LIMITS ON MASSLESS PARTICLES RECONSIDERED

1980

THE WEINBERG - WITTEN THEOREM

REASONABLE ASSUMPTION :

GRAVITATIONAL ENERGY IS MEASURED BY LONG-DISTANCE GRAVITATIONAL FIELD

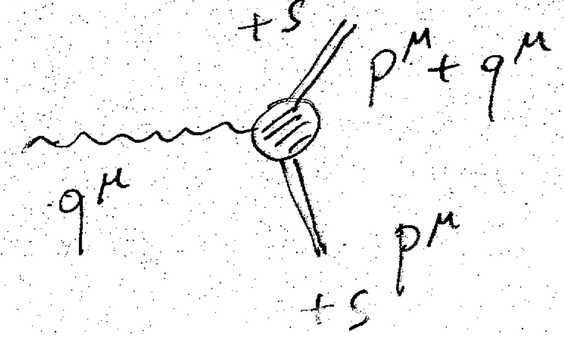
$$\phi \sim \frac{GE}{r}$$

W&W?

NO

N&E!

FOR MASSLESS PARTICLES THIS IS THE SAME AS



$q^2 > 0$

$q \to 0$

IN FORMULAS

$$\lim_{\substack{q^\mu \to 0 \\ q^2 > 0}} \langle +s, p+q | T_{\mu\nu} | +s, p \rangle = C\, p_\mu p_\nu$$

- NO HELICITY FLIP (ELASTIC SCATTERING)

- PRINCIPLE OF EQUIVALENCE:

$$C = 1$$

- ON SHELL:

$$p^2 = (p+q)^2 = 0$$

$$q^2 \neq 0 \quad \text{(OFF-SHELL,}$$

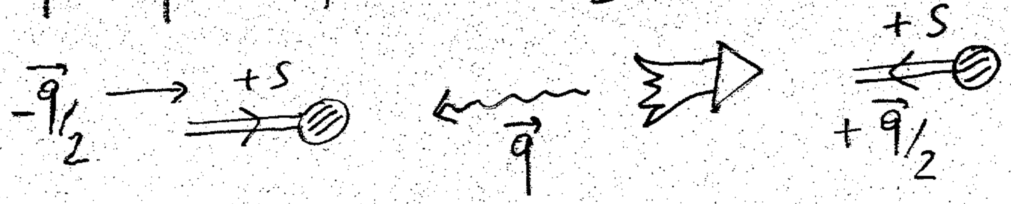
AS IN NEWTONIAN POTENTIAL

$$\sim \quad G p^0 / q^2 \equiv G E / q^2 \equiv G M / q^2$$

SPECIAL KINEMATICAL CONFIGURATION :

$$p^\mu = (|q|/2, -\vec{q}/2) \qquad q^\mu = (0, \vec{q})$$

$$p^\mu + q^\mu \equiv p'^\mu = \left(\frac{|q|}{2}, \vec{q}/2\right)$$



$$\langle +s, \vec{q}/2 | R^\dagger T_{\mu\nu} R | +s, -\vec{q}/2 \rangle$$

$$= \langle +s, \vec{q} | e^{i\theta s} T_{\mu\nu} e^{i\theta s} | +s, -\vec{q}/2 \rangle$$

$$= \langle +s, \vec{q}/2 | e^{ig\theta} T_{\mu\nu} | +s, -\vec{q}/2 \rangle$$

$$g = 0, 1, 2 \quad (T_{00}, T_{0i}, T_{ij}.$$

IF $\quad 2s > \max g = 2 \quad$ THEN

$$\langle +s, \vec{q}/2 | T_{\mu\nu} | +s, -\vec{q}/2 \rangle = 0 \quad \forall \mu\nu$$

BOOST TO (p, p', q)

YET : 1976

S. FERRARA, D.Z. FREEDMAN
P. VAN NIEUWENHUIZEN
PHYS. REV. D13, 3214

S. DESER, B. ZUMINO
PHYS. LETT. 62B, 325

GRAVITY + SPIN $\frac{3}{2}$ ----
... A COUNTER-EXAMPLE
PRECEEDING A THEOREM
BY FOUR YEARS ?

SUBTLETY: WE ASSUMED THAT

$$\langle +s, p | T_{\mu\nu} | +s, p' \rangle$$

TRANSFORMS AS A TENSOR:
IS IT TRUE?

CONSIDER SPIN $s = n + \frac{1}{2}$

ON SHELL STATE REPRESENTED BY

$$v_{\mu_1 \ldots \mu_n}(p)$$

WITH:

$$p^2 = 0 \qquad \not{p}\, v_{\mu_1 \ldots \mu_n}(p) = 0$$

$$\gamma^\mu v_{\mu \mu_1 \ldots \mu_{n-1}}(p) = 0$$

STILL TOO MANY COMPONENTS.
NEED ALSO

$$v_{0 \mu_1 \ldots \mu_{n-1}} = 0$$

LORENTZ TRANSF: (6

$$\omega_0{}^i\, v_{i\mu_1\dots\mu_{n-1}} + \partial_{[0}\,\epsilon_{\mu_1\dots\mu_{n-1}]} = 0$$

COMPENSATING GAUGE TRANSF.

BECAUSE OF IT

$$\delta \langle +s,p|\, T_{\mu\nu}\, |+s,p'\rangle = \omega_{(\mu}{}^{\lambda} \langle +s,p|\, T_{\lambda\nu)}\, |+s,p'\rangle$$

$$+\, \delta_\epsilon \langle +s,p|\, T_{\mu\nu}\, |+s,p'\rangle$$

HOMOGENEOUS TERM

INHOMOGENEOUS TERM.

MAKES MATRIX ELEMENT NONZERO IN BOOSTED FRAME EVEN WHEN MATRIX ELEMENT IS ZERO IN BRICK-WALL FRAME.

SO, IS ANYTHING POSSIBLE NOW?

NO. $S = 3/2$ IS THE LIMIT

($S=2$ FOR INTEGER SPIN)

— o — o — o —

MANIFESTLY LORENTZ-INVARIANT FORMALISM : DROP CONSTRAINT

$$v_{0\mu_1 \cdots \mu_{n-1}} = 0$$

INTRODUCE ON-SHELL GAUGE INVARIANCE TO ELIMINATE EXTRA SPURIOUS POLARIZATIONS

$$v_{\mu_1 \cdots \mu_n} \longrightarrow v_{\mu_1 \cdots \mu_n} + p_{(\mu_1} \epsilon_{\mu_2 \cdots \mu_n)}$$

$$\gamma^\mu \epsilon_{\mu\mu_1 \cdots \mu_{n-2}} = 0 \qquad \not{p}\, \epsilon_{\mu_1 \cdots \mu_{n-1}} = 0$$

ON-SHELL GAUGE INVARIANCE :

$$\delta_\epsilon \langle +s, p \,|\, T_{\mu\nu} \,|\, +s, p' \rangle \propto$$ GRAVITON EQUATIONS OF MOTION

(COMPENSATE VARIATION BY CHANGE OF GRAVITON FIELD)

SOME REASONABLE ASSUMPTIONS

1) SMOOTHNESS AT $q \to 0$

$$\langle s, p | T_{\mu\nu} | s, p' \rangle = G \, p_\mu p_\nu + F_{\mu\nu\lambda}(q, p) q^\lambda + F_{\mu\nu\rho\sigma}(q, p) q^\rho q^\sigma$$

NON-SINGULAR FUNCTIONS

2) NO STATE OF SPIN $S-1$ TRANSFORMING AS

$$\delta u_{\mu_1 \ldots \mu_{n-1}} \propto \epsilon_{\mu_1 \ldots \mu_{n-1}}$$

THIS IS A STÜKELBERG FIELD THAT SIGNALS GAUGE SYMMETRY BREAKING

I.E. : SPIN S FIELD MASSIVE !

3) SPIN S STATE IS NOT THE STÜKELBER OF A HIGHER-SPIN STATE

THE COVARIANT CONSTRUCTION

$$\langle s,p'|T_{\mu\nu}|s,p\rangle = 2C\, p_{(\mu}\bar{v}^{\mu_1\cdots\mu_n}\gamma_{\nu)}u_{\mu_1\cdots\mu_n}$$

$$+ F\, q_{(\mu}\bar{v}^{\mu_1\cdots\mu_n}\gamma_{\nu)}u_{\mu_1\cdots\mu_n} +$$

$$q^\rho[\,A\,\bar{v}^{\mu_1\cdots\mu_{n-1}}{}_{\mu}\gamma_\nu u_{\rho\mu_1\cdots\mu_{n-1}} +$$

$$B\,\bar{v}^{\mu_1\cdots\mu_{n-1}}{}_{\mu}\gamma_\rho u_{\nu\mu_1\cdots\mu_{n-1}} +$$

$$D\,\bar{v}^{\mu_1\cdots\mu_{n-1}}{}_{\rho}\gamma_\mu u_{\nu\mu_1\cdots\mu_{n-1}} + \mu \leftrightarrow \nu\,]$$

$$+ E\, p_{(\mu}\bar{v}_{\rho\mu_1\cdots\mu_{n-1}} q^\rho \gamma_{\nu)} u_{\sigma\mu_1\cdots\mu_{n-1}} q^\sigma$$

$$+ \text{TERMS BEGINNING AT } O(q^3)$$

NOTICE: HELICITY $+s$ MEANS

$$\gamma^5 u_{\mu_1\cdots\mu_n} = u_{\mu_1\cdots\mu_n}$$

$$\gamma^5 v_{\mu_1\cdots\mu_n} = v_{\mu_1\cdots\mu_n}$$

WE WROTE THE MOST GENERAL EXPANSION UP TO $O(q^2)$

PRINCIPLE OF EQUIVALENCE: $C = 1$

CONSERVATION OF $T_{\mu\nu}$ UP TO $O(q^3)$:

NEED TO USE REPEATEDLY :

$$\not{p}\, u_{\mu_1 \cdots \mu_n} = 0 \qquad \not{p}\, v_{\mu_1 \cdots \mu_n} = -\not{q}\, v_{\mu_1 \cdots \mu_n}$$

$$p \cdot q = -q^2/2$$

$$F = C \qquad\qquad D = -A$$

(TERM $\propto B$ IS ZERO ON-SHELL)

MATRIX ELEMENT MUST ALSO BE PROPORTIONAL TO E.O.M. OF GRAVITON WHEN

$$u_{\mu_1 \cdots \mu_n} = p_{(\mu_1}\, \epsilon_{\mu_2 \cdots \mu_n)}$$

VARIATION GIVES TO $O(q)$:

$$C\, p_\mu q_\rho\, \bar{v}^{\rho \mu_1 \cdots \mu_{n-1}} \gamma_\nu\, \epsilon_{\mu_1 \cdots \mu_{n-1}}$$

$$-\frac{A}{n}\, p_\mu q_\rho\, \bar{v}^{\rho \mu_1 \cdots \mu_{n-1}} \gamma_\nu\, \epsilon_{\mu_1 \cdots \mu_{n-1}}$$

$$+\ \mu \leftrightarrow \nu \ + O(q^2)$$

TERM $O(q)$ NOT PROPORTIONAL TO E.O.M. OF GRAVITON, WHICH IS $O(q^2)$. MUST CANCEL:

$$A = n\,C$$

NOW COLLECT TERMS $O(q^2)$

$$C\Big\{ \tfrac{1}{2}\, q_\mu\, \bar{v}^{\mu_1\cdots\mu_n}\gamma_\nu\, q_{\mu_1}\epsilon_{\mu_2\cdots\mu_n}$$

$$+\, q\cdot p\; \bar{v}^{\mu_1\cdots\mu_{n-1}}{}_\mu\,\gamma_\nu\,\epsilon_{\mu_1\cdots\mu_{n-1}}$$

$$+\,(n-1)\,\bar{v}^{\mu_1\cdots\mu_{n-1}}{}_\rho\,\gamma_\mu q^\rho q_{\mu_1}\epsilon_{\nu\mu_2\cdots\mu_{n-1}}$$

$$-\,(n-1)\,\bar{v}^{\mu_1\cdots\mu_{n-1}}{}_\mu\,\gamma_\nu\, q_{\mu_1} q^\rho\epsilon_{\rho\mu_2\cdots\mu_{n-1}}$$

$$+\;\mu\leftrightarrow\nu\Big\} + O(q^3)$$

USE $p\cdot q = -\,q^2/2$ TO REWRITE BLUE TERMS AS

$$\tfrac{C}{2}\underbrace{(q_\mu q_{\mu_1} - q^2\eta_{\mu\mu_1})}_{\text{GRAVITON E.O.M.}}\,\bar{v}^{\mu_1\cdots\mu_n}\gamma_\nu\,\epsilon_{\mu_2\cdots\mu_n}$$

$h^{\mu_1}{}_\nu : h^{\mu_1}{}_{\mu_1} = 0$

TERMS $\propto (n-1)$ DEPEND ON
RANK-4 TENSOR

$$\bar{\psi}^{\alpha\beta\mu_1\cdots\mu_{n-2}}\,\gamma^{\nu}_{\rho}\,\epsilon_{\sigma\mu_1\cdots\mu_{n-2}} = q^{\alpha\beta}{}_{\rho\sigma}$$

DO NOT VANISH ON THE EINSTEIN EQUATION SHELL. COUPLING INCONSISTENT FOR $n>1$

$n=1$ IS ARAGONE-DESER NO-GO

OUR ANALYSIS SHOWS THIS PROBLEM IS NOT DUE TO $s>\frac{3}{2}$ FUNDAMENTAL INSTEAD OF BOUND STATE.

FOR ALL $s>\frac{3}{2}$ ONLY CONSISTENT "COUPLING" IS

$$C=0$$

" SUPERGRAVITY IS THE LIMIT "

HIGH-SPIN BOSONS

POLARIZATION VECTOR:

$$U_{\mu_1\cdots\mu_n} : \quad p^\mu U_{\mu\mu_1\cdots\mu_{n-1}} = 0$$

$$U^\lambda{}_{\lambda\mu_1\cdots\mu_{n-2}} = 0$$

P.O.E.

$$\langle s q' | T_{\mu\nu} | s p \rangle = 2 p_\mu p_\nu U^{\mu_1\cdots\mu_n} V_{\mu_1\cdots\mu_n} +$$

$$\Big[A U^{\mu_1\cdots\mu_n} p_\mu q_\nu V_{\mu_1\cdots\mu_n} + B U^{\mu_1\cdots\mu_n} q_\mu q_\nu V_{\mu_1\cdots\mu_n}$$

$$+ D q^\sigma U_\sigma{}^{\mu_1\cdots\mu_{n-1}} V_{\mu_1\cdots\mu_{n-1}\mu} p_\nu + E q^\sigma U_{\mu_1\cdots\mu_{n-1}\mu} V_{\sigma\mu_1\cdots\mu_{n-1}}$$

$$+ F q^\sigma U_{\sigma\mu_1\cdots\mu_{n-1}} V^{\mu_1\cdots\mu_{n-1}}{}_\mu q_\nu + G q^\sigma U_{\mu_1\cdots\mu_{n-1}\mu} V_{\sigma\mu_1\cdots\mu_{n-1}} q$$

$$+ H q^\sigma q^\rho U_{\rho\sigma\mu_1\cdots\mu_{n-2}} V^{\mu_1\cdots\mu_{n-2}}{}_{\mu\nu} + I q^\sigma q^\rho U_{\mu\nu\mu_1\cdots\mu_{n-2}} V_{\rho\sigma}{}^{\mu_1\cdots\mu_n}$$

$$+ J q^\rho q^\sigma U_{\rho\mu\mu_1\cdots\mu_{n-2}} V_{\sigma\nu}{}^{\mu_1\cdots\mu_{n-2}} + \mu \leftrightarrow \nu \Big]$$

$$+ O(q^3)$$

MUST IMPOSE CONSERVATION OF CURRENT UP TO $O(q^3)$ AND CANCEL DIFFERENT TENSOR STRUCTURES: $U^{\mu_1\cdots\mu_n} V_{\mu_1\cdots\mu_n}$, $(q\cdot U)^{\mu_1\cdots\mu_{n-1}} (q\cdot V)_{\mu_1}$

(12'')

$A = 1$, $B = \frac{1}{4}$ $E = -D$

$F = \frac{D}{2}$ $G = -D/2$ $H = -\frac{J}{2}$ $I = -\frac{J}{2}$

TWO FREE PARAMETERS,

D, J.

NEXT, GAUGE INVARIANCE MUST BE IMPOSED UP TO $O(q^3)$:

WHEN

$$V_{\mu_1 \dots \mu_n} = P_{(\mu_1} \epsilon_{\mu_2 \dots \mu_n)}$$

MATRIX ELEMENT MUST BE $\propto$ TO GRAVITON'S E.O.M.

TO $O(q)$ $\delta_\epsilon \langle s, p' | T_{\mu\nu} | s, p \rangle$ IS

$$-2 q^\sigma U_{\sigma \mu_1 \dots \mu_{n-1}} P_\mu P_\nu \epsilon^{\mu_1 \dots \mu_{n-1}} +$$

$$\frac{2D}{n} q^\sigma U_{\sigma \mu_1 \dots \mu_{n-1}} P_\mu P_\nu \epsilon^{\mu_1 \dots \mu_{n-1}}$$

$$\Longrightarrow \quad D = n$$

TO $O(q^2)$:

$$-U_{\sigma\mu_1\cdots\mu_{n-1}}\, p_\mu\, q_\nu\, q^\sigma\, \epsilon^{\mu_1\cdots\mu_{n-1}}$$

$$-D\,\frac{n-1}{n}\, q^\sigma q^\rho\, U_{\rho\sigma\mu_1\cdots\mu_{n-2}}\, \epsilon_\mu{}^{\mu_1\cdots\mu_{n-1}}\, p_\nu$$

$$+\frac{E}{n}\, p\cdot q\, U_{\mu_1\cdots\mu_{n-1}\mu}\, \epsilon^{\mu_1\cdots\mu_{n-1}}\, p_\nu$$

$$-\frac{n-1}{n}\, E\, q^\sigma q^\rho\, U_{\mu_1\cdots\mu_{n-2}\mu\rho}\, \epsilon_\sigma{}^{\mu_1\cdots\mu_{n-2}}\, p_\nu$$

$$+\frac{F}{n}\, q^\sigma\, U_{\sigma\mu_1\cdots\mu_{n-1}}\, \epsilon^{\mu_1\cdots\mu_{n-1}}\, p_\mu q_\nu\ +$$

$$+\frac{2}{n} H\, q^\sigma q^\rho\, U_{\rho\sigma\mu_1\cdots\mu_{n-2}}\, \epsilon^{\mu_1\cdots\mu_{n-2}}{}_\mu\, p_\nu$$

$$+\frac{J}{n}\, q^\sigma q^\rho\, U_{\rho\mu\mu_1\cdots\mu_{n-2}}\, \epsilon_\sigma{}^{\mu_1\cdots\mu_{n-2}}\, p_\nu$$

$$+\ \mu \leftrightarrow \nu$$

TERMS MULTIPLYING G $\phi_{\mu\nu} = U_{\mu\mu_1\cdots\mu_{n-1}}\, \epsilon^{\mu_1\cdots\mu_{n-1}}\, p_\nu$
$+\ \mu \leftrightarrow \nu$

PROPORTIONAL TO EINSTEIN EQS. O.K

TERMS MULTIPLYING

$\phi_{\mu\nu\rho\sigma} = U_{\mu_1\cdots\mu_{n-2}\mu\rho}\, \epsilon_\sigma{}^{\mu_1\cdots\mu_{n-2}}\, p_\nu\ +\ \mu \leftrightarrow \nu$

MUST CANCEL.

THEY DO FOR $E = -n$ $F = \frac{n}{2}$ $G = -\frac{n}{2}$

$H = n(n-1)/2$ $J = n(n-1)/2$

TO $O(q^3)$ WE FIND TWO CLASSES OF TERMS

A) TERMS MULTIPLYING

$$\psi_{\mu\nu} = U_\mu{}^{\mu_1 \dots \mu_{n-1}} \epsilon_{\mu_1 \dots \mu_{n-1}} q_\nu + \mu \leftrightarrow \nu$$

OR

$$\theta_{\mu\nu} = q^\sigma U_{\mu\sigma\mu_1 \dots \mu_{n-2}} \epsilon^{\mu_1 \dots \mu_{n-2}}{}_\nu + \mu \leftrightarrow \nu$$

THEY ARE PROPORTIONAL TO GRAVITON'S E.O.M. O.K.

B) TERMS MULTIPLYING

$$\psi_{\mu\nu\rho\sigma} = q^\lambda U_{\mu\nu\lambda\mu_1 \dots \mu_{n-3}} \epsilon_{\rho\sigma}{}^{\mu_1 \dots \mu_{n-3}}$$

THEY ARE <u>NOT</u> PROPORTIONAL TO GRAVITON E.O.M.

CANNOT BE CANCELED BY ADDING TERMS $O(q^3)$ IN $\langle s, p' | T_{\mu\nu} | s, p \rangle$, SINCE GAUGE VARIATION IS EITHER $O(q^4)$ OR $\propto p_{(\mu} V_{\nu)} \sim O(q^3)$

THEY ARE MULTIPLIED BY (12v)

$(n-2)(n-1)$

SO, ABSENT FOR SPIN 1 &
SPIN 2:
IT IS O.K. TO COUPLE MASSLESS PARTICLES TO GRAVITY UP TO SPIN 2. NO HIGHER SPIN

THIS RESULT CAN BE STRENGTHENED PROBABLY, SINCE IN OUR ANALYSIS WE COULD HAVE SEVERAL SPIN-2 MASSLESS PARTICLES INSTEAD OF JUST THE GRAVITON.

ANYWAY... MASSLESS PARTICLES OF SPIN >2 CANNOT COUPLED CONSISTENTLY TO GRAVITY.
AGAIN: SUPERGRAVITY IS THE LIMIT.

REMARKS:

- WE CAN COUPLE MASSLESS HIGHER SPIN PARTICLES TO GRAVITY IN AdS SPACE
- WE CAN COUPLE MASSIVE PARTICLES TO GRAVITY IN AdS AND FLAT SPACE

IN BOTH CASES, INSTEAD OF DOWNRIGHT INCONSISTENCIES WE FIND STRONG COUPLING PROBLEMS

$$g_{EFF} \sim \left(E/\Lambda^{\frac{1}{2}}\right)^{\#} \left[\text{OR } \left(\frac{E}{m}\right)^{\#}\right]$$

SPIN 2 IS THE HIGHEST POSSIBLE FOR A MASSLESS PARTICLE IN FLAT SPACE

$\Lambda \sim M^2$ SO, IN AdS HIGHER SPIN MASSLESS STATES ARE POSSIBLE

IN FLAT SPACE, MASSLESS $S > 2$ WOULD REQUIRE A (PROBABLY) UNACCEPTABLE BREAKDOWN OF LOCALITY

ANYWAY, THIS NO-GO SIGNALS ANOTHER ASPECT IN WHICH SPIN 2, $\frac{3}{2}$ ARE SPECIAL

LIMITS ON MASSLESS ①
PARTICLES, RECONSIDERED

OR

HOW SUPERGRAVITY
SURVIVED A YET-TO-BE
NO GO THEOREM

YET : 1976

PHYS. REV. D13, 3214

FERRARA, FREEDMAN &
VAN NIEUWENHUIZEN

EXHIBIT A THEORY WITH
MASSLESS SPIN $\frac{3}{2}$ INTERACTING
WITH GRAVITY

---- A COUNTER EXAMPLE
PRECEEDING A THEOREM BY
FOUR YEARS?

SUPERGRAVITY
EVADED A THEOREM YET TO BE FOUND.
IT TOOK COURAGE AND LUCK FOR SERGIO, DAN AND PETER TO GO AGAINST THE MANY ODDS & OBSTACLES STANDING IN FRONT OF THEM.

FOR THIS ALONE, TODAY'S RECOGNITION HAS BEEN FULLY EARNED BY SERGIO.

FOR THE FUTURE, I WISH HIM TO FIND YET MORE NO-GO THEOREMS TO EVADE......

INCLUDING THIS ONE!