THEORETICAL and PHENOMENOLOGICAL ASPECTS OF QCD at HIGH-ENERGY COLLIDERS

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- Particle Physics has entered the LHC era.

  The goal of the LHC:

  proton-proton collisions → NEW and Standard Model PHYSICS at the TeV energy scale

  $\sqrt{s} = 14$ TeV

- Despite the goal of new physics, the LHC is a "QCD machine"

  ⇒ accurate QCD predictions for high-multiplicity final states are mandatory

  both
  - to explore QCD in a new high-energy regime
  - to evaluate signal and background processes for new physics searches

- This talk:
  - not a general review of perturbative QCD
  - selection* of results on recent progress in the field

(* oriented towards LHC physics)

QCD HARD SCATTERING

- consider INCLUSIVE HARD scattering process

hard scale $Q = M, p_\perp, \dots \gg M_{hadron} \sim 1\,\mathrm{GeV}$

$$h_1(p_1) + h_2(p_2) \longrightarrow F(\text{"}Q\text{"}) + X$$

INCLUSIVE (unobserved final state)

final-state system of high mass M or $p_\perp$ (jets, vector bosons, heavy quarks, Higgs bosons, SUSY particles, ...)

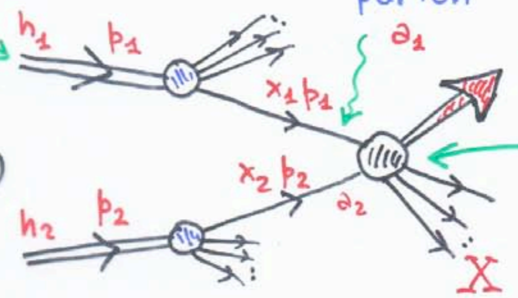
- QCD approach to hard scattering is based on

FACTORIZATION of (Theorem) $\left\{ \begin{array}{l} \text{long-distance (hadronic, } M_{had.}) \\ \text{vs.} \\ \text{short-distance (partonic, } Q \gg M_{had.}) \end{array} \right.$ physics

hadron

parton density $f_{a_1/h_1}(x_1, \mu_F^2)$

momentum fraction



parton a_1

"Q" $\sim M, p_\perp, \dots$

partonic cross section $\hat{\sigma}^{part.}_{a_1 a_2}$

$$\sigma(p_1, p_2) \sim \sum_{a_1, a_2 = q, \bar{q}, g} \int_0^1 dx_1\, dx_2\, f_1(x_1, \mu_F^2)\, f_2(x_2, \mu_F^2)\, \hat{\sigma}^{part.}_{a_1 a_2}(x_1 p_1, x_2 p_2; \mu_F^2, \mu_R^2)$$

μ_F: factorization scale — μ_R: renormalization scale

$\mu_R \sim \mu_F \sim Q$ hard scale

$\hat{\sigma}^{part.}$ computable as a perturbative series in the QCD coupling α_s

$$\hat{\sigma}^{part.} \sim \alpha_s^n \left[\hat{\sigma}^{(0)} + \alpha_s \hat{\sigma}^{(1)} + \alpha_s^2 \hat{\sigma}^{(2)} + \dots \right]$$

N.B. factorization is not exact, but corrections are $\sim O\left((M_{had}/Q)^p \right)$ power suppressed

$p = 1, 2, \dots$

- QCD predictions require :

specific (process-dependent) theor. calculations + universal (process-independent) inputs

inputs : - value of α_s
- parton densities $f_a(x, Q_0^2)$ ($a = q, \bar{q}, g$) at a scale $Q_0 \sim M_{had} \sim 1\,\text{GeV}$

alternatively, having th. calculations,
→ "inputs" can be extracted from comparison with exp. data

- nowadays,
both inputs are pretty well known!

↳ important point for present and future QCD / particle physics

THE QCD RUNNING COUPLING

the MOST important PREDICTION of (perturbative) QCD

→ ASYMPTOTIC FREEDOM

distinctive feature of non-abelian gauge theories

$$\alpha_s(Q^2) \sim \frac{1}{\beta_0 \ln \frac{Q^2}{\Lambda^2_{QCD}}}$$

QCD coupling is logarithmically small at large scales Q
$(12\pi\beta_0 = 11\, N_c - 2N_f)$

from perturbative RENORMALIZATION GROUP

QCD fundamental scale

- owing to high-precision { theory (e.g. NNLO calculations) + exps. (LEP, SLC, HERA, fixed-target DIS, ...)

in the LEP era ,

AF verified to high accuracy ($\sim 2 \div 3\%$)
over about 2 orders of magnitude

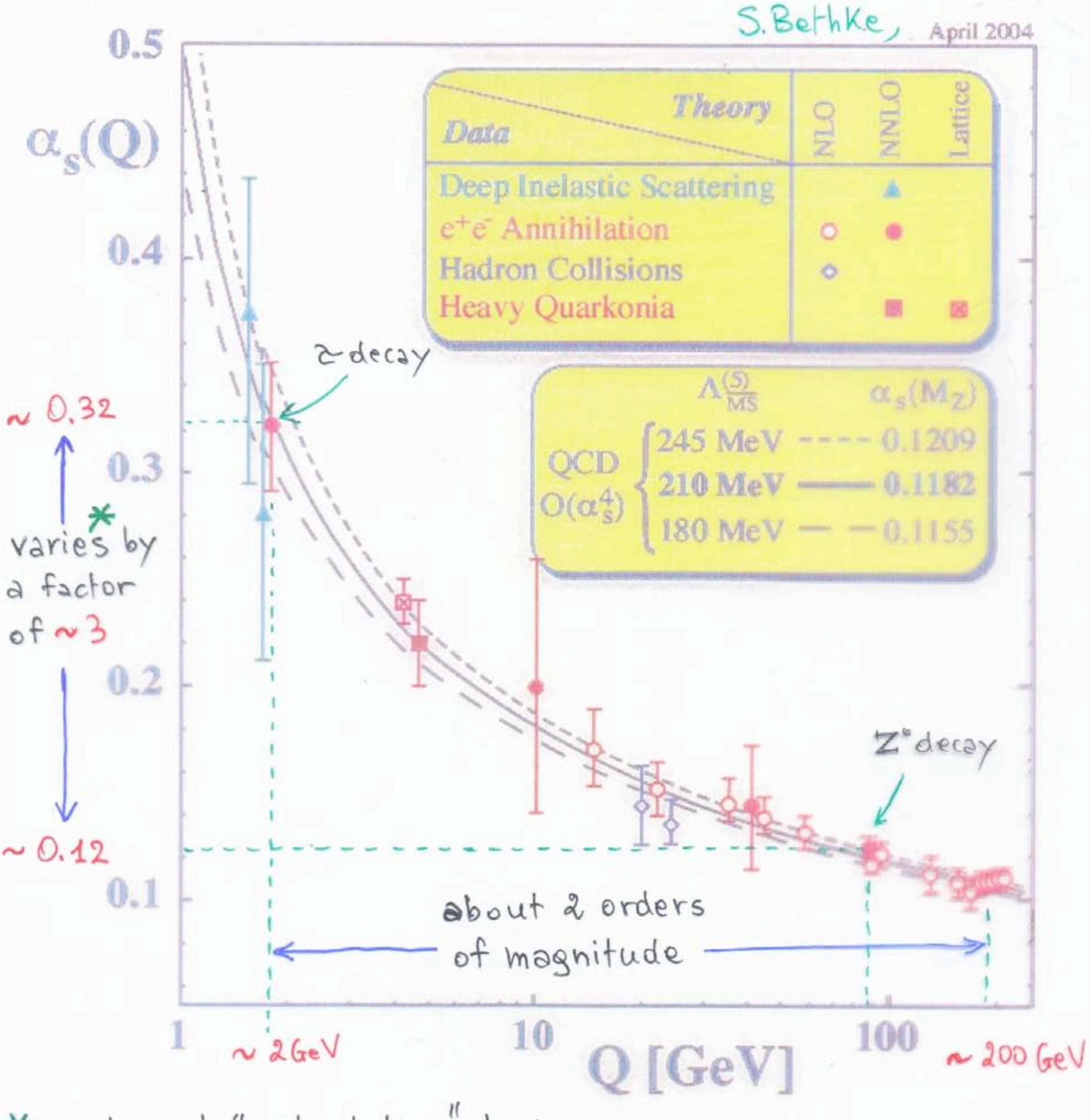
e.g. compilation by S. Bethke '04

$$\alpha_s(M_Z) = 0.118 \pm 0.003 \;(\text{th.} + \text{exp.})^*$$

[* ~2/3 of error is th.; though th. error is difficult to define/estimate because of strong/unknown correlations]

determinations of $\alpha_s(Q)$
from many different measurements
in several different processes ⇝ Fig.

Summary of $\alpha_s(Q^2)$ determinations



* non-trivial "perturbative" physics

equivalently, $\alpha_s(M_Z) \sim \dfrac{\alpha_s(M_\tau)}{1 + \underbrace{\beta_0\, \alpha_s(M_\tau) \ln M_Z^2/M_\tau^2}}$

effective expansion parameter ~ 2

PARTON DENSITY FUNCTIONS (pdf)

- $\{f_a(x,Q^2)\}_{a=q,\bar{q},g}$ describe how hadron momentum is distributed among partons at the resolution scale Q

- though pdf originate from non-perturbative dynamics, scale evolution predicted/computable in pQCD

$$\underbrace{\frac{d f_a(x,Q^2)}{d \ln Q^2}}_{\neq 0 \text{ : violation of Bjorken scaling}} = \sum_{b=q,\bar{q},g} \int_x^1 \frac{dz}{z} \underbrace{P_{ab}(\alpha_s(Q^2), z)}_{\text{AP Kernel} \sim \text{series in } \alpha_s} \; f_b\left(\frac{x}{z}, Q^2\right)$$

DGLAP evolution equations

$Q_0 \sim 1\,\mathrm{GeV}$

→ need only initial condition $f_a(x,Q_0^2)$

- determinations of pdf: $f(x,Q_0^2)$
  - in principle, computable by non-pert. methods (hadronic matrix element of non-local twist-2 operators)
  - in practice, extracted from global fit to data (DIS fixed-target + HERA, DY production, jet data at Tevatron)

MRST
CTEQ
GRV
Alekhin
ZEUS

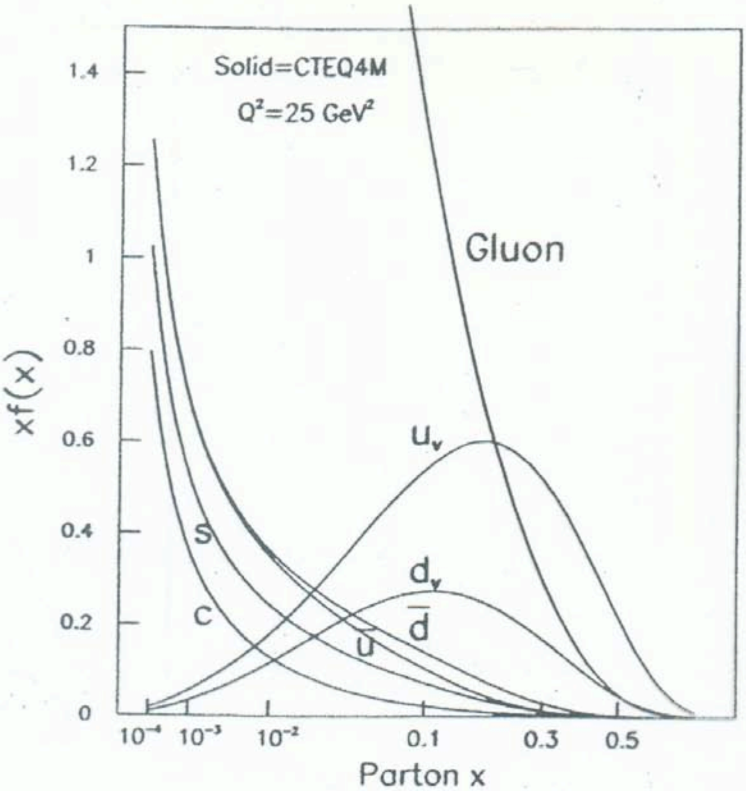
typical behaviour ⇝ Fig.

- uncertainties on pdf:
  - depend on x, Q
  - exp. error → from data; th. error → more difficult to define/quantify

very roughly speaking when $\begin{cases} x \sim 10^{-2} \div 10^{-3} \\ Q \sim 100\ \mathrm{GeV} \div 1\ \mathrm{TeV} \end{cases}$ ⇒ quark densities ~ few percent, gluon density $\lesssim 10\%$

uncertainty increases when $\begin{cases} Q: \text{smaller} \\ x: \text{smaller or larger} \end{cases}$

- typical behaviour of parton densities of the proton



- all densities decrease at large x, where valence quarks dominate

- at small x:

  valence quarks vanish

  strong rise of gluon (dominant)

  typical non-abelian feature

  self-coupling of spin 1 massless partons

  sea quarks increase (driven by gluon:

  sea $q\bar{q}$ pairs produced by gluon splitting

- quark densities are not flavour symmetric:

  $u_v \neq d_v$

  (also $\bar{u} \neq \bar{d}$)

pQCD THEORY : FIXED-ORDER EXPANSION

- perturbative computation of partonic subprocess

power series expansion in α_S

$$\hat{\sigma}^{part.}(Q^2) \sim \alpha_S^n \left[\hat{\sigma}^{(0)} + \alpha_S \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \dots \right]$$

hard-scattering scale

LO result — NLO result — NNLO result

well defined and systematic expansion

but, due to the perturbative nature of the QCD running : $\alpha_S(Q^2) \simeq \alpha_S(Q_1^2)\left[1+\beta_0 \alpha_S(Q_1^2) \ln \frac{Q_1^2}{Q^2} + \dots\right]$

$$\hat{\sigma}_{LO}^{part.}(Q^2) = \alpha_S^n(Q^2)\, \hat{\sigma}^{(0)} \sim \alpha_S^n(Q_1^2)\left[\hat{\sigma}^{(0)} + O(\alpha_S)\right]$$

performing this replacement

$$\hat{\sigma}_{LO}^{part.}(Q^2) = \alpha_S^{n}(\,?\,)\, \hat{\sigma}^{(0)}$$

LO calculation gives only order of magnitude of the cross section

- shortly, LO ⇝ only order of magnitude of $\hat{\sigma}$

NLO ⇝ "reliable" estimate of central value of $\hat{\sigma}$

NNLO ⇝ "reliable" estimate of uncertainty on $\hat{\sigma}$

e.g. $(\Delta\hat{\sigma})_{th.} = \hat{\sigma}^{NNLO} - \hat{\sigma}^{NLO}$

high-precision pQCD starts at

- present overall status :

NLO : Today's standard — non-trivial ; feasible, in practice, with general (process-independent) methods

Giele-Glover-Kosower '93
Frixione-Kunszt-Signer '96
Seymour + S.C. '97

NNLO : Today's frontier — NNLO results available only for few quantities, due to dedicated calculations

PARTON EVOLUTION at NNLO

- scale evolution of pdf controlled by

AP Kernels (splitting functions)

$$P_{ab}(\alpha_s, z) = \underset{\text{LO}}{\alpha_s P_{ab}^{(0)}(z)} + \underset{\text{NLO}}{\alpha_s^2 P_{ab}^{(1)}(z)} + \underset{\text{NNLO}}{\alpha_s^3 P_{ab}^{(2)}(z)} + \dots$$

LO: well known since early days of QCD

e.g. $P_{qq}^{(0)}(z) = C_F \frac{1+z^2}{1-z}$

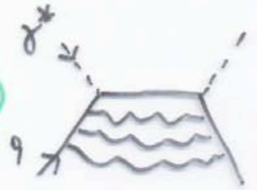
NLO: computed 25 years ago

Gonzalez-Arroyo, Lopez, Yndurain '79
Curci-Furmanski-Petronzio '80
Floratos-Kounnas-Lacaze '81

- NNLO: computation completed in Spring '04
Moch-Vermaseren-Vogt

- ~4 year project carried out with aid of dedicated computer algebra + new mathematics

- monumental computation
fully analytic
evaluation of 9607 diagrams at 3 LOOPS



⇝ Fig.

- NNLO evolution: quantitative impact

NNLO effects are

- small (< 5%) for $x \gtrsim 10^{-3}$
- increasing for smaller x
(e.g. $\delta q \sim 10\%$ at $x \sim 10^{-4}$)

⇝ Fig.

⇓

VERY GOOD CONVERGENCE OF PERTURBATIVE EXPANSION

(apart from very small x)
↳ BFKL physics
improvements possible

Ciafaloni-Colferai-Salam-Stasto
Altarelli-Ball-Forte '04

NNLO singlet splitting functions

$P_{ab}^{(2)}(z)$

$H_{-1,0}(z) = \ln z \ln(1+z) + Li_2(-z)$

$\zeta_2 = \frac{\pi^2}{6}$

$P_{qq}^{(2)}$

$P_{qg}^{(2)}$

$P_{gq}^{(2)}$

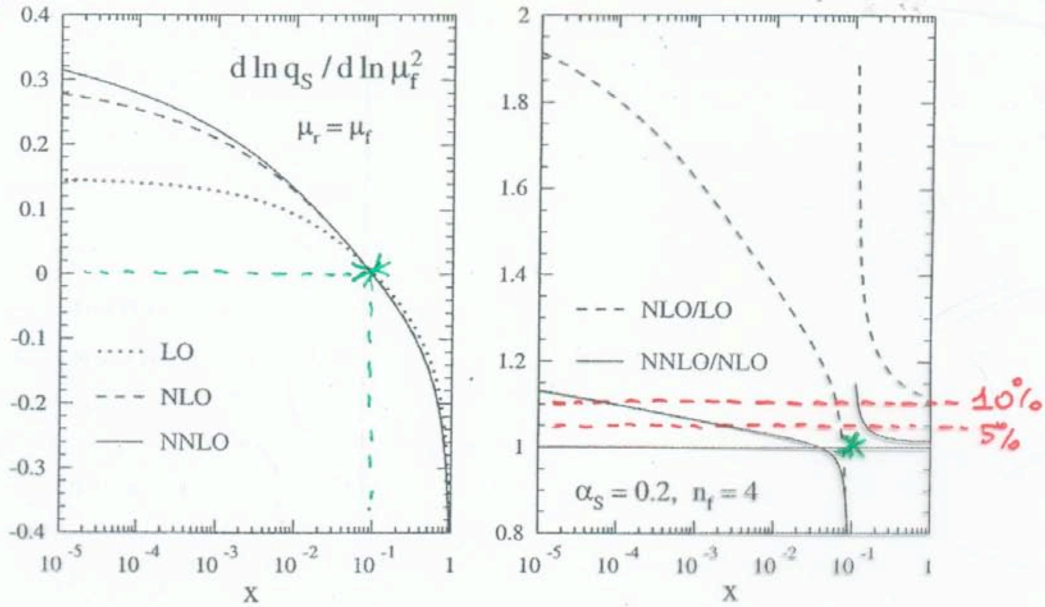
$P_{gg}^{(2)}$

- quantitative effect of NNLO Kernels (vs. LO, NLO)

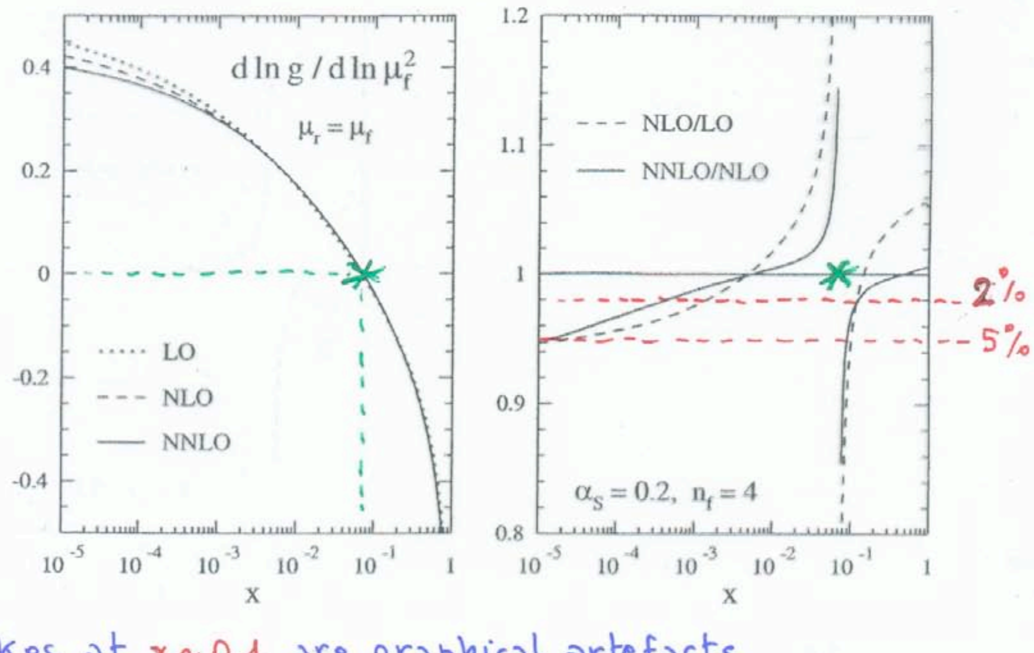
set $\mu_f^2 = 30\,\mathrm{GeV}^2$

singlet quark density $q_S(\mu_f^2) = \sum_f (q_f + \bar{q}_f)$

Moch-Vermaseren-Vogt



gluon density $g(\mu_f^2)$



✳ spikes at $x \sim 0.1$ are graphical artefacts

(simply due to $\frac{df}{d\ln\mu^2} = 0$ at LO and NLO)

- NNLO evolution: new quantum phenomena

besides its quantitative importance,
NNLO evolution reveals new physical phenomena

e.g. strange-antistrange asymmetry in the nucleon

$$S(x,Q^2) \neq \bar{s}(x,Q^2)$$

← definite (though qualitative) prediction of QCD

Rodrigo-de Florian-Vogelsang + S.C. '04

physical consequence of: pQCD charge asymmetry + valence content of the nucleon

splitting

$$q \to q' + \bar{X} \neq q \to \bar{q}' + \bar{X}$$

different flavours

new quantum effect starting from 3 loops (i.e. $O(\alpha_s^3)$)

$$\frac{d(\Delta s(Q^2))}{d \ln Q^2} = (P_{qq} - P_{\bar{q}q})\,\Delta s + (P_{q'q} - P_{\bar{q}'q})\,q_v \neq 0$$

even if $\Delta s = 0$

$\Delta s \equiv s - \bar{s}$

$q_v \equiv (u - \bar{u}) + (d - \bar{d})$

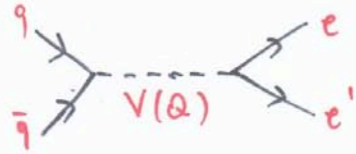
(quantitatively, $\frac{\Delta s}{s} \sim \alpha_s^3 \frac{q_v}{s} \sim$ few percent)

NNLO CROSS SECTIONS : DY PROCESS

- hadroproduction of lepton pairs with high invariant mass Q : $h_1 + h_2 \to ee'(Q) + X$

from decay of vector boson $V = \gamma^*, Z^0, W^\pm$

partonic subprocess

LO 

Drell-Yan mechanism
$q\bar{q}$ annihilation

- NNLO (partonic) cross section known

total cross section σ — Hamberg-Van Neerven-Matsuura '91, Harlander-Kilgore '02

rapidity distribution $d\sigma/dy$ — Anastasiou-Dixon-Melnikov-Petriello '03

→ Fig.

- W and Z production

typical theory uncertainty	from NNLO	from different pdf
Tevatron	$\pm 3 \div 4\%$	$\pm 2 \div 5\%$
LHC	$\pm 1 \div 2\%$	$\pm 2 \div 5\%$

→ good candidate as (parton) luminosity monitor at the LHC

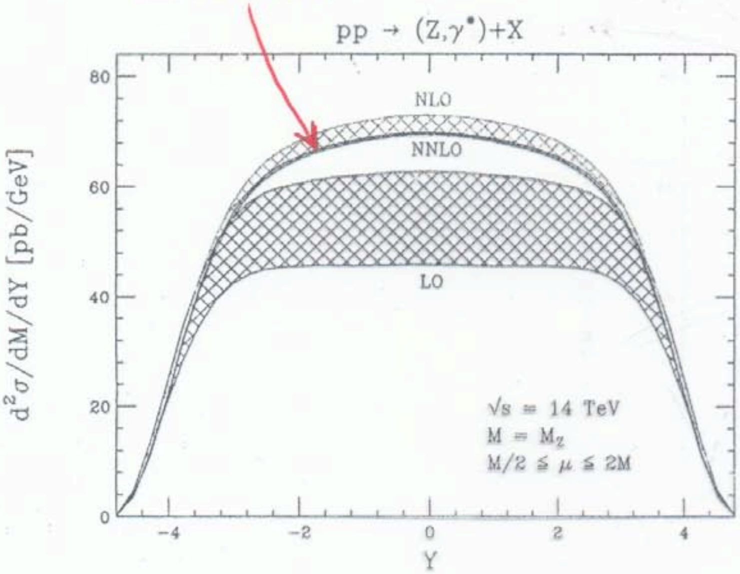
Dittmar-Pauss-Zürcher '97

- Z production at the LHC: rapidity distribution

bands from scale variations
$\frac{M_Z}{2} < \mu_R = \mu_F < 2M_Z$

Anastasiou, Dixon, Melnikov, Petriello

NNLO band



(i) scale dependence at NNLO ~ ±1%

(ii) differences NNLO vs. NLO ~ ± 1÷2 %

(iii) NLO vs. NNLO: overlapping bands

⇓

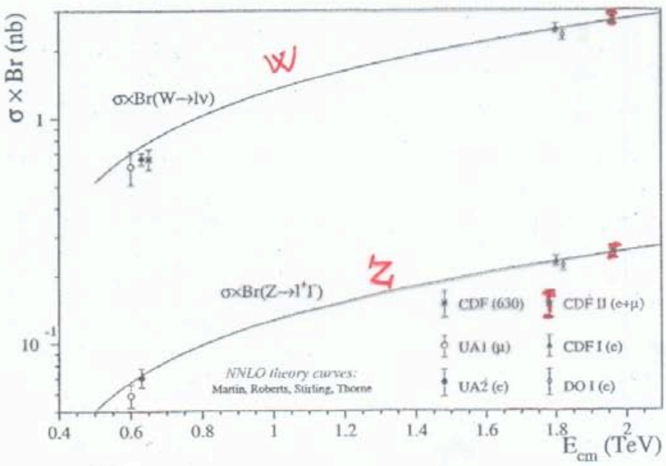
NICE CONVERGENCE OF PERTURBATIVE EXPANSION

- W and Z production: comparison
hadron collider data ⟷ NNLO predictions

MRST 2002 pdf

$Sp\bar{p}S$
Tevatron (630 GeV)
Tevatron Run I (1.8 TeV)
" Run II (1.96 TeV)

NICE AGREEMENT



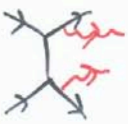
QCD at NNLO: GENERAL METHODS

- several groups are actively working to develop general (process-independent) methods to compute physical (IR finite) σ at NNLO
(combine analytically/numerically REAL radiation ⊕ VIRTUAL 2-loop terms)

separately IR divergent

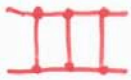
Anastasiou-Melnikov-Petriello
De Ridder-Gehrmann-Glover
Heinrich-Binoth
Kosower, Weinzierl
Frixione-Grazzini
Del Duca-Somogyi-Trocsanyi

e.g.

$[\sigma(2\to 2\,\text{jets})]_{NNLO} \sim \int$  $+\int$  $+\int$ 

phase space integral — DOUBLE REAL — REAL 1-loop — 2-loop VIRTUAL

- amplitudes a 2 LOOPS:
  - no amplitude with 4 (or more) external legs was known before 2000
  - breakthrough: evaluation of planar and non-planar double boxes

 Smirnov '99

 Tausk '99

triggered intense activity + fast amazing progress

all 4-leg amplitudes computed

[3 massless + 1 massless/massive]

q, g, γ — γ^*, W, Z, H

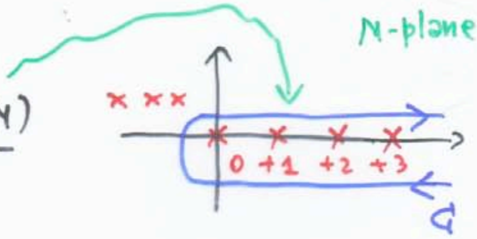
Bern-Dixon-Ghinculov '01
Anastasiou-Glover-Oleari-Tejeda Yeomans
Bern-Freitas-Dixon
Garland-Gehrmann-Glover-Koukoutsakis-Remiddi '02
Moch-Uwer-Weinzierl

- to evaluate multileg 2-LOOP amplitudes: new mathematics has been 'rediscovered', invented, developed

examples:

(i) Mellin-Barnes representation

$$\frac{1}{(x+y)^\nu} = \int_C \frac{dN}{2\pi i} \frac{y^N}{x^{\nu+N}} \frac{\Gamma(\nu+N)\Gamma(-N)}{\Gamma(\nu)}$$



- introduced by Smirnov to evaluate (ε expansion, $d=4-2\varepsilon$ space-time dimensions) 2-loop Feynman integrals
- related to Sommerfeld-Watson representation used in old times (partial wave expansion, Regge pole physics)

(ii) Harmonic PolyLogarithms (and generalizations)

$H_{m_1, m_2, \ldots, m_k}(x)$ $\quad$ $m_i = 0, \pm 1$ index
$\quad$ k: weight

- basis of functions needed to express results of 2-loop integrals (their algebra, $H \cdot H = \Sigma H$, fully studied)

Remiddi-Vermaseren '99
Gehrmann-Remiddi '01

- generalization (EXTENSION, if weight $k \geq 4$) of logarithms, polylogarithms ($Li_2(x), \ldots, Li_n(x), \ldots$), Nielsen polylogarithms $S_{n,p}(x)$
- related to "Multiple Polylogarithms," independently introduced by mathematicians

Goncharov '98

pQCD THEORY : SOFT-GLUON RESUMMATION

recall : perturbation theory at fixed order (LO, NLO, NNLO, ..) is not always sufficient / reliable

- hard processes are often multiscale processes :

  several hard scales
  $Q_1, Q_2, \ldots \gg \Lambda_{QCD}$

  e.g. $\begin{cases} Q_1 = M \\ Q_2 = Q_\perp \end{cases}$ of $W^\pm$, Z, Higgs, ...

when hard scales are very different
$Q_1 \gg Q_2$
$L \equiv \ln \frac{Q_1^2}{Q_2^2} \gg 1$ $\longrightarrow$ $\frac{\hat{\sigma}^{(n)}}{\hat{\sigma}^{(0)}} \underset{L\gg 1}{\sim} \alpha_s^n L^{2n}$

large double logs (due to soft gluons) spoil convergence of f.o. expansion
($\alpha_s L^2 \sim O(1)$ if $L \gg 1$)

$\Downarrow$

perturbative summation to all orders is necessary

- resummation studies started in the eighties

Dokshitzer, Dyakonov, Troian, Parisi, Petronzio, Amati, Bassetto, Ciafaloni, Marchesini, Veneziano, Curci, Greco, Srivastava, Mueller, Collins, Soper, Sterman, kodaira, Trentadue, Stirling, Webber, S.C., ...
⋮

in many cases, double logs exponentiate

(because of gauge invariance, unitarity, inclusiveness)

$$\hat{\sigma} \sim \hat{\sigma}^{(0)} \exp\left\{ L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \dots \right\}$$

$$= \hat{\sigma}^{(0)} \exp\left\{ \sum_{n=1}^{+\infty} \left[\underbrace{A_n \alpha_s^n L^{n+1}}_{LL} + \underbrace{B_n \alpha_s^n L^n}_{NLL} + \underbrace{C_n \alpha_s^n L^{n-1}}_{NNLL} + \dots \right] \right\}$$

at fixed $\alpha_s L$: systematic expansion as in f.o.

$O(\alpha_s)$ $O(\alpha_s)$

LL (leading logs)

NLL (next-to-leading logs)

NNLL

Today's standard

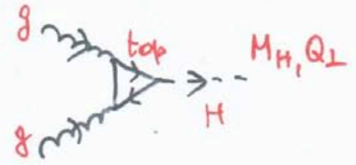
feasible with general methods:
Kidonakis, Oderda, Sterman, Laenen '97
Bonciani, Mangano, Nason, + S.C. '98
Banfi, Salam, Zanderighi '02
Marchesini, Dokshitzer '05

presently available only in few cases; needed for high-precision QCD

NNLL CROSS SECTIONS : HIGGS PRODUCTION

- production of Standard Model Higgs boson at hadron colliders

partonic subprocess

LO 

consider gluon fusion: dominant* production mechanism at the Tevatron and the LHC

(*other production mechanisms, with high $S/\sqrt{B}$ ratio, are also relevant for Higgs boson search)

- NNLO results available for σ_{TOT} (including h.o. soft-gluon effects) and rapidity distribution

Harlander-Kilgore '02
Anastasiou-Melnikov '02
Ravindran-Smith-Van Neerven '03
de Florian-Grazzini-Nason+S.C. '03
Anastasiou-Melnikov-Petriello '04

- $Q_\perp$ distribution:

when $Q_\perp \ll M_H$, resummation of large logs $L = \ln M_H^2/Q_\perp^2$ is needed (mandatory):

presently, NNLL + NLO
(small $Q_\perp$) (large $Q_\perp$)

Bozzi-deFlorian-Grazzini+S.C. '03

$$\left[\text{N.B.} \int_0^\infty dQ_\perp \left(\frac{d\sigma}{dQ_\perp}\right)_{NNLL+NLO} = (\sigma_{TOT})_{NNLO} \right]$$

$\rightsquigarrow$ Fig.

$\Rightarrow$ quantitative results:

(i) well-behaved results as $Q_\perp \to 0$;

(ii) stable perturbative predictions, with uniform accuracy ($\sim \pm 10\%$) from small to intermediate $Q_\perp$

(~ 10 GeV) (~ 80 GeV)

– Higgs boson $Q_\perp$-spectrum at the LHC

bands: $\frac{M_H}{2} \le \mu_R, \mu_F \le 2M_H$

Bozzi-deFlorian-Grazzini + S.C.

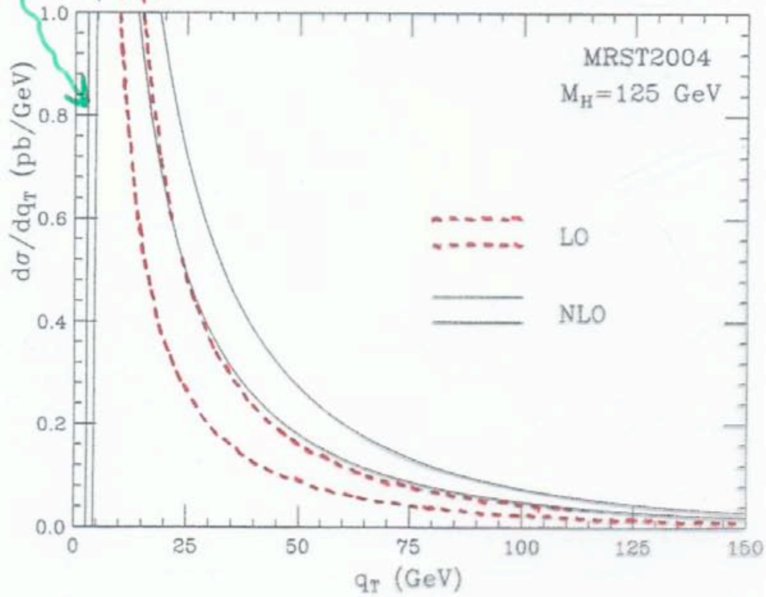
NLO → $-\infty$
$Q_\perp \to 0$

LO → $+\infty$
$Q_\perp \to 0$

NLO: unphysical peak due to compensation between dominant and subdominant logs

A) fixed-order expansion ($Q_\perp \neq 0$)

- small $Q_\perp$:
  LO, NLO, ...
  f.o. divergences as $Q_\perp \to 0$; removed after resummation
- intermediate $Q_\perp$:
  poor convergence (bands do not overlap)



B) resummed expansion

NNLL vs. NLL

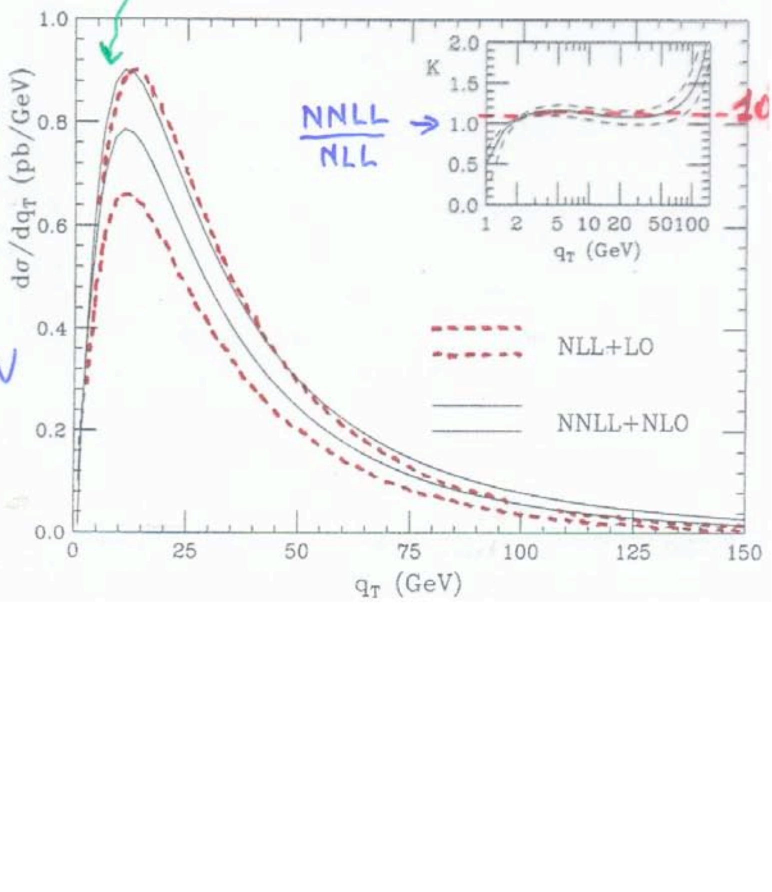
- scale dependence:
  NLL ~ ±15% → ±20%
  (peak) ($Q_\perp \sim 100$ GeV)
  NNLL ~ ±8% → ±20%
  ↓
  smaller at NNLL
- overlapping bands when $10 \lesssim Q_\perp \lesssim 80$ GeV

⇓

stability of resummed (expansion) predictions

- physical behaviour when $Q_\perp \to 0$
- NNLL slightly harder than NLL

$\frac{NNLL}{NLL}$ → 1.0



NEW METHODS IN QCD AND PERTURBATIVE GAUGE THEORIES

- a great deal of recent activity triggered by Witten's paper on "topological string theory in twistor space"

hep-th/0312171

- new interpretation of scattering amplitudes in twistor space
- proposal of weak-weak duality between a (topological) string theory and QCD ($N=4$ SUSY YM)

2 years and a half

- ~ 160 papers so far (rate: ~2 papers per week)
  (only ~40 by "QCD people")
- many new results

→ would deserves a fully dedicated talk

→ in the following,
a very brief "QCD-ist" overview of the field

See
introductory lecture notes
Cachazo-Svrček
hep-th/0504194

GAUGE THEORY AMPLITUDES

- naively, very cumbersome expressions of their

degrees of freedom $\begin{cases} \text{colours} & a_i \\ \text{momenta} & p_i^\mu \\ \text{spins (helicities)} & h_i = \pm 1 \end{cases}$

↳ expressions simplify by using "right variables":

colour decomposition$^{(1)}$ + helicity basis$^{(2)}$
(strip off colour)

(1) e.g. n gluon, tree level — Mangano-Parke-Xu '88

$$M_n^{\text{tree}}(\{p_i, h_i, a_i\}) \sim \sum_{\substack{\text{non-cyclic} \\ \text{perms.}}} \underbrace{\mathrm{Tr}\left(t^{a_1} \cdots t^{a_n}\right)}_{\substack{\text{Chan-Paton} \\ \text{factor in} \\ \text{open string} \\ \text{theory}}} A_n^{\text{tree}}(p_1, h_1; \ldots; p_n, h_n)$$

$i = 1, \ldots, n$ $\quad a_i$

t^a_{ij} : $SU(N_c)$ generators in fundamental representation

colour subamplitude (depends on momenta and helicities)

(2) spinor helicity formalism

Xu-Zhang-Chang '87
Berends-Kleiss-Causmaecker-Gastmans-Wu '81
Gunion-Kunszt '85

use Dirac (Weyl) spinors

$$\boxed{\begin{array}{ll} \text{right-handed} & u_\alpha^{(+)}(p_i) \equiv (\lambda_i)_\alpha \\ \text{left-handed} & u_{\dot\alpha}^{(-)}(p_i) \equiv (\tilde\lambda_i)_{\dot\alpha} \end{array}} \quad \begin{array}{l} \alpha = 1,2 \\ \dot\alpha = 1,2 \end{array}$$

gluon (physical) polarizations

$$\varepsilon_\mu^{(\pm)}(p_i) = \pm \frac{\bar u^{(\mp)}(n)\, \gamma_\mu\, u^{(\mp)}(p_i)}{\sqrt{2}\, \bar u^{(\mp)}(n)\, u^{(\pm)}(p_i)}$$

n^μ ($n^2 = 0$): reference vector (arbitrary, because of gauge invariance)

- eventually, results fully expressed in terms of

"angle-bracket" $\langle ij\rangle \equiv \bar{u}^{(-)}(p_i)\, u^{(+)}(p_j) = \varepsilon^{\alpha\beta}(\lambda_i)_\alpha (\lambda_j)_\beta$

"square-bracket" $[ij] \equiv \bar{u}^{(+)}(p_i)\, u^{(-)}(p_j) = \varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta}$

$$\left(\left.\begin{matrix}\langle ij\rangle \\ [ij]\end{matrix}\right\} = \sqrt{2p_i\cdot p_j}\left\{\begin{matrix} e^{i\phi_{ij}} \\ e^{-i(\phi_{ij}+\pi)}\end{matrix}\right. \right)$$

i.e. helicity $\sim \pm\sqrt{\text{momentum}}$

$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ = "vector rep."

- remarkable simplifications (tree level):

$A_n(\overbrace{1^+,2^+,\ldots,n^+}^{+}) = 0$ all-plus helicities

$A_n(1^-,\overbrace{2^+,\ldots,n^+}^{+}) = 0$ all-plus helicities but one

(vanish because of tree-level helicity conservation)

$$A_n(1^-,2^-,\overbrace{3^+,\ldots,n^+}^{+}) = i\,\frac{(\langle 12\rangle)^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Parke-Taylor '86 (conjecture)

Berends-Giele '87 (proof by recursive approach)

first non-vanishing amplitude

$\Rightarrow$ named: A_n^{MHV}

Maximal Helicity Violating

very simple!

[depends only on λ_i and not on $\tilde\lambda_i$

$\rightarrow$ "holomorphic function"]

TWISTOR SPACE AND GAUGE THEORY AMPLITUDES

- start in spinor (helicity) space : amplitude $A(\{p_i, h_i\}) = A(\{\lambda_i, \tilde{\lambda}_i\})$

↳ Twistor transform = "half Fourier transform"

twistor space obtained by Fourier transform w.r.t. positive helicities $\tilde{\lambda}_i$ (but not λ_i) Penrose '67

spinor space $\lambda^\alpha, \tilde{\lambda}^{\dot{\alpha}} \xrightarrow{\text{F.T.}} \lambda^\alpha, \mu^{\dot{\alpha}}$ twistor space

$$\tilde{A}(\lambda_i, \mu_i) = \int \prod_k \frac{d^2\tilde{\lambda}_k}{(2\pi)^2} \exp\Big\{ i \sum_j \mu_j^{\dot{\alpha}} \tilde{\lambda}_{j\dot{\alpha}} \Big\} A(\lambda_i, \tilde{\lambda}_k)$$

- Witten's observation :

A_n^{MHV} has support on a line in twistor space

simply follows from : $\begin{cases} A_n^{\text{MHV}} \text{ independent of } \tilde{\lambda} \\ \text{momentum conservation constraint } \big((p_i)_{\alpha\dot{\alpha}} = (\lambda_i)_\alpha (\tilde{\lambda}_i)_{\dot{\alpha}}\big) \end{cases}$ spinorial representation

$$\Big[A^{\text{MHV}}(\lambda_j)\, \delta^{(4)}\big(\textstyle\sum_i p_i\big)\Big]_{TS} = A^{\text{MHV}}(\lambda_j) \Big[\delta^{(4)}\big(\textstyle\sum_i p_i\big)\Big]_{TS} \qquad \delta^{(4)}(\,) = \int d^4x \exp\Big\{-i\, x^{\alpha\dot{\alpha}} \sum_j (\lambda_j)_\alpha (\tilde{\lambda}_j)_{\dot{\alpha}}\Big\}$$

$$\Rightarrow \int d^4x \prod_i \delta^{(2)}\big((\mu_i)^{\dot{\alpha}} - x^{\alpha\dot{\alpha}} (\lambda_i)_\alpha\big)$$

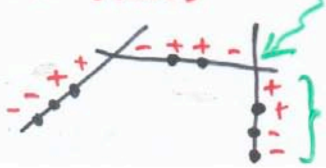
at fixed x : $\mu = x\lambda$ is a line

- Witten's conjecture (inspired by string theory interpretation) :

next-to-MHV amplitudes next-to-next MHV and so forth

have support on intersection of 2 lines / 3 lines

intersection in twistor space = propagator in momentum space



line in twistor space = vertex in momentum space

- twistor spinoffs :

Witten's conjecture has lead to HIGHLY SIMPLIFIED "Feynman rules" for tree-level amplitudes

➡ RECURSION FORMULAE FOR ON-SHELL (*) AMPLITUDES

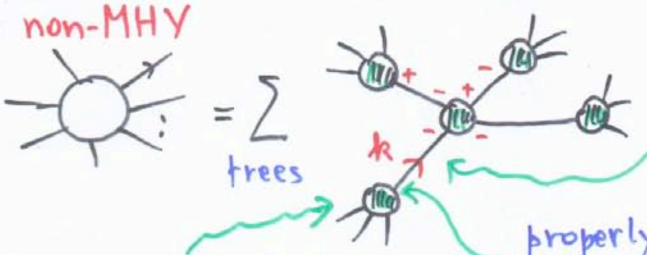
2 different/alternative set of rules

A) MHV rules

Cachazo-Svrček-Witten '04

effective vertices (A^{MHV}) joined by scalar propagators

non-MHV



simple scalar propagator : $\frac{1}{k^2}$

MHV amplitude = multi-particle vertex

properly defined on-shell projection of off-shell momentum k

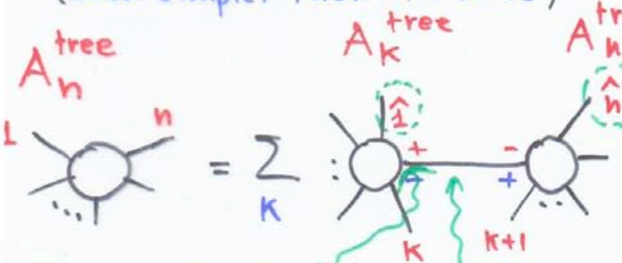
$(\tilde{k}^\mu = k^\mu - k^2 n^\mu / k \cdot n)$

B) ON-SHELL rules

(even simpler than MHV rules)

argued { Roiban-Spradlin-Volovich '04, Britto-Cachazo-Feng

Britto-Cachazo-Feng-Witten '05



on-shell projection of off-shell momentum

scalar propagator

properly defined momentum (shifted by complex amount)

PROOF: simply follows from ANALYTICITY (no twistors, exotics,..)

↓

the r.h.s.

(i) has the correct residue at any multiparticle poles

(ii) is unique

(*) KEY SIMPLIFICATION (w.r.t. Feynman graphs) :

on-shell amplitudes directly obtained from on-shell amplitudes with fewer external legs

- recursion formulae (MHV rules / on-shell rules) at the tree level extended to include fermions, vector bosons, Higgs bosons

Georgiou-Glover-Khoze '04
Wu-Zhu
Dixon-Glover-Khoze-Badger
Bern-Forde-Kosower-Mastrolia
Luo-Wen '05
Badger-Glover-Khoze-Svrcek

- loop amplitudes: much ongoing activity to generalize twistor-inspired methods at loop level
(most promising route seems: loop recursion formulae, i.e. get loops from trees via UNITARITY / DISPERSION RELATIONS)

partial list of the many contributing authors
Brandhuber, Spence, Travaglini, Cachazo, Svrcek, Witten, Britto, Feng, Bidder, Bjerrum-Bohr, Dixon, Dunbar, Perkins, Bern, Kosower, Del Duca, ...

$\Rightarrow$ much progress expected

remark:
besides their practical spinoffs (scattering amplitudes), this research activity may reveal new <u>general</u> properties, features, hidden symmetries, ... of perturbative gauge field theories

apologies for not having discussed recent progress on many other topics :

- theory / phenomenology of heavy-quark (especially, b-quark) production
  Cacciari, Nason, Mangano, Frixione, Ridolfi, Melnikov, Mitov, Corcella, ...

- BFKL, small-x QCD, high-energy behaviour
  Ciafaloni, Colferai, Salam, Stasto, Altarelli, Ball, Forte, ...

- QCD physics and (implementation in) Monte Carlo event generators
  Krauss, Kuhn, Webber, S.C., Frixione, Nason, Mangano, Moretti, Piccinini, Pittau, Mrenna, Richardson, Collins, Hautmann, Soper, Krämer, Nagy, ...

- interface pert./non-pert. QCD, power corrections, renormalons, ...
  Dokshitzer, Marchesini, Webber, Korchemsky, Sterman, Salam, Dasgupta, Beneke, Zakharov, Mueller, Gardi, Magnea, ...

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SUMMARY

- theoretical progress on QCD at high momentum transfer is HARD / SLOW but STEADY
  - sometimes boosted, speeded up

  e.g. in this talk
  - high-precision QCD
  - new methods in QCD and perturbative gauge theories

  two topics on which progress has been / is expected to be faster

- concluding remark:

  owing to
  - intense theor. activity, new ideas
  - continuous interactions (inputs/outputs) with experiments at high-energy colliders (Tevatron, HERA, RHIC, ..)
  - many new, YOUNG people in the field

  QCD is on track for the LHC