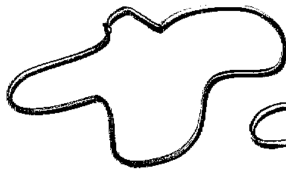
Sept '05

„Geometry" at ∞ curvature

1. Gauge/String duality.

Gauge theory: Flux lines observables

$q \qquad \bar{q}$



C

$$W(C) = \langle \mathrm{Tr}\, P \exp \oint_C A_\mu dx^\mu \rangle$$

Alternative observables: „words" The alphabet:

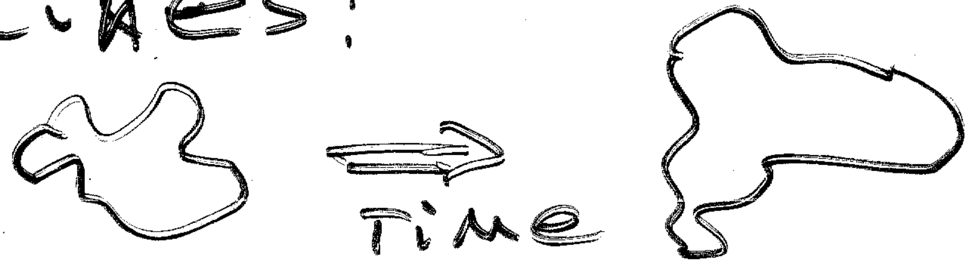
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu A_\nu]$$

$$\nabla_\lambda F_{\mu\nu} = \partial_\lambda F_{\mu\nu} + [A_\lambda F_{\mu\nu}]$$

$$\Omega \sim (\nabla^k F)^\ell ; \quad \boxed{\mathrm{Tr}(\Omega_{i_1} \ldots \Omega_{i_n})}$$

String theory

Evolution of flux lines:



$X^\mu = X^\mu(\sigma,\tau) \equiv X^\mu(\xi^1, \xi^2)$

The action

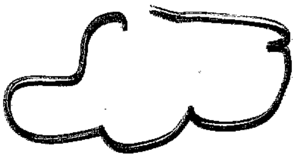
$$S \sim M^2 \int \sqrt{g}\, g^{ab}\, \partial_a X^\mu \partial_b X^\mu \cdot d^2\xi$$

$M^2 \equiv \frac{1}{4\pi\alpha'}$ - string tension

$g_{ab} = e^{\varphi} \delta_{ab}$

an independent metric (2d)

[redacted]

In critical dimensions 26, 10, .. strings are φ - independent /Weyl symmetry/.
There are ∞ number of excited states of  or described by the wave functionals

$$\Psi_n = \Psi_n[x^\mu(\sigma)] = e^{ip_\mu x^\mu} \cdot |\Phi_n\rangle$$

x^μ – center of mass
$|\Phi_n\rangle$ – oscillators of excited string.

[redacted]

Scattering of such objects described by the Veneziano formula. There are ∞ number of both open and closed states

$$\alpha'_{open} = 2\,\alpha'_{closed}$$

Massless open states $\Rightarrow$ <u>gauge fields</u>

Massless closed states $\Rightarrow$ <u>gravitons</u>

Non-critical strings ($D = 4, \dots$).

The field $\varphi(\xi)$ becomes dynamical

Hence $\Psi_n = \Psi_n(x^\mu(\sigma), \varphi(\sigma))$

These strings propagate in the $5D$ space which is warped in general. Effective metric is

$$ds^2 = d\varphi^2 + a^2(\varphi)(d\vec{x})^2$$

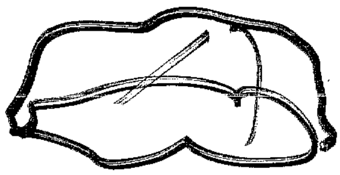
Random surface in 4 d described by string in 5d

[redacted]

Gauge / string relatio

Wilson Loop = Wave functi

of closed string:

$W[C] =$  $=$

$$= \sum_{(S_c)} e^{-\text{Area}(S_c)}$$

How to locate 4d contour C in 5d (x, φ) space?

It must be placed so that the open strings reproduce gauge th.

For that we need

$\alpha'_{closed} \sim const$

$\alpha'_{open} \to 0$.

/In this case open strings have only massless excitations (gluons)/

All massive are swept to ∞ at the Boundary

But we have a $\boxed{\text{blue shift}}$:

$$\alpha'_{open} \sim 1/a^2(\varphi_*) \cdot \alpha'_{closed}$$

~~hence~~

[redacted]

5) If the field theory has zero β-function
$a^2(\varphi) \propto e^{\alpha\varphi}$
($\Leftrightarrow$ conformal symmetry)
In this case we have AdS_5-space:

$$ds^2 \propto \frac{d\vec{x}^2 + dy^2}{y^2} \qquad y \sim e^{-\alpha\varphi}$$

If it is confining $a^2(\varphi)$ ~~flows up~~ doesn't vanish at
$\varphi \to -\infty$

6) The limit of free gauge fields correspond to ∞ curvature of AdS_5.

Sigma models

Describe interaction of Goldstone's particles.

For AdS_5

$$\mathcal{L} = \frac{1}{2\gamma_0} (\partial \vec{N})^2$$

$$\vec{N}^2 = -1$$

For a sphere:

$$\mathcal{L} = \frac{1}{2\gamma_0} (\partial \vec{n})^2$$

$$\vec{n}^2 = +1.$$

Without fermions

S^5 – $\vec{n}$ becomes massive

AdS_5 – $\vec{N}$ becomes free

[redacted]

We need <u>non-trivial conformal points</u>.

This happens if we ~~pass to the spinning~~ ~~st~~ add spin degrees of freedom / RNS or GS fermions /.

This amounts in going from $\vec{n} = \frac{SO(6)}{SO(5)}$

To ~~sup~~ various super-cosets.

The known examples are

1) $\frac{OSp(1|2)}{SO(2)}$; 2) $\frac{OSp(2|4)}{SO(2)\times SO(4)}$

3) $\frac{SU(4|2)}{SO(2)\times SO(5)}$ 4) $\frac{PSU(4|4)}{SO(5)\times SO(5)}$

A typical Lagrangian (II)

$$\mathcal{L} = \frac{1}{\gamma_0}\left\{ (B_\mu^a)^2 + \bar{\Psi}_\mu \gamma_3 \Psi_\nu \epsilon_{\mu\nu} \right\}$$

/ in OSp(1|2) case,
all other similar /

$(B_\mu^a,\ \Psi_\mu,\ A_\mu^{ab})$ –

– zero curvature connexion
of OSp(1|2)

Bethe – ansatz:

Introduce (λ, φ) multiplet

$$\mathcal{L} = \epsilon^{\alpha\beta} \lambda_\alpha^f \partial_{\bar z} \lambda_\beta^f + \varphi^f \partial_{\bar z} \varphi^f$$

$\uparrow$ B $\quad$ $\uparrow$ F

Couple to superconnection

$$\mathcal{L} \sim \underline{\Psi}^f (\partial + A) \underline{\Psi}^f \qquad \Psi = (\lambda, \varphi)$$

If $N_f \to \infty$ enforces zero curvature

An example of the strong coupling limit

$$\mathcal{L} = \frac{1}{2\gamma_0}(\partial\vec{n})^2 + \vec{\Psi}_-\partial_+\vec{\Psi}_-$$

with $\vec{n}^2 = 1 \quad \vec{\Psi}_-\vec{n} = 0$

The β - function $\neq 0$

$$\beta(\gamma_0) \sim \gamma_0^2 + \dots$$

But there is IR fixed point

$$\mathcal{L} = \frac{1}{2\gamma_0}(\partial\vec{n})^2 + \vec{\Psi}_-\partial_+\vec{\Psi}_-$$

$$+ \xi_+(\vec{\Psi}_-\vec{n}) + \lambda(\vec{n}^2 - 1)$$

In the IR!

$$\sim \frac{1}{\lambda}\,\xi_+\partial_-\xi_+$$

$$\mathcal{L}_{IR} = \xi_+\partial_-\xi_+ + \vec{\Psi}_-\partial_+\vec{\Psi}_-$$

$$\vec{n}_{IR} = \xi_+\vec{\Psi}_-$$

The moral:

at ∞ coupling

1) $(\partial \vec{n})^2$ term is dropped, 2) constraints $\Phi_- \vec{n} = 0$, $\vec{n}^2 = 1$ are relaxed

3) $\vec{n}_{IR}$ is expressed in terms of fermions

In case of $AdS_5 \times S_5$ we have

$$\vec{Z} = \vec{n} \oplus \vec{N} \subset R^{10,2}$$

$$L_{GS} = \bar{\Theta} \left(\delta_{\alpha\beta} + \gamma_{12} \epsilon_{\alpha\beta} \right) \cdot \left(\vec{\gamma} \partial_\alpha \vec{Z} \right) \partial_\beta \Theta$$

$$\gamma_{12} \left(\vec{\gamma} \vec{n} \right) \left(\vec{\gamma} \vec{N} \right) \Theta = \Theta$$

If the constraints are relaxed, ~~the~~ the sigma model acquires $SO(10,2)$ symmetry which must be present in $N=4$ YM. theory at zero coupling