LATTICE QCD AND FLAVOR PHYSICS

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OUTLINE

1. Motivations for flavor physics
2. Lattice QCD and quark masses
3. CKM matrix
   a) The Cabibbo angle and the first row unitarity test
   b) The Unitarity Triangle Analysis and CP violation

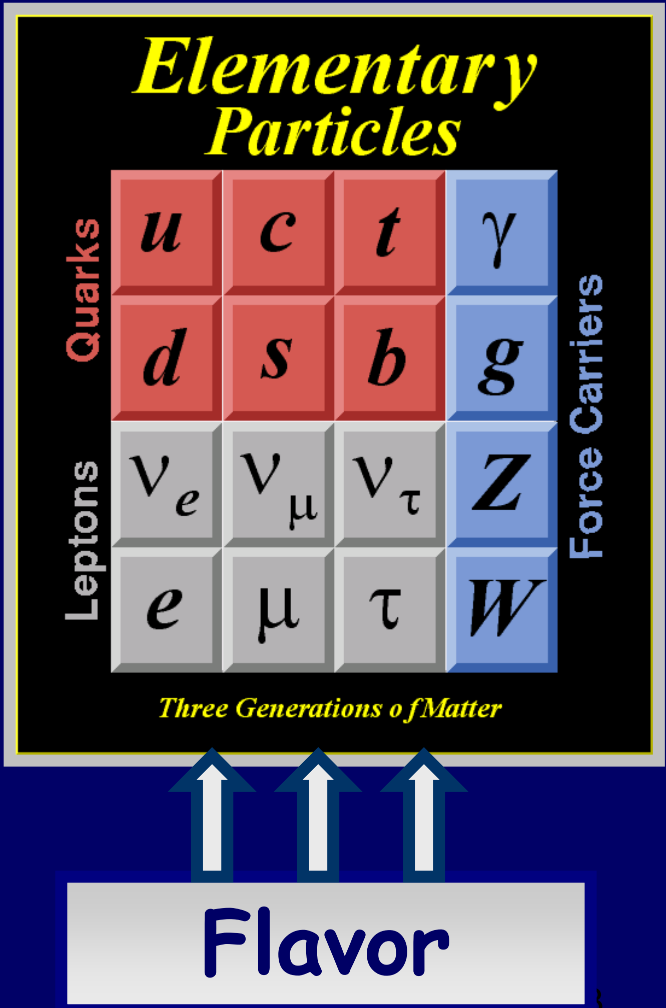
①

MOTIVATIONS FOR FLAVOR PHYSICS

Flavor physics is (well) described but not explained in the Standard Model:

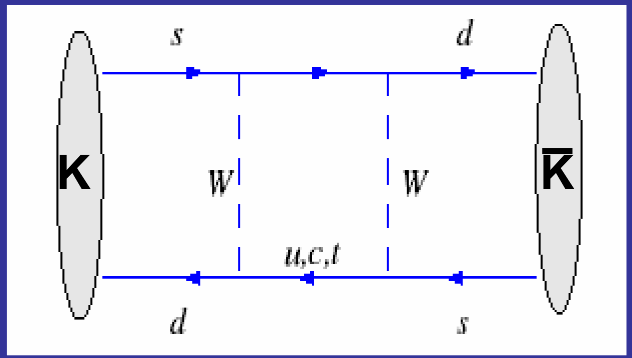
A large number of **free parameters** in the flavor sector (10 parameters in the quark sector only, 6 m_q + 4 CKM)

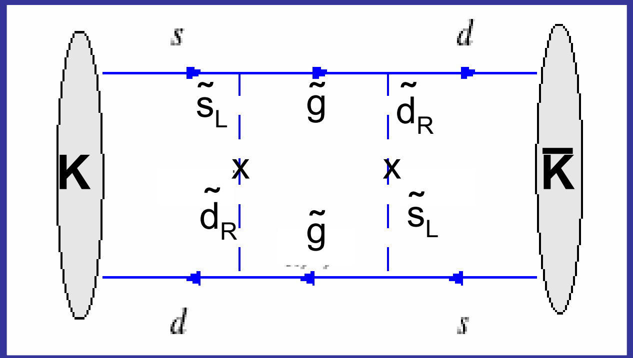
- Why **3 families**?
- Why the **spectrum** of quarks and leptons covers 5 orders of magnitude? ($m_q \sim G_F^{-1/2}$...)
- What give rise to the pattern of **quark mixing** and the magnitude of **CP violation**?



Flavor physics is an open window on New Physics: FCNC, CP asymmetries,…

E.g. K^0-$\bar{K}^0$ mixing:





New Physics can be conveniently described in terms of a **low energy effective theory**:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda_{NP}} O_i^{(5)} + \sum_i \frac{c_i}{\Lambda_{NP}^2} O_i^{(6)} + \ \ldots$$

E.g.: $\Lambda_{K^0\text{-}K^0} \sim 100$ TeV

The flavor problem:

$$\Lambda_{NP} >> \Lambda_{EWSB} \sim O(1\ \text{TeV})$$

New Physics must be very special !!

DETERMINATION OF THE SM FLAVOR PARAMETERS

THE PRECISION ERA OF FLAVOR PHYSICS

EXPERIMENTS

$\varepsilon_K = 2.280\ 10^{-3} \pm 0.6\%$

$\Delta m_d = 0.502\ ps^{-1} \pm 1\%$

$\sin(2\beta) = 0.687 \pm 5\%$

THEORY

We need to control the theoretical input parameters at a comparable level of accuracy !!

Challenge for LATTICE QCD

②

LATTICE QCD AND QUARK MASSES

♦ QUARK MASSES CANNOT BE DIRECTLY MEASURED IN THE EXPERIMENTS, BECAUSE QUARKS ARE CONFINED INSIDE HADRONS

♦ BEING FUNDAMENTAL PARAMETERS OF THE STANDARD MODEL, QUARK MASSES CANNOT BE DETERMINED BY THEORETICAL CONSIDERATIONS ONLY.

⇒ QUARK MASSES CAN BE DETERMINED BY COMBINING TOGETHER A THEORETICAL AND AN EXPERIMENTAL INPUT. E.G.:

$$[M_{HAD}(\Lambda_{QCD}, m_q)]^{TH.} = [M_{HAD}]^{EXP.}$$

⇒ LATTICE QCD

LATTICE DETERMINATION OF QUARK MASSES

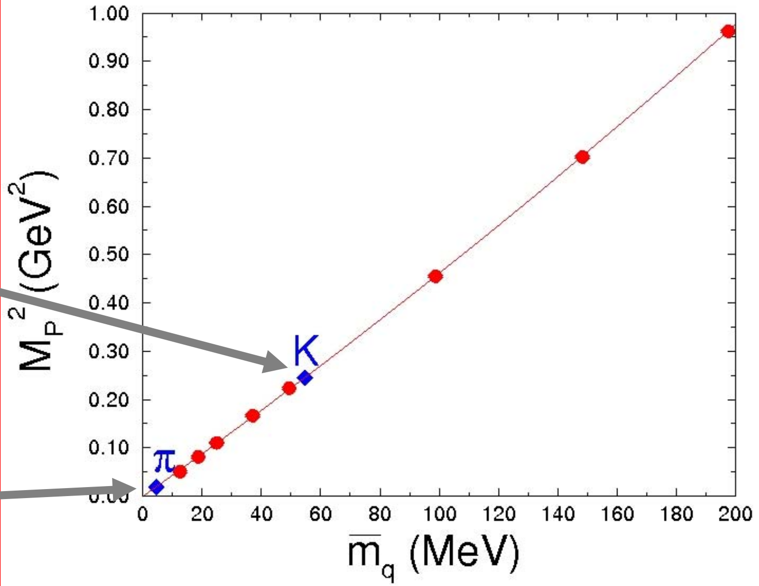
$$\hat{m}_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL
$M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY OR
NON-PERTURBATIVE METHODS

Extrapolation
to $m = m_s$

Extrapolation
to $m = m_{u,d}$



SYSTEMATIC ERRORS

$$m_q(\mu) = m_q(a)\, Z_m(\mu a)$$

ADJUSTED UNTIL $M_H^{LATT} = M_H^{EXP}$

PERTURBATION THEORY

O(a)

O(α²)

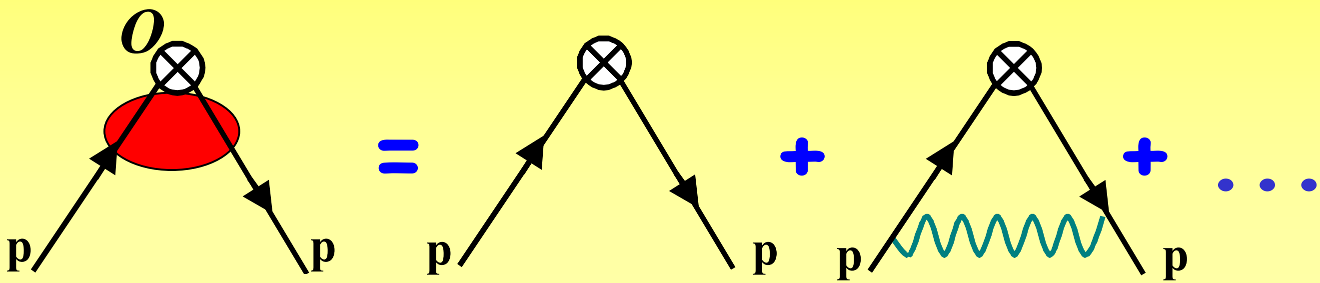
IMPROVED ACTIONS:

$$S_{LATT} = S_{QCD} + \cancel{a\,S_1} + a^2 S_2 + \dots$$

NON-PERTURBATIVE RENORMALIZATION

TWO IMPORTANT THEORETICAL TOOLS

NON-PERTURBATIVE RENORMALIZATION
THE RI-MOM METHOD

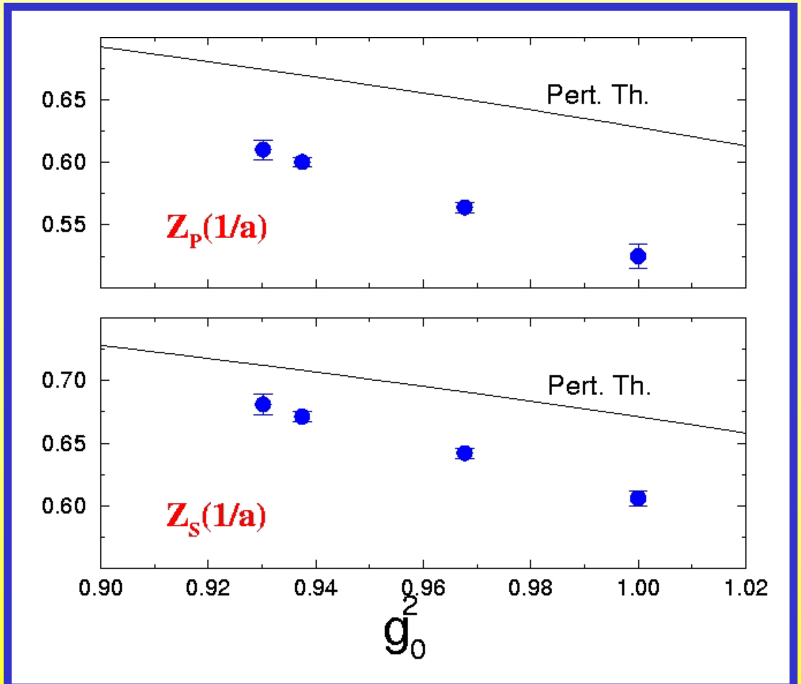


$\Gamma_O(p^2)$

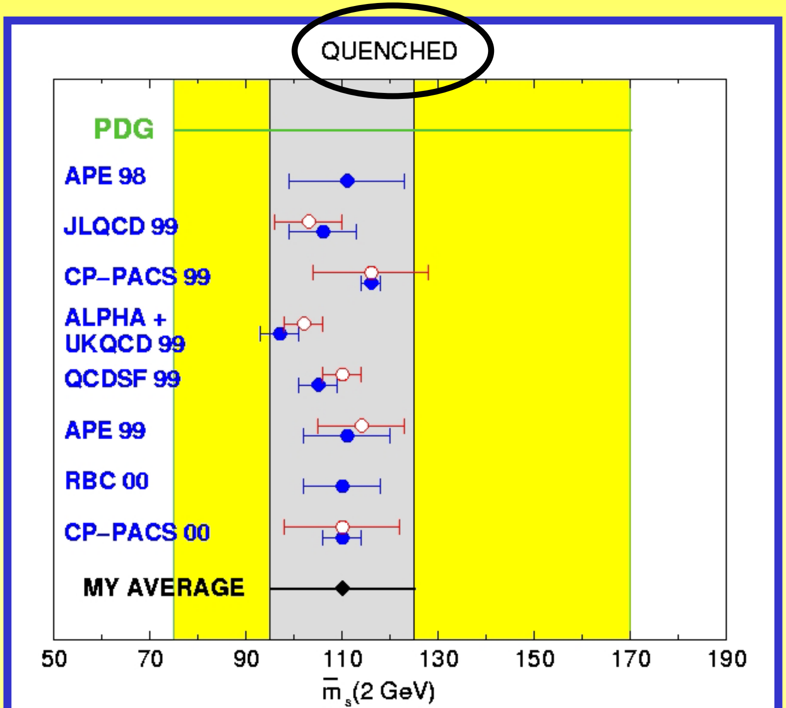
The (non-perturbative) renormalization condition:

$$Z_O(a\mu)\,\Gamma_O(p^2)|_{p^2=\mu^2} = \Gamma_{\text{Tree-Level}}$$

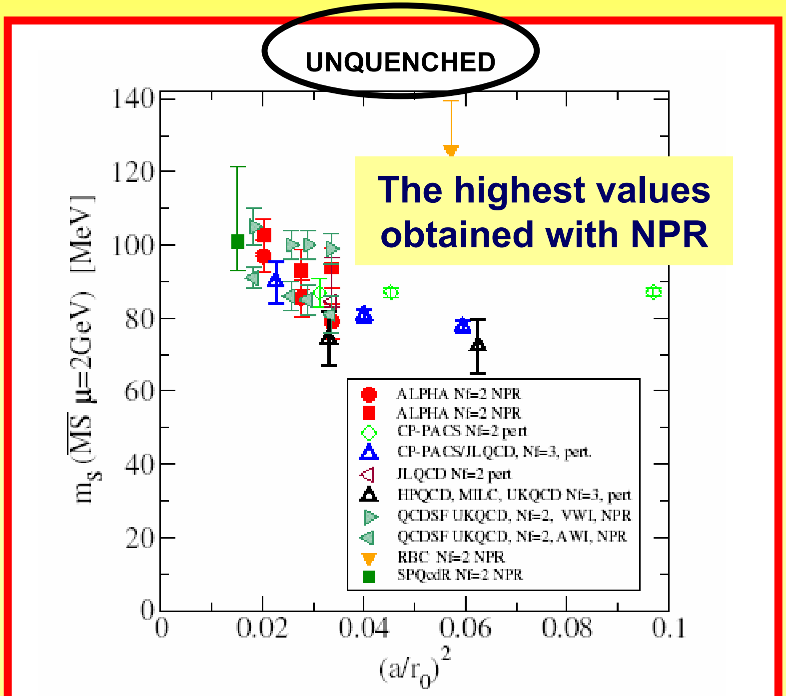
Several NPR techniques have been developed: Ward Identities, Schrodinger functional, X-space



THE STRANGE QUARK MASS



V.Lubicz, plenary at Lattice 2000



T.Izubuchi, plenary at Lattice 2005

$$\overline{m}_s\,(2\text{ GeV}) = (105 \pm 15 \pm 20)\text{ MeV}$$

[PDG 2002: $\overline{m}_s = (120 \pm 40)$ MeV]

Particle Data Group 2004

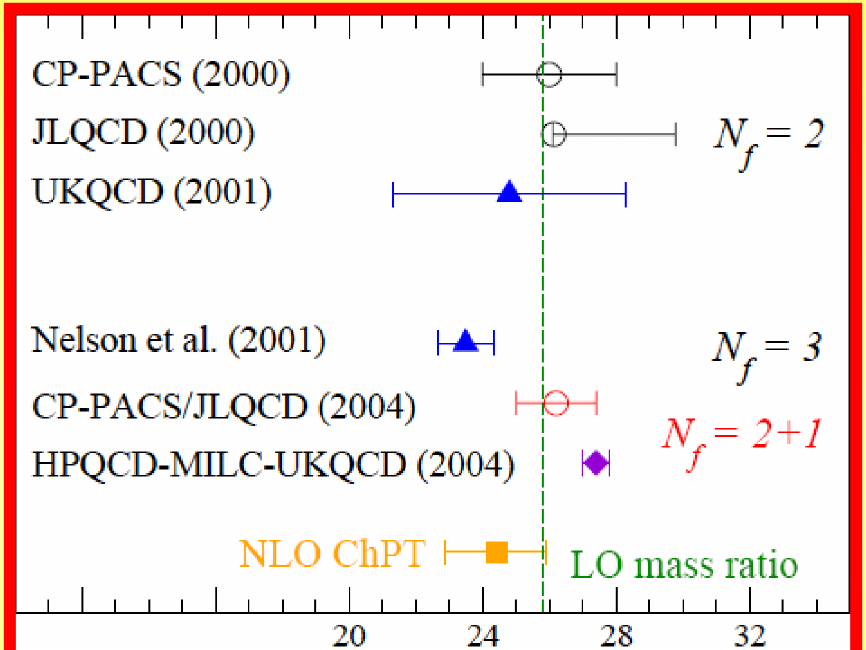
THE AVERAGE UP/DOWN QUARK MASS

RATIOS OF LIGHT QUARK MASSES ARE PREDICTED ALSO BY CHIRAL PERTURBATION THEORY:

$$\frac{m_u}{m_d} = 0.553 \pm 0.043$$

$$\frac{m_s}{(m_u + m_d)/2} = 24.4 \pm 1.5$$

From S. Hashimoto ICHEP 2004



$2\, m_s/(m_u+m_d)$

Good agreement with the ChPT prediction

③

CKM MATRIX

a) THE CABIBBO ANGLE AND THE "FIRST ROW" UNITARITY TEST

The most stringent unitarity test:
$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

PDG 2004 quotes a 2σ deviation from unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0029 \pm 0.0015$$

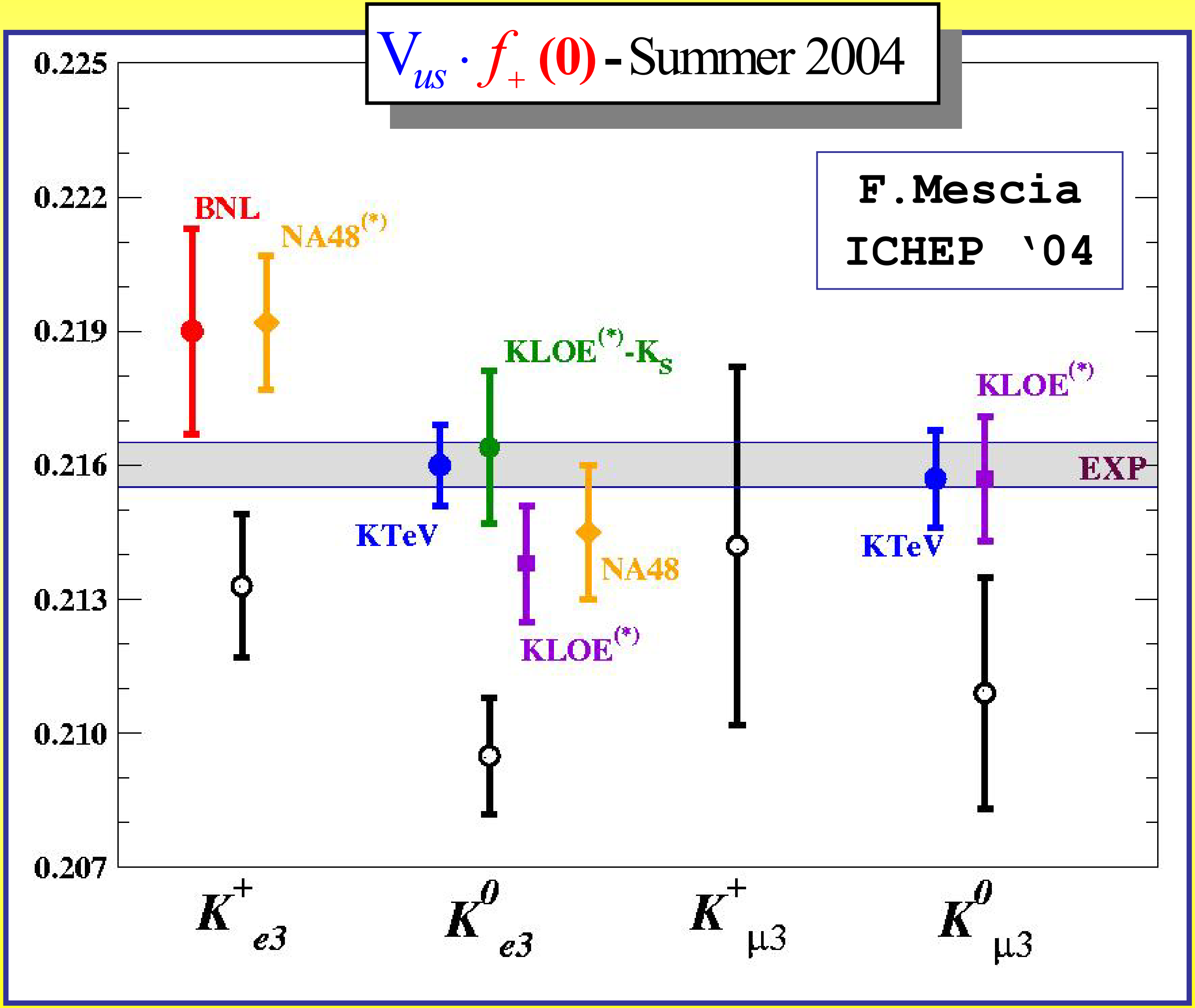
$|V_{ud}| = 0.9740 \pm 0.0005$ SFT and neutron β-decay

$|V_{us}| = 0.2200 \pm 0.0026$ $K \to \pi l \nu$ (BNL-E865 + old exps)

$|V_{ub}| = 0.0037 \pm 0.0005$ $b \to u$ incl. and excl. ($|V_{ub}|^2 \approx 10^{-5}$)

BUT: the PDG average for $|V_{us}|$ is superseded by NEW experimental and theoretical results

Kl3: the NEW experimental results



BNL-E865
PRL 91, (2003) 261802

KTeV
PRL 93, (2004) 181802

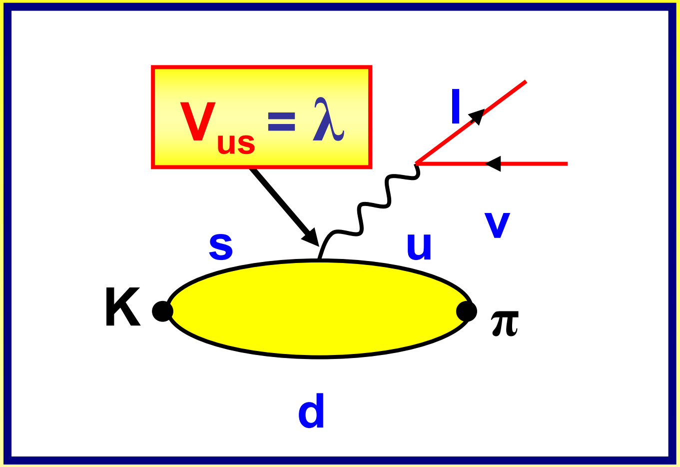
NA48
PLB 602, (2004) 41

KLOE
hep-ex/0508027

Kl3 theory

$$\Gamma(K\to\pi l\nu(\gamma)) = \frac{G_F^2 M_K^5}{192\pi^3} \cdot$$

$$\cdot\, C_K^2 |V_{us}|^2 \, |f_+^{K^0\pi^-}(0)|^2 I_l S_{ew} (1+\delta_l)^2$$



Ademollo-Gatto: $f_+(0) = 1 - O(m_s - m_u)^2$ ⟸ $O(1\%)$. But represents the largest theoret. uncertainty

ChPT

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -0.023$
Independent of L_i
(Ademollo-Gatto)

THE LARGEST UNCERTAINTY

"Standard" estimate:
Leutwyler, Roos (1984)
(QUARK MODEL)
$f_4 = -0.016 \pm 0.008$

f_4: the ChPT calculation...

$$f_4 = \Delta_{loops}(\mu) - \frac{8}{F_\pi^4}\left[C_{12}(\mu) + C_{34}(\mu)\right](M_K^2 - M_\pi^2)^2$$

Post and Schilcher, Bijnens and Talavera

$C_{12}(\mu)$ and $C_{34}(\mu)$ can be determined from the slope and the curvature of the scalar form factor. But experimental data are not accurate enough

... and model estimates

(μ = ???)

Leutwyler and Roos, $f_4^{LOC} = -0.016 \pm 0.008$ [Quark model]

Jamin et al., $f_4^{LOC} = -0.018 \pm 0.009$ [Dispersive analysis]

Cirigliano et al., $f_4^{LOC} = -0.002 \pm 0.008$ [1/Nc+Low resonance]

Lattice QCD: VERY CHALLENGING

A PRECISION OF O(1%) MUST BE REACHED ON THE LATTICE!!

The first Lattice QCD calculation

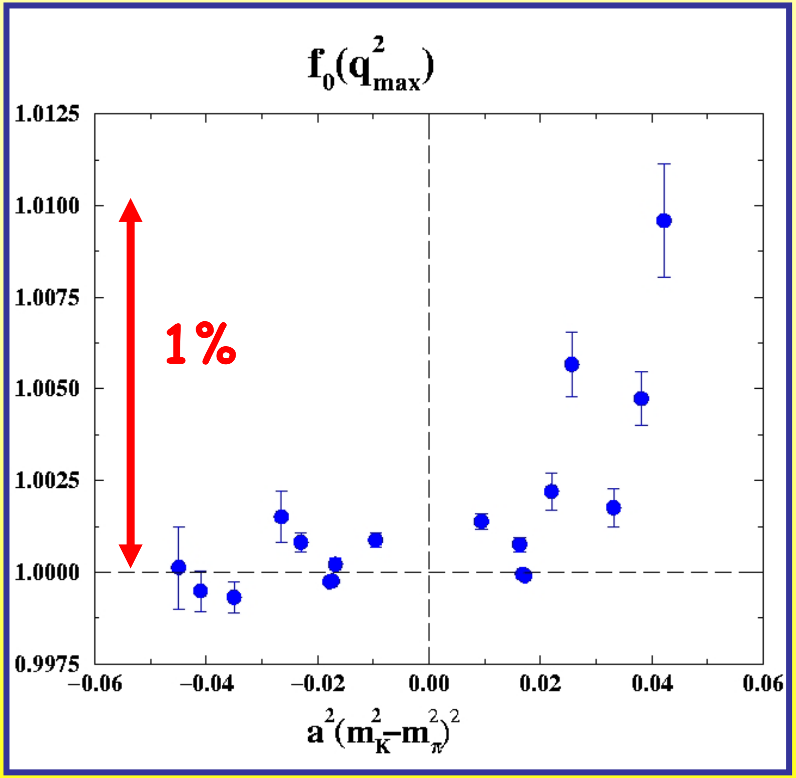
D.Becirevic, G.Isidori, V.L., G.Martinelli, F.Mescia, S.Simula, C.Tarantino, G.Villadoro. [NPB 705,339,2005]

1) Evaluation of $f_0(q^2_{MAX})$

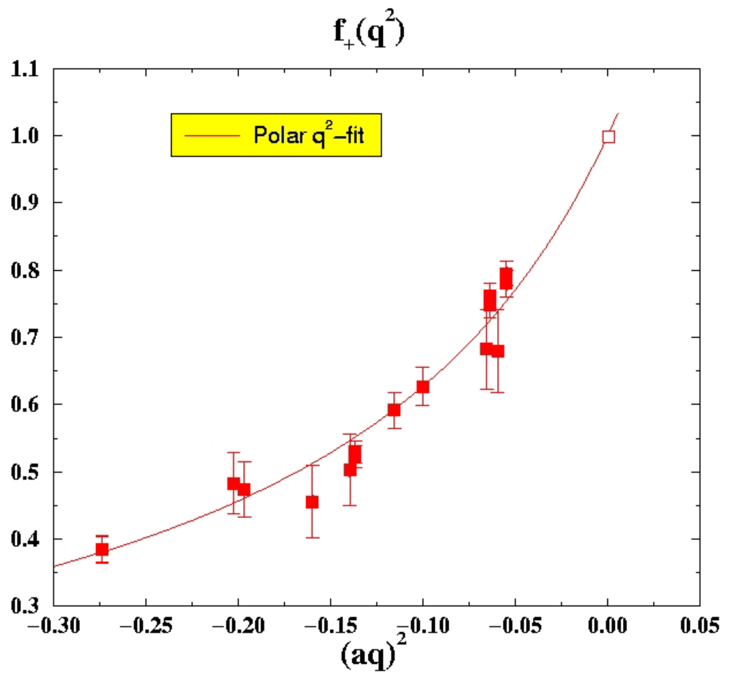
The basic ingredient is a **double ratio** of correlation functions:

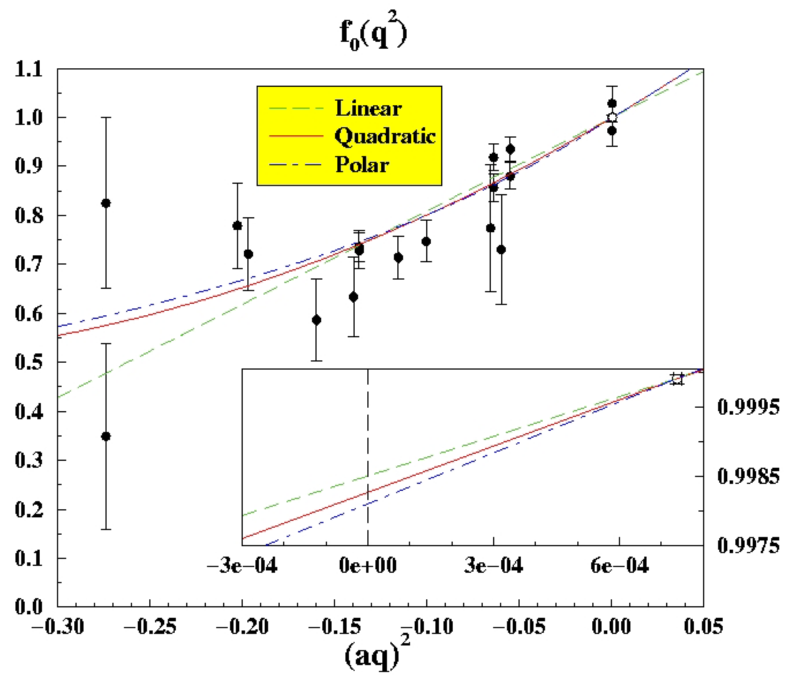
$$R = \frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle\langle K|\bar{u}\gamma_0 s|\pi\rangle}{\langle\pi|\bar{u}\gamma_0 u|\pi\rangle\langle K|\bar{s}\gamma_0 s|K\rangle}$$

$$= \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} f_0(q^2_{max})^2$$



2) Extrapolation of $f_0(q^2_{MAX})$ to $f_0(0)$





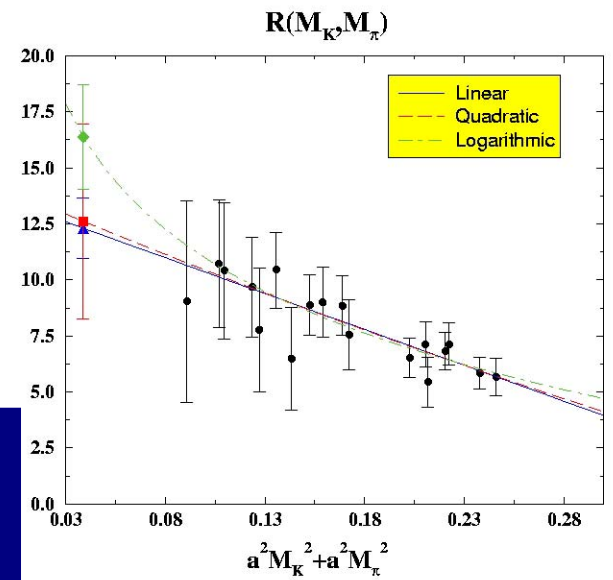
LQCD PREDICTION !!

Comparison of polar fits:

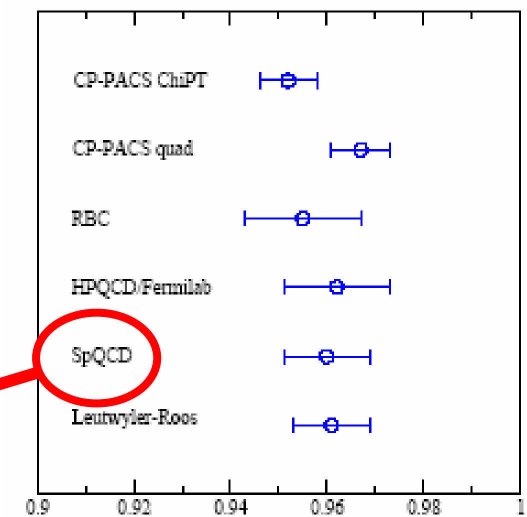
LQCD: $\lambda_+ = (25 \pm 2)\, 10^{-3}$ $\lambda_0 = (12 \pm 2)\, 10^{-3}$

KTeV: $\lambda_+ = (24.11 \pm 0.36)\, 10^{-3}$ $\lambda_0 = (13.62 \pm 0.73)\, 10^{-3}$

3) Extrapolation to the physical masses



Preliminary unquenched results have been also presented

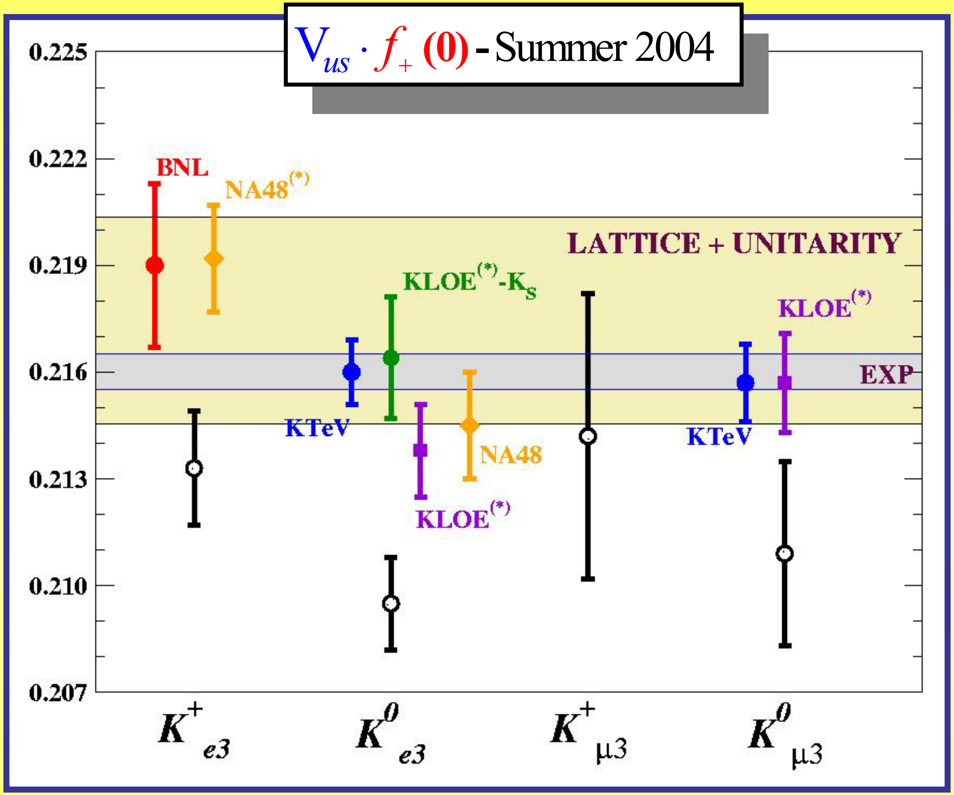


C.Dawson, plenary at Lattice 2005

$f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005 \pm 0.007$

(In agreement with LR !!)

FIRST ROW UNITARITY



$$[V_{us} \cdot f_+(0)]_{EXP} = \\ = 0.2250 \pm 0.0021$$

$$f_+(0) = 0.960 \pm 0.009$$

$$V_{us} = 0.2250 \pm 0.0021$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0007 \pm 0.0014$$

3

CKM MATRIX:

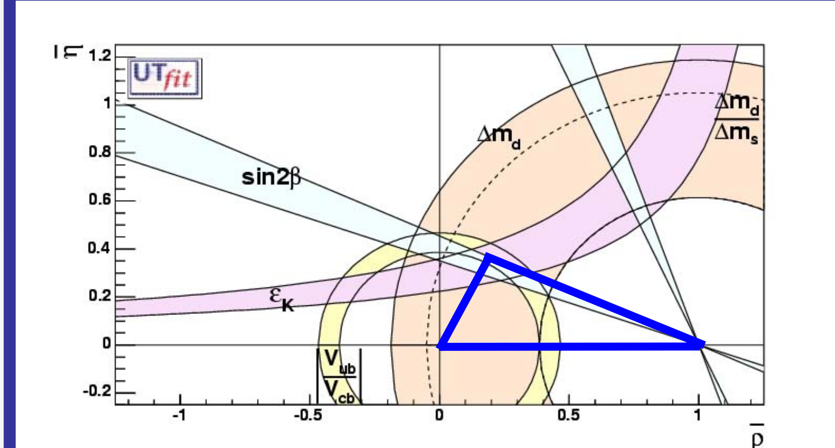
b) THE UNITARITY TRIANGLE ANALYSIS AND CP VIOLATION

THE UNITARITY TRIANGLE ANALYSIS

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CP violation



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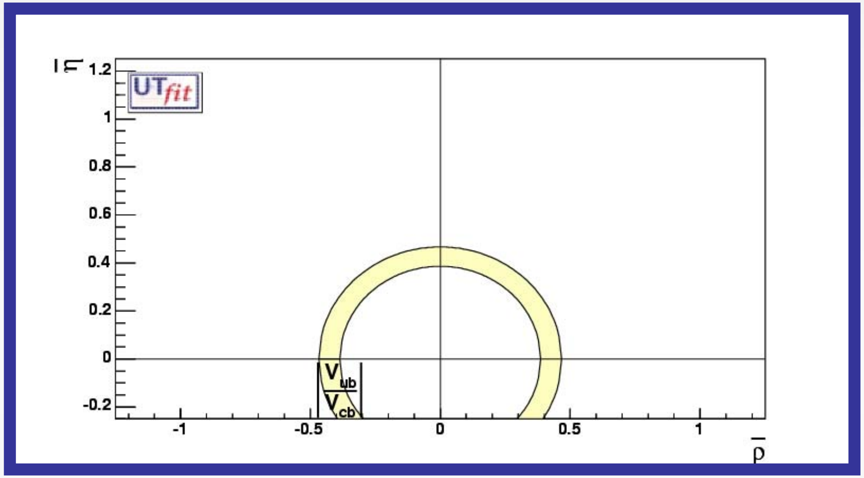
CP violation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$(b\to u)/(b\to c)$	$\bar\rho^2 + \bar\eta^2$	$f_+, F(1), \ldots$

5 CONSTRAINTS

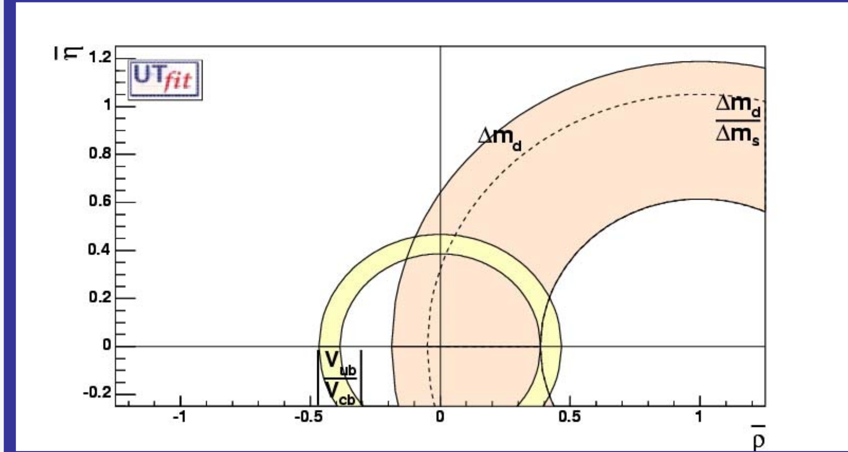
Hadronic matrix elements from LATTICE QCD

THE UNITARITY TRIANGLE ANALYSIS

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CP violation



(b→u)/(b→c)	$\bar\rho^2 + \bar\eta^2$	f_+,F(1),…
Δm_d	$(1-\bar\rho)^2 + \bar\eta^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d/\Delta m_s$	$(1-\bar\rho)^2 + \bar\eta^2$	ξ

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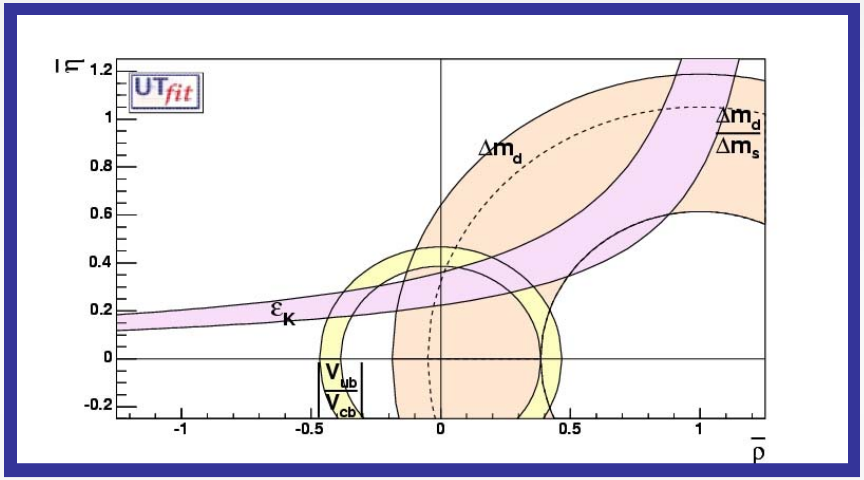


(b→u)/(b→c)	$\bar\rho^2 + \bar\eta^2$	f_+,F(1),…
Δm_d	$(1-\bar\rho)^2 + \bar\eta^2$	$f^2_{B_d} B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1-\bar\rho)^2 + \bar\eta^2$	ξ
ε_K	$\bar\eta[(1-\bar\rho) + P]$	B_K

5 CONSTRAINTS

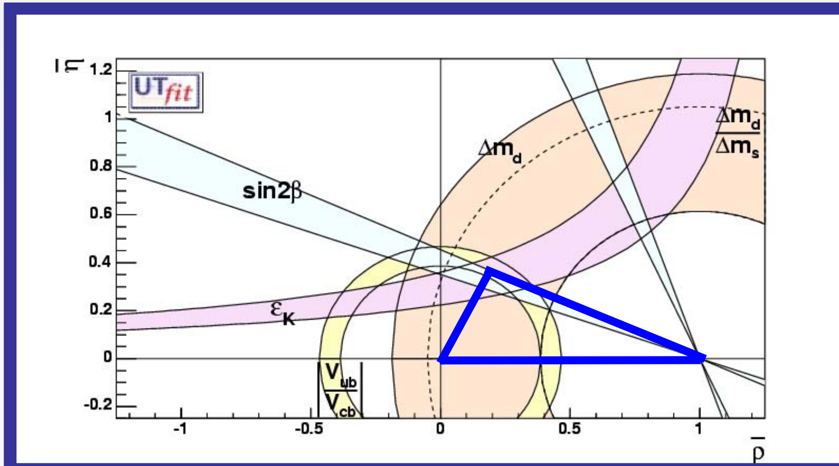
Hadronic matrix elements from LATTICE QCD

THE UNITARITY TRIANGLE ANALYSIS

$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CP violation

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

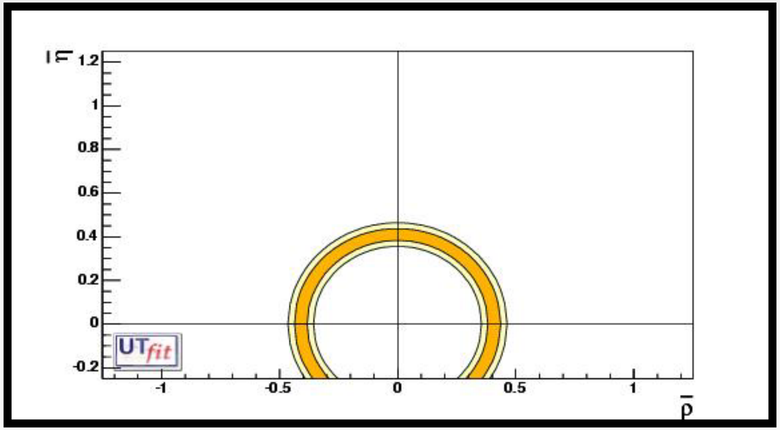


$(b\to u)/(b\to c)$	$\bar\rho^2 + \bar\eta^2$	$f_+, F(1), \ldots$
Δm_d	$(1-\bar\rho)^2 + \bar\eta^2$	$f^2_{B_d} B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1-\bar\rho)^2 + \bar\eta^2$	ξ
ε_K	$\bar\eta[(1-\bar\rho) + P]$	B_K
$A(J/\psi K_S)$	$\sin 2\beta(\bar\rho, \bar\eta)$	—

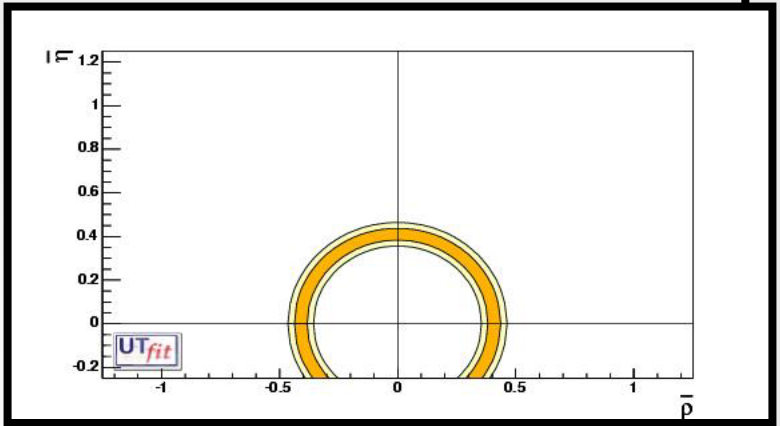
5 CONSTRAINTS

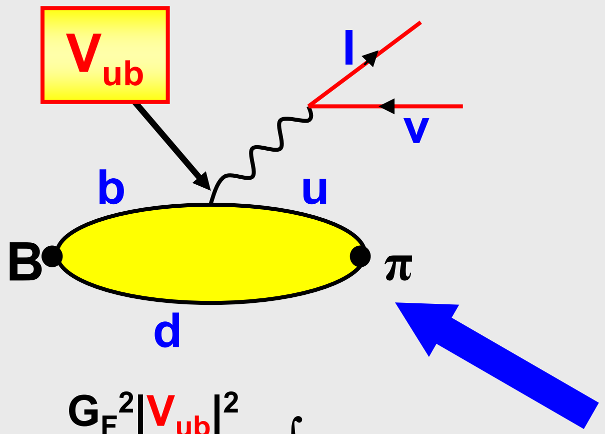
Hadronic matrix elements from LATTICE QCD

V_{ub} and V_{cb} from semileptonic decays

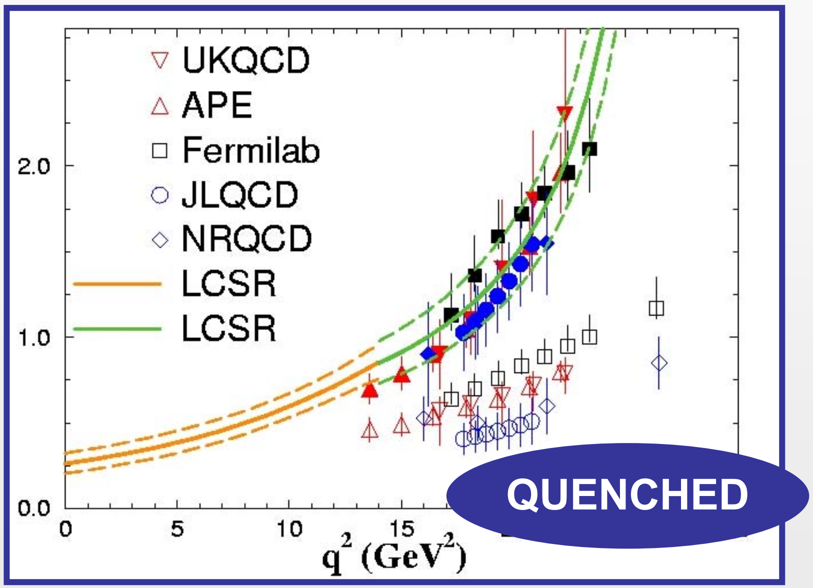


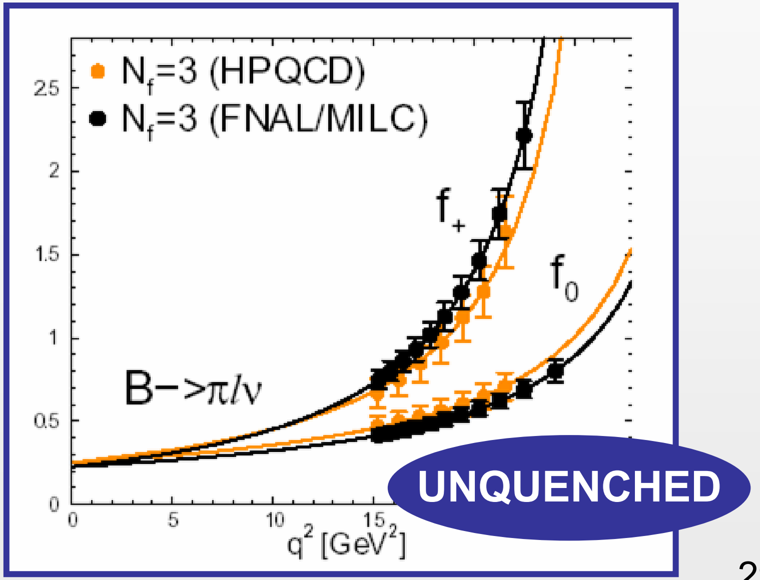
V_{ub} and V_{cb} from semileptonic decays



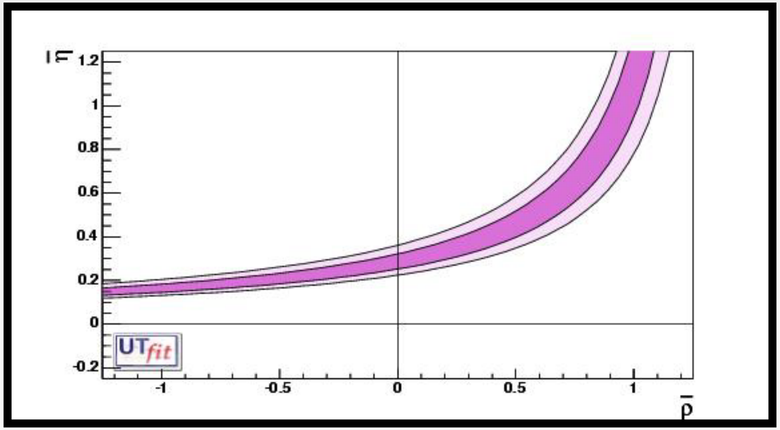


$$\Gamma(B \rightarrow \pi l \nu) = \frac{G_F^2 |V_{ub}|^2}{192\,\pi^3} \int dq^2\, \lambda(q^2)^{3/2}\, |f_+(q^2)|^2$$





$K-\bar{K}$ mixing: ε_K and B_K



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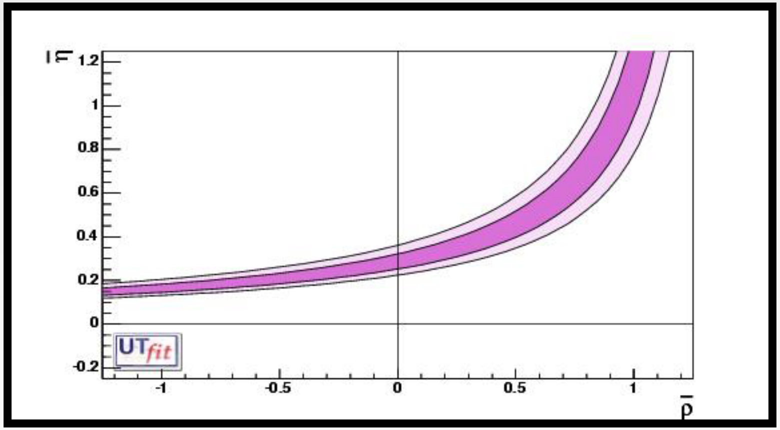
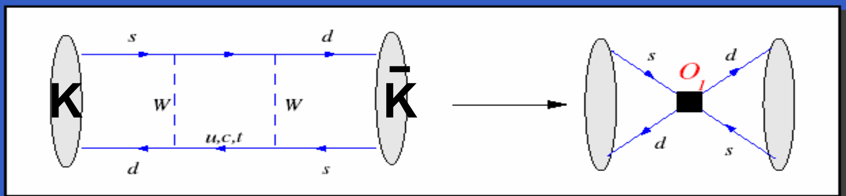


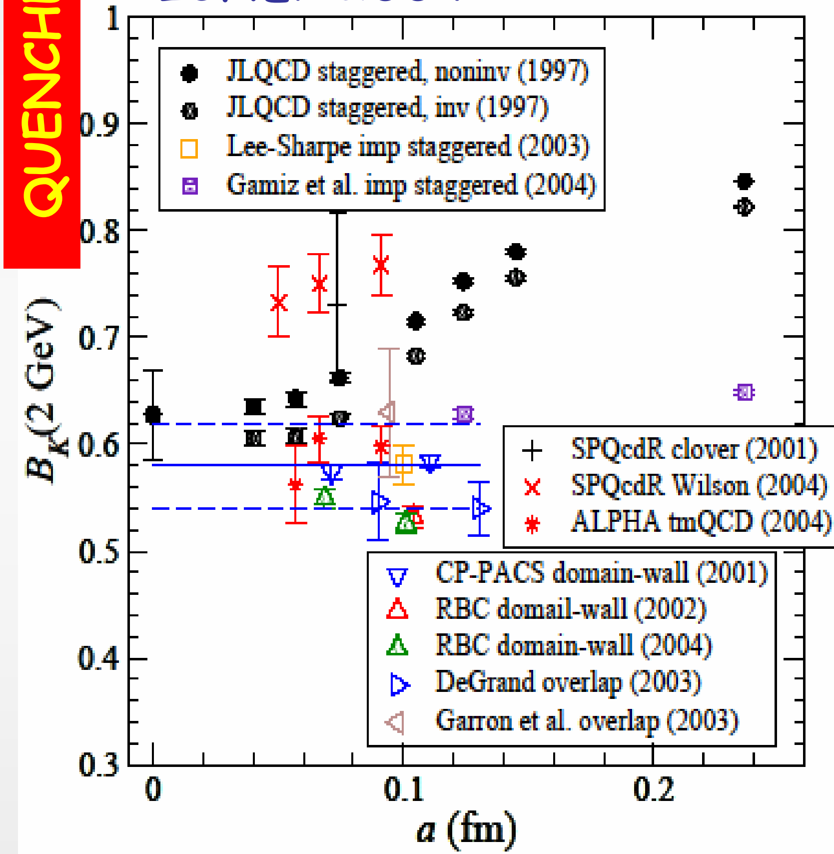


$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

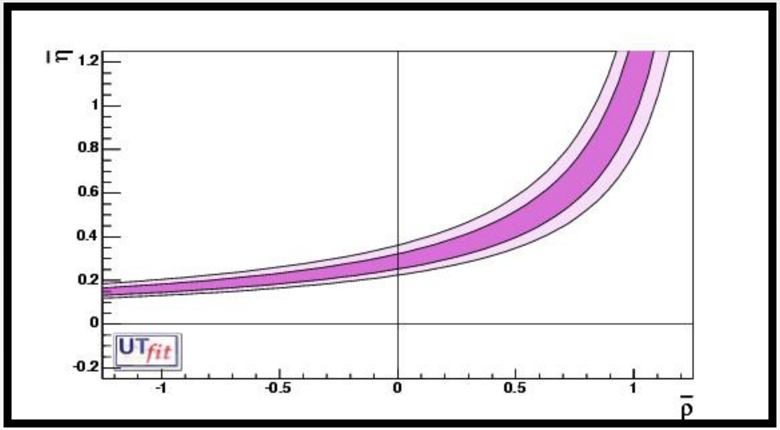
$\hat{B}_K = 0.79 \pm 0.04 \pm 0.09$

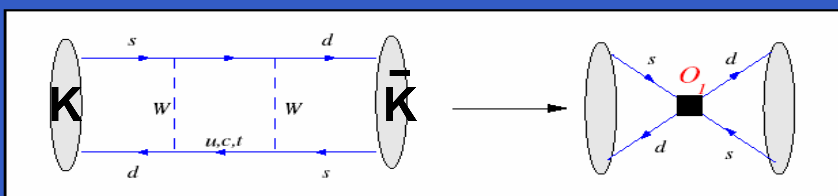
QUENCHED

From S. Hashimoto
ICHEP 2004

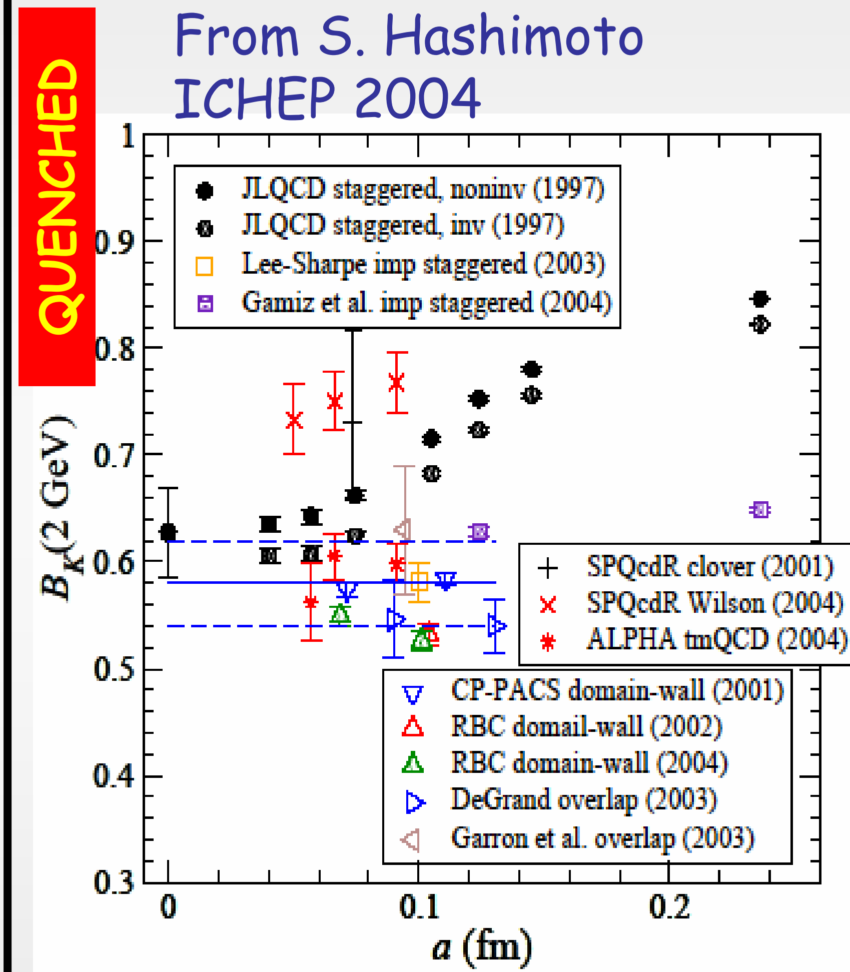


K-$\bar{K}$ mixing: ε_K and B_K





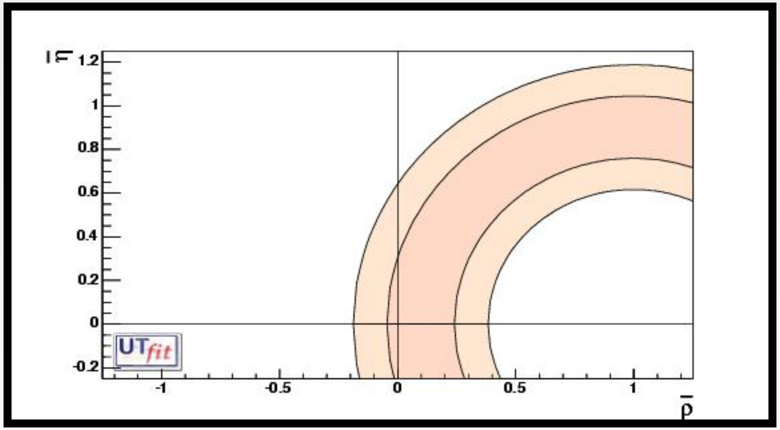
$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

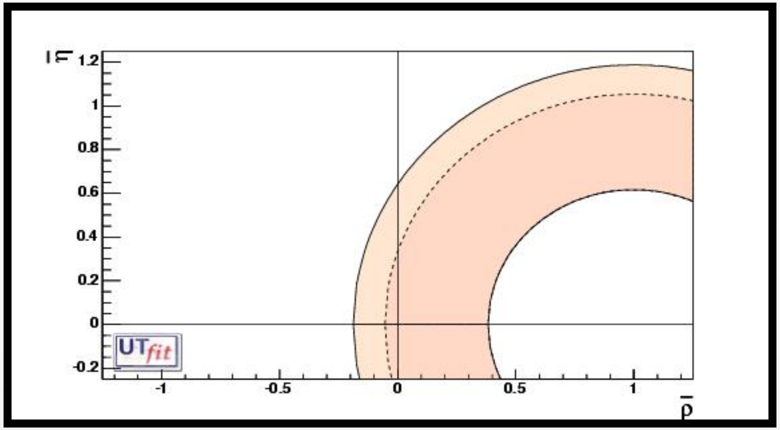


$\hat{B}_K = 0.79 \pm 0.04 \pm 0.09$

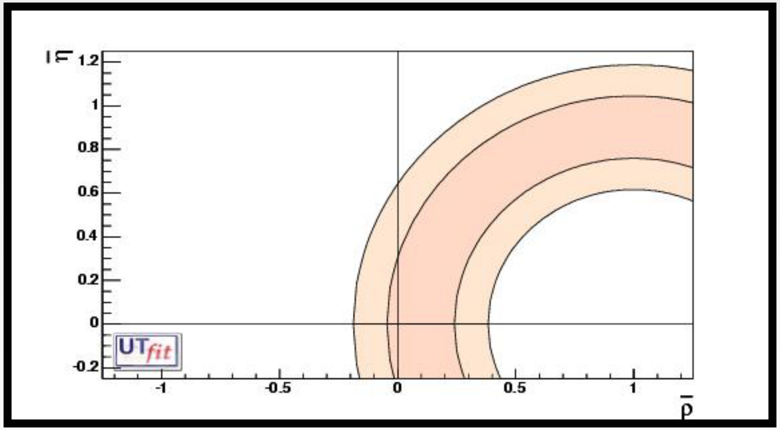
LATTICE PREDICTION (!) $\hat{B}_K = 0.90 \pm 0.20$ [Gavela et al., 1987]

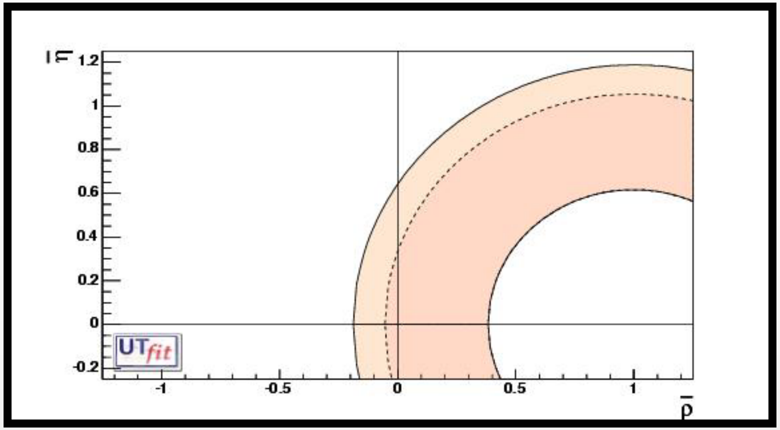
B_d and B_s mixing: $f_B\sqrt{B_B}$





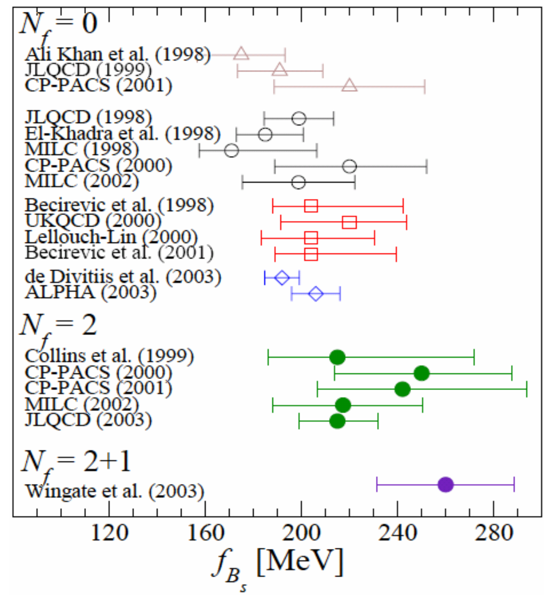
B_d and B_s mixing: $f_B\sqrt{B_B}$



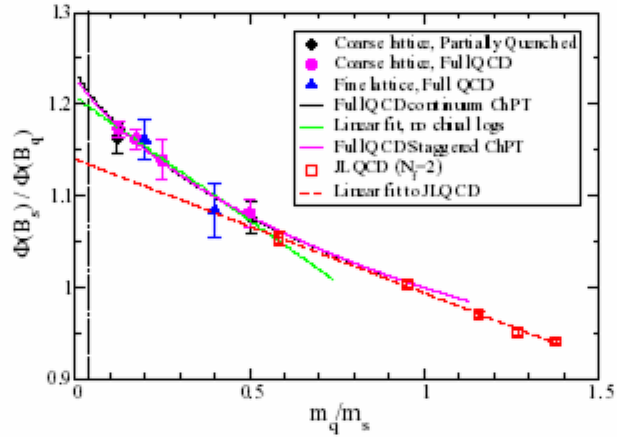


$f_{Bs}\sqrt{B_{Bs}}$ = 276 ± 38 MeV,

ξ = 1.24 ± 0.04 ± 0.06



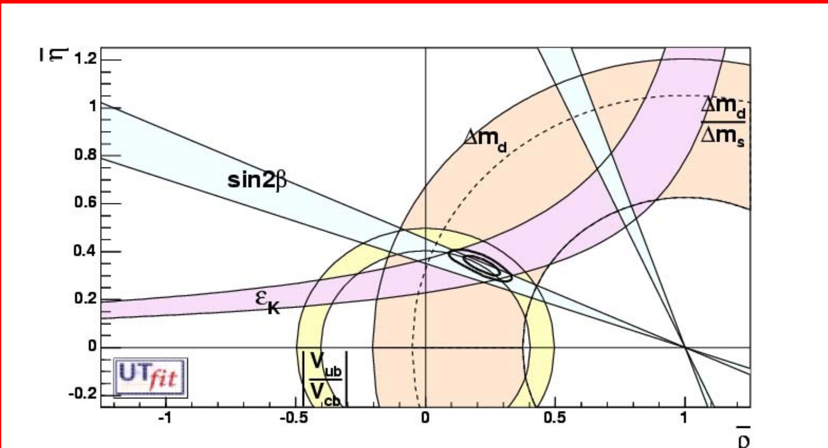
From S. Hashimoto
ICHEP 2004



UT-FIT RESULTS

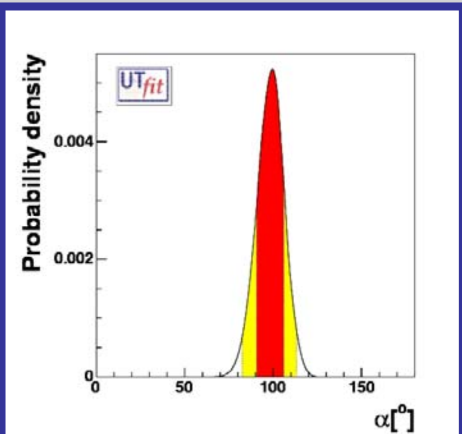
UTfit Collaboration

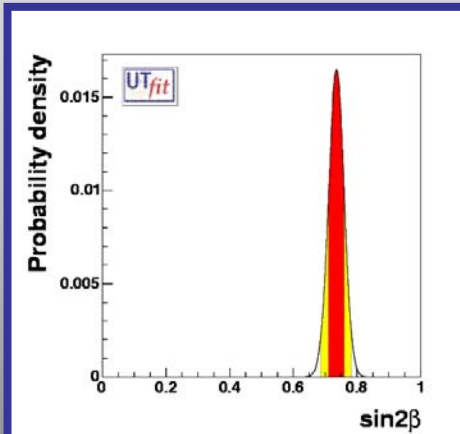
[JHEP 0507 (2005) 028]

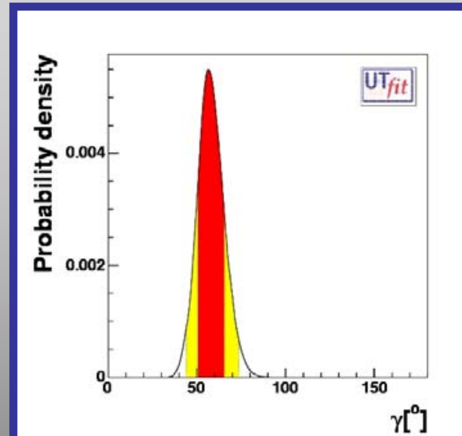


$\bar{\rho}$ = 0.214 ± 0.047

$\bar{\eta}$ = 0.343 ± 0.028







$\alpha = (98.2 \pm 7.7)^\circ$

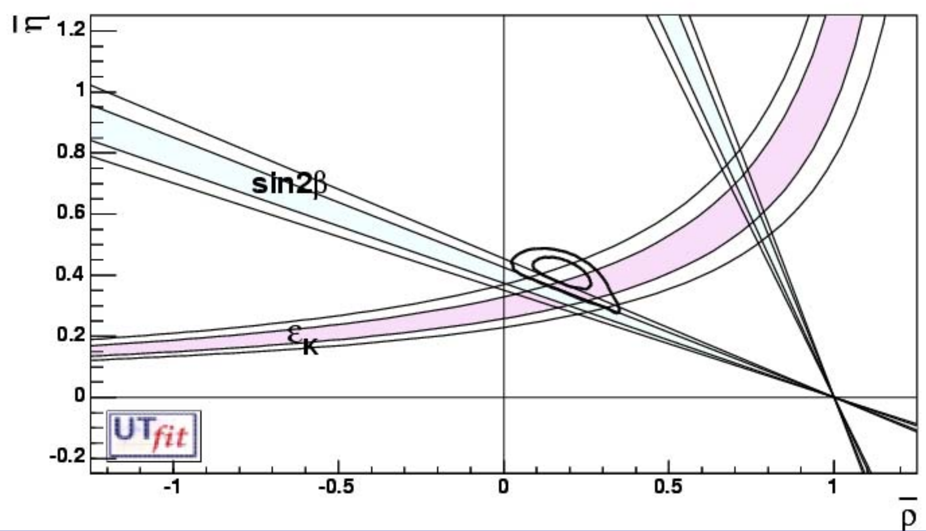
$\sin 2\beta = 0.734 \pm 0.024$

$\gamma = (57.9 \pm 7.4)^\circ$

INDIRECT EVIDENCE OF ~~CP~~
A CRUCIAL TEST OF THE SM

3 FAMILIES:

- Only 1 phase
- Angles from the sides



$\mathbf{Sin2\beta_{UT\ Sides} = 0.793 \pm 0.033}$

$\mathbf{Sin2\beta_{J/\psi\ Ks} = 0.687 \pm 0.032}$

INDIRECT EVIDENCE OF ~~CP~~
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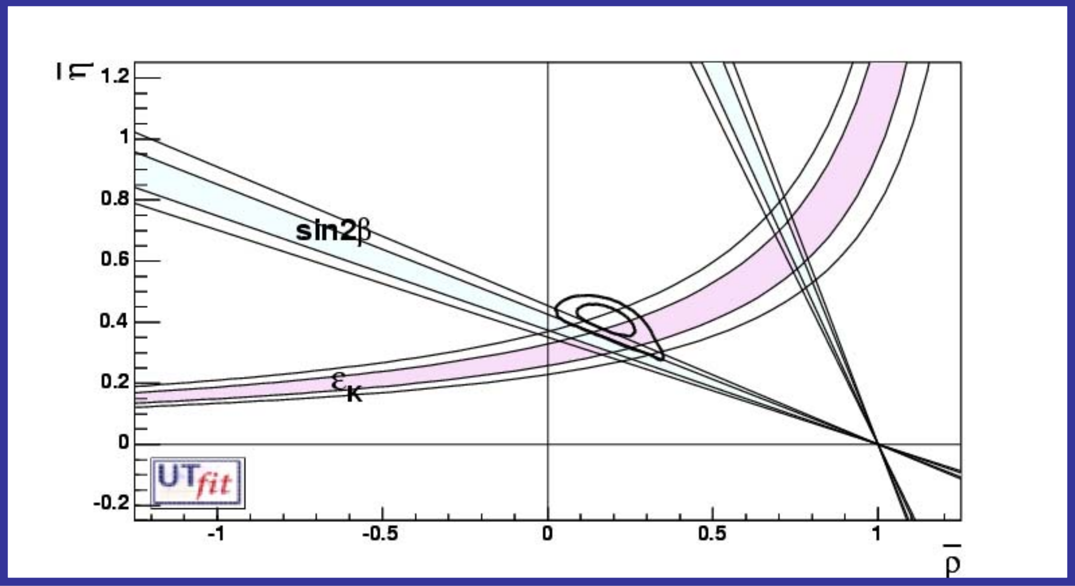
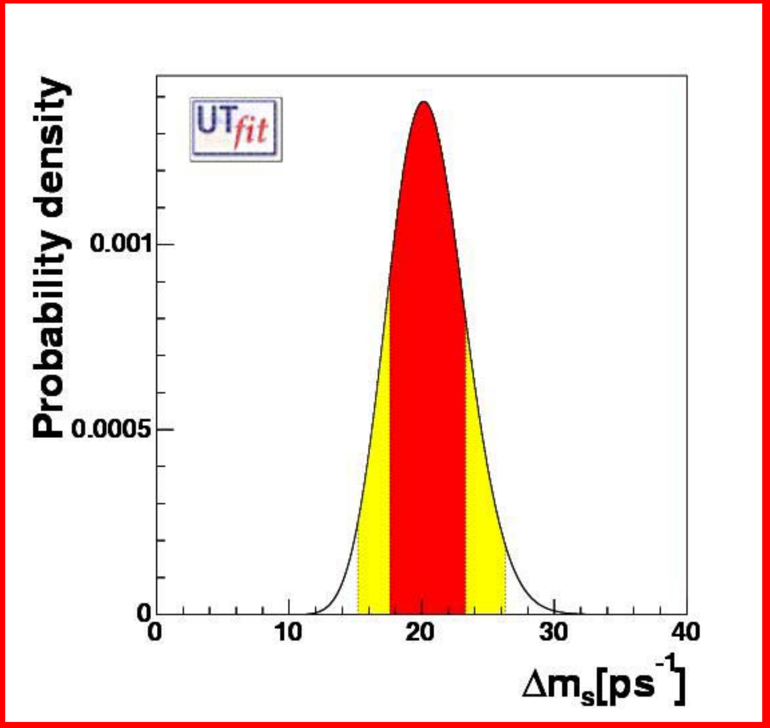


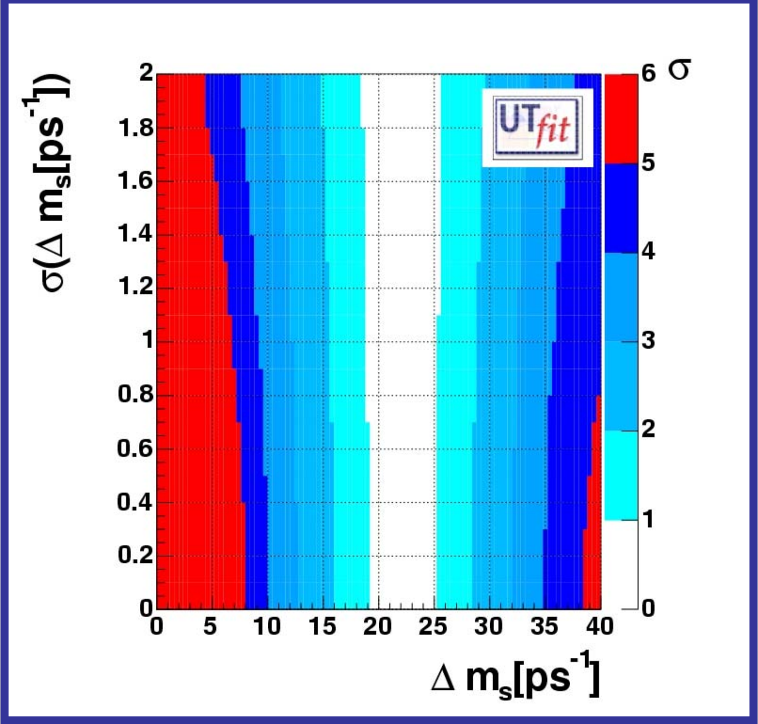
$\sin 2\beta_{\text{UT Sides}} = 0.793 \pm 0.033$

$\sin 2\beta_{J/\psi\, K_s} = 0.687 \pm 0.032$

Prediction (Ciuchini et al., 2000): **$\sin 2\beta_{\text{UTA}} = 0.698 \pm 0.066$**

PREDICTION FOR Δm_s

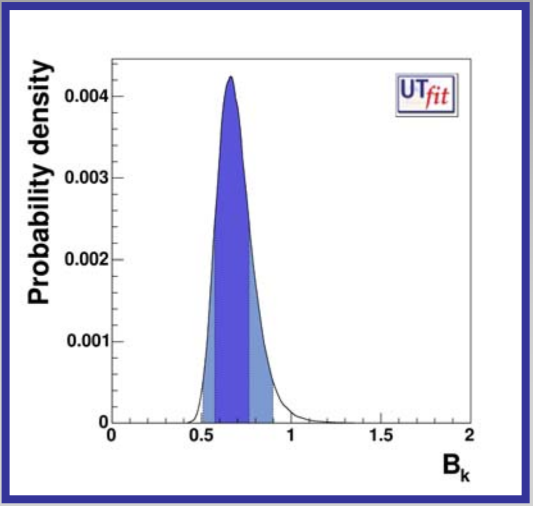


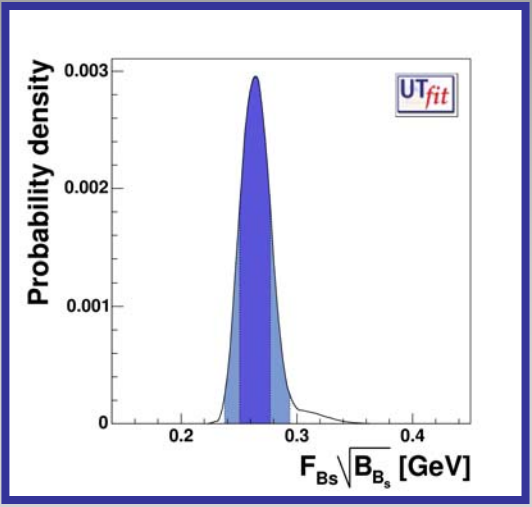


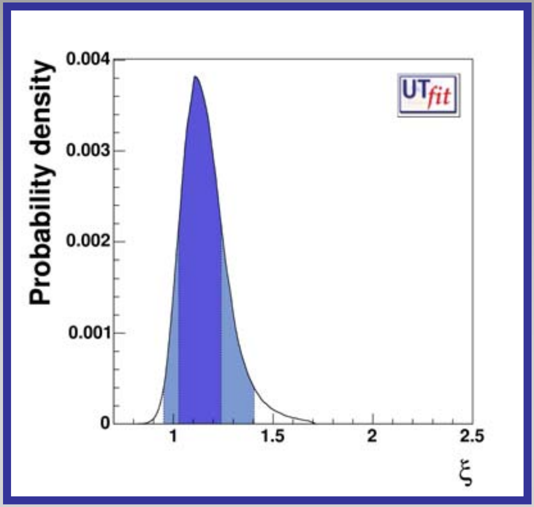
$\Delta m_s = (22.2 \pm 3.1)\ ps^{-1}$

DIRECT MEASUREMENT: $\Delta m_s > 14.5\ ps^{-1}$ @ 95% C.L.

LATTICE QCD vs UT FITS

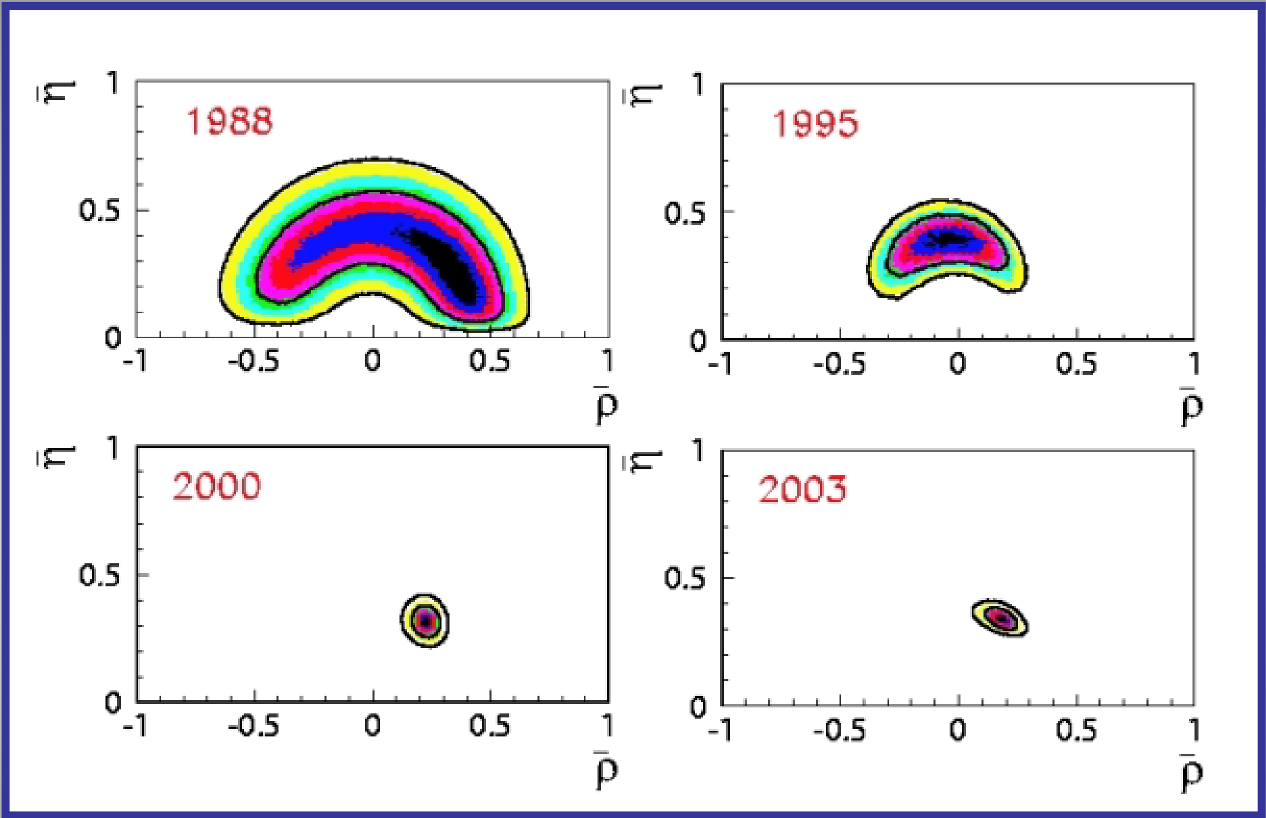






	Lattice QCD	UT Fits
$\hat{B}_K$	0.79 ± 0.04 ± 0.09	0.69 ± 0.10
$f_{Bs}\sqrt{B_{Bs}}$	276 ± 38 MeV	265 ± 13 MeV
ξ	1.24 ± 0.04 ± 0.06	1.15 ± 0.11

15 YEARS OF ($\bar{\rho}$-$\bar{\eta}$) DETERMINATIONS



Such a progress would have not been possible without LATTICE QCD calculations !!