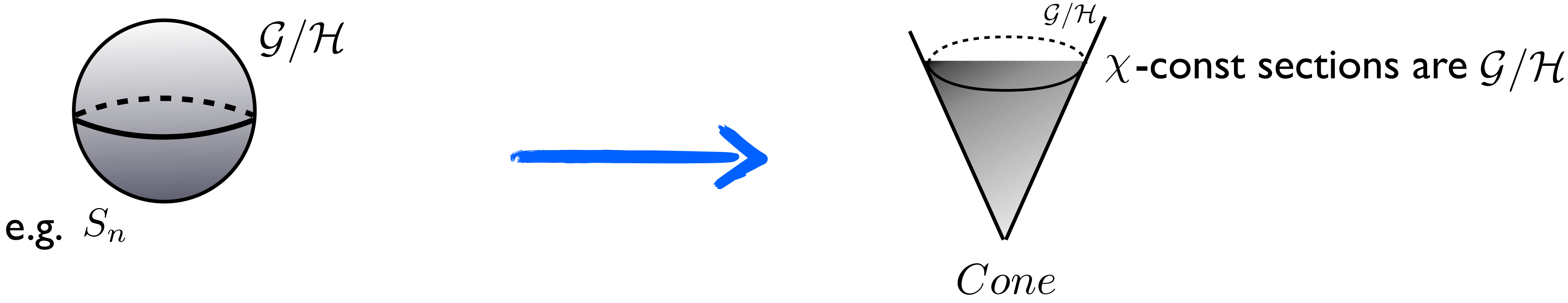

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“speed of light”

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

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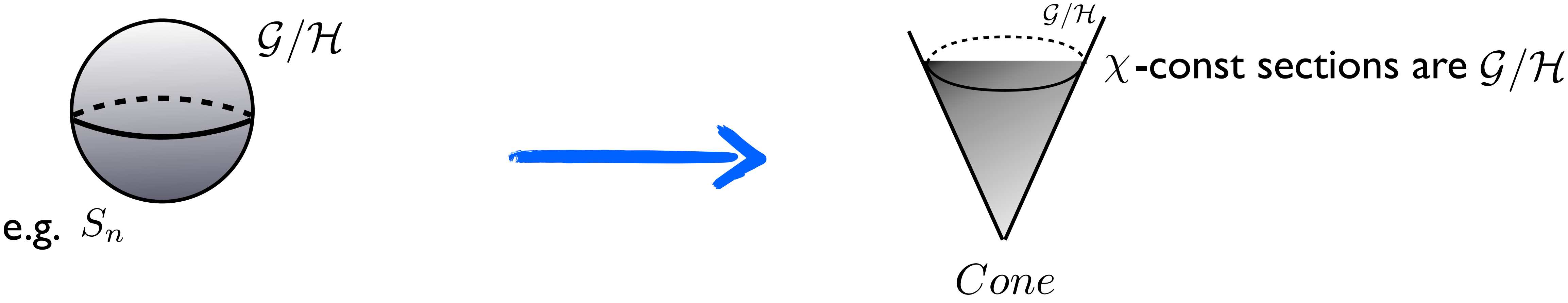

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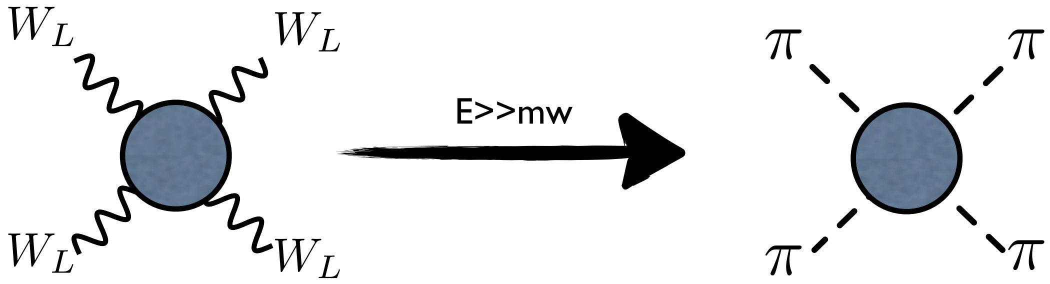
1) non-compact (dilations): no scale

2) singular at the apex where cut-off=0

WW-SCATTERING



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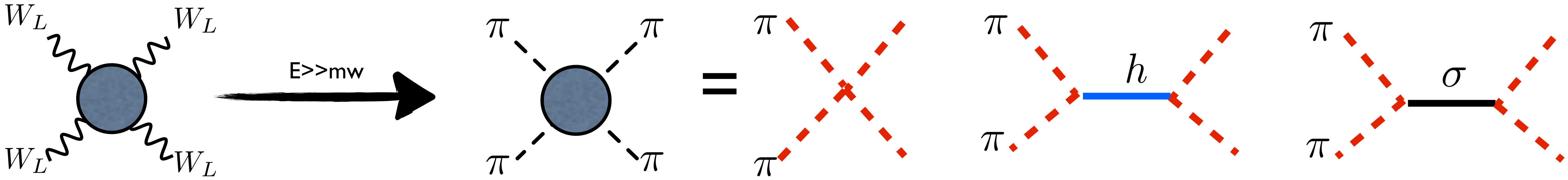

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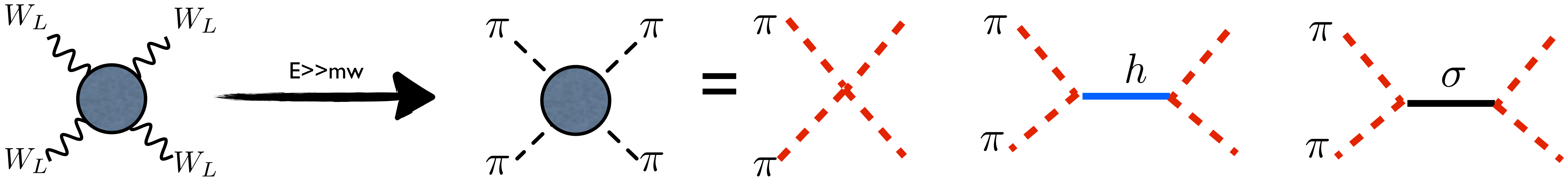

$$SO(n+1)/SO(n) = S_n \longrightarrow A(\pi\pi \to \pi\pi) \sim \underline{\frac{s}{f_\pi^2} - \frac{s}{f^2}}$$

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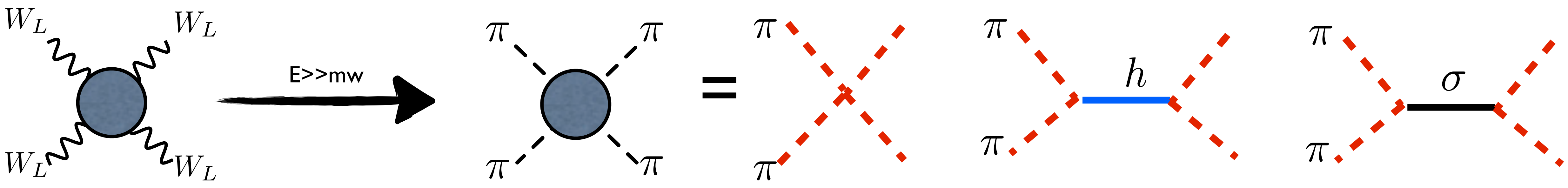


$$SO(n+1)/SO(n) = S_n \longrightarrow A(\pi\pi \to \pi\pi) \sim \underline{\frac{s}{f_\pi^2} - {\color{red}\frac{s}{f^2}}}$$

Limit: $f = f_\pi$

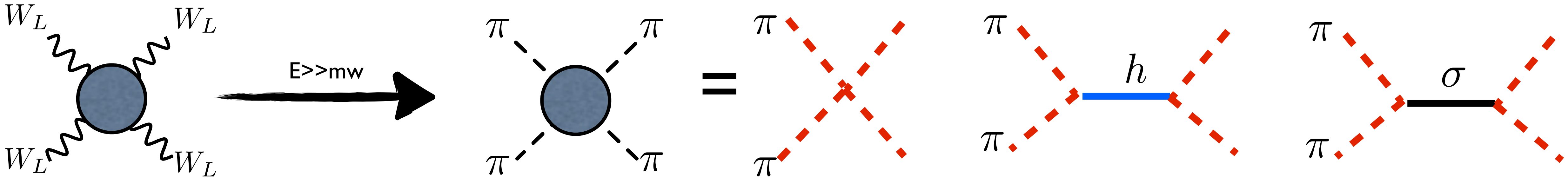
$$A(\pi\pi \to \pi\pi) \sim \left(\frac{s}{f_\pi^2} - {\color{red}\frac{s}{f^2}}\right) \to 0$$

$$A(\pi\pi \to hh) \sim \left(\frac{s}{f_\pi^2} - {\color{red}\frac{s}{f^2}}\right) \to 0$$

$$A(\pi\pi \to \sigma\sigma) \to 0$$

... all amplitudes vanish!!

WW-SCATTERING



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E.g.: the Higgs-like dilaton $\underbrace{\text{SO(4)/SO(3)}}$?

$$(f_\pi \equiv)\, v = f\ ?$$

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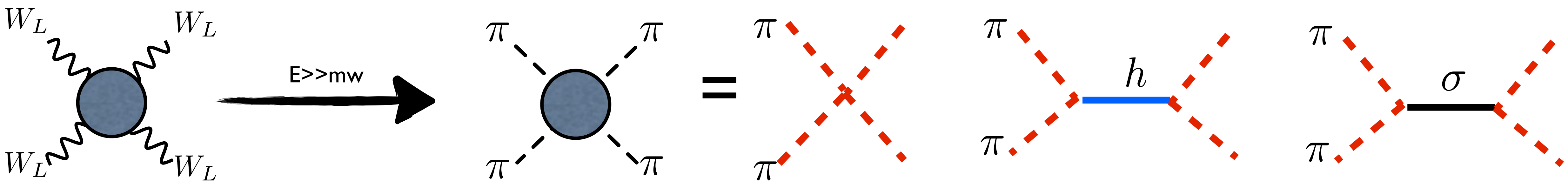

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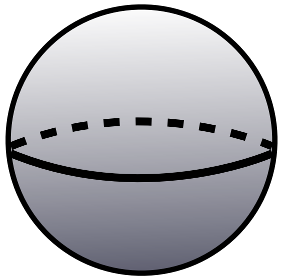
$$(f_\pi \equiv)\, v = f\ ?$$

- symmetry, tuning or dynamics?
- is it actually weakly coupled?

$\Lambda \sim 4\pi f\ ?$ $\quad \Lambda \gg 4\pi f\ ?$

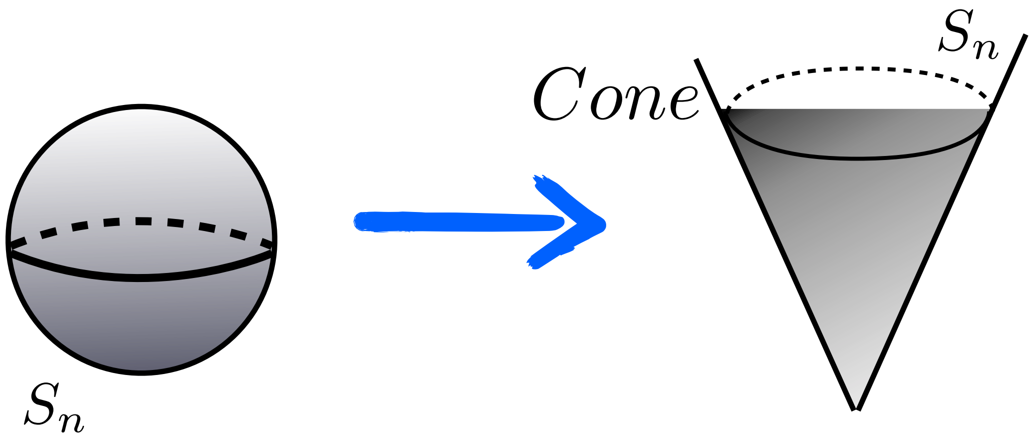
Cone vs Plane



S_n

$$|\Phi|^2 = f_\pi^2$$

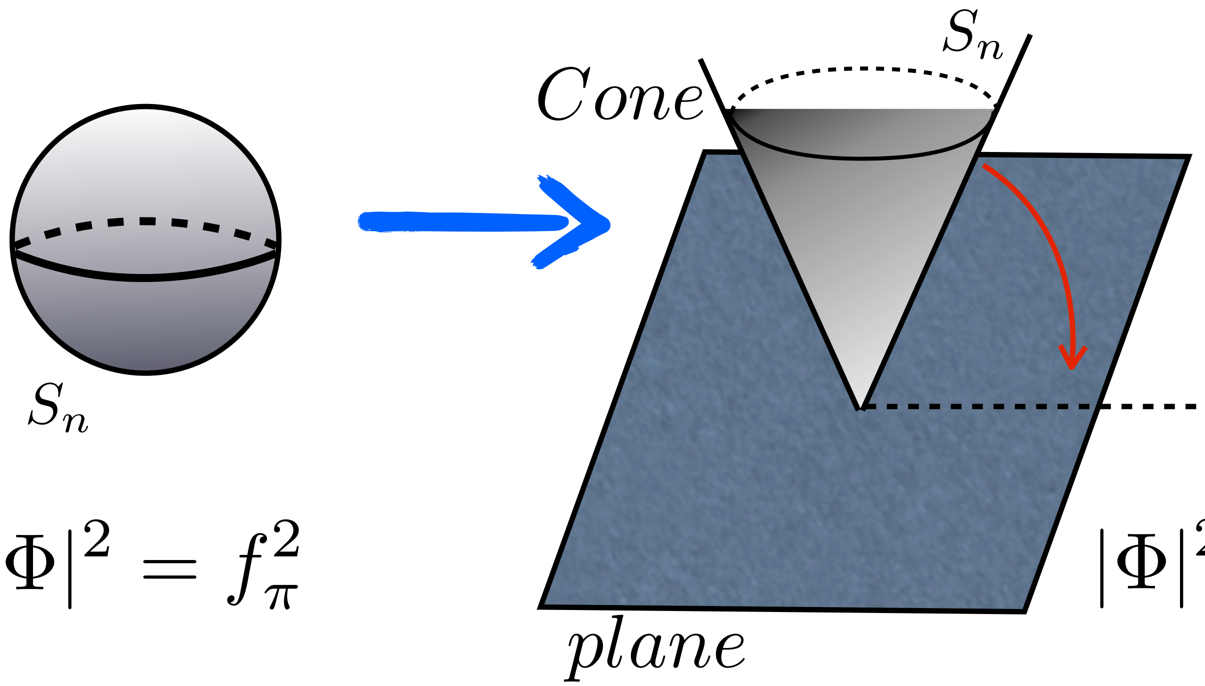
Cone vs Plane



$$|\Phi|^2 = f_\pi^2$$

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

CONE VS PLANE



the cone unfolds into a plane

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"speed of light"= ∞

$$f_\pi = f$$

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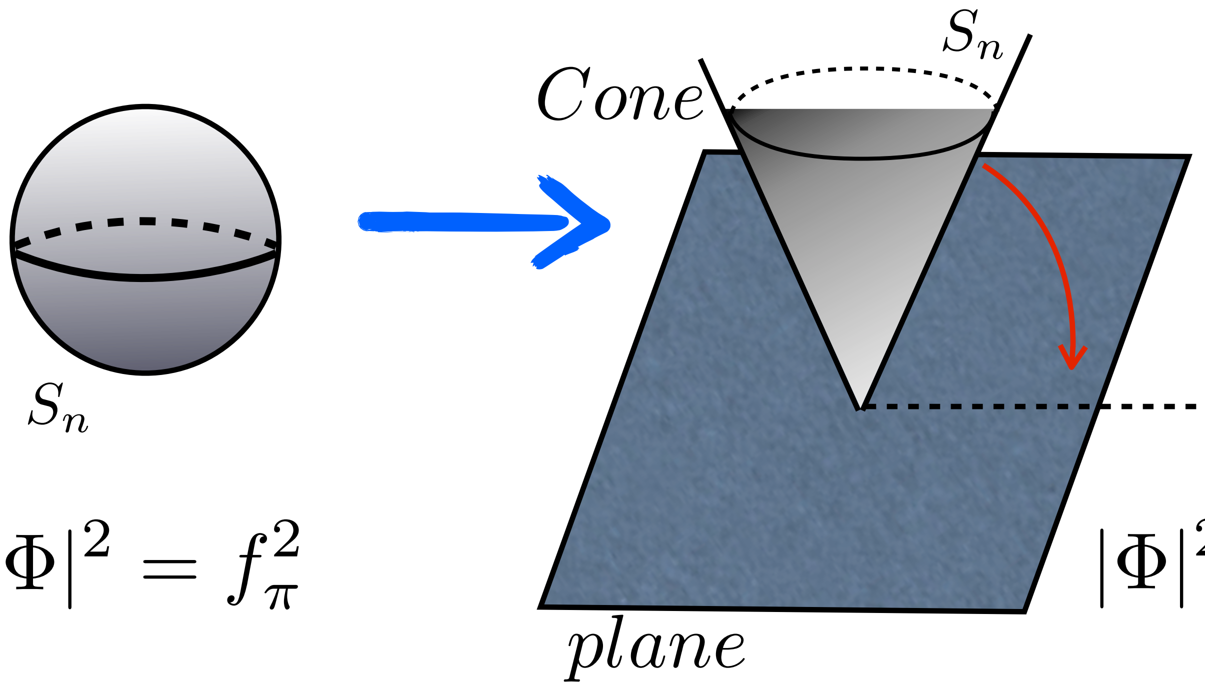

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radius goes to 1

radial coordinates of a plane

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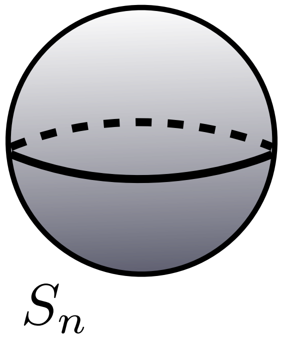

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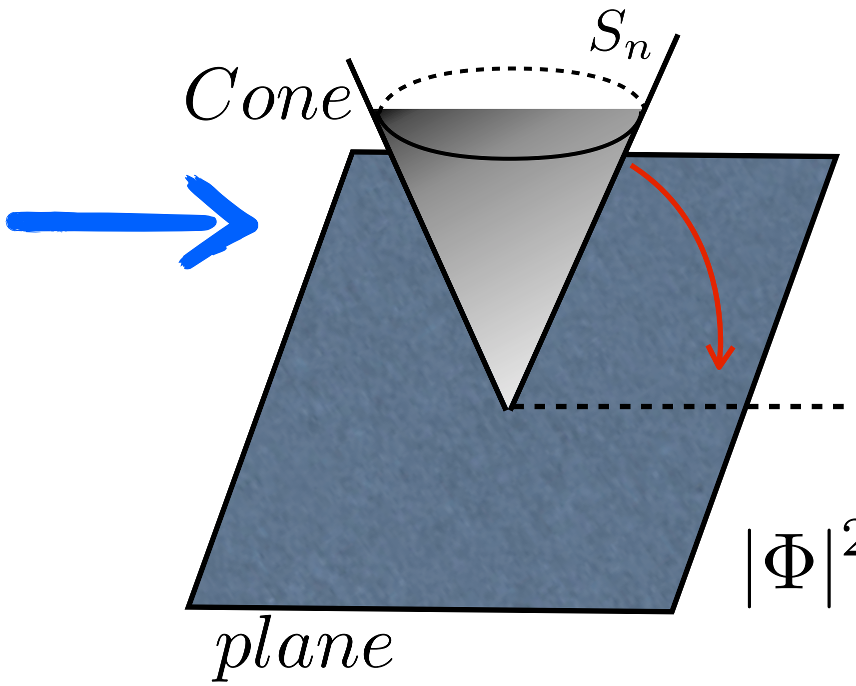

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$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu \varphi^a \partial_\mu \varphi^a$$

free theory at O(p^2) in euclidean coordinates

all amplitudes are trivially vanishing (at this order)

HIGHER ORDERS?

CCWZ notation: $e^{-i\pi}\partial_\mu e^{i\pi} = i d_\mu^{\hat{a}} T^{\hat{a}} + i E_\mu^a T^a$

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only pions: $(\mathrm{Tr}[d_\mu d^\mu])^2$, $\mathrm{Tr}[d_\mu d^\nu]\mathrm{Tr}[d_\mu d^\nu]$, $\mathrm{Tr}[E_{\mu\nu}E^{\mu\nu}]$

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a-anomaly Komargodski-Schwimmer

by e.o.m enters in pi-pi scattering

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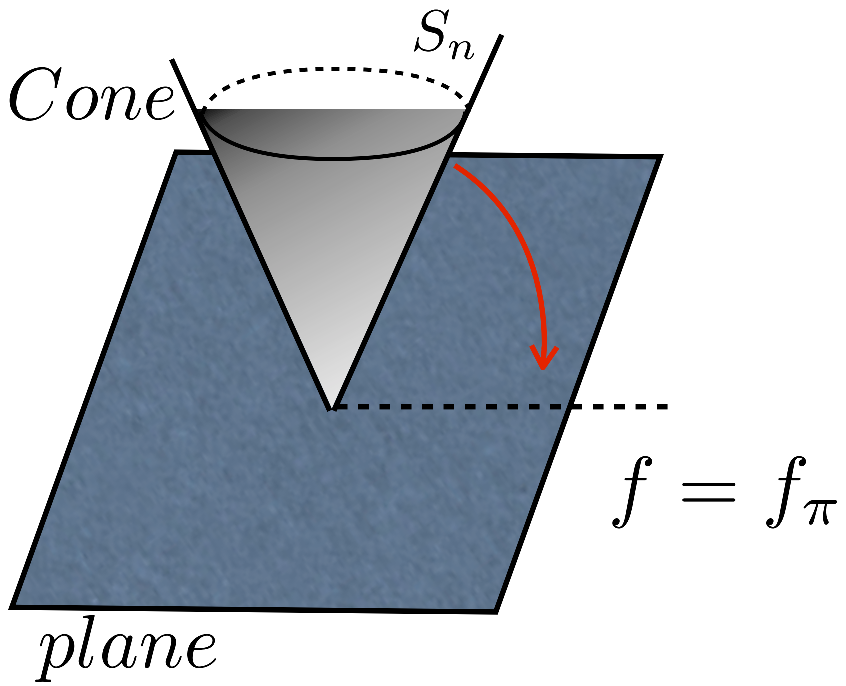
no reasons to expect cancellations

$$\boxed{A(\pi\pi \to \pi\pi) \sim E^4}$$

Are we sensitive to E^4 vs E^0 in WW-scattering?

ACCIDENTAL SYMMETRY



plane: invariant ISO(n+1)=SO(n+1)+translations

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu\varphi^a\partial_\mu\varphi^a \qquad \varphi^a \to \varphi^a + c^a$$

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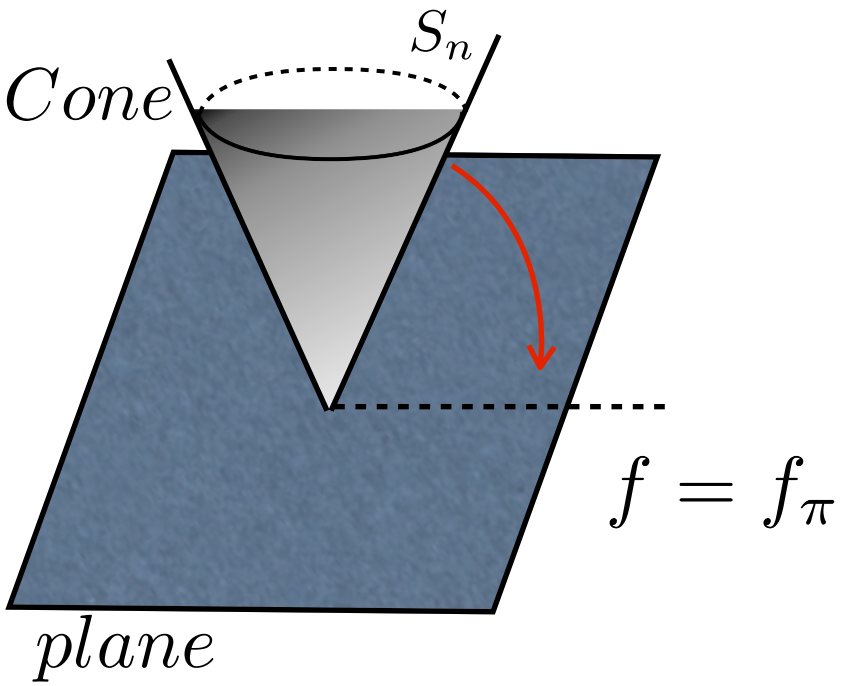

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(true sym. only SO x CFT)

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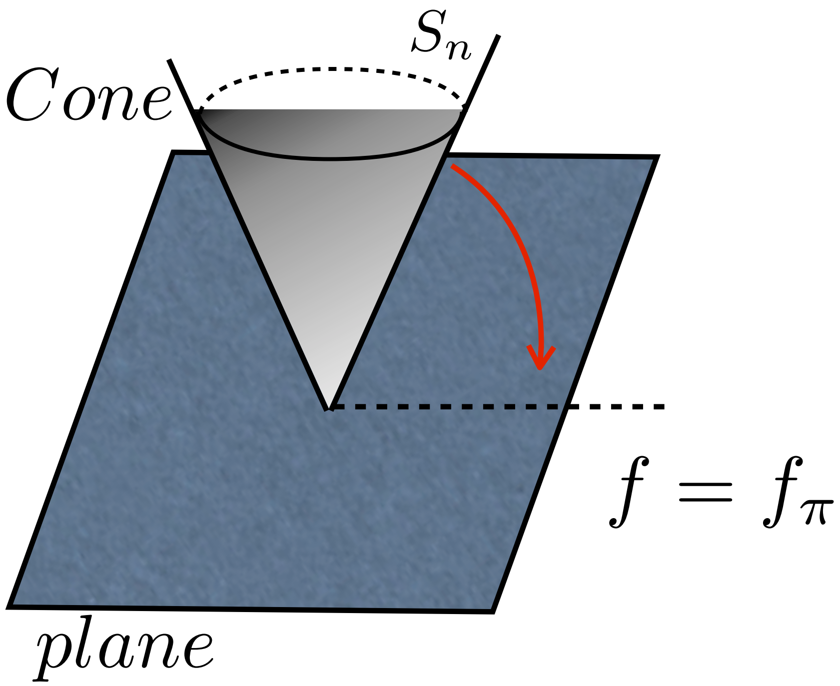

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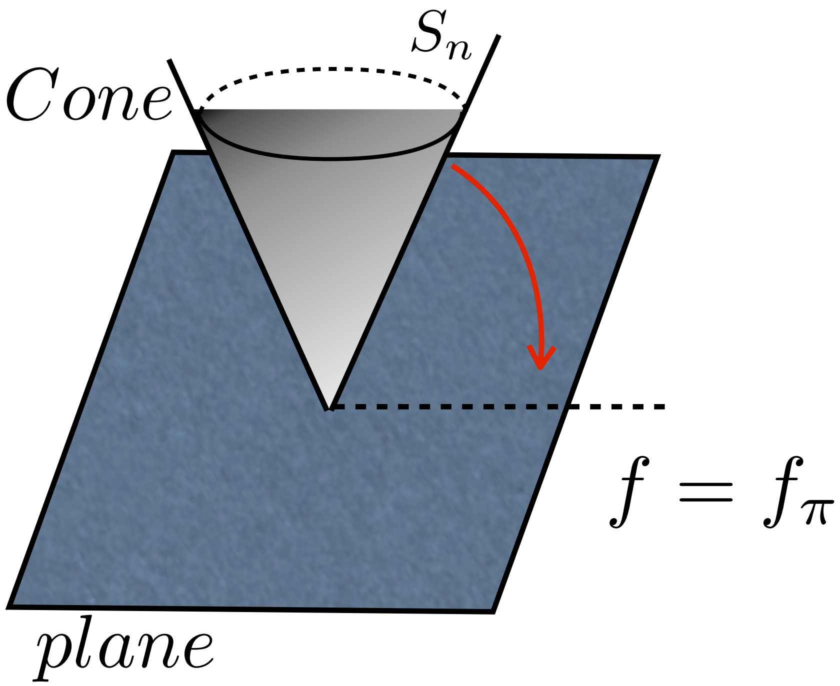

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step 2) make it marginal: divide by~ $\varphi^a\Box\varphi^a$?

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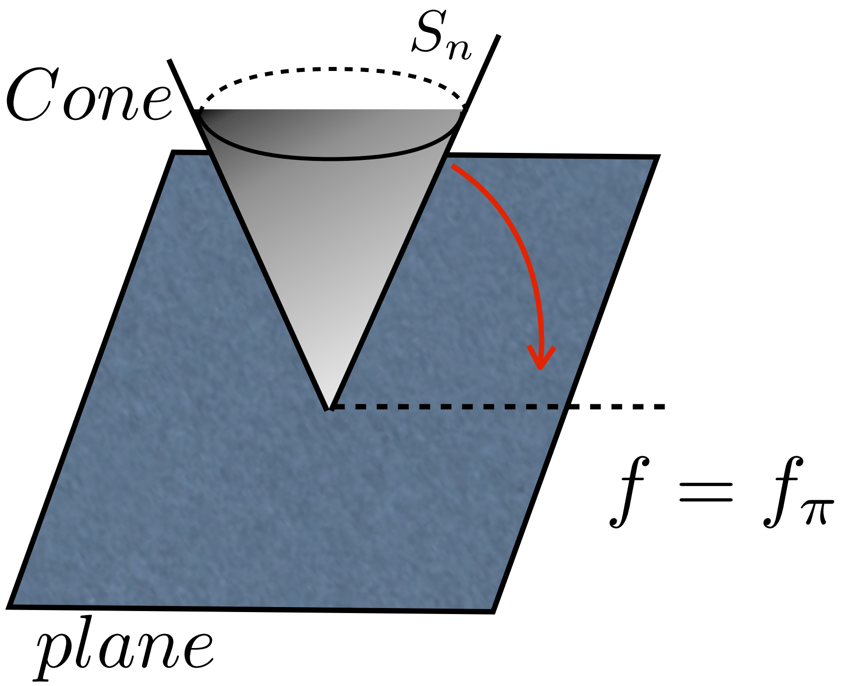

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$$\mathcal{L}^{(4)} \sim \frac{a}{\varphi^a\Box\varphi^a}(\partial_\mu\varphi^a)^4 + \frac{b}{(\dots)}(\partial_\mu\varphi^i\partial_\nu\varphi^i)^2 + \dots$$

non-locality forced by translations+dilations!

ACCIDENT VS SYMMETRY

Accident vs Symmetry

Accident

$$\begin{tikzcd} SO(n+1) \times CFT \arrow[d] \\ SO(n) \times Poincare' \end{tikzcd}$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^4$$

resonances at $\Lambda = 4\pi f$

strongly coupled

ACCIDENT VS SYMMETRY

barring non-locality (=no extra massless fields)

Accident

$$SO(n+1) \times CFT$$

$$\Big\downarrow$$

$$SO(n) \times Poincare'$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^4$$

resonances at $\Lambda = 4\pi f$

strongly coupled

Symmetry

$$ISO(n+1) \times CFT$$

$$\Big\downarrow$$

$$SO(n) \times Poincare'$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^0$$

cut-off can be at $\Lambda = \infty$

weakly coupled

HIERARCHY PROBLEM?

Symmetry

$$ISO(n+1) \times CFT$$

↓

$$SO(n) \times Poincare'$$

weakly coupled

breakings translations

new scale breaking CFT

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi^a \partial_\mu \varphi^a + \epsilon_{I\cancel{S}O} \times M^2_{C\cancel{F}T}\, \varphi^a \varphi^a + \dots$$

the relevant operator is small by symmetry

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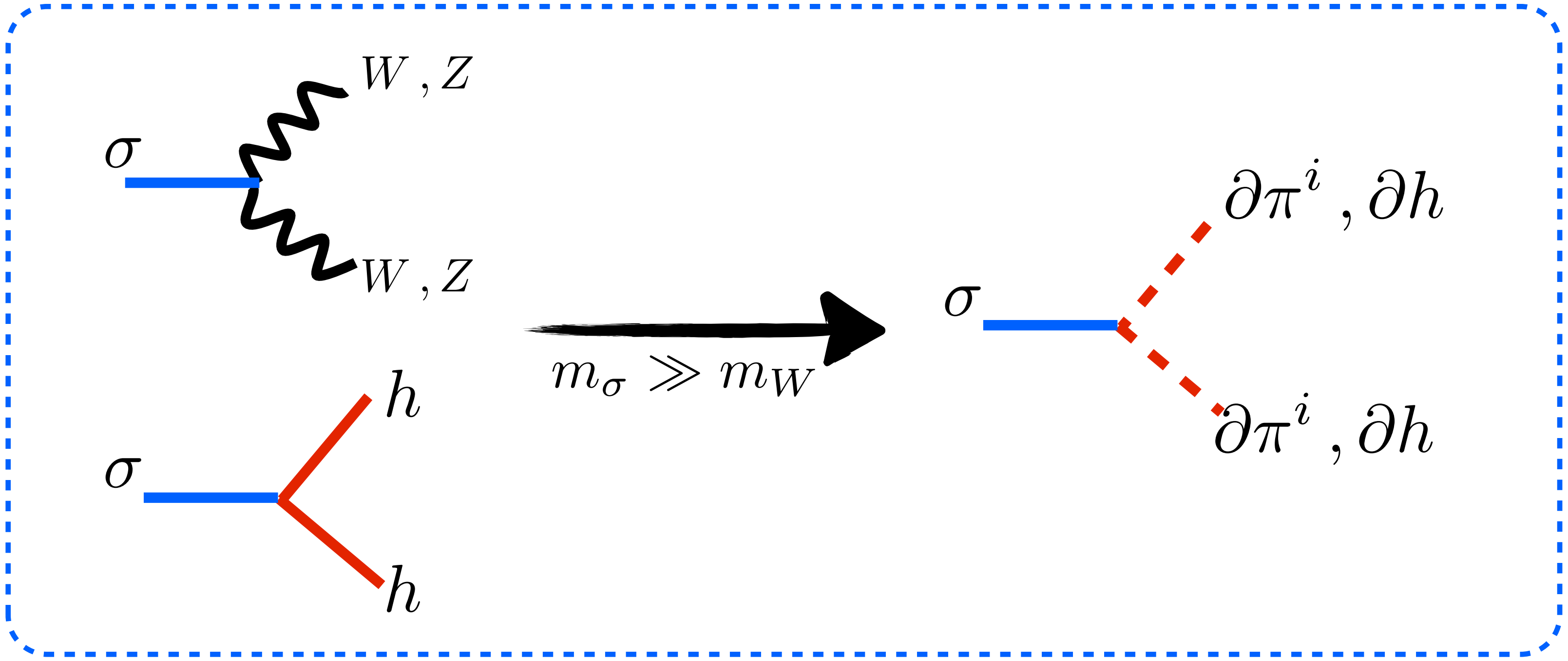
$$\boxed{f^2 = \frac{M^2_{C\not{F}T}}{4\lambda^2} \qquad m^2_\sigma \propto \epsilon f^2 \qquad \Delta = \frac{f^2}{v^2} \gg 1}$$

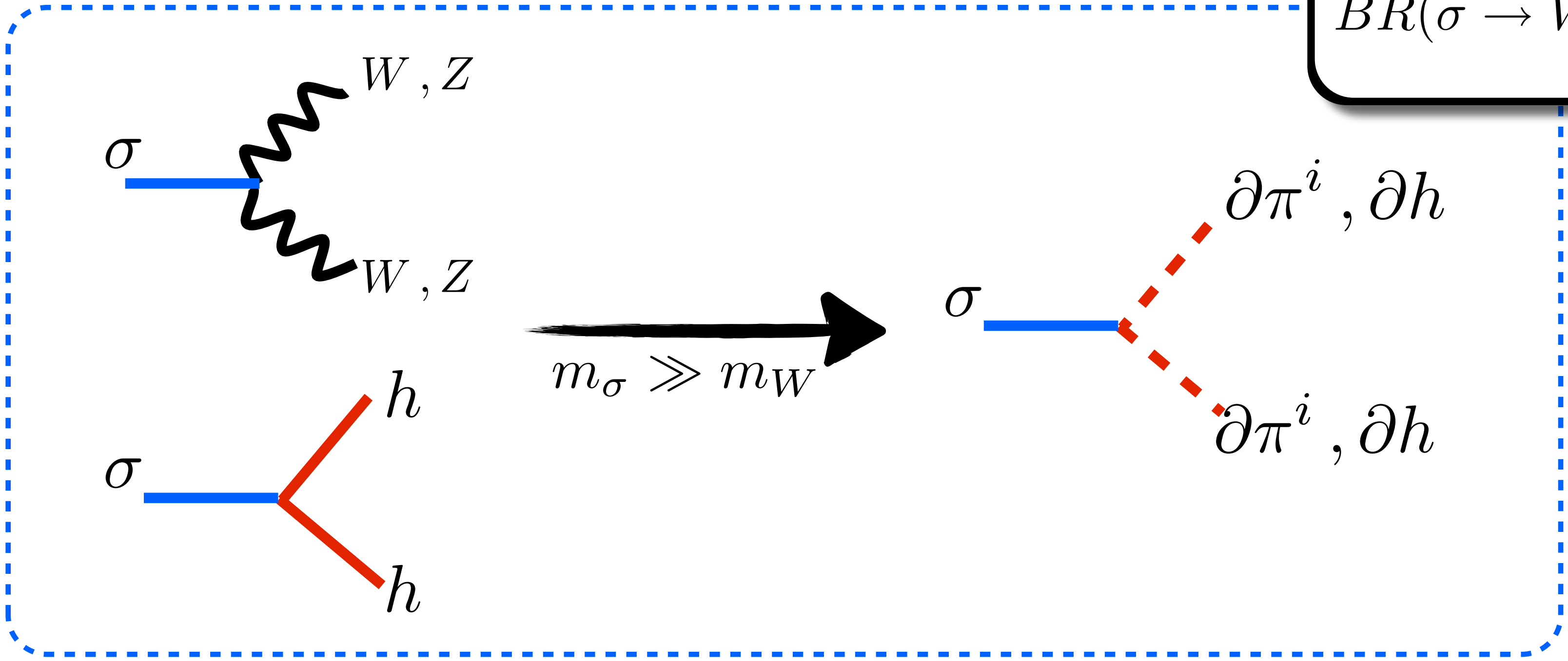
generically big tuning!

Dilaton decays



DILATON DECAYS

$$\boxed{BR(\sigma \to WW) \simeq 2BR(\sigma \to ZZ) \simeq 2BR(\sigma \to hh)}$$



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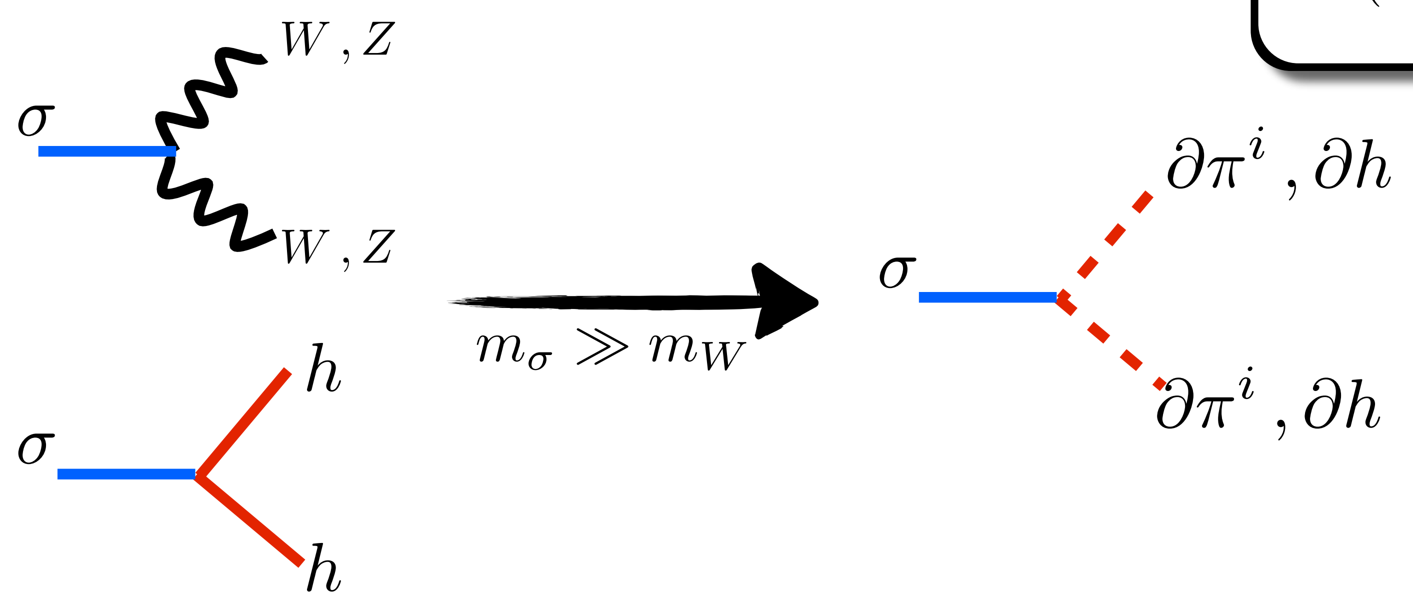

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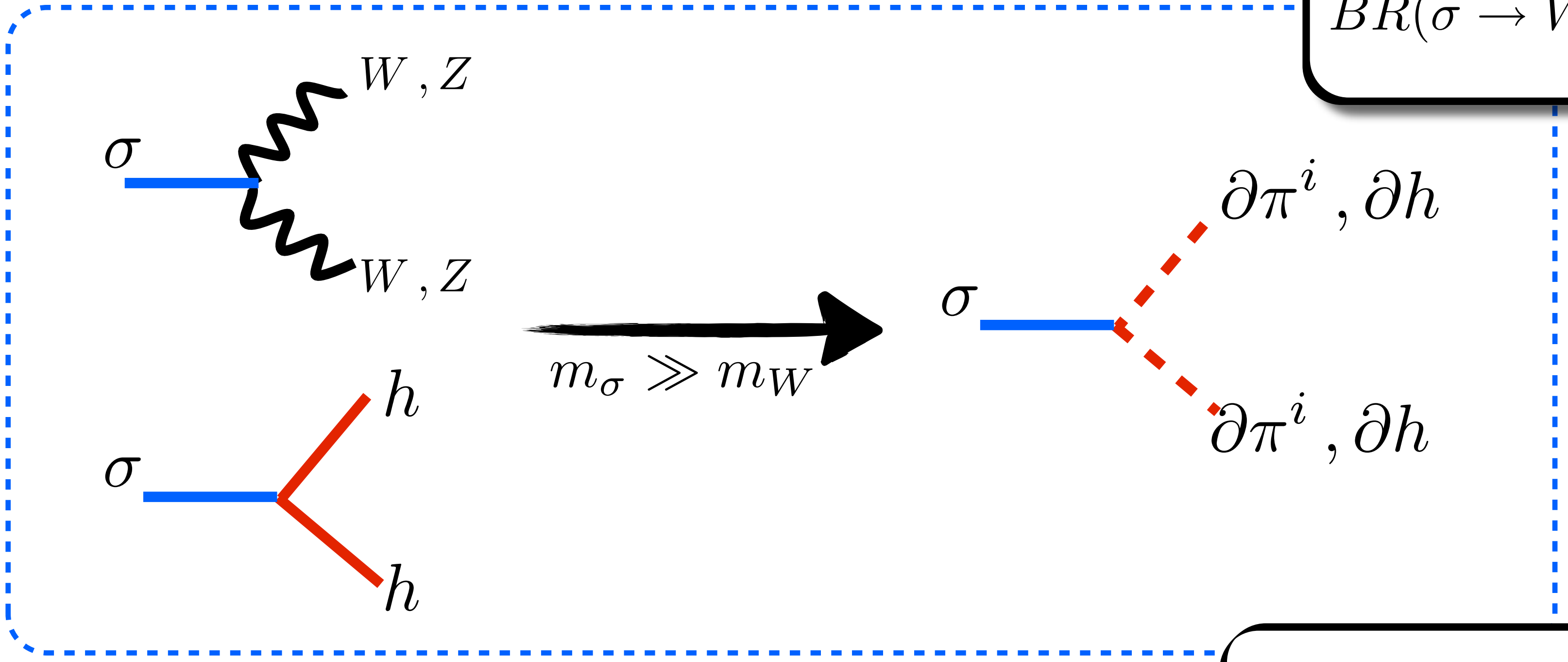

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$$|\nabla_\mu H|^2 = |\partial_\mu - \Delta_H \partial_\mu \sigma) H|^2$$

Giudice, Rattazzi, Wells hep-ph/0002178;
L.Vecchi 1002.1721 [hep-ph]

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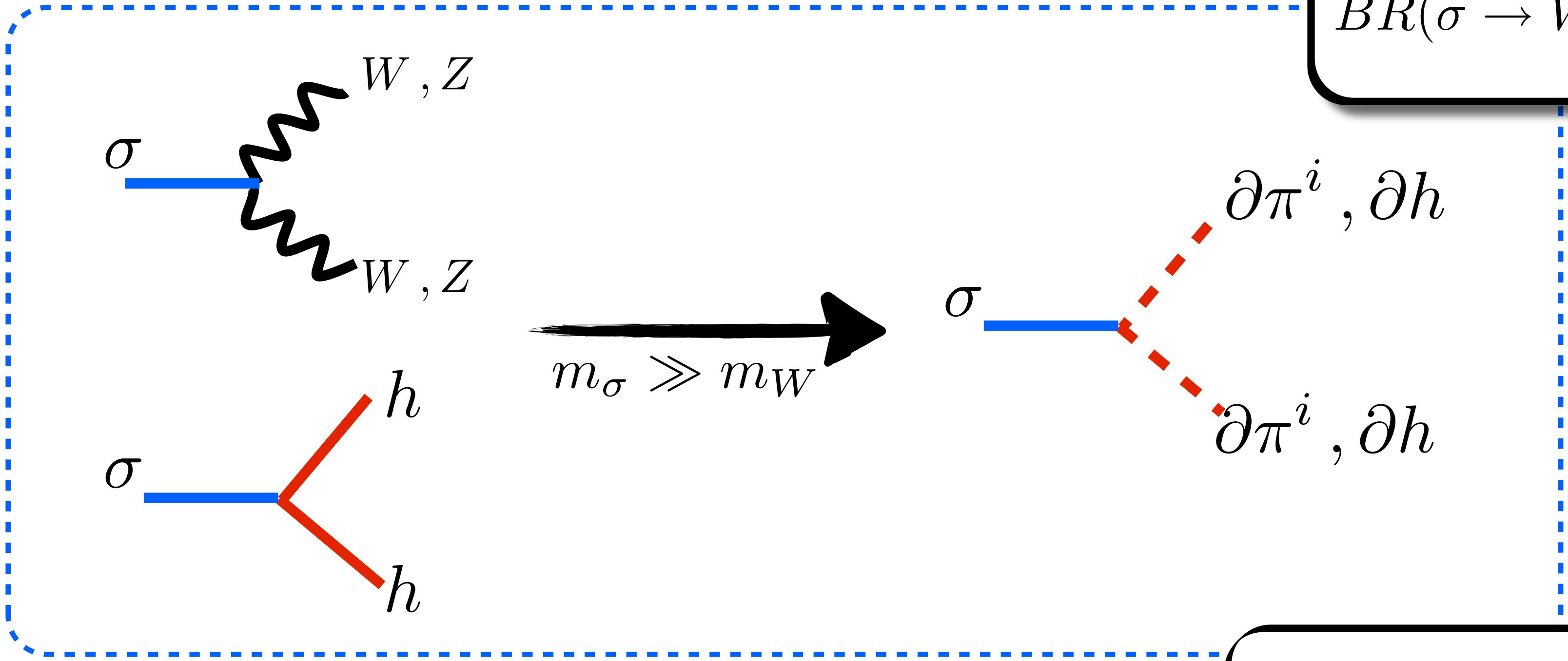

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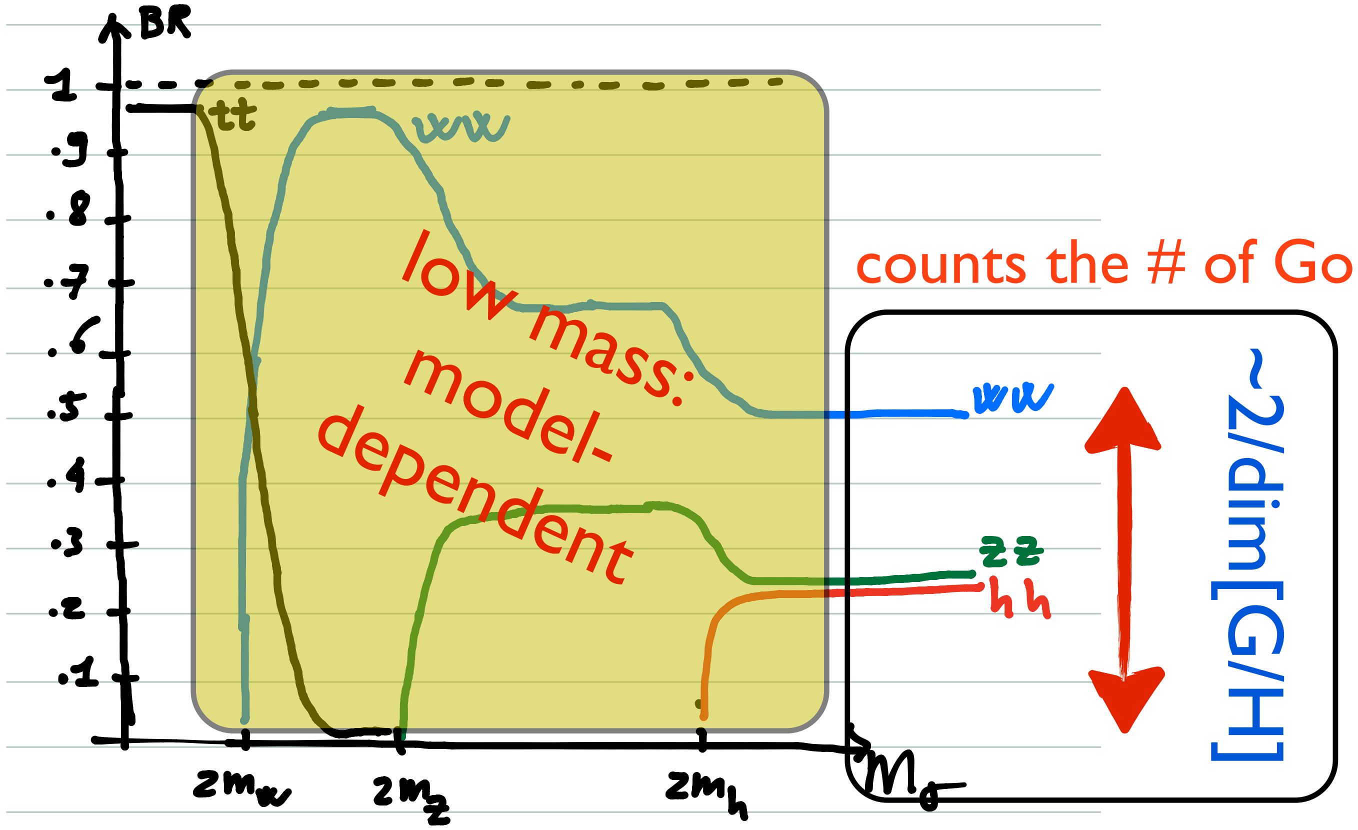
counts the # of Goldstones



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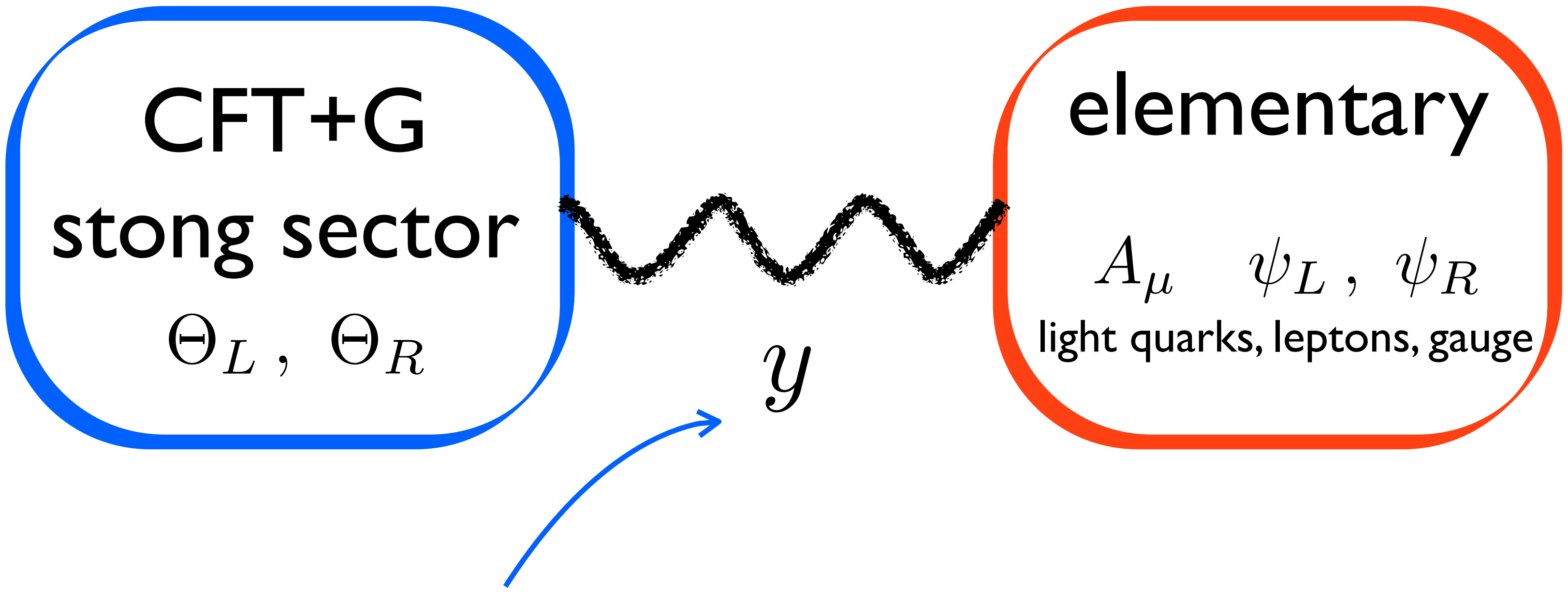
Other dilaton couplings



CFT+G

stong sector

$\Theta_L\ ,\ \Theta_R$

y

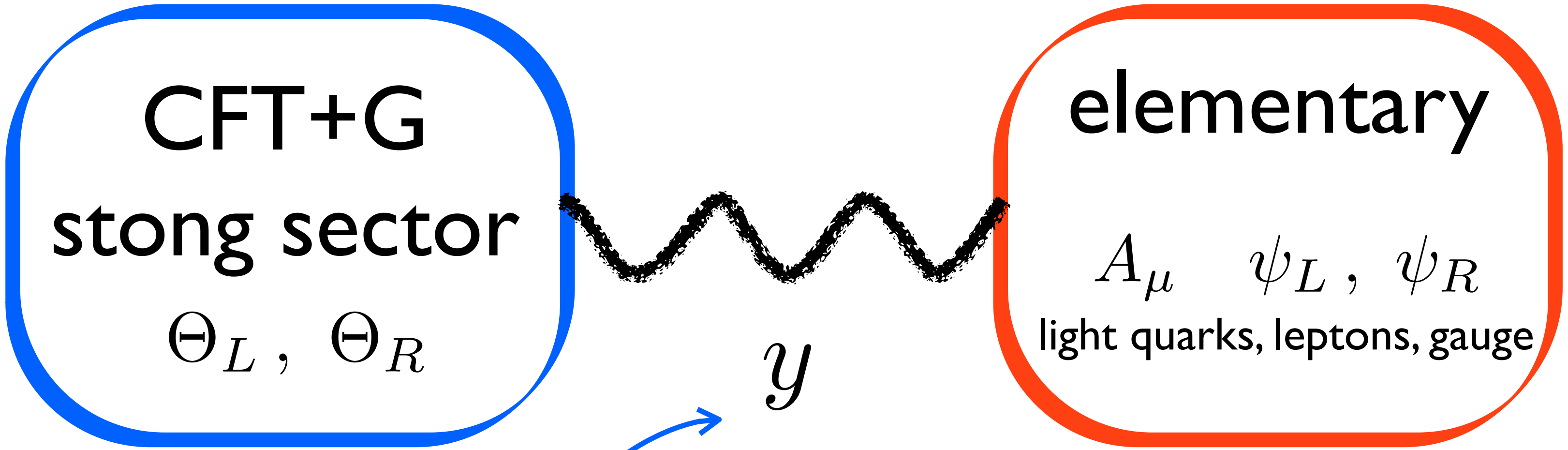
elementary

$A_\mu \quad \psi_L\ ,\ \psi_R$

light quarks, leptons, gauge

Partial Compositeness: explicitly break CFT & G

OTHER DILATON COUPLINGS



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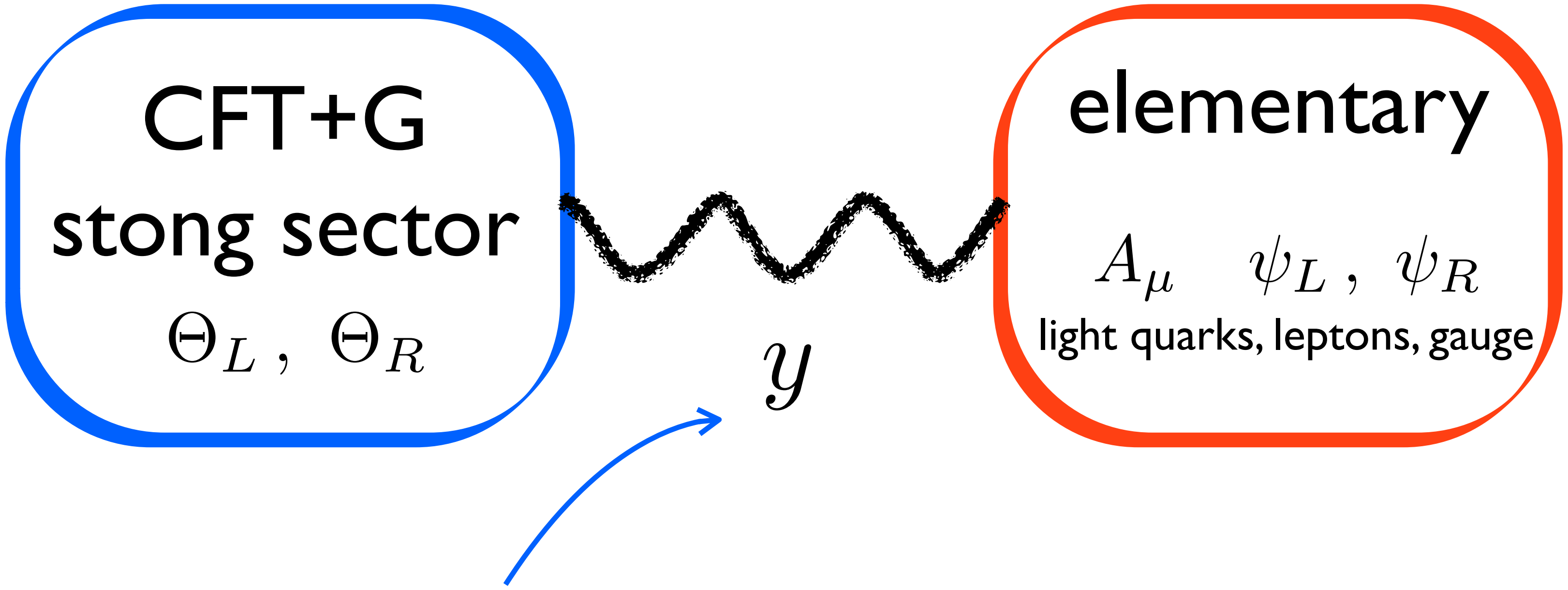
spurions carry both G-indexes and scale dimension

$$\mathcal{L}_{mix} = y_L\psi_L\Theta_R + \underbrace{y_R}_{[y_{R,L}] = -\gamma_{L,R}}\underbrace{\psi_R}_{3/2}\underbrace{\Theta_L}_{5/2+\gamma_R}$$

OTHER DILATON COUPLINGS



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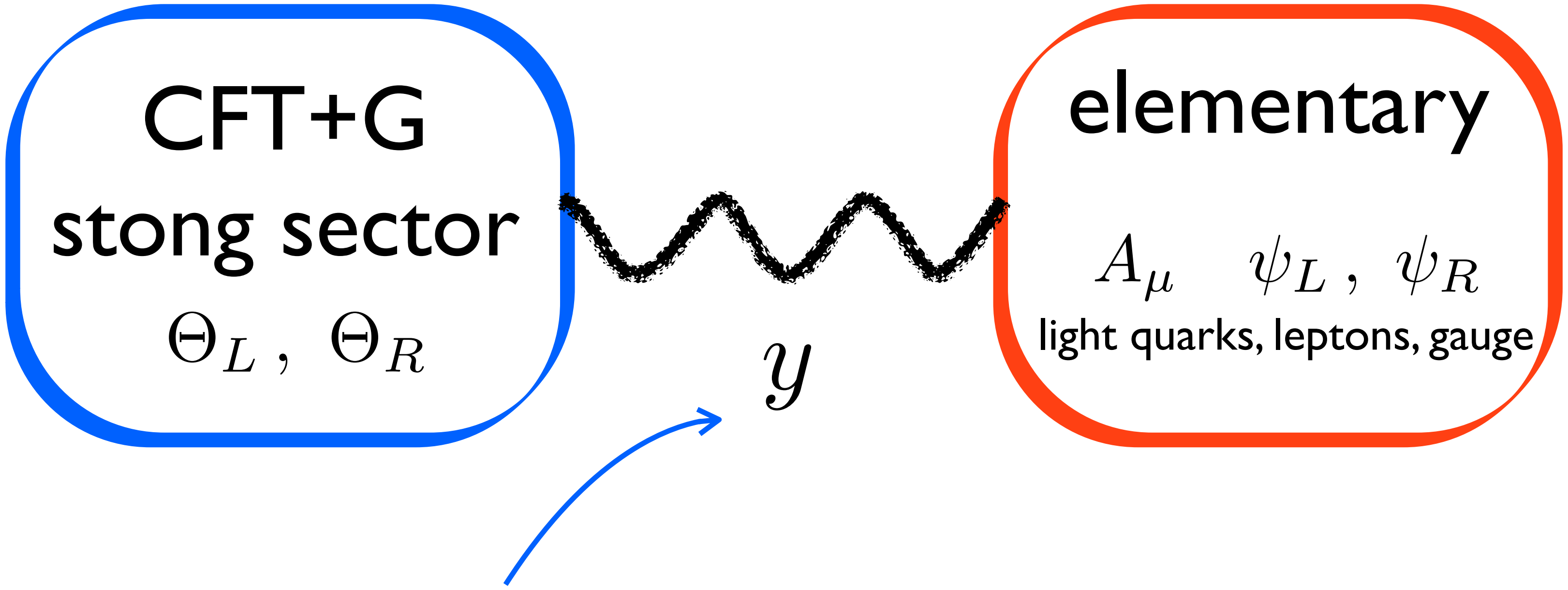

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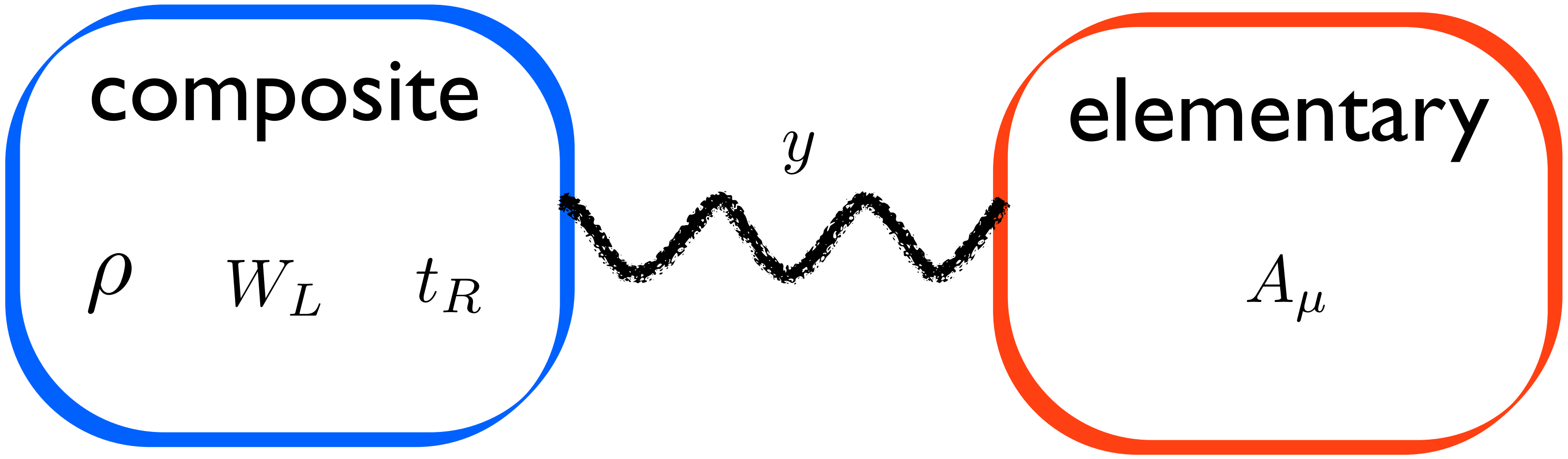
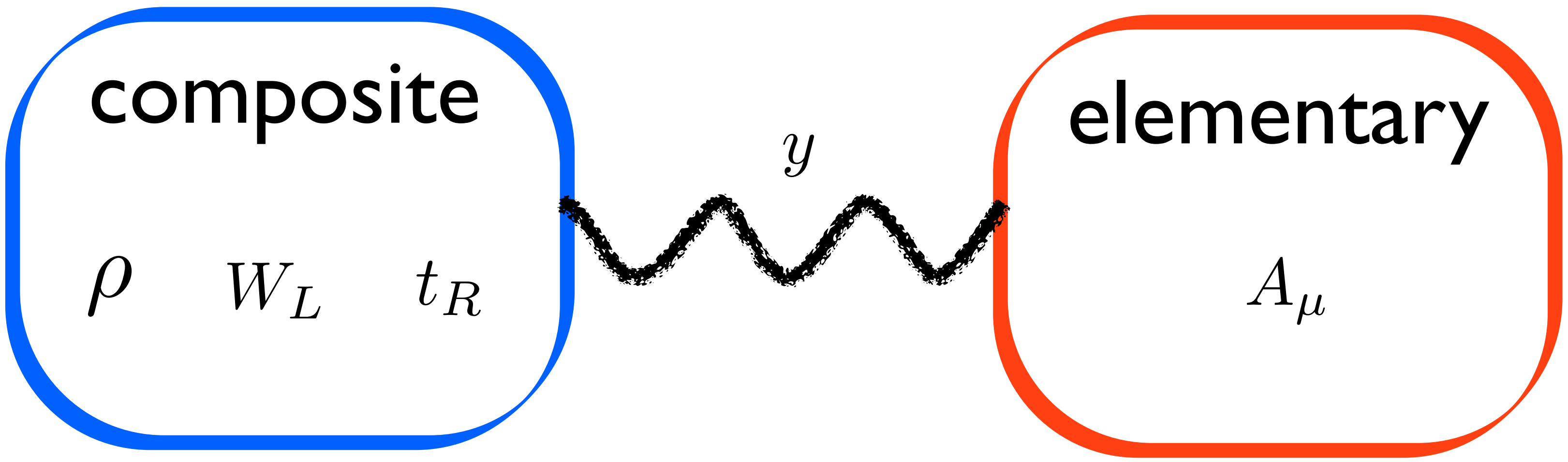
$$\mathcal{L} \supset m_\psi\,\psi_L\psi_R\left[1+\frac{\sigma}{f}(1+\gamma_L+\gamma_R)\right]$$

PHOTON AND GLUON COUPLINGS



$$\mathcal{L}_{mix} \supset -\frac{1}{4g^2}F^2_{\mu\nu} + A_\mu \mathcal{J}^\mu$$

PHOTON AND GLUON COUPLINGS



composite

ρ W_L t_R

y

elementary

A_μ

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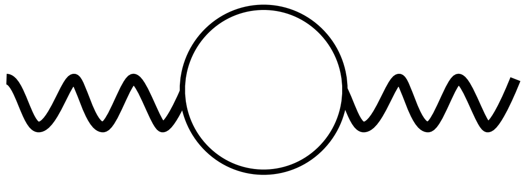
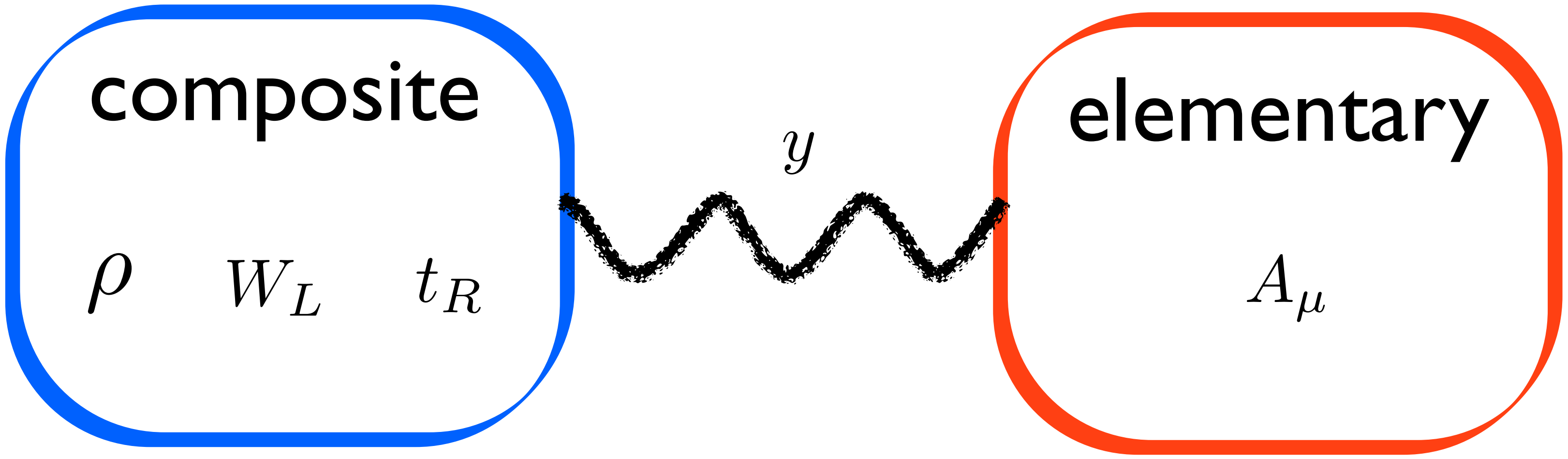
integrate out the CFT: $-\frac{1}{4g^2(\mu)} F_{\mu\nu}^2$



$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} - \frac{b_{UV}^{CFT}}{8\pi^2} \log \frac{\Lambda}{f} - \frac{b_{IR}^{comp}}{8\pi^2} \log \frac{f}{\mu} - \frac{b_{elem}}{8\pi^2} \log \frac{\Lambda}{\mu}$$

PHOTON AND GLUON COUPLINGS



composite

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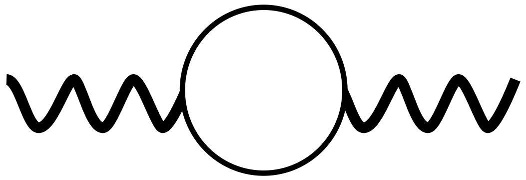

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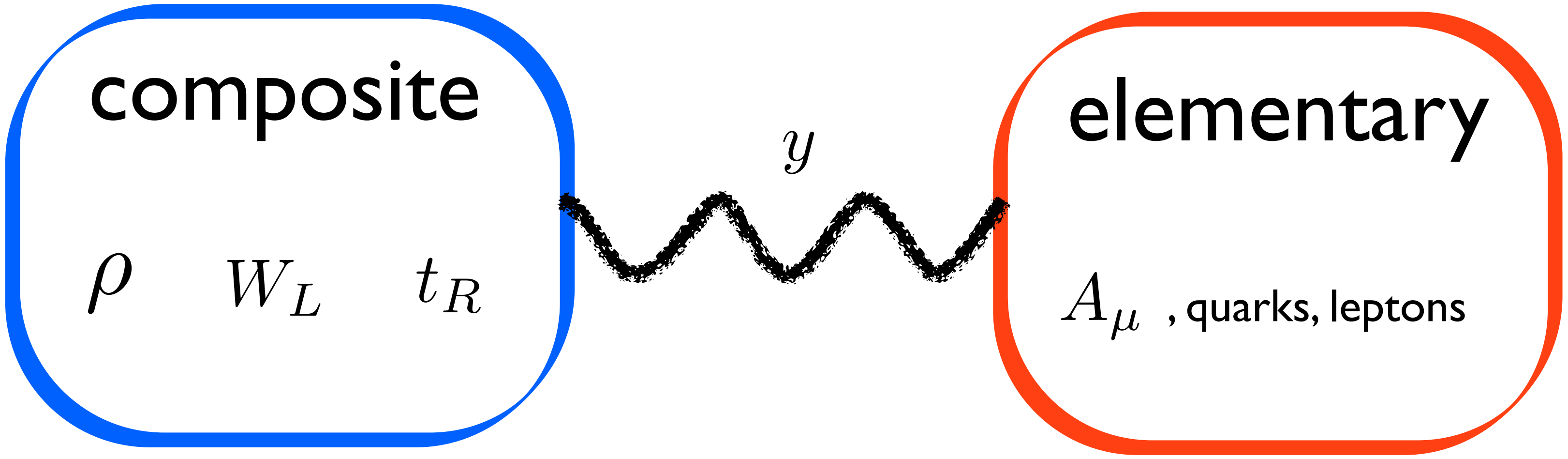
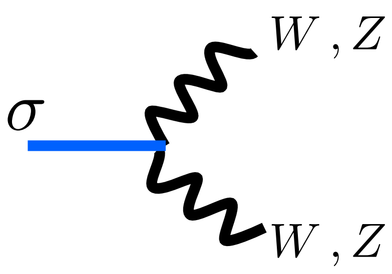
compensate: $f \longrightarrow f\chi = fe^{\sigma/f}$ $\longrightarrow$

$$\mathcal{L} = -\frac{1}{2}\left(\frac{\beta_{IR}^{comp}}{g} - \frac{\beta_{UV}^{CFT}}{g}\right)\frac{\sigma}{v}F^2_{\mu\nu}$$

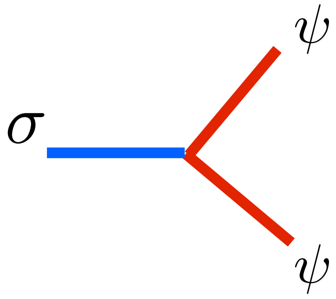
DILATON COUPLINGS: SUMMARY



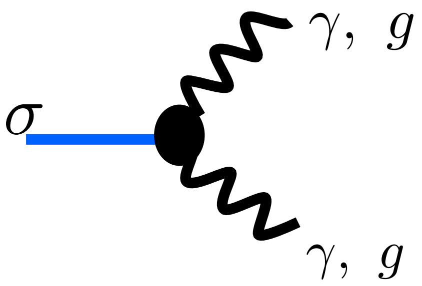
$$\mathcal{L} = \overbrace{\frac{v}{f}}^{\text{overall rescaling}}\sigma\left\{\left[2m_W^2 W_\mu^2 + m_Z^2 Z^2 + m_\psi \psi \overbrace{(1+\gamma)}^{\text{anomalous dim.}}\psi \ldots\right] + 2\overbrace{(\beta_{UV}^{CFT} - \beta_{IR}^{comp})}^{\text{beta-functions}}/g\, F_{\mu\nu}^2\right\}$$



$SM \times \frac{v}{f}$

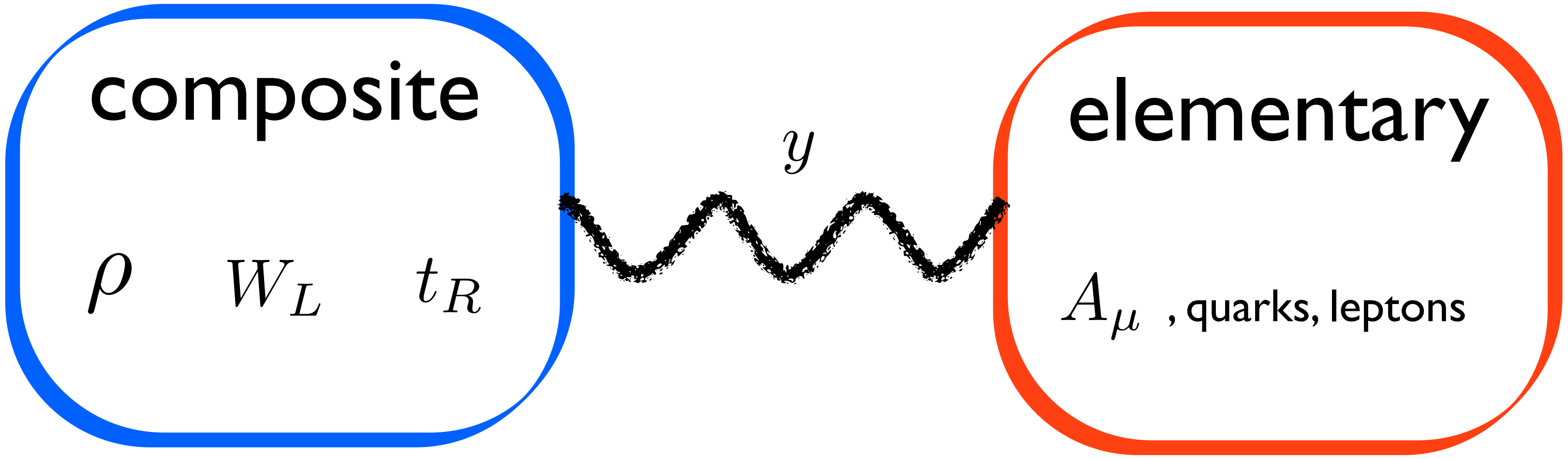


$SM \times \frac{v}{f}(1+\gamma)$

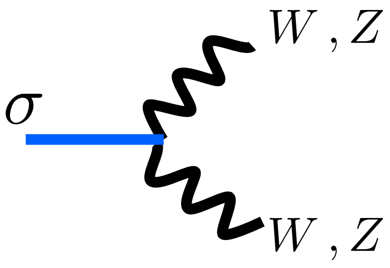


$\frac{v}{f}(\beta_{UV}^{CFT} - \beta_{IR}^{comp})$

DILATON COUPLINGS: SUMMARY



$$\mathcal{L} = \overbrace{\frac{v}{f}}^{\text{overall rescaling}}\sigma\left\{\left[2m_W^2 W_\mu^2 + m_Z^2 Z^2 + m_\psi\psi\overbrace{(1+\gamma)}^{\text{anomalous dim.}}\psi\ldots\right] + 2\overbrace{(\beta_{UV}^{CFT} - \beta_{IR}^{comp})}^{\text{beta-functions}}/g\,F_{\mu\nu}^2\right\}$$



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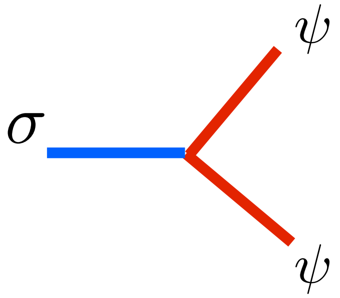

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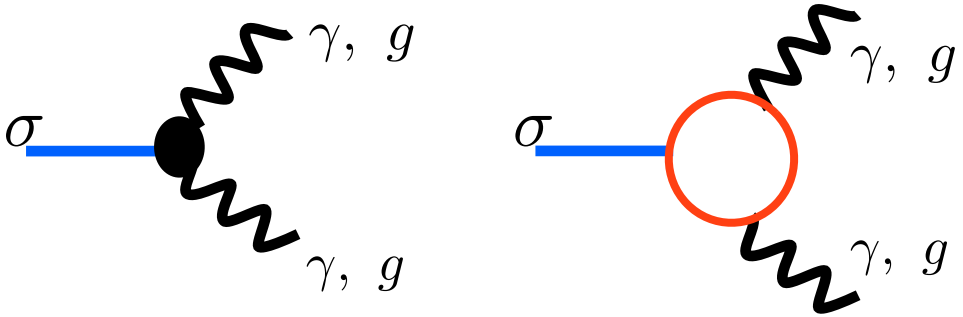
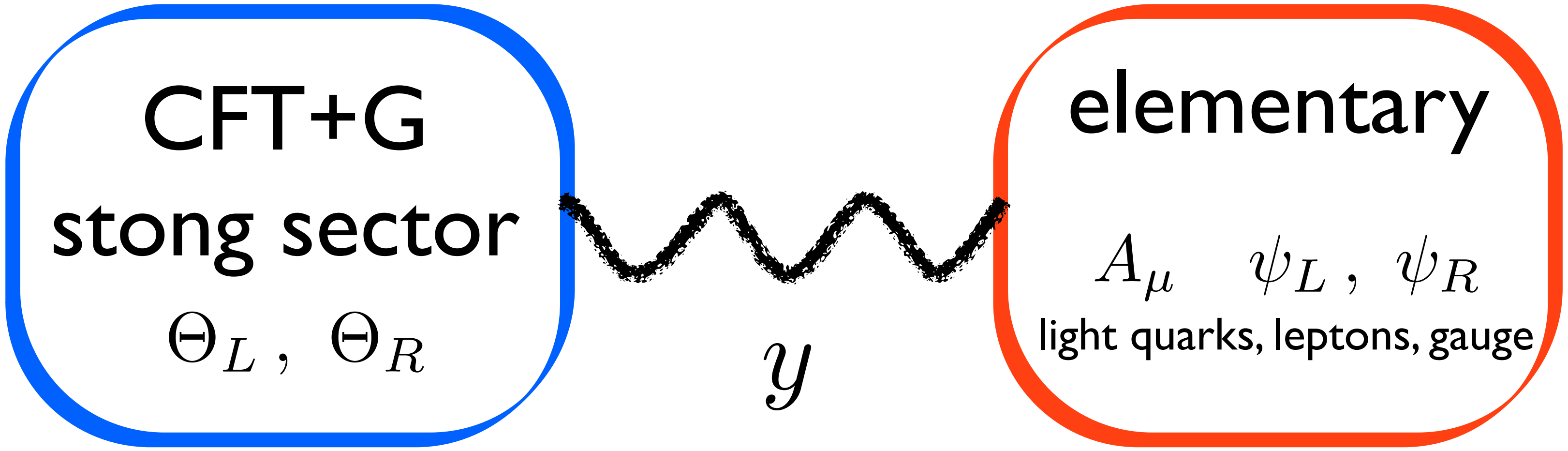


$\frac{v}{f}(\beta_{UV}^{CFT} - \beta_{IR}^{comp} + loops)$

example w/ composite top-right for Higgs-like Dilaton: $\frac{v}{f}(\beta_{UV}^{CFT} + \beta_{SM}^{\gamma} - \beta_{t_R,W_L}^{\gamma})$

Dilaton & Higgs potentials



CFT+G

stong sector

$\Theta_L \, , \; \Theta_R$

y

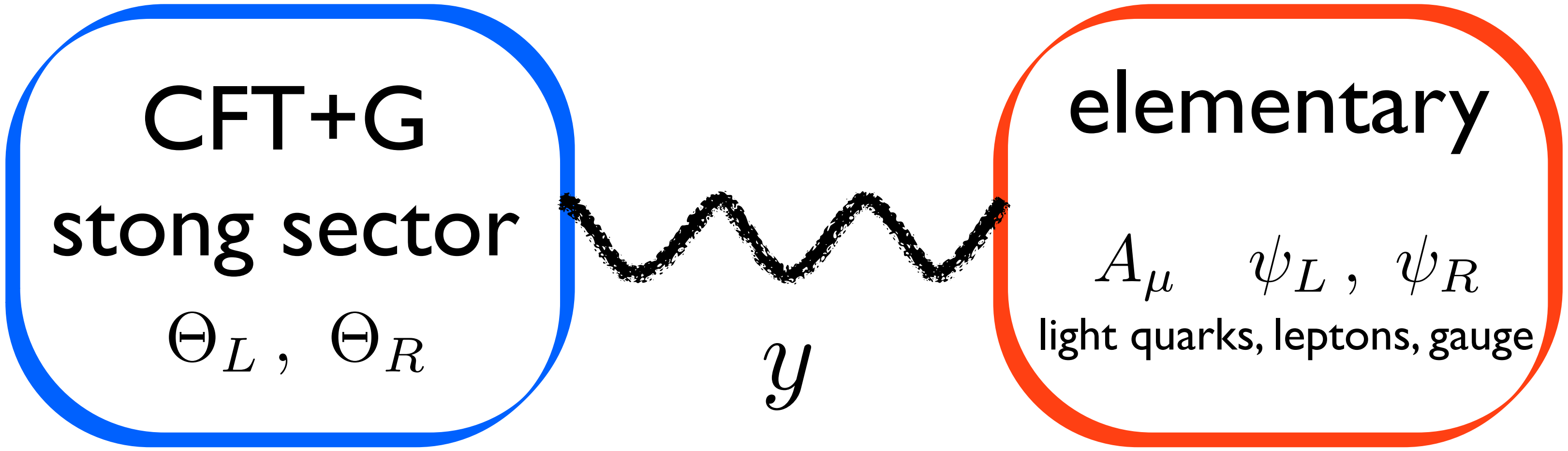
elementary

$A_\mu \quad \psi_L \, , \; \psi_R$

light quarks, leptons, gauge

Integrate-out CFT (all orders)

DILATON & HIGGS POTENTIALS



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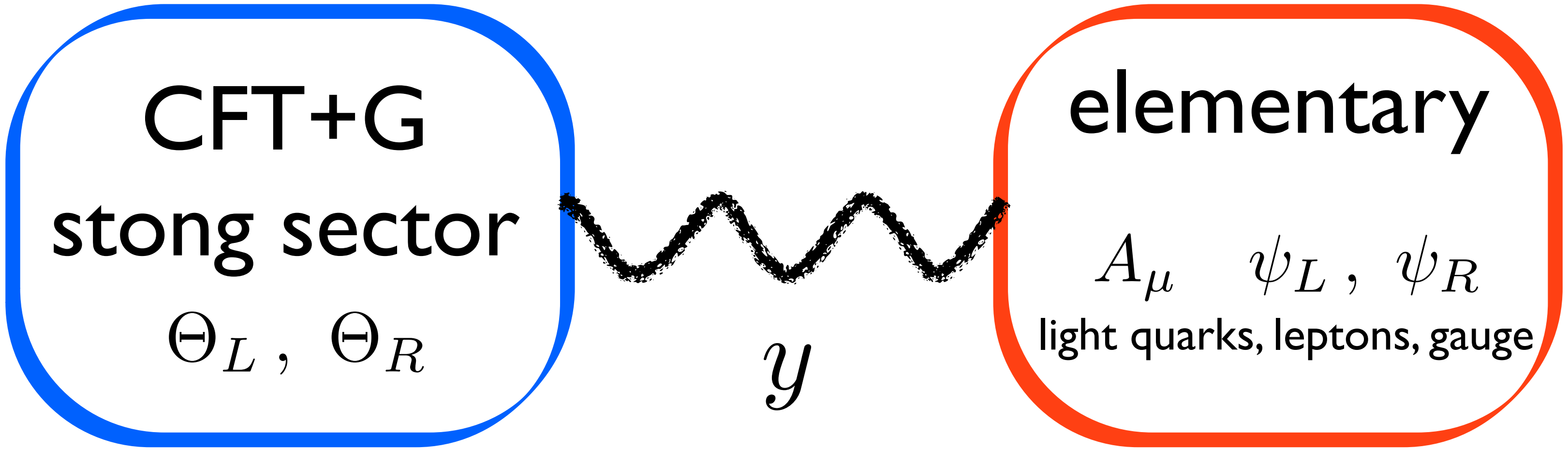
form factors

DILATON & HIGGS POTENTIALS



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1-loop of elem. fields: Coleman-Weinberg! $V(\pi,\chi) = \sum_i \int \frac{d^4p}{(2\pi)^4} \log \Pi_i(p^2,\Phi)$

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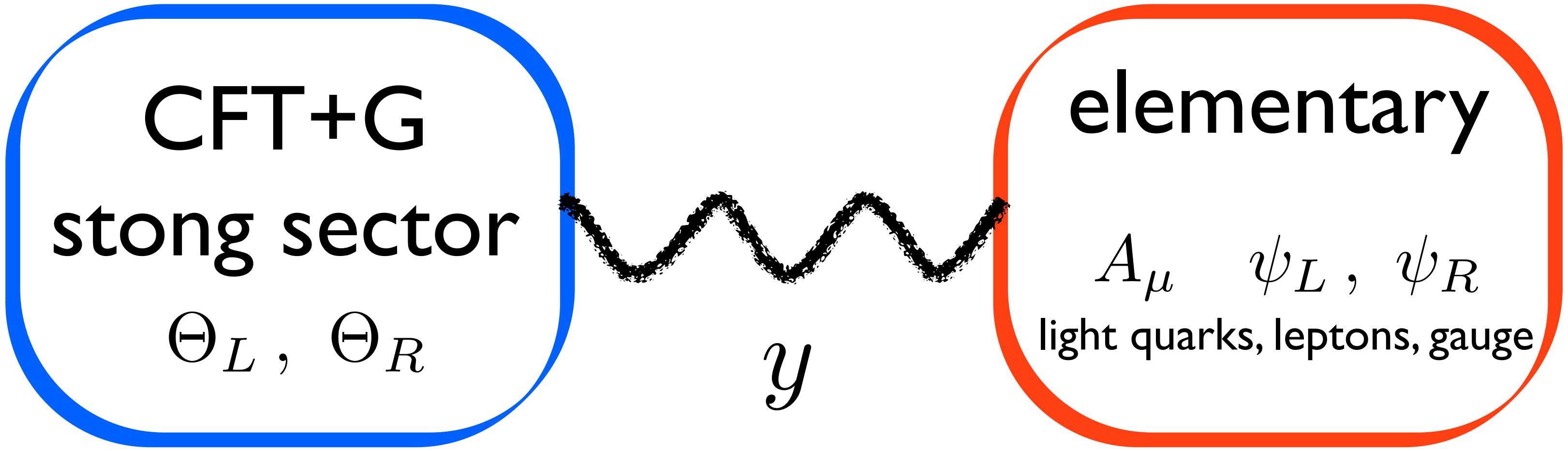

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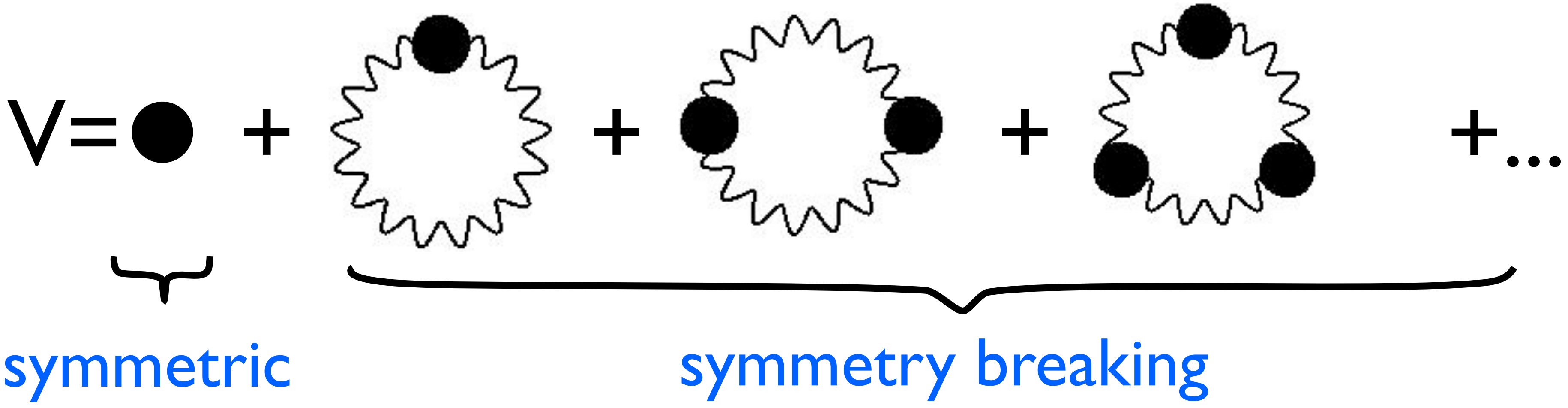

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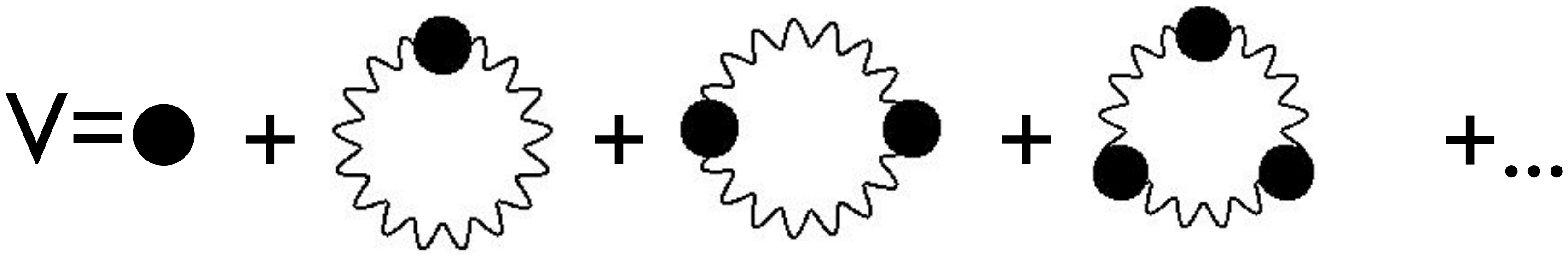

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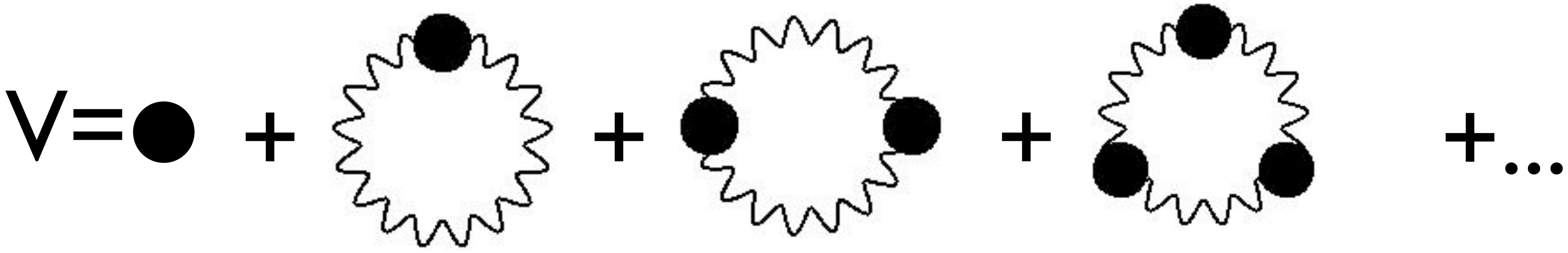
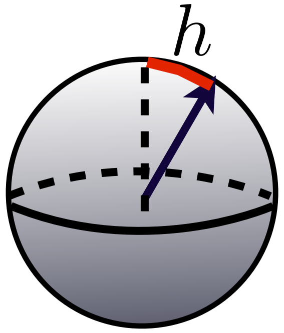
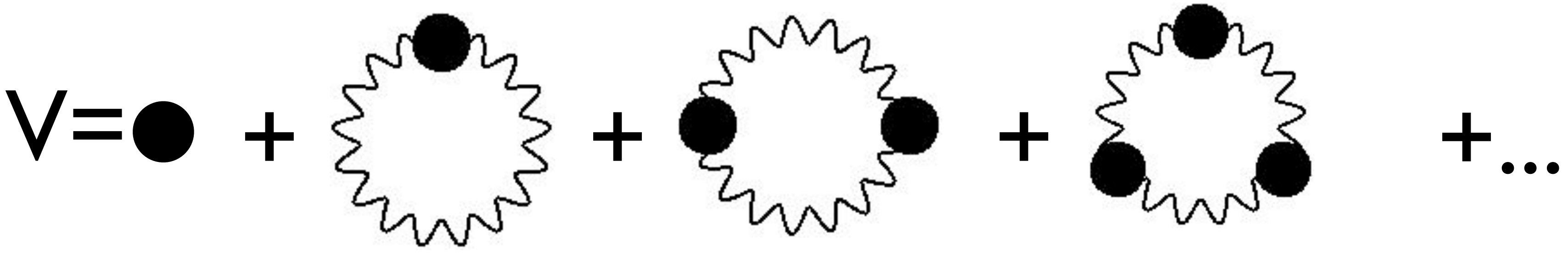


Potential on the sphere



$$v = f_\pi \sin h$$

DILATON & HIGGS POTENTIALS



$$V = \kappa + y^2\left(\Lambda_1 + A\sin^2 h + B\sin^4 h\right)$$

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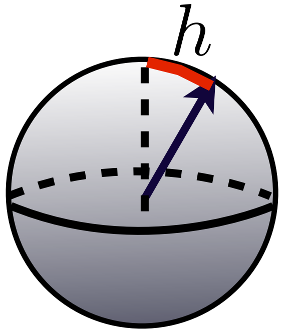

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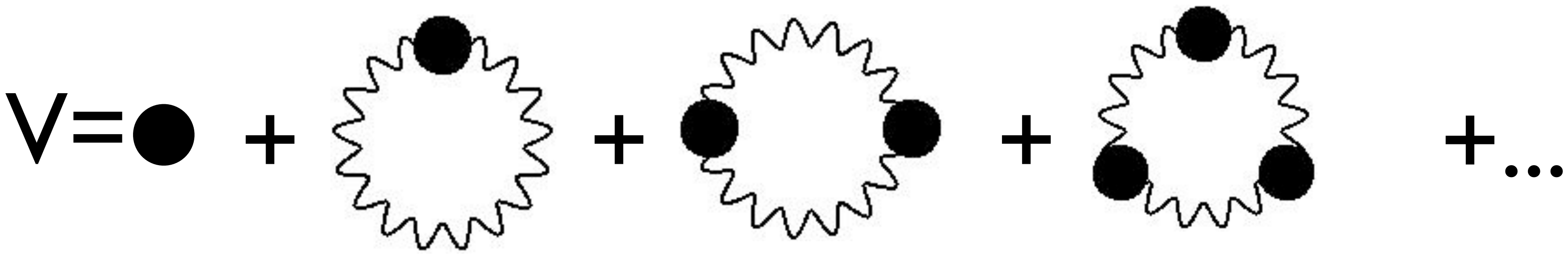

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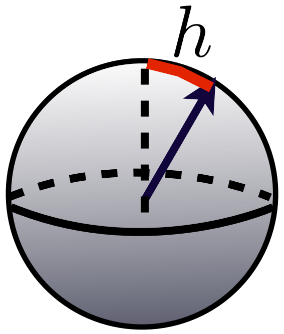

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$$V = \left(\frac{\chi}{f}\right)^4\left[\kappa + y^2\left(\frac{\chi}{f}\right)^{2\gamma}\left(\Lambda_1 + A\sin^2 h + B\sin^4 h\right)\right] = \chi^4 F(y(\chi), \sin h)$$

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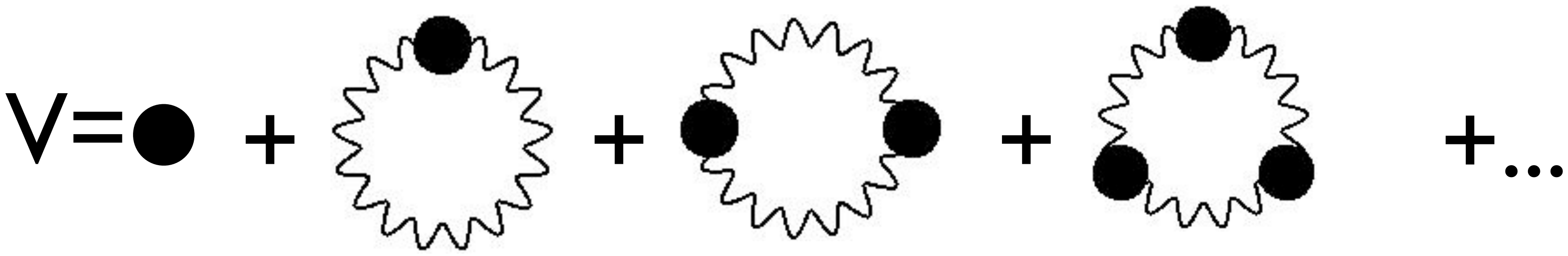

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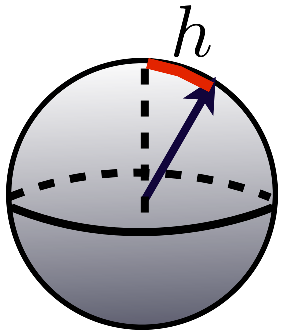

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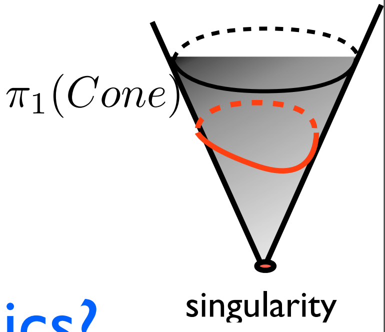
5 parameters: trade for $m_\sigma \quad m_h \quad v/f_\pi \quad f \quad m_t$

Predictions (e.g. amplitudes) all in terms of physical quantities

CONCLUSIONS & QUESTIONS

- The Higgs has been discovered and it can well be a pNGB
- The CFT broken spontaneously in the IR gives a light dilaton in the spectrum
- Chiral lagrangian for Composite Higgs+Dilaton is quite interesting
  - ★ Funny geometrical structure (btw, is the cone homotopy trivial?)
  - ★ f=fpi by symmetry ISO(n), but weakly coupled, what about dynamics?
  - ★ Clear Dilaton BRs: can we count the Goldstone bosons=dim[G/H]?
  - ★ Curious WW-scattering: can we see E^4 behavior? strong vs weak dynamics, dynamics vs symmetry)
  - ★ Higgs and Dilaton potential are related
  - ★ Can we distinguish it from another Higgs (2HDM?) or extra pNGB?



THANK YOU!

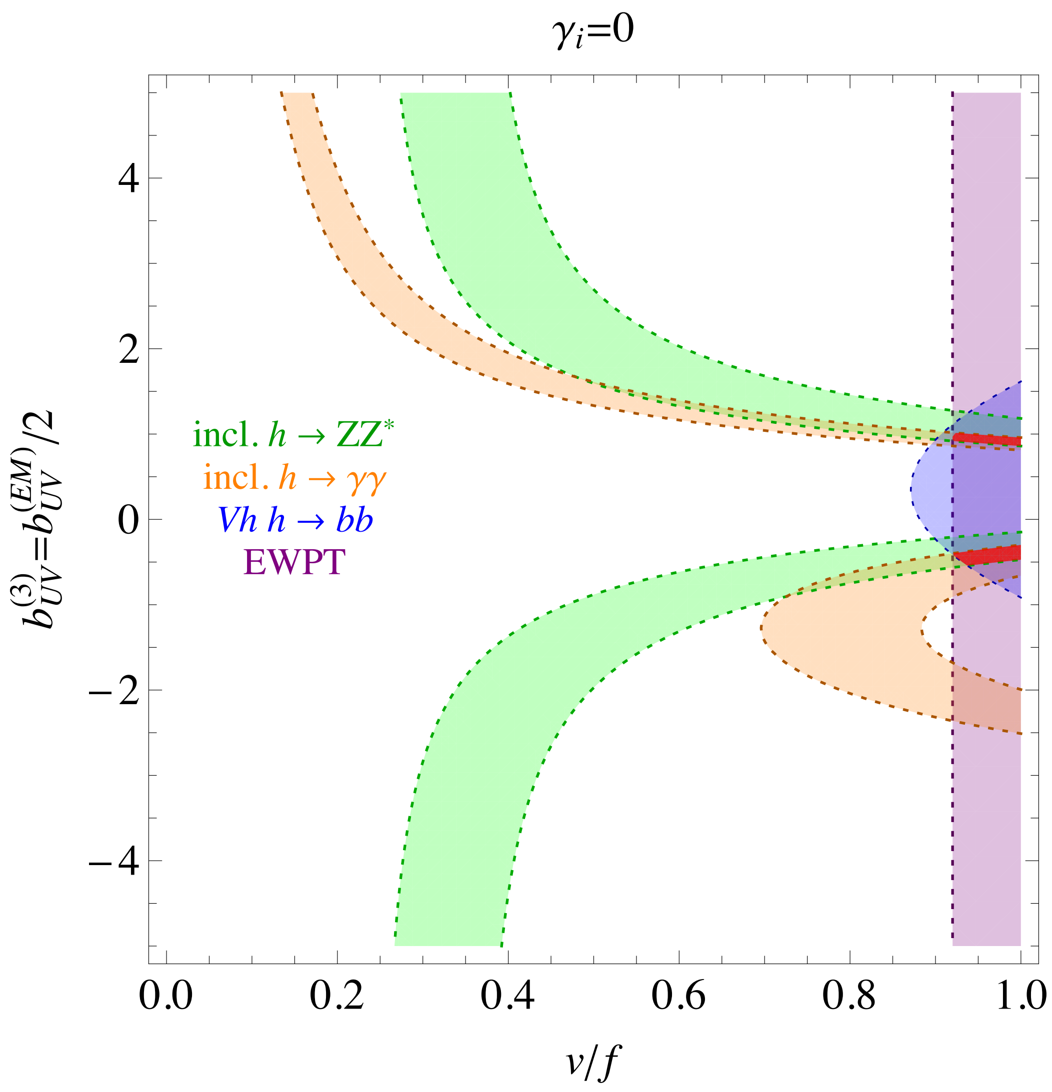
backup slides

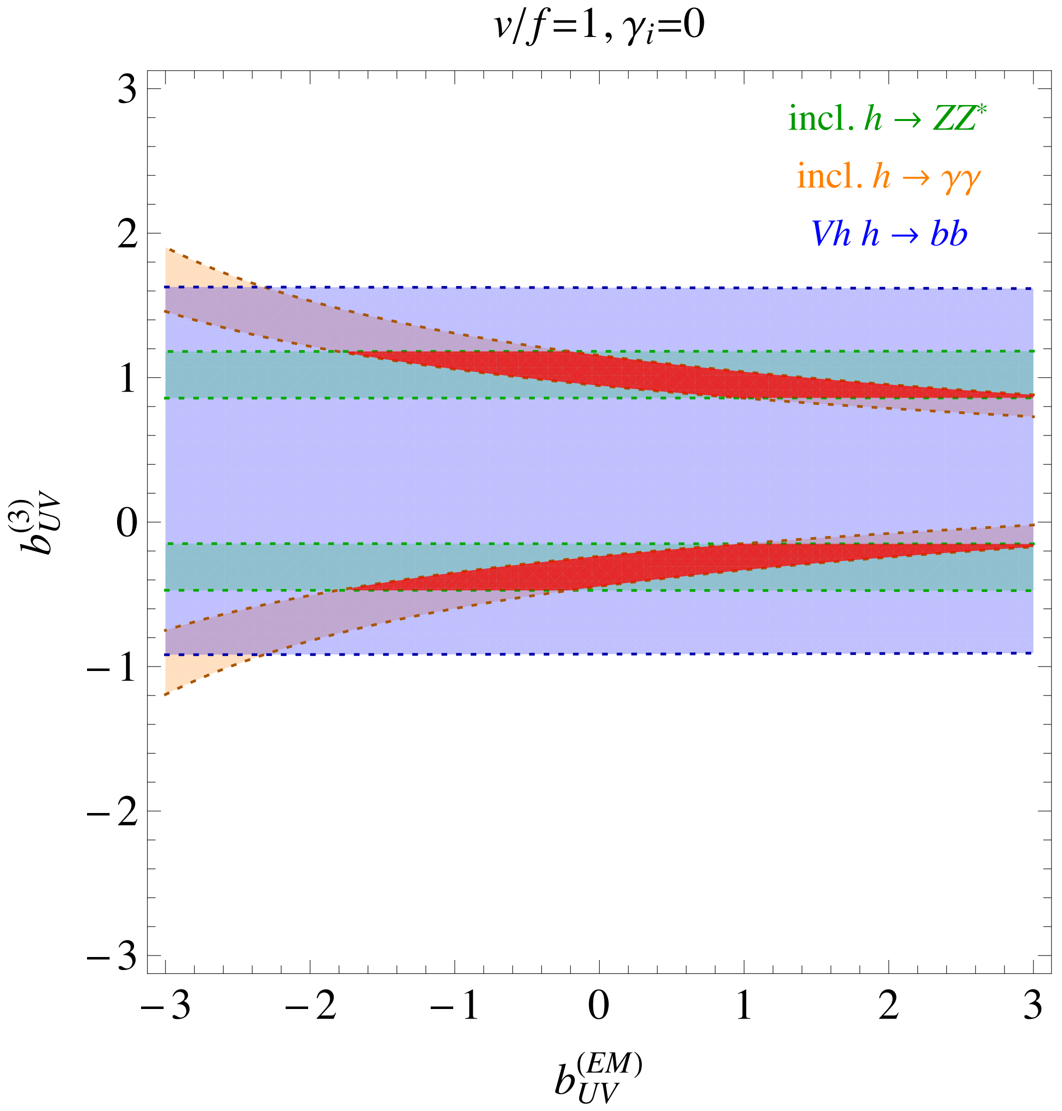
HIGGS-LIKE DILATON: FITTING DATA

rates scale with: v^2/f^2

$$R = \frac{\sigma}{\sigma_{SM}} \times \frac{BR}{BR_{SM}}$$

easily fit data if v~f
&
m=125 GeV





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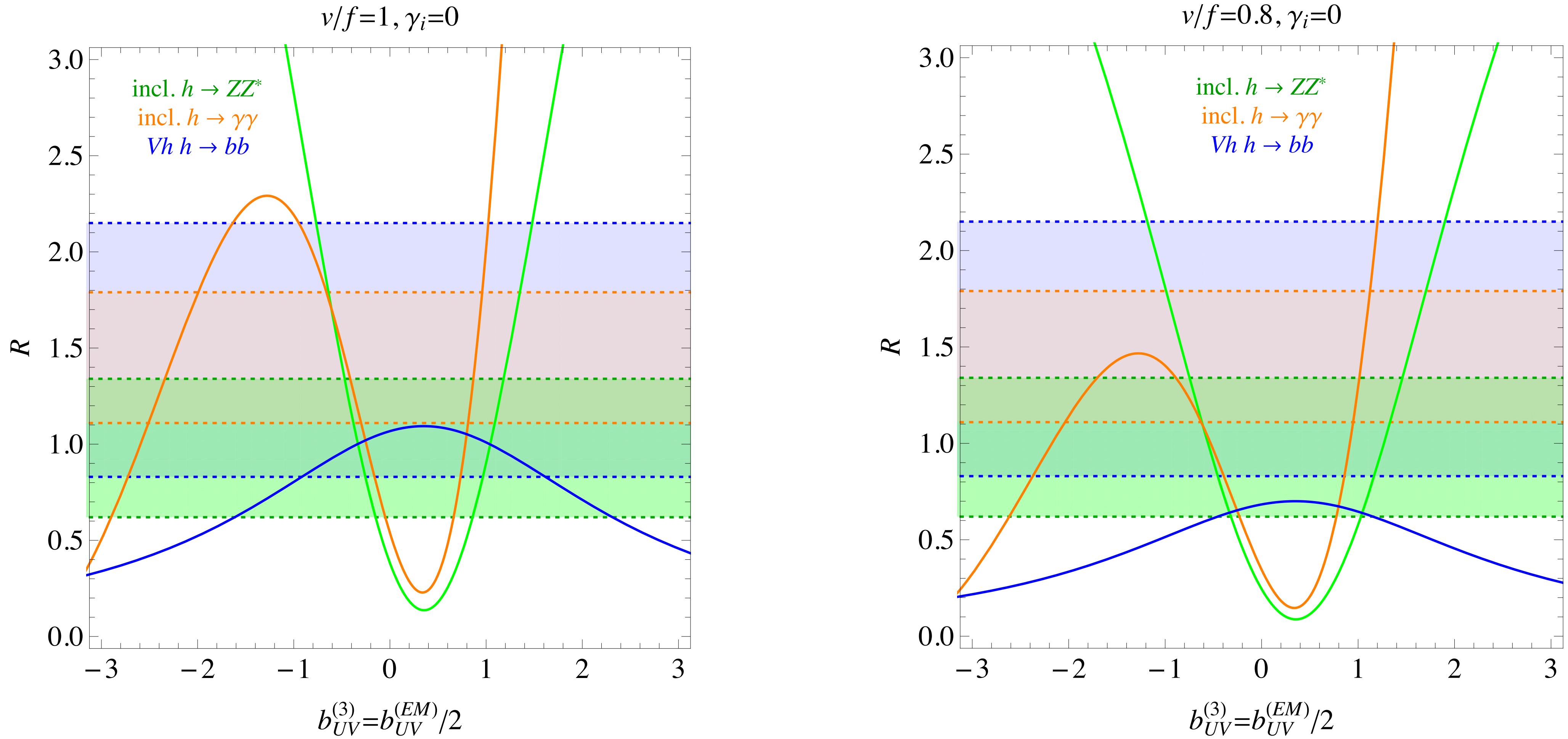
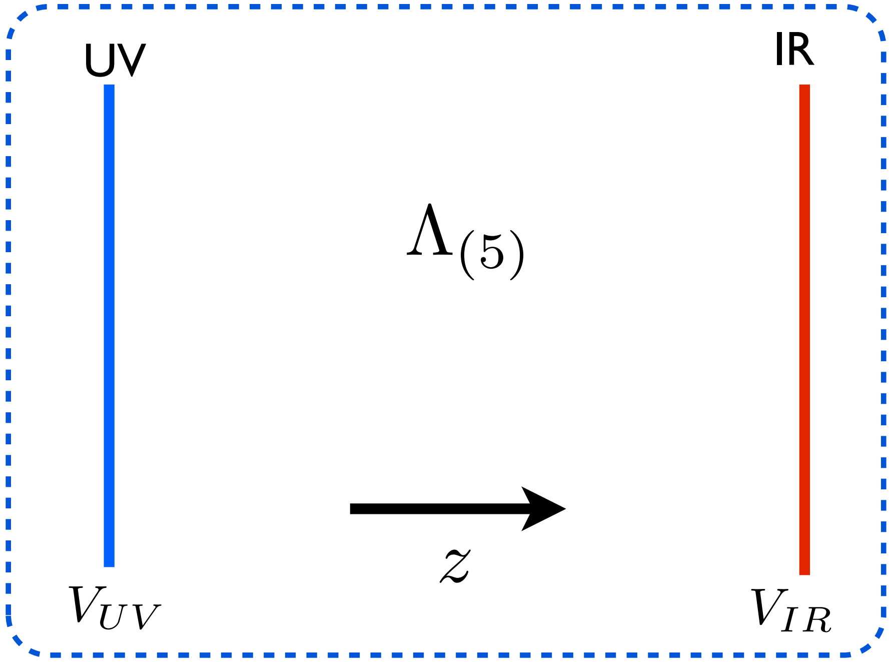


THE RS STORY



$$ds^2 = \frac{L^2}{z^2}\left(dx^2 - dz^2\right)$$

$x \to \lambda x\,,\ z \to \lambda z$

$z_{IR} \to \lambda z_{IR}$

breaks it spontaneously

$f = z_{IR}^{-1}$ the radion

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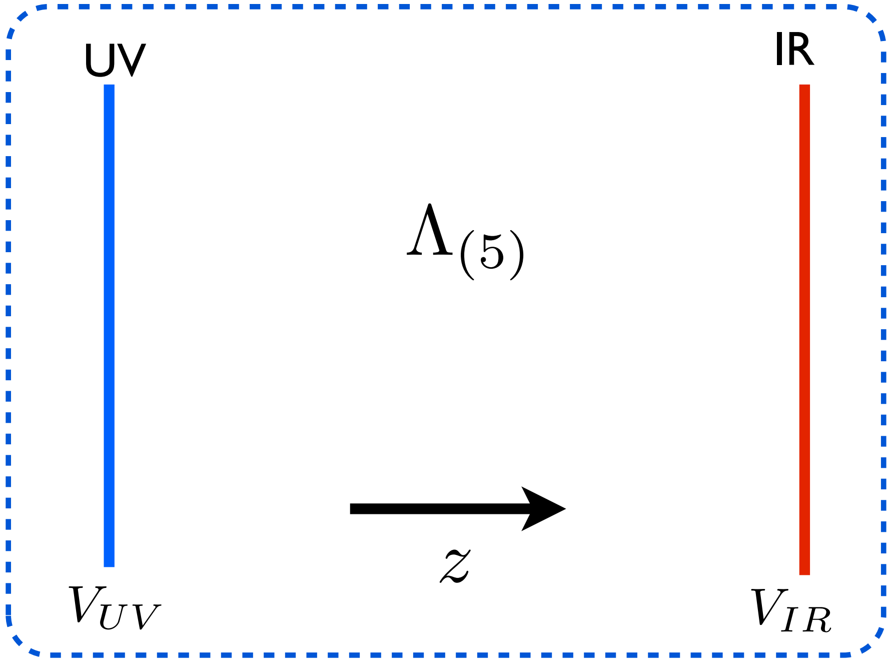

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$$S = -\int_{z=L} \underbrace{\sqrt{g_{(4)}}\, V_{UV}}_{\text{UV-tension}} + \int \underbrace{\sqrt{g_{(5)}}\left(2M_*^3 R_{(5)} - \Lambda_{(5)}\right)}_{\text{bulk}} - \int_{z=z_{IR}} \underbrace{\sqrt{g_{(4)}}\, V_{IR}}_{\text{IR-tension}}$$

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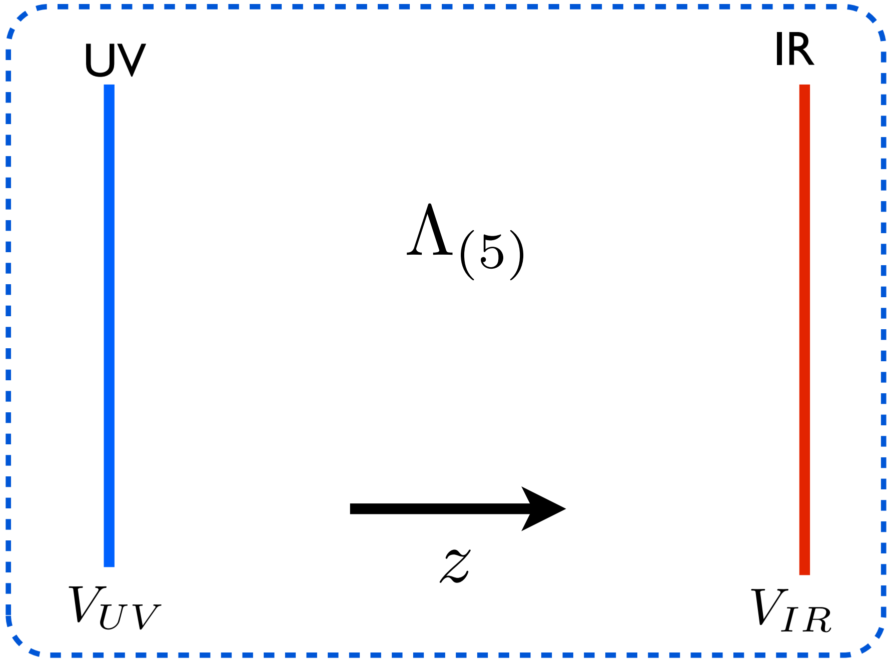

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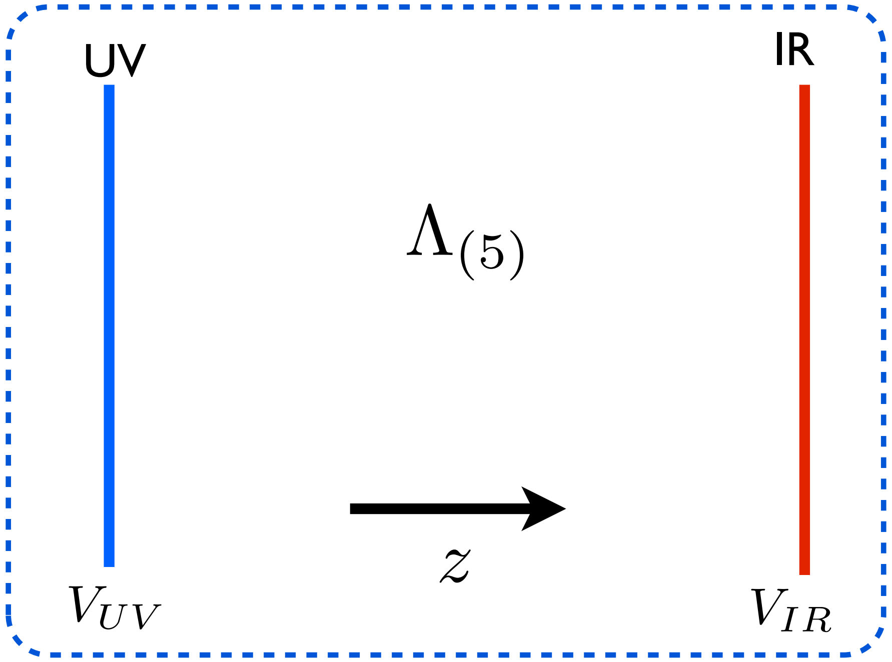
floating IR-brane →

$$V_{eff} = \overbrace{\left(V_{UV} + \Lambda_{(5)}L\right)}^{\Lambda_{(4)}=0} + \frac{L^4}{z_{IR}^4}\overbrace{\left(-\Lambda_{(5)}L + V_{IR}\right)}^{a=0} = \Lambda_{(4)} + a\chi^4$$

vanishing 4d CC
FT-1

vanishing quartic
FT-2

THE RS STORY



$$L_{eff} = -\Lambda_{(5)} L^5 (\partial\chi)^2/2 - \chi^4 \left(-\Lambda_{(5)} L^5 + V_{IR} L^4\right)$$

NDA:
$$\begin{cases} \delta a_{bulk} = -\Lambda_{(5)} L^5 \sim \dfrac{12^{5/2}}{24\pi^3} = \mathcal{O}(1) \\ \delta a_{IR=} V_{IR} L^4 = V_{IR} \left(\dfrac{L}{z_{IR}}\right)^4 z_{IR}^4 \sim 16\pi^2 \end{cases}$$

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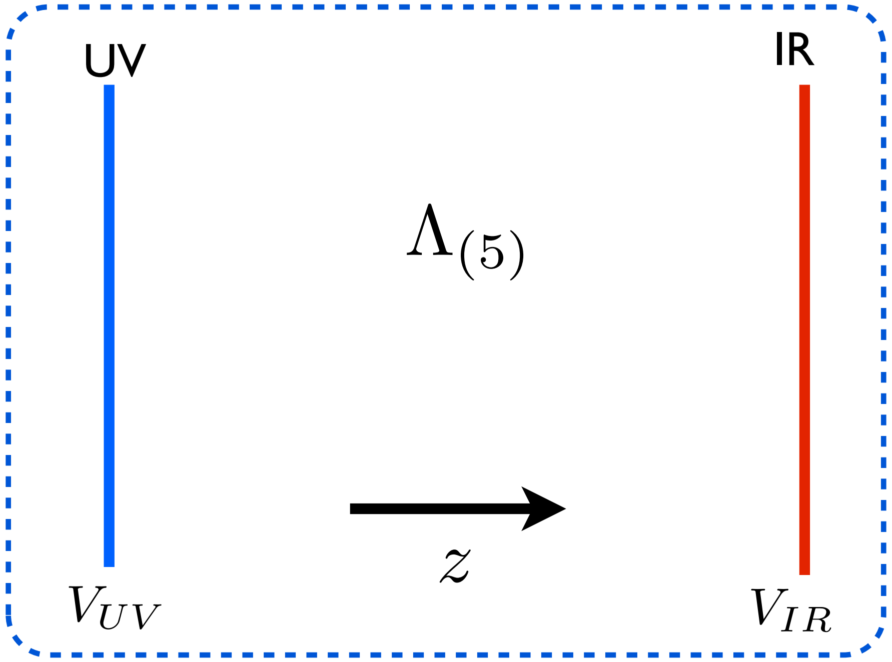

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1) assume the RS tuning: vanishing/small quartic

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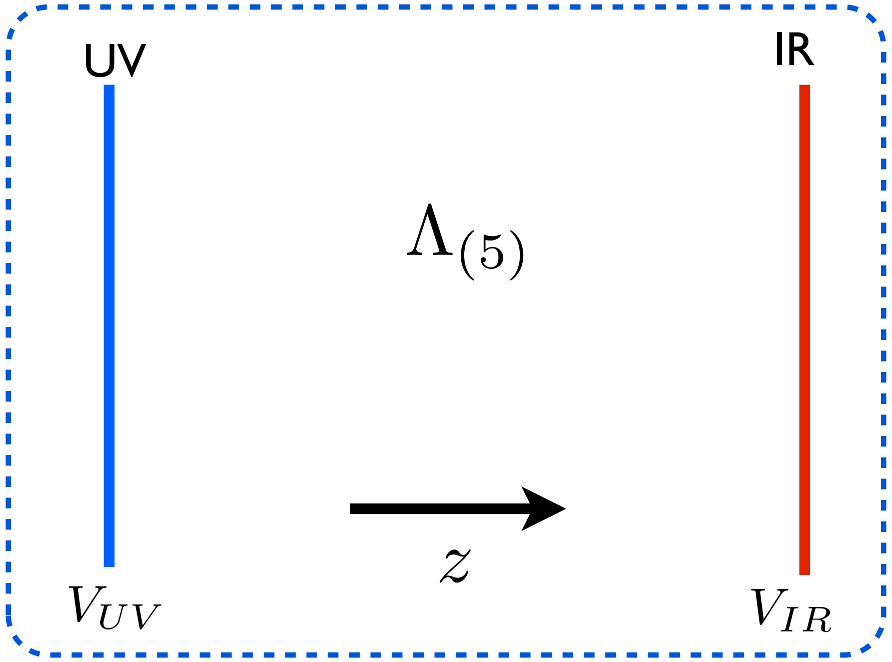

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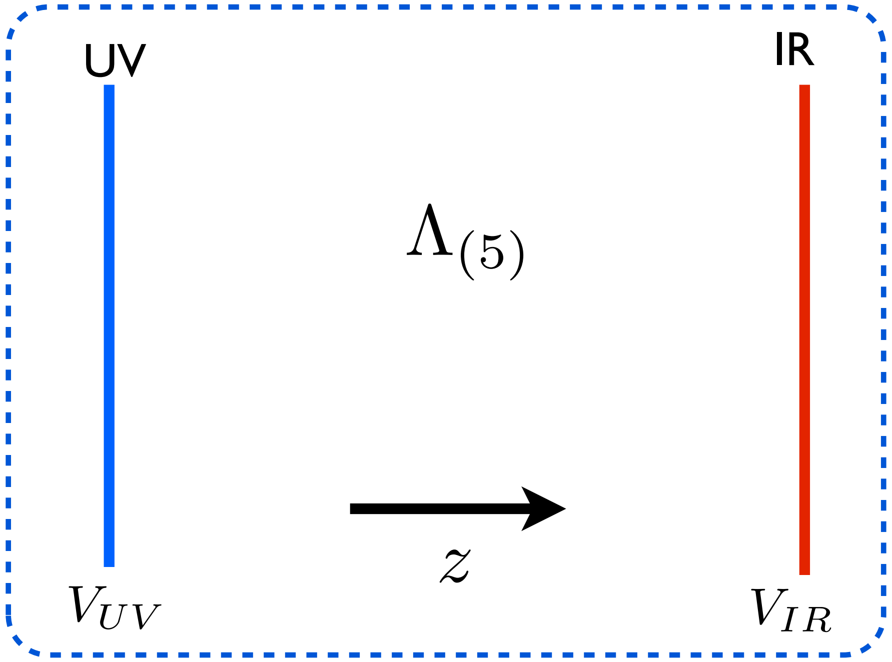

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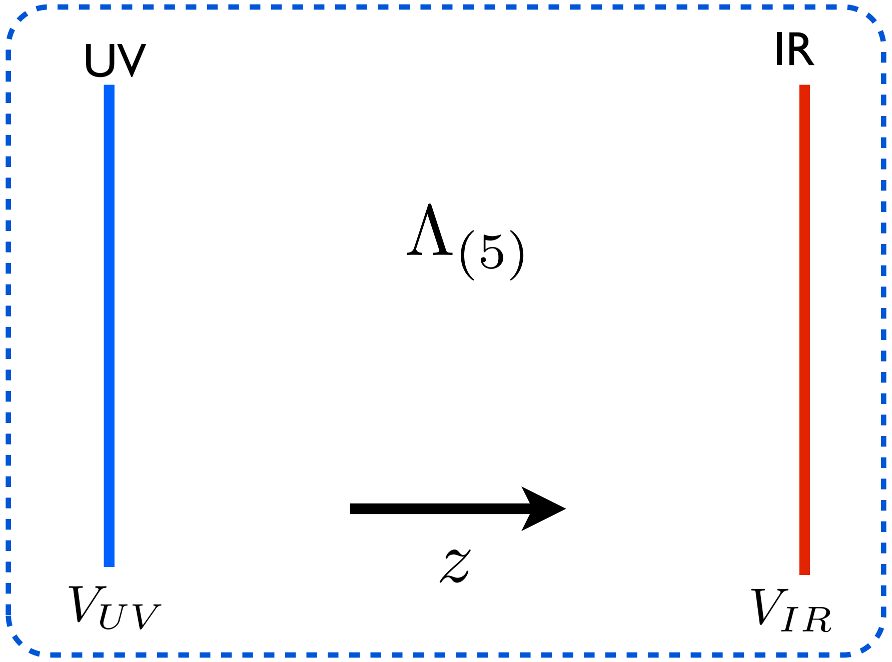

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from large K.T.

not a good candidate

THE SM-HIGGS IS A FINE-TUNED DILATON

Higgs potential

$$V = \lambda\left(H^2 - \frac{v^2}{2}\right)^2$$

$$\lambda \approx 0.1$$

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2) scale invariance $x \rightarrow x' = (1 - \epsilon) x$

$$\Phi(x) \rightarrow \Phi'(x') = (1 + \epsilon \underset{\substack{\uparrow \\ \text{scaling dim.}}}{\Delta}) \Phi(x)$$

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After EWSB:
only the *diagonal* symmetry survives

$$h \to (1 + \epsilon) h(x(x')) + \epsilon v$$

non-linearly like a Goldstone boson

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w/o the Higgs (no sym.) $S_{IR} = \int d^4x \, [m_t \bar{t}_L t_R + \ldots] \to \int d^4x \, [m_t (1 - \epsilon) \bar{t}_L t_R + \ldots]$

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scaling dim.

After EWSB:
only the *diagonal* symmetry survives

$$h \to (1+\epsilon)h(x(x')) + \epsilon v$$

non-linearly like a Goldstone boson

w/o the Higgs (no sym.) $$S_{IR} = \int d^4x\,[m_t\bar{t}_L t_R + \ldots] \to \int d^4x\,[m_t(1-\epsilon)\bar{t}_L t_R + \ldots]$$

$$m_t(\frac{h}{v} + \epsilon)\bar{t}_L t_R$$

w/ the Higgs $$S_{IR} = \int d^4x\,[m_t\bar{t}_L t_R + \frac{h}{v}(m_t\bar{t}_L t_R) + \ldots]$$

sym.is restored!

SUSY Example: 3-2 model

	gauge			
	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\mathbf{3}$	$\mathbf{2}$	$1/3$	1
L	$\mathbf{1}$	$\mathbf{2}$	-1	-3
$\overline{U}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-4/3$	-8
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$$g_i(\Lambda_i) \approx 4\pi$$

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runaway direction →

push the fields to large vevs

$$V_{eff} \approx \frac{\Lambda_3^{14}}{f^{10}} + \lambda\frac{\Lambda_3^7}{f^3} + \lambda^2 f^4 \qquad f \approx \frac{\Lambda_3}{\lambda^{1/7}} \gg \Lambda_3$$

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$$g_i(f) \ll 1$$

no large breaking of conformality

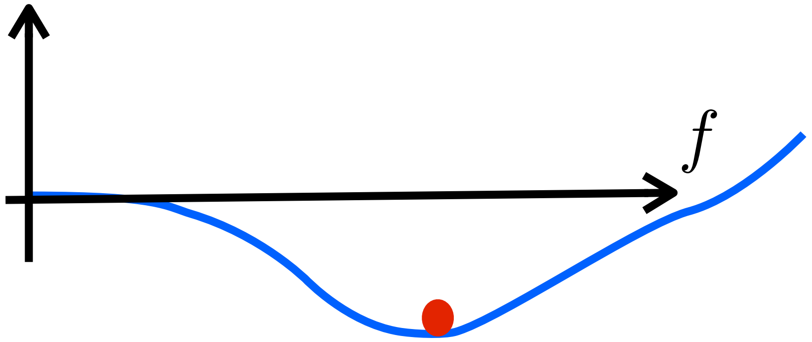
$V \approx \lambda^{10/7}\Lambda_3^4$ small quartic

$m_{dil} \approx \lambda f \approx \lambda^{6/7}\Lambda_3$ light dilaton

DILATON POTENTIAL

$$\mathcal{L}_{CFT} + \lambda\mathcal{O}$$

$$\boxed{V = \chi^4 F(\lambda(\chi))}$$



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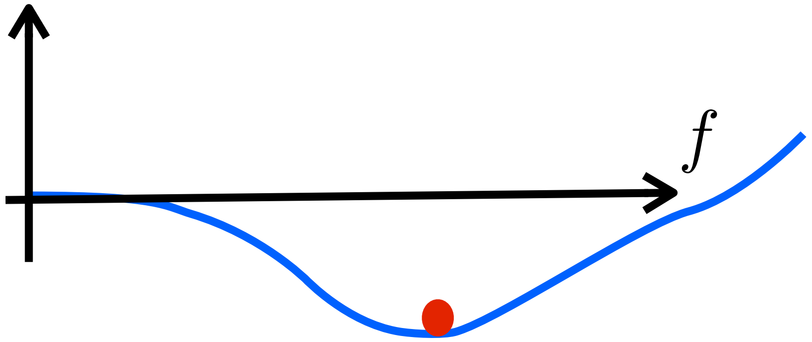

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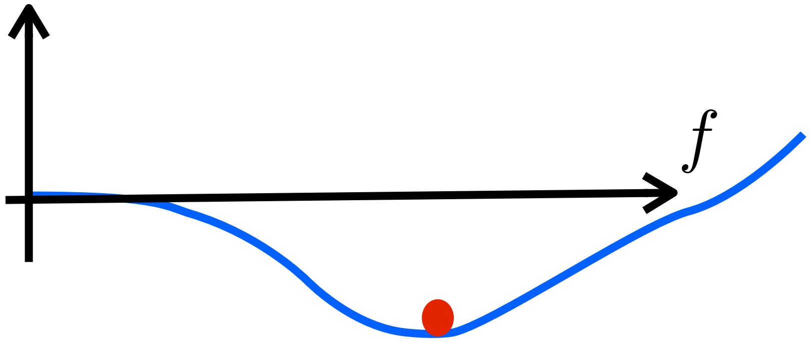

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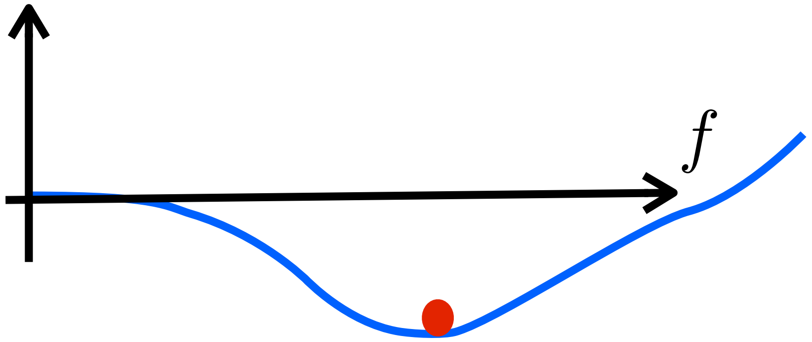

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is the dilaton naturally light?

not quite

LIGHT DILATON?

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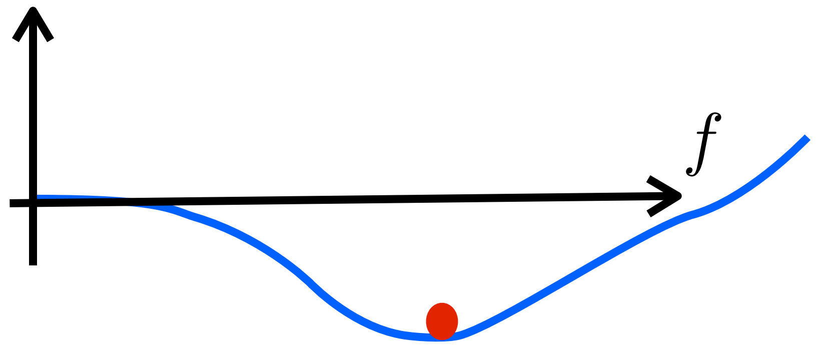
F is the vacuum energy in units of f



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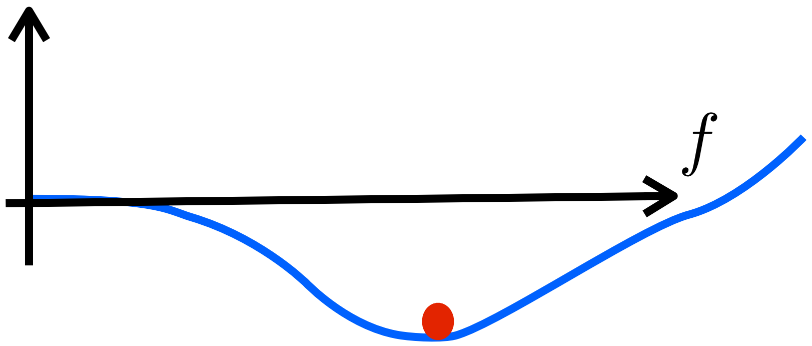
generically very steep potential!



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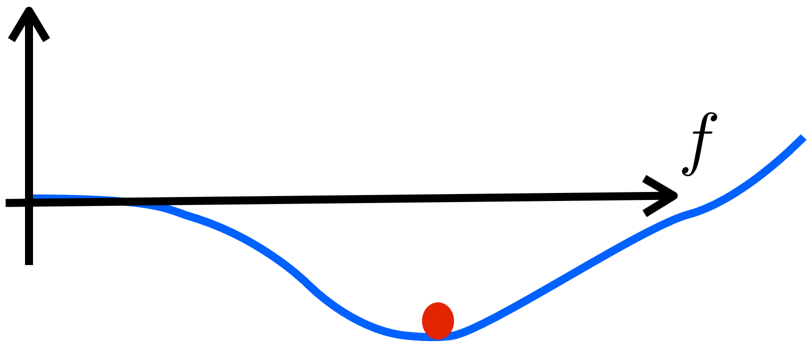

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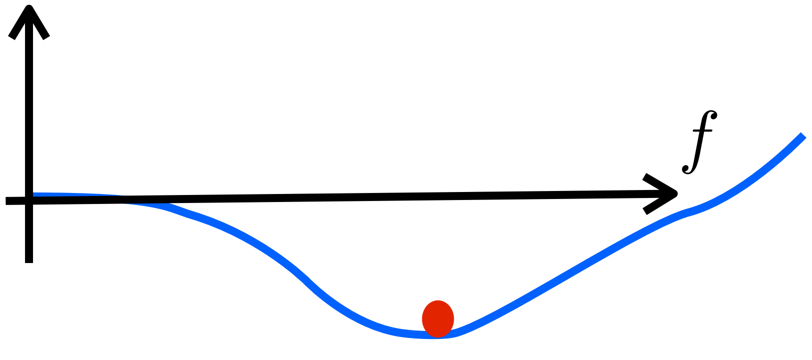

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to establish f<<UV-cutoff beta(IR) must be big
the CFT(IR) and the light dilaton are lost

or

start with a ~flat direction; no large vacuum energy
(natural only in SUSY?)

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$$F(\lambda) = \underset{\substack{\uparrow \\ \text{sym}}}{a} + \underset{\substack{\uparrow \\ \text{sym breaking}}}{\delta F(\lambda)} = 16\pi^2 \left[c_0 + c_1 \frac{\lambda}{4\pi} + \ldots \right]$$

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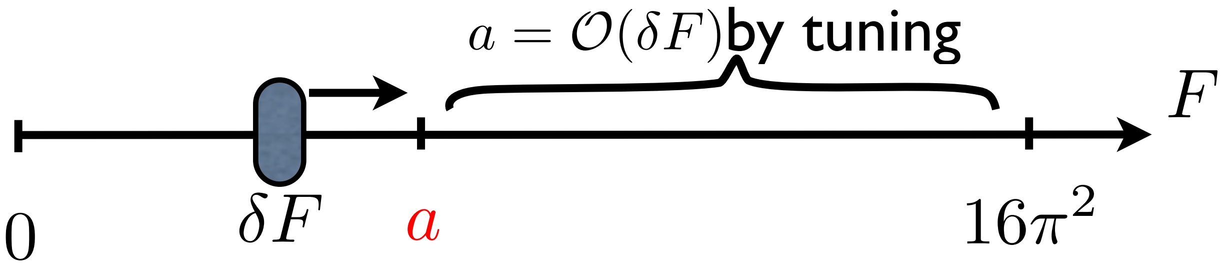
1) small vacuum energy $a \ll 16\pi^2$



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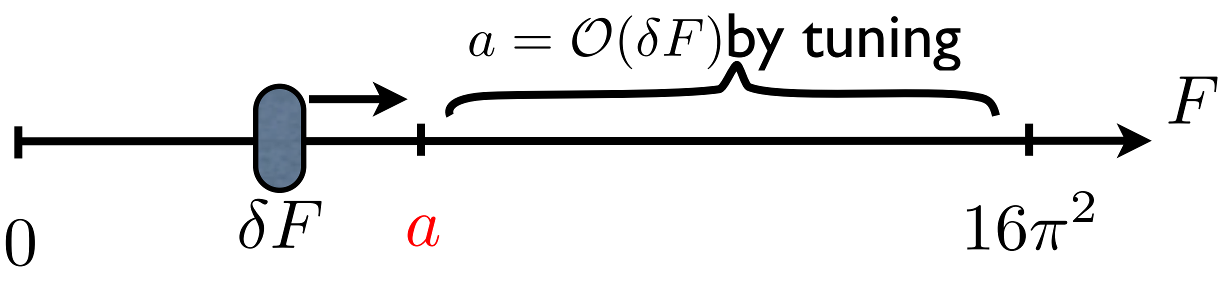

2) δF dynamically cancels vs a

$$a + \delta F(f) \simeq 0 \longrightarrow f = \Lambda_{UV}\left(\frac{-4\pi c_0}{\lambda(M)c_1}\right)^{1/\epsilon}$$

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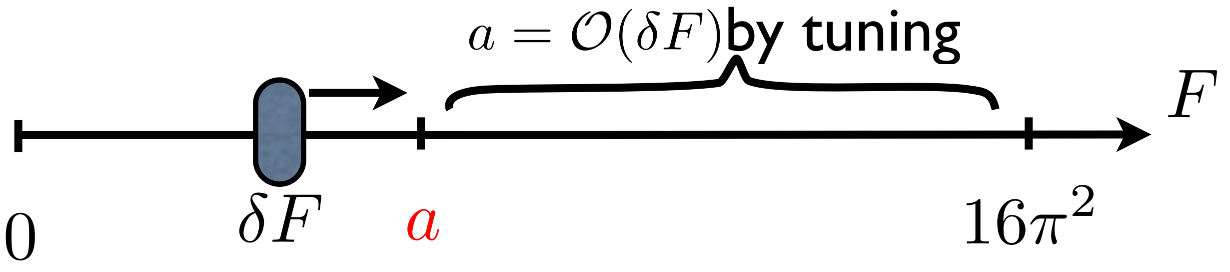

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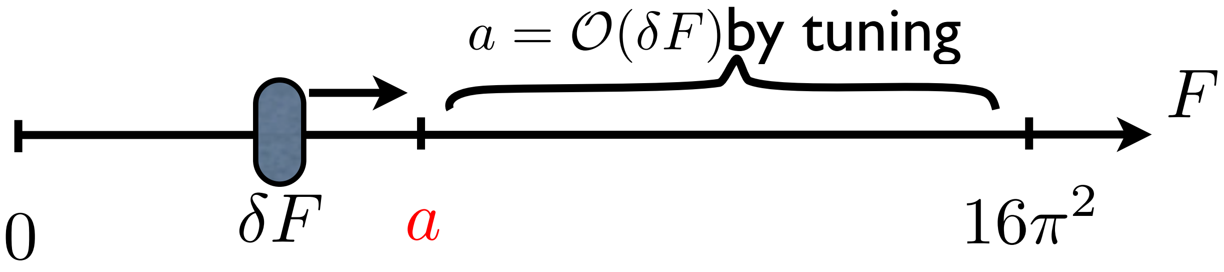

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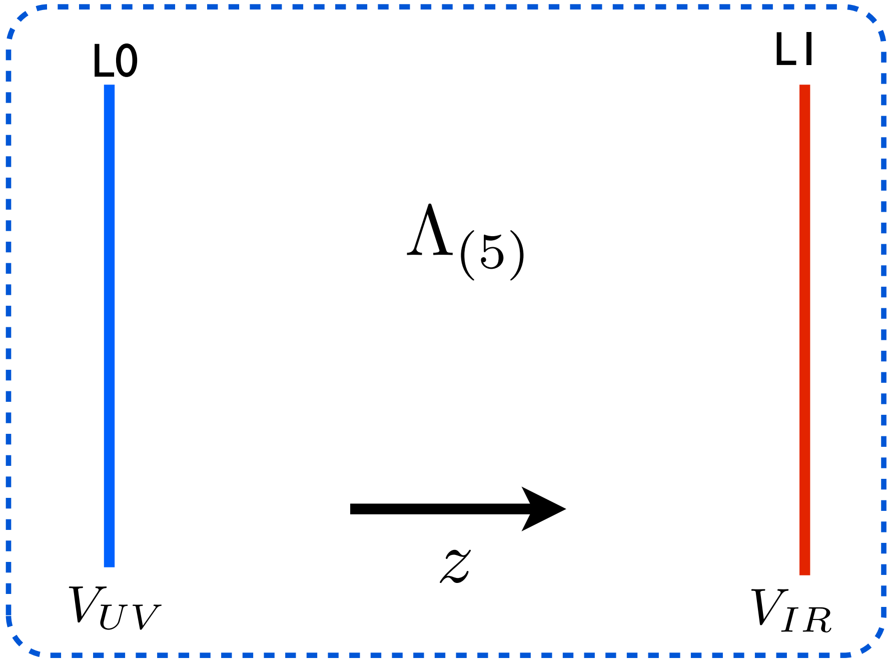
way out? largish coupling but small beta $\beta = \epsilon\left(\lambda + b_1 \frac{\lambda^2}{4\pi} + \ldots\right) \ll 1$ dual to a pNGB in 5D?

Bellazzini, Csaki, Hubisz, Serra, Terning, 1305.3919
Coradeschi, Lodone, Pappadopulo, Rattazzi, & Vitale 1306.4601

Contino, Pomarol, Rattazzi, at Planck09

G	H	N_G	NGBs rep.$[H]$ = rep.$[\mathrm{SU}(2)\times\mathrm{SU}(2)]$
SO(5)	SO(4)	4	$\mathbf{4}=(\mathbf{2},\mathbf{2})$
SO(6)	SO(5)	5	$\mathbf{5}=(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{2})$
SO(6)	SO(4) × SO(2)	8	$\mathbf{4}_{\mathbf{+2}}+\bar{\mathbf{4}}_{\mathbf{-2}}=2\times(\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	6	$\mathbf{6}=2\times(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{2})$
SO(7)	G_2	7	$\mathbf{7}=(\mathbf{1},\mathbf{3})+(\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × SO(2)	10	$\mathbf{10}_{\mathbf{0}}=(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{2},\mathbf{2})$
SO(7)	$[\mathrm{SO}(3)]^3$	12	$(\mathbf{2},\mathbf{2},\mathbf{3})=3\times(\mathbf{2},\mathbf{2})$
Sp(6)	Sp(4) × SU(2)	8	$(\mathbf{4},\mathbf{2})=2\times(\mathbf{2},\mathbf{2}),(\mathbf{2},\mathbf{2})+2\times(\mathbf{2},\mathbf{1})$
SU(5)	SU(4) × U(1)	8	$\mathbf{4}_{-5}+\bar{\mathbf{4}}_{\mathbf{+5}}=2\times(\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	14	$\mathbf{14}=(\mathbf{3},\mathbf{3})+(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$

Holography



$$ds^2 = \left(\frac{L_0}{z}\right)^2 \left(dx^2 - dz^2\right)$$

$$S = -\int d^5x\sqrt{g}\left(M_*^3 R + \Lambda_{(5)}\right) - \int d^5x\sqrt{g}\frac{1}{4g_5^2}F^{MN}F_{MN}$$

calculable gauge theory $m_{KK} \sim \pi/L_1 \ll 24\pi^3/g_5^2 \times (L_0/L_1) \longrightarrow \frac{g_5^2}{24\pi^2 L_0} \ll 1$

calculable gravity $\pi/L_1 \ll (24\pi^3)^{1/3} M_* \times (L_0/L_1) \longrightarrow 24(M_* L_0)^3 \gg 1$

large N theory

$$f^2 = 12(M_* L_0)^3/L_1^2$$

$$f_\pi^2 = \frac{4}{g_5^2}\frac{L_0}{L_1^2}$$

with $g_5^2/L_0 \gtrsim g_{eff}^2 \log(L_1/L_0) \simeq 16$

$$\frac{L_0}{g_5^2}\log(L_1/L_0) + \frac{1}{g_{UV}^2} + \frac{1}{g_{IR}^2} = \frac{1}{g_{eff}^2} \longrightarrow \boxed{\frac{f_\pi^2}{f^2} \ll 1}$$

in AdS5