Beyond the Static Mean-Field Approach

Symmetry Restoration and Configuration Mixing

Nuclear Many-Body Correlations

short-range
(hard repulsive core of the NN-interaction)

long-range
nuclear resonance modes (giant resonances)

...vary smoothly with nucleon number!
Can be included implicitly in an effective Energy Density Functional.

collective correlations
large-amplitude soft modes:
(center of mass motion, rotation, low-energy quadrupole vibrations)

...sensitive to shell-effects and strong variations with nucleon number!
Cannot be included in a simple EDF framework.

Restoration of Broken Symmetries

A self-consistent mean-field (SCMF) wave function breaks necessarily several symmetries of the nuclear Hamiltonian (translational, rotational).

EXAMPLE: the only translational invariant product wave functions are products of plane waves, but they cannot be used to describe strong correlations between nucleons and their clustering into a finite nucleus.

A. Symmetry of a Hamiltonian and Broken Symmetry

→ symmetry of the hamiltonian of the system:

$$UHU^{+} = H \qquad [H, U] = 0$$

-unitary transformation: preserves the norm of state vectors and the matrix elements of observables.

-symmetry group of H: the eigenvectors of H are classified according to the irreducible representation of the symmetry group.

State of broken symmetry (deformed state): cannot be classified according to an irreducible representation of the symmetry group of the Hamiltonian H.

$U(\alpha)$

→ set of unitary operators (representing the symmetry group of the Hamiltonan). The parameter α can be discrete or continuous.

$$|\Phi\alpha\rangle = U(\alpha)|\Phi\rangle$$

$$\begin{aligned} \langle\Phi\alpha|H|\Phi\alpha\rangle &= \langle\Phi|U^{+}(\alpha)HU(\alpha)|\Phi\rangle \\ &= \langle\Phi|H|\Phi\rangle \qquad \forall\alpha \end{aligned}$$

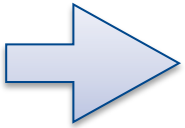
⇒ all deformed states |Φ α > are DEGENERATE.

B. Symmetries of the Hartree-Fock field

|Φ> → independent-particle state with the associated single-particle density ρ

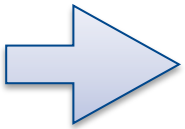
$$\rho_{ij} = \langle \Phi | a_j^+ a_i | \Phi \rangle \equiv \langle \Phi | \rho | \Phi \rangle$$

→ consider a unitary transformation: $|\overline{\Phi}\rangle = U|\Phi\rangle$



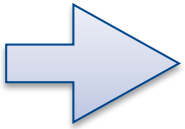
$$\bar{\rho}_{ij} \equiv \langle \overline{\Phi} | a_j^+ a_i | \overline{\Phi} \rangle$$

→ with: $U^+ a_j^+ U = \sum_k U_{jk}^* a_k^+$ $\qquad$ $U^+ a_i U = \sum_k U_{ik} a_k$



$$\bar{\rho} = U \rho U^+$$

→ in the HF approximation: $E[\rho] = \langle \Phi | H | \Phi \rangle$

→ if $UHU^+ = H$  $E[\bar{\rho}] = E[\rho]$

$$E[\bar{\rho}] = E[\rho] \quad \Rightarrow \quad h[\bar{\rho}] = U h[\rho] U^{+}$$

transformation of the Hartree-Fock hamiltonian

1) the density matrix ρ is invariant under the transformation U:

$$\bar{\rho} = \rho \quad \Longrightarrow \quad h[\bar{\rho}] = h[\rho]$$

U represents a ***self-consistent symmetry*** of the HF hamiltonian.

2) $\bar{\rho} \neq \rho \qquad [h[\rho], U] \neq 0$

U represents a ***broken symmetry*** of the HF hamiltonian.

Example: translational symmetry is always broken by the HF potential of a bound finite system.

C. Symmetries in the presence of pairing fields

In the Hartree-Fock-Bogoliubov (HFB) approximation the quasiparticle vacuum is characterized by the generalized density matrix:

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & \mathbb{1} - \rho^* \end{pmatrix}$$

→ unitary transformation: $\boxed{|\overline{\Phi}\rangle = U|\Phi\rangle}$

$$\begin{aligned} \bar{\rho}_{ij} &= \langle\overline{\Phi}|a_j^+ a_i|\overline{\Phi}\rangle = (U\rho U^+)_{ij} \\ \bar{\kappa}_{ij} &= \langle\overline{\Phi}|a_j a_i|\overline{\Phi}\rangle = (U\kappa\tilde{U})_{ij} \end{aligned}$$

$$\bar{\mathcal{R}} = \mathcal{U}\mathcal{R}\mathcal{U}^+ \qquad \mathcal{U} = \begin{pmatrix} U & 0 \\ 0 & U^+ \end{pmatrix}$$

If U represents a symmetry of the Hamiltonian H ⇒ $E[\bar{\mathcal{R}}] = E[\mathcal{R}]$

$$\mathcal{H}[\bar{\mathcal{R}}] = \mathcal{U}\mathcal{H}[\mathcal{R}]\mathcal{U}^{+}$$

transformation of the HFB Hamiltonian

1) self-consistent symmetry of the HFB Hamiltonian

$$[\mathcal{R},\mathcal{U}] = 0 \qquad \bar{\mathcal{R}} = \mathcal{R} \qquad \mathcal{U}\mathcal{H}\mathcal{U}^{+} = \mathcal{H}$$

2) broken symmetry

$$[\mathcal{R},\mathcal{U}] \neq 0 \qquad \bar{\mathcal{R}} \neq \mathcal{R} \qquad \mathcal{U}\mathcal{H}\mathcal{U}^{+} \neq \mathcal{H}$$

The pairing field breaks the invariance with respect to the transformation induced by the operator:

$$U = e^{iN\phi} \qquad N = \sum_i a_i^{+} a_i$$

$$\bar{\mathcal{R}} = \mathcal{U}\mathcal{R}\mathcal{U}^{+} = \begin{pmatrix} \rho & \kappa e^{2i\phi} \\ -\kappa^{*} e^{-2i\phi} & \mathbb{1} - \rho^{*} \end{pmatrix}$$

D. Broken symmetries in finite systems

In ***finite systems*** broken symmetries arise only as a result of approximations (variational principle applied to a restricted set of trial wave functions).

A ***broken symmetry*** implies a ***degeneracy*** of the solutions of variational equations.

$$|\Phi\alpha\rangle \equiv U(\alpha)|\Phi\rangle$$

SYMMETRY RESTORATION → the new trial wave function is a linear superposition of the degenerate deformed states.

$$|\psi\rangle = \int d\alpha f(\alpha)|\Phi\alpha\rangle$$

The ***minimization of the energy*** with respect to the expansion coefficients f(α) is equivalent to the ***projection*** of states of good symmetry from the deformed state |Φ >. The resulting states can be classified according to the ***irreps*** of the symmetry group.

EXAMPLE: ***parity*** - discrete broken symmetry

|Φ > a normalized state that is not an eigenstate of the **parity operator Π**. **[H,Π]=0** implies that the linearly independent states **|Φ > and Π|Φ > are degenerate.**

→ new trial function: $|\psi(\lambda)\rangle = |\Phi\rangle + \lambda\Pi|\Phi\rangle$

→ parameter to be evaluated by minimizing the energy expectation value

$$E(\lambda) = \frac{\langle\Psi(\lambda)|H|\Psi(\lambda)\rangle}{\langle\Psi(\lambda)|\Psi(\lambda)\rangle} = \langle H\rangle \frac{1+\lambda^2+2\lambda\langle H\Pi\rangle/\langle H\rangle}{1+\lambda^2+2\lambda\langle\Pi\rangle}$$

$$\frac{dE(\lambda)}{d\lambda} = 0 \quad \Longrightarrow \quad \boxed{[\langle H\Pi\rangle - \langle H\rangle\langle\Pi\rangle](1-\lambda^2) = 0}$$

If |Φ > is not an eigenstate of the parity operator:

$$\langle H\Pi\rangle \neq \langle H\rangle\langle\Pi\rangle \quad \Longrightarrow \lambda = \pm 1$$

→ parity eigenstates:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(1 \pm \Pi)|\Phi\rangle \quad \Longrightarrow \quad \boxed{\Pi|\Psi_\pm\rangle = \pm|\Psi_\pm\rangle}$$

The same states are obtained by simply acting with the projection operators:

$$\boxed{P_\pm = \frac{1}{\sqrt{2}}(1 \pm \Pi)}$$

on the deformed state |Φ >. The degeneracy of the deformed states |Φ > and Π|Φ > has been removed:

$$E_+ - E_- = 2\frac{\langle H\Pi\rangle - \langle H\rangle\langle\Pi\rangle}{1 - \langle\Pi\rangle^2}$$

E. Non-conservation of particle number

|Φ > a normalized state, not an eigenstate of the **particle number operator N**.

$$[H, N] = 0 \quad \Longrightarrow \quad e^{-i\alpha N}|\Phi\rangle \quad \alpha \in [0, 2\pi]$$

degenerate states!

→ new trial function:

$$|\Psi\rangle = \int_0^{2\pi} \frac{d\alpha}{2\pi} f(\alpha) e^{-i\alpha \hat{N}} |\Phi\rangle \equiv \int_0^{2\pi} \frac{d\alpha}{2\pi} f(\alpha) |\Phi\alpha\rangle$$

def. $\hat{N} = N - \bar{n}, \qquad |\Phi\alpha\rangle \equiv e^{-i\alpha \hat{N}} |\Phi\rangle$

The projection on states with good particle number is equivalent to the requirement that the energy:

$$E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

is stationary with respect to variations of ***f*(α)*** and ***f(α)***.

$$= \frac{\int_0^{2\pi} d\alpha \int_0^{2\pi} d\alpha' f^*(\alpha) \langle\Phi\alpha|H|\Phi\alpha'\rangle f(\alpha')}{\int_0^{2\pi} d\alpha \int_0^{2\pi} d\alpha' f^*(\alpha) \langle\Phi\alpha|\Phi\alpha'\rangle f(\alpha')}$$

$$\int_0^{2\pi} \frac{d\alpha'}{2\pi} \langle \Phi | e^{i\hat{N}(\alpha-\alpha')} (H - E) | \Phi \rangle f(\alpha') = 0$$

Hill-Wheeler equation

the solutions are eigenstates of the particle number operator!

Fourier transform:

$$f(\alpha) = \sum_{n=0}^{\infty} f_n e^{i(n-\bar{n})\alpha} \quad \Longrightarrow \quad f_n = \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{-i(n-\bar{n})\alpha} f(\alpha)$$

HW equation:

$$\sum_{n=0}^{\infty} f_n \langle \Phi | (H - E) P_n | \Phi \rangle e^{i(n-\bar{n})\alpha} = 0$$

where:

$$P_n \equiv \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{-i(N-n)\alpha}$$

→ operator projecting onto states with particle number n.

The Hill-Wheeler equation is valid for all angles α:

$$f_n \langle \Phi | (H - E) P_n | \Phi \rangle = 0$$

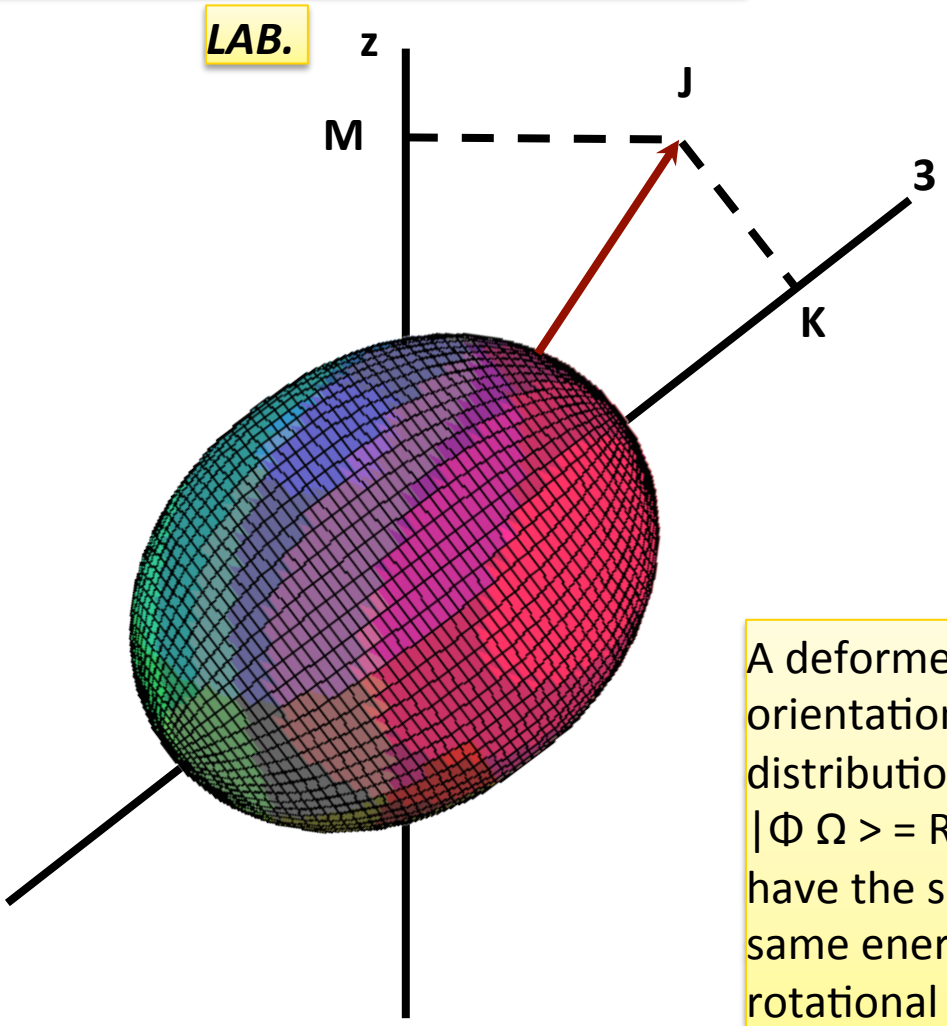
nonvanishing coefficients exist only if the energy E equals:

$$E_n = \frac{\langle \Phi | H P_n | \Phi \rangle}{\langle \Phi | P_n | \Psi \rangle}$$

The solution of the HW equation is the projected state:

$$|\Psi\rangle = f_n |\Psi_n\rangle, \qquad |\Psi_n\rangle \equiv P_n |\Phi\rangle$$

F. Angular momentum projection



Symmetry violation in many-particle wave functions can be related to ***collective motion***.

A deformed wave function $|\Phi>$ defines a fixed orientation in space (principal axes of the mass distribution). All wave functions:
$|\Phi\ \Omega> = R(\Omega)\ |\Phi>$
have the same internal structure and yield the same energy expectation value ⇒ collective rotational motion approximately preserves the intrinsic structure.

Angular momentum operators in the laboratory and intrinsic frames

INT. $\hat{e}_a \quad (a = 1, 2, 3): \quad \hat{e}_a \cdot \hat{e}_b = \delta_{ab}, \quad \hat{e}_a \times \hat{e}_b = \epsilon_{abc}\hat{e}_c$

LAB. $\hat{u}_i \quad (i = x, y, z): \quad \hat{e}_a = \tilde{\mathcal{R}}_{ai}(\Omega)\hat{u}_i, \quad \Omega = \{\alpha, \beta, \gamma\}$

Euler angles

The Euler angles are dynamical variables which specify the orientation of the intrinsic frame.

Def. intrinsic angular momentum operators: $I_a = \hat{e}_a \cdot \vec{J}$

$$[I_a, J_i] = 0 \quad \forall a, i$$

$$[I_a, I_b] = -i\epsilon_{abc} I_c$$

$$\vec{I}^{\,2} = \sum_a I_a^2 = \sum_i J_i^2 = \vec{J}^{\,2}$$

$$\vec{I}^{\,2} = \vec{J}^{\,2}, I_3, J_z$$

set of commuting operators,
can be diagonalized simultaneously

$$\begin{aligned}
\vec{I}^{\,2}|IKM\rangle &= I(I+1)|IKM\rangle \\
I_3|IKM\rangle &= K|IKM\rangle \quad -I \le K \le I \\
J_z|IKM\rangle &= M|IKM\rangle \quad -I \le M \le I
\end{aligned}$$

$$\sum_{IKM} |IKM\rangle\langle IKM| = 1, \qquad \langle IKM|I'K'M'\rangle = \delta_{II'}\delta_{KK'}\delta_{MM'}$$

The states |IKM > can be represented by the wave functions < Ω|IKM >, which depend on the Euler angles Ω = {α,β,γ}. With the definition of the state |Ω >:

$$|\Omega\rangle = \mathcal{R}(\Omega)|\Omega = 0\rangle$$

$$\begin{aligned}
\langle\Omega|IKM\rangle &= \langle\Omega = 0|\mathcal{R}^+(\Omega)|IKM\rangle \\
&= \sum_{I'K'M'} \langle\Omega = 0|I'K'M'\rangle\langle I'K'M'|\mathcal{R}^+(\Omega)|IKM\rangle
\end{aligned}$$

The rotation does not change the intrinsic angular momenta => K=K' and I=I'. If the Euler angles are chosen in such a way that the INT and LAB frames coincide for Ω=0:

$$\langle \Omega = 0 | IKM \rangle = c_I \delta_{KM} \qquad c_I = \sqrt{(2I+1)/8\pi^2}$$

$$\begin{aligned} \langle \Omega | IKM \rangle &= \sqrt{(2I+1)/8\pi^2} D^I_{MK}(\Omega) \\ &= \sqrt{(2I+1)/8\pi^2} e^{i\alpha M} d^I_{MK}(\beta) e^{i\gamma K} \end{aligned}$$

$$\begin{aligned} \langle IKM | I'K'M' \rangle &= \frac{2I+1}{8\pi^2} \int d\Omega D^{I'}_{M'K'}(\Omega) D^{I*}_{MK}(\Omega) \\ &= \delta_{II'} \delta_{KK'} \delta_{MM'} \end{aligned}$$

Variational principle and angular momentum projection

Deformed state |Φ> , not an eigenstate of $\vec{J}^{\,2}, \;\; J_3$

$$\mathcal{R}(\alpha,\beta,\gamma) = e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z}$$

$$[H,\mathcal{R}] = 0 \quad \Longrightarrow \quad |\Phi\Omega\rangle = \mathcal{R}(\Omega)|\Phi\rangle$$ degenerate states

→ new trial function: $$|\Psi\rangle = \int d\Omega f(\Omega)|\Phi\Omega\rangle \equiv \int d\Omega f(\Omega)\mathcal{R}(\Omega)|\Phi\rangle$$

The weight function f(Ω) is determined by requiring that the energy expectation:

$$E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$ is stationary with respect to variations of f* and f.

Hill-Wheeler equation

$$\int d\Omega' \langle\Phi\Omega|H - E|\Phi\Omega'\rangle f(\Omega') = 0$$

The solutions of the HW equation are eigenstates of the operators $\vec{J}^{\,2},\ J_3$

$$f(\Omega)=\sum_{IMK}\frac{2I+1}{8\pi^2}f^I_{MK}D^I_{MK}(\Omega)\Rightarrow f^I_{MK}=\int d\Omega f(\Omega)D^{I^*}_{MK}(\Omega)$$

$$\Rightarrow\ |\Psi\rangle=\sum_{IMK}f^I_{MK}P^I_{MK}|\Phi\rangle$$

where:

$$P^I_{MK}=\frac{2I+1}{8\pi^2}\int d\Omega D^I_{MK}(\Omega)\mathcal{R}(\Omega)$$

$$\left(P^I_{MK}\right)^+=P^I_{KM}\qquad \left(P^I_{MK}\right)^+P^{I'}_{M'K'}=\delta_{II'}\delta_{MM'}P^I_{KK'}$$

not quite a projector!

with: $[H,P^I_{MK}]=0$ ⇒

HW equation:

$$\sum_{K'} \langle \Phi | (H - E) P^I_{K K'} | \Phi \rangle f^I_{M K'} = 0$$

⇒ eigenvalues determined by the equation:

$$\det \left[\langle \Phi | (H - E) P^I_{K K'} | \Phi \rangle \right] = 0$$

a) the HW equation is equivalent to the diagonalization of the hamiltonian in the basis $P^I_{M K} | \Phi \rangle$
b) H does not connect states with I≠I', and the eigenvalues do not depend on M

⇒ eigenstates:

$$| \Psi_{I M} \rangle = \sum_K f^I_{M K} P^I_{M K} | \Phi \rangle$$

G. Projection before and after variation

How do we determine the deformed (symmetry-violating) intrinsic function |Φ > ?

i) Variation before the projection (VBP)

|Φ > is determined by the variational principle:

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

The deformed solution is a superposition of eigenstates of the corresponding symmetry operator (for example, angular momentum). The wave function:

$$|\Psi_I\rangle = P^I |\Phi\rangle$$

is no longer a product wave function, but a complicated superposition of Slater determinants. It contains many more correlations than the function |Φ >.

This method violates the variational principle, because we do not vary the projected wave function. It does not allow for changes in the self-consistent mean-field for different values of I (within a rotational band).

ii) Variation after projection (VAP)

$$\delta \frac{\langle \Psi_I | H | \Psi_I \rangle}{\langle \Phi_I | \Psi_I \rangle} = \delta \frac{\langle \Phi | P^I H P^I | \Phi \rangle}{\langle \Phi | P^I P^I | \Phi \rangle} = 0$$

⇒ minimize the expectation value of the projected energy $P^I H P^I$ within the set of product wave functions |Φ >.

This method corresponds to a double variation, using the ansatz:

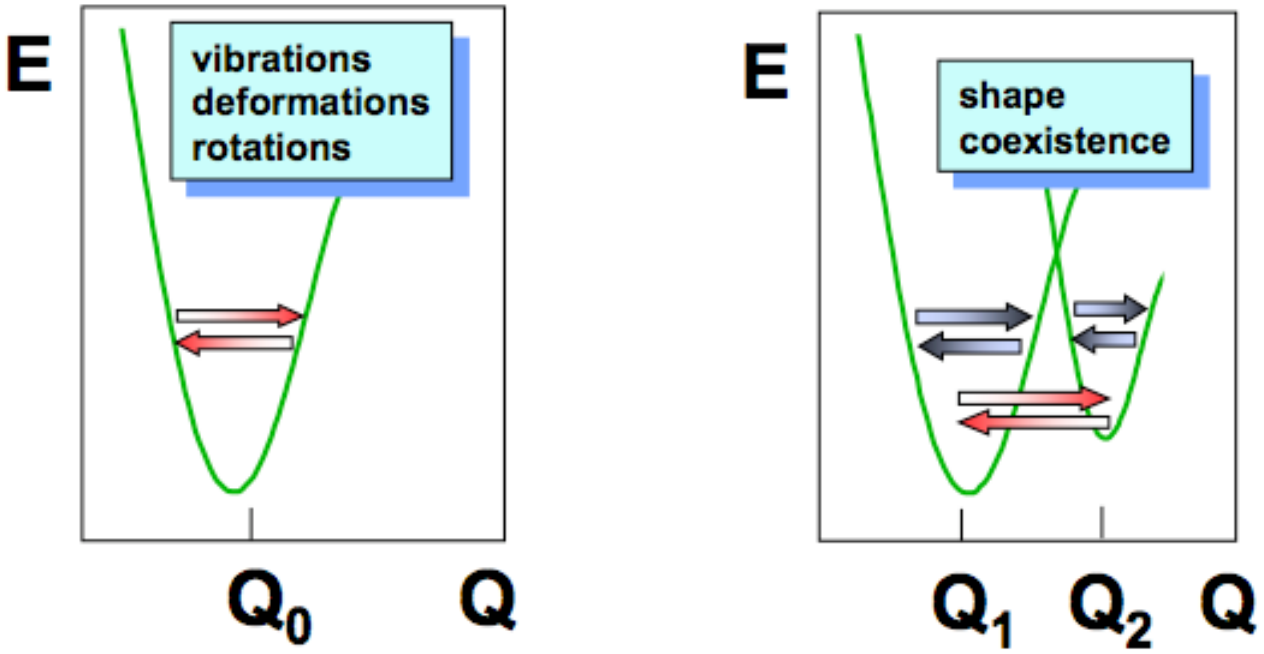
$$|\Psi\rangle = \int d\Omega f(\Omega) \mathcal{R}(\Omega) |\Phi\rangle$$

and varying the energy with respect to both the weight function f(Ω) and the generating function |Φ >.

Much more complicated than VBP!

(1) must repeat the variation for each value of I
(2) $P^I H P^I$ is a multi-particle operator

Configuration Mixing



The most important correlation effects in nuclear structure stem from large amplitude collective motion. Low-lying excited states are admixed into the mean-field ground state. These admixtures can be removed by configuration mixing: superposition of several mean-field states.

Correlations include nuclear surface vibrations (low-lying excitations) and zero-energy modes (translation, rotation, ...) related to restoration of symmetries which are broken by the mean-field ground state.

A. The Generator Coordinate Method

→ starting from a set of mean-field states |Φ(q) > that depend on the collective coordinate q, approximate eigenstates of the Hamiltonian H are obtained by GCM configuration mixing:

$$|\Psi_k\rangle = \int dq\, |\Phi(q)\rangle\, f_k(q)$$

Weight functions

Generator coordinate (collective variable)

Intrinsic (e.g. HFB) wave functions

The weight functions $f_k(q)$ are found by requiring that the expectation value:

$$E_k = \frac{\langle \Psi_k | \hat{H} | \Psi_k \rangle}{\langle \Psi_k | \Psi_k \rangle}$$ is stationary with respect to an arbitrary variation δf_k.

Hill-Wheeler equation

$$\int dq' \left[\mathcal{H}(q,q') - E_k \mathcal{I}(q,q') \right] f_k(q')$$

$$\mathcal{H}(q,q') = \langle \Phi(q)|\hat{H}|\Phi(q')\rangle \qquad \mathcal{I}(q,q') = \langle \Phi(q)|\Phi(q')\rangle$$

Hamiltonian kernel

overlap kernel

→ for any operator O:

$$\mathcal{O}(q,q') = \langle \Phi(q)|\hat{O}|\Phi(q')\rangle$$

The weight functions are not orthonormal and they cannot be interpreted as collective wave functions for the variable q. This role is assigned to the functions:

$$g_k(q) = \int dq'\, \mathcal{I}^{1/2}(q,q')\, f_k(q')$$

The matrix element of any operator between two GCM states can be expressed in terms of the g_k's as:

$$\langle \Psi_k|\hat{O}|\Psi_l\rangle = \iint dq\, dq'\, g_k^*(q)\, \tilde{\mathcal{O}}(q,q')\, g_l(q')$$

with: $$\tilde{\mathcal{O}}(q,q') = \iint dq''\, dq'''\, \mathcal{I}^{1/2}(q,q'')\, \mathcal{O}(q'',q''')\, \mathcal{I}^{1/2}(q''',q')$$

The GCM energies E_k and functions g_k are the eigenvalues and eigenvectors of the hermitian integral operator

$$\int dq'\, \tilde{\mathcal{H}}(q,q')\, g_k(q') = E_k g_k(q)$$

Gaussian Overlap Approximation: the overlap kernel is replaced by a Gaussian function of the form:

$$\mathcal{I}(q,q') \simeq \mathcal{I}_G(q,q') = \exp\left\{ -\frac{1}{2} \left[\frac{(q-q')}{a(\bar{q})} \right]^2 \right\}$$

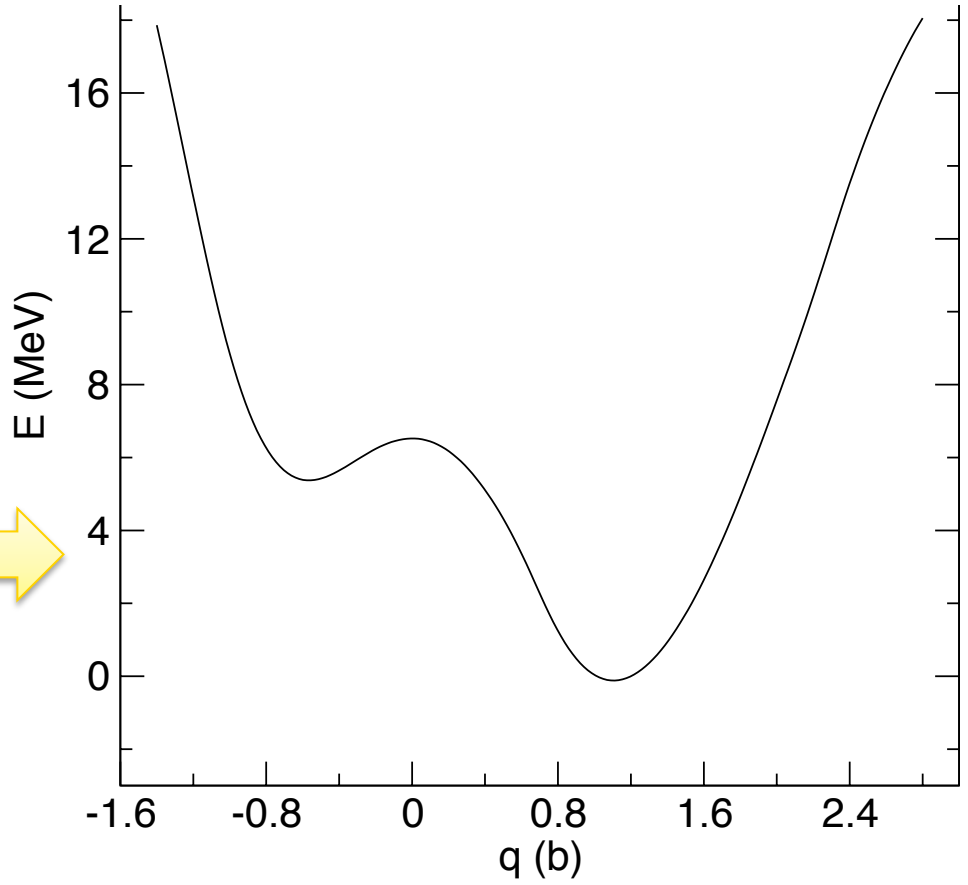
based on the rapid decrease of the matrix elements between wave functions corresponding to different values of the collective variable.

B. Choice of the collective coordinate

1. RESTORATION OF BROKEN SYMMETRIES: the family of wave functions $|\Phi(q)>$ is generated by the symmetry operations: rotation in coordinate space for angular momentum, rotation in gauge space for particle number. The functions $f_k(q)$ are a priori determined by the properties of the symmetry operator.

2. SHAPE DEGREES OF FREEDOM:
the collective space is generated by constrained mean-field calculations. The generating function is unknown and has to be determined by the diagonalization of the Hill-Wheeler equation.

The starting point is usually a constrained HFB calculation of the potential energy surface with the mass quadrupole components as constrained quantities.

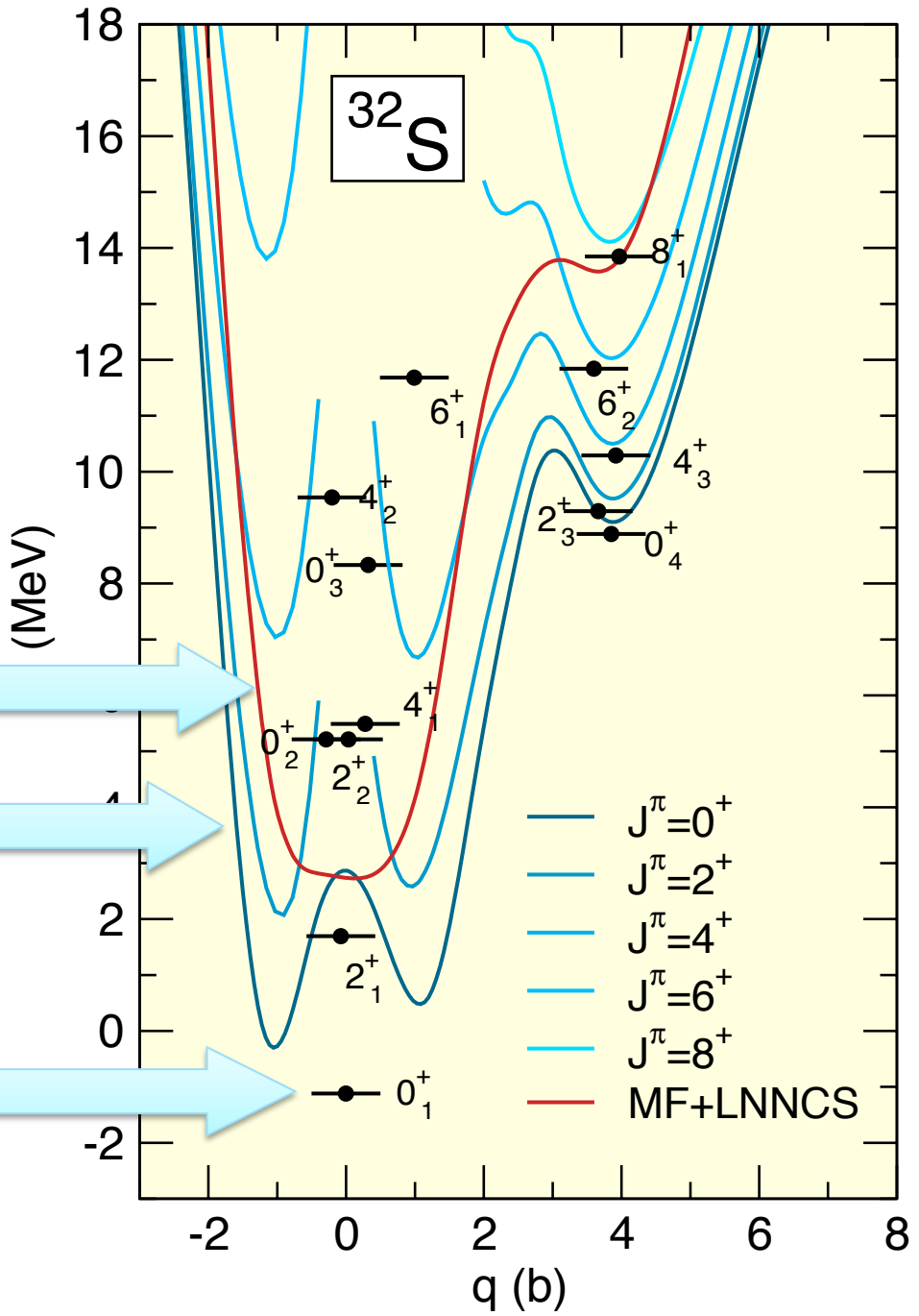


Example: Self-consistent mean-field calculation which includes correlations related to restoration of broken symmetries (rotational, particle number) and to fluctuations of collective variables (quadrupole deformation).

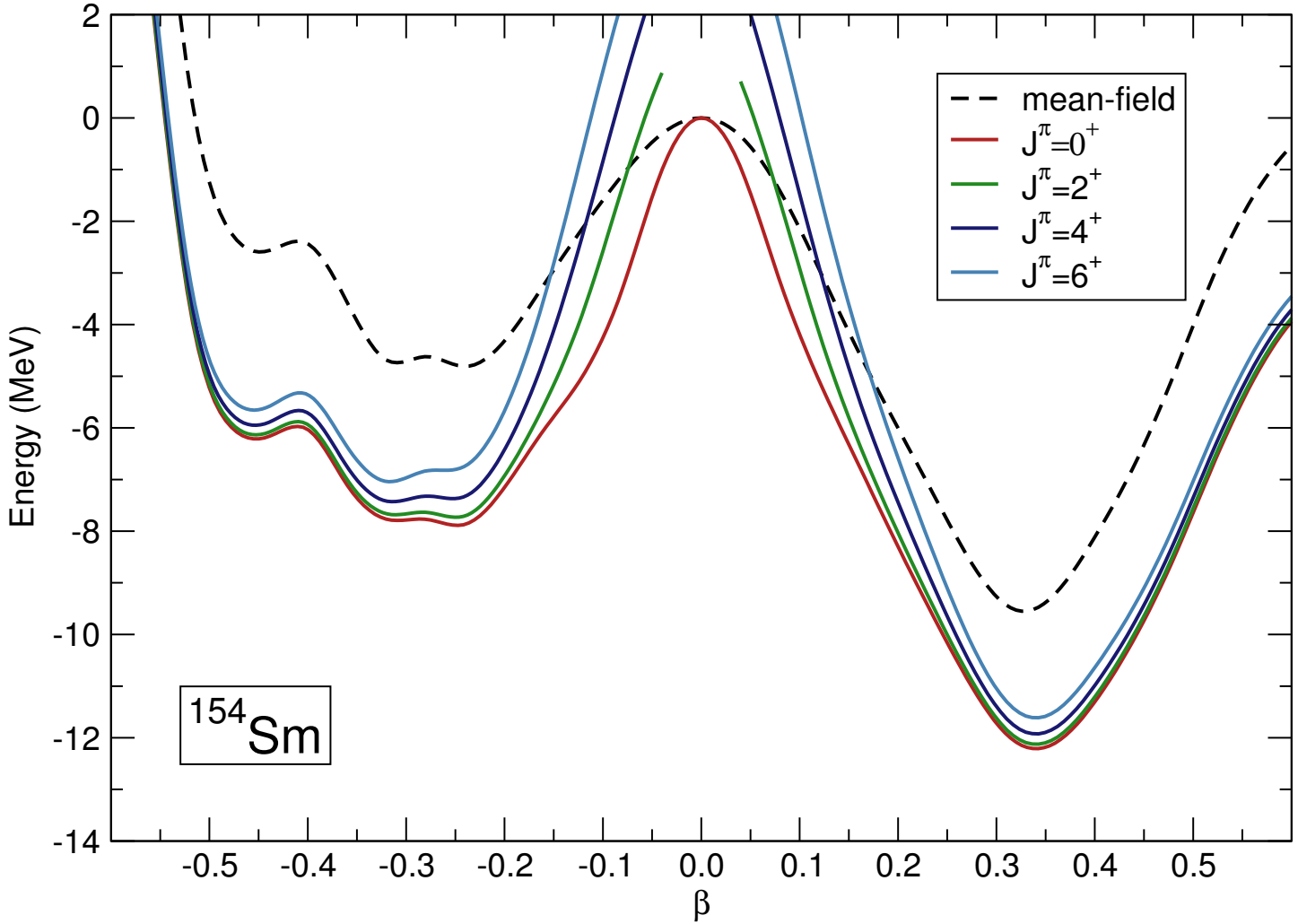
1. Mean-field potential energy curve calculated with a constraint on the quadrupole moment.

2. Angular-momentum and particle -number projected energy curves.

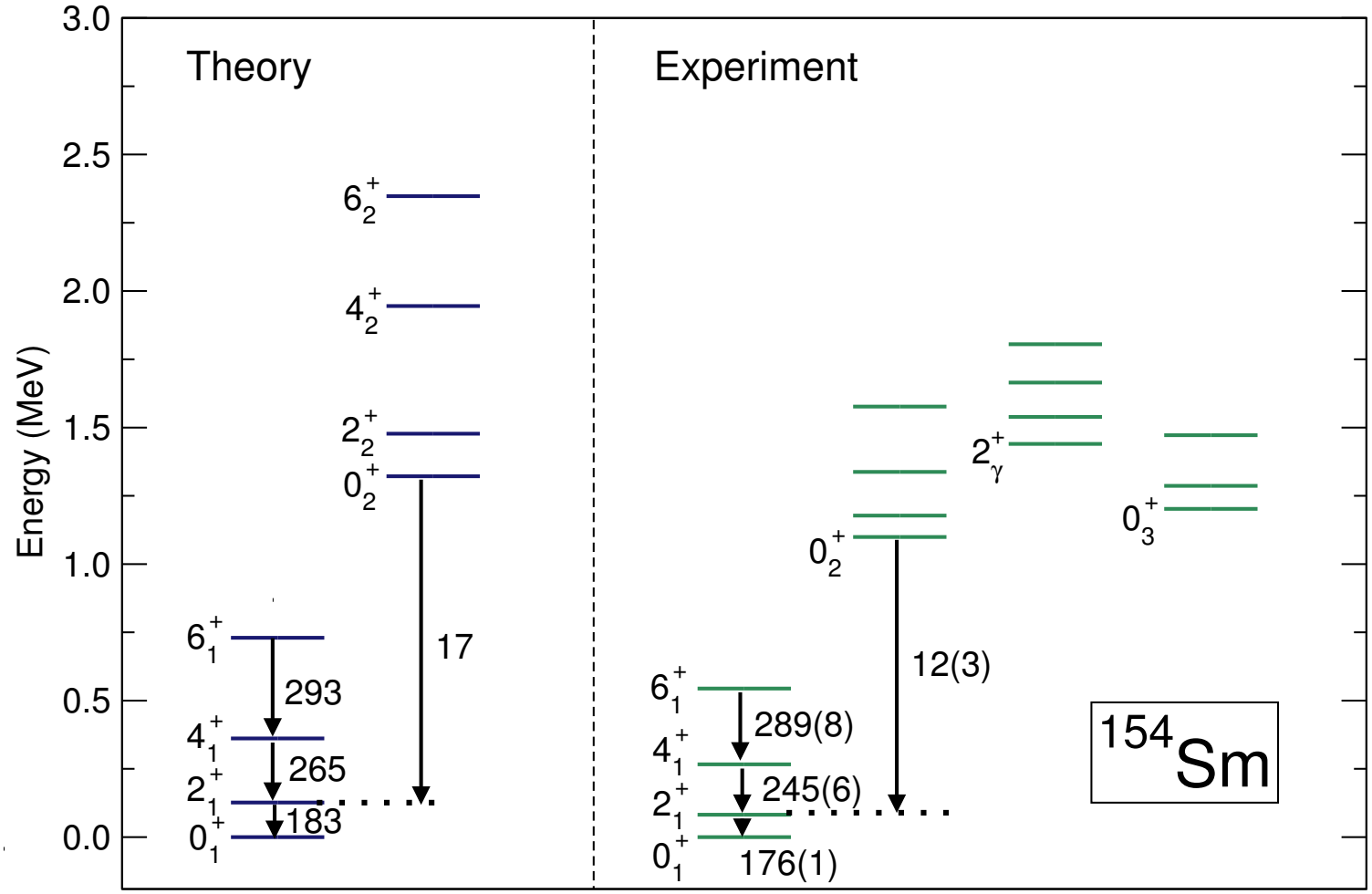
3. The Hamiltonian is diagonalized within each of the collective sub-spaces of the nonorthogonal bases $|J, q\rangle$ by using the Generator Coordinate Method.



C. Angular momentum projection and configuration mixing: ^{154}Sm



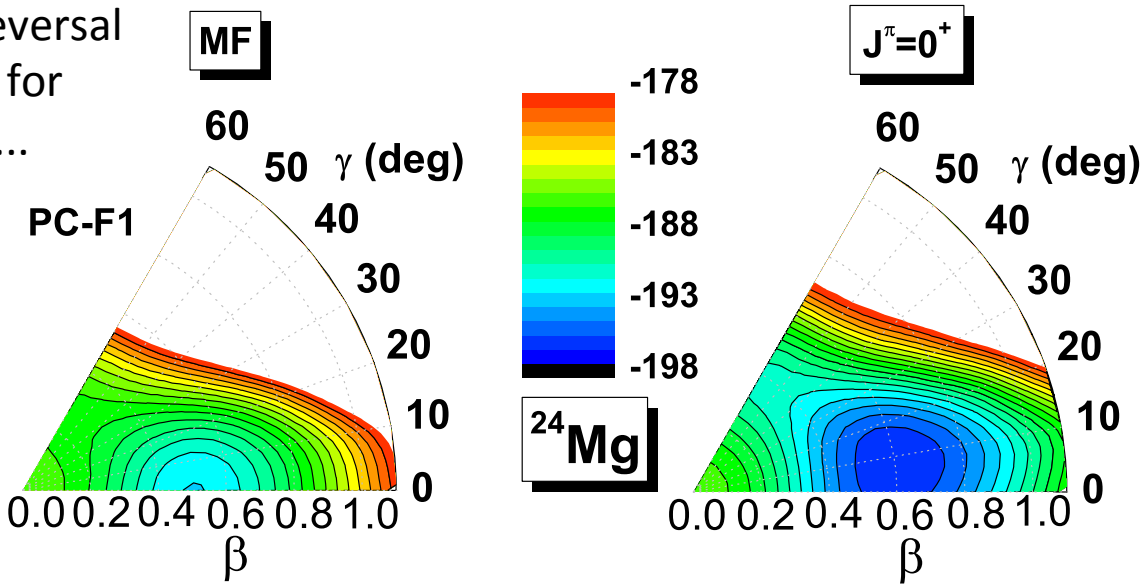
Men-field energy curve of ^{154}Sm (dashed), and the corresponding angular-momentum projected (J = 0+; 2+; 4+ , and 6+) energy curves, as functions of the axial deformation β.



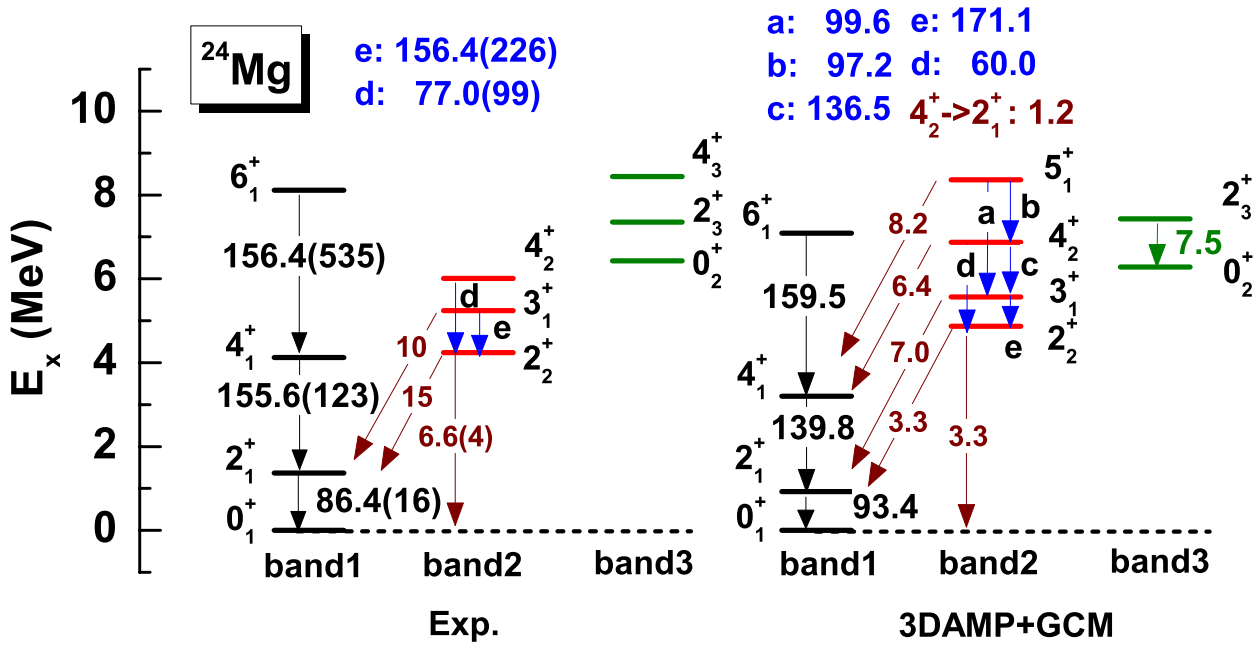
Angular-momentum projected GCM results for the excitation energies and B(E2) values (in Weisskopf units) of the lowest two bands in ^{154}Sm, in comparison to data.

→ larger variational space for projected GCM calculations!

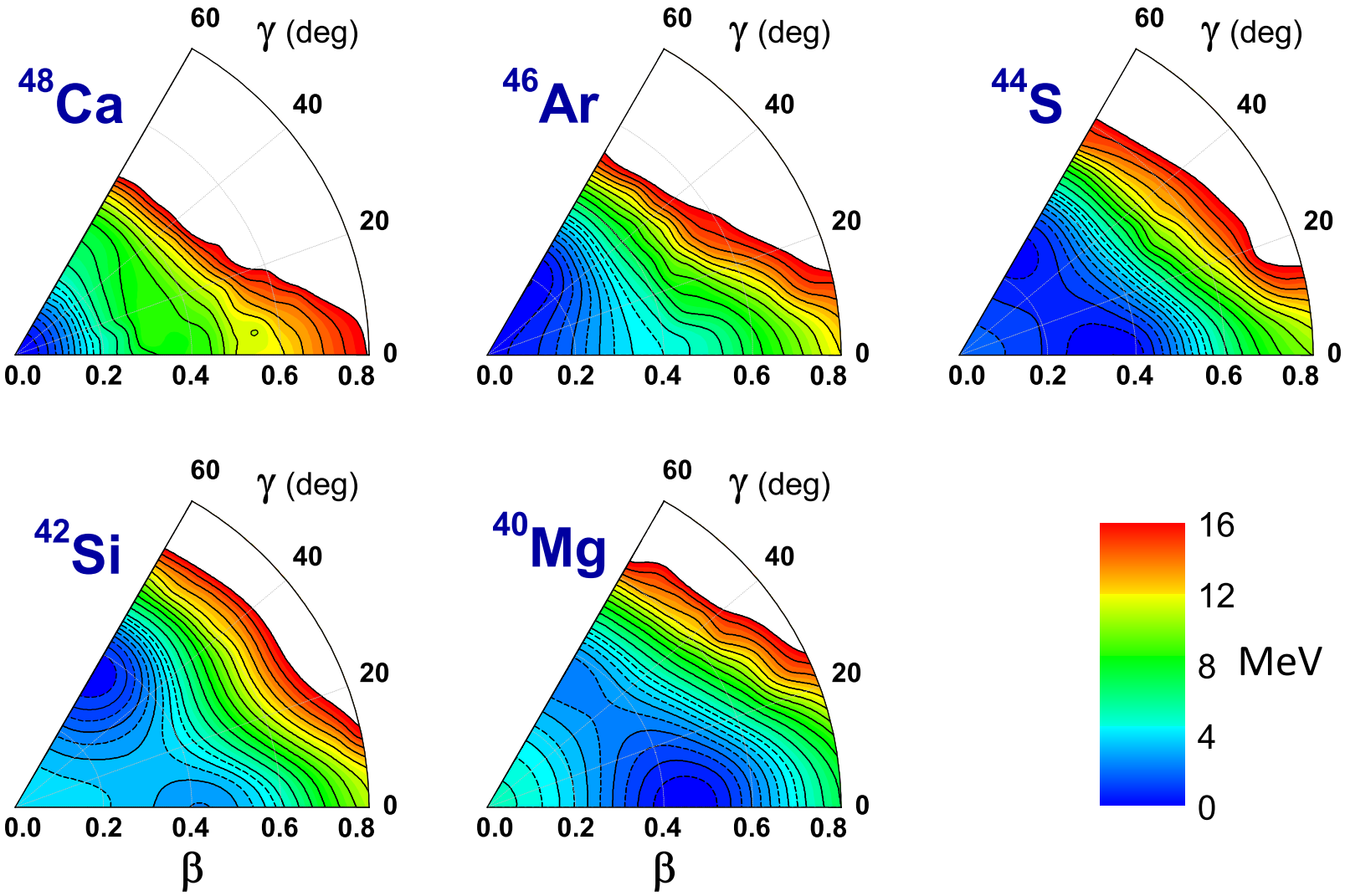
→ triaxial shapes, breaking time-reversal invariance, different deformations for proton and neutron distributions, ...

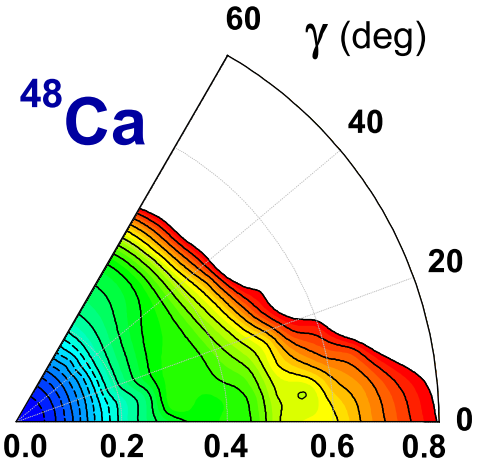


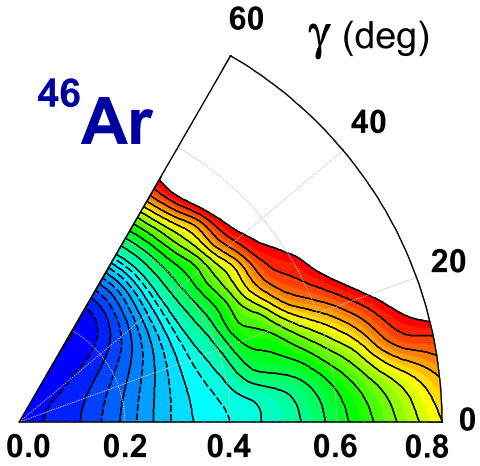
3D AMP+GCM model

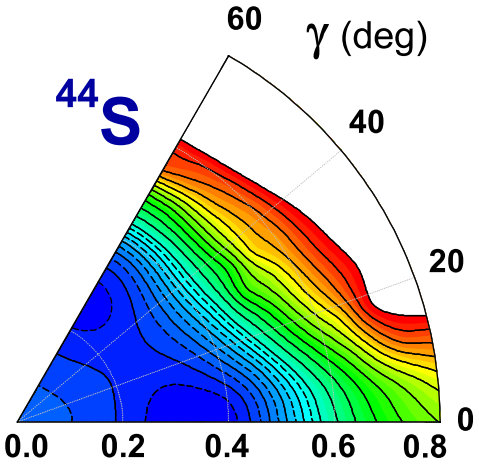


D. Coexisting shapes in the N=28 isotones



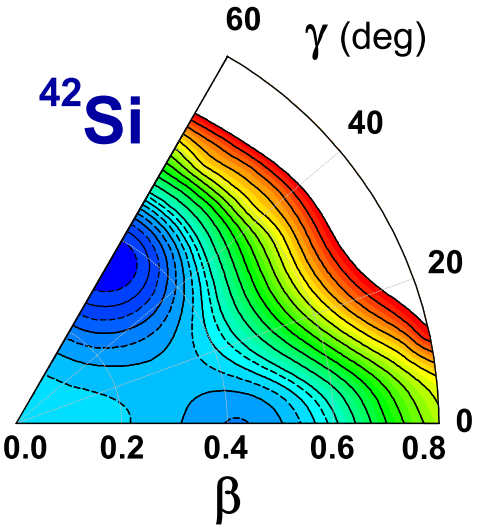


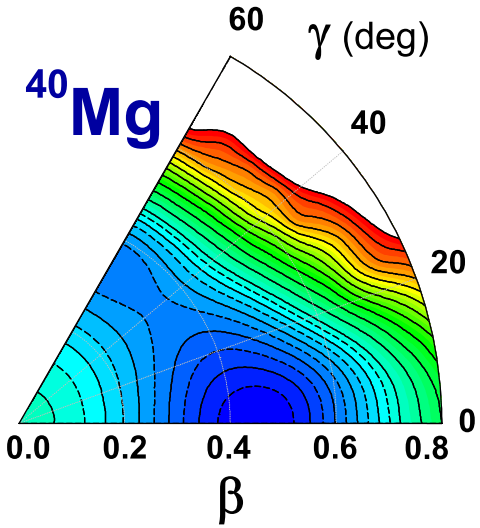




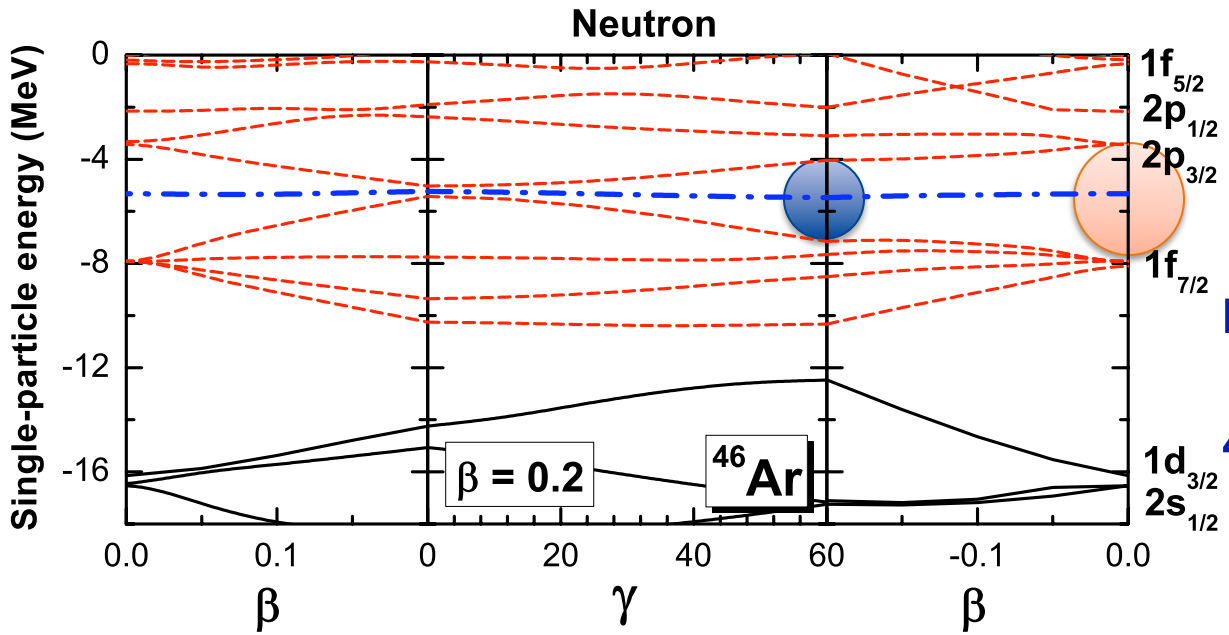
Neutron N=28 spherical shell gaps

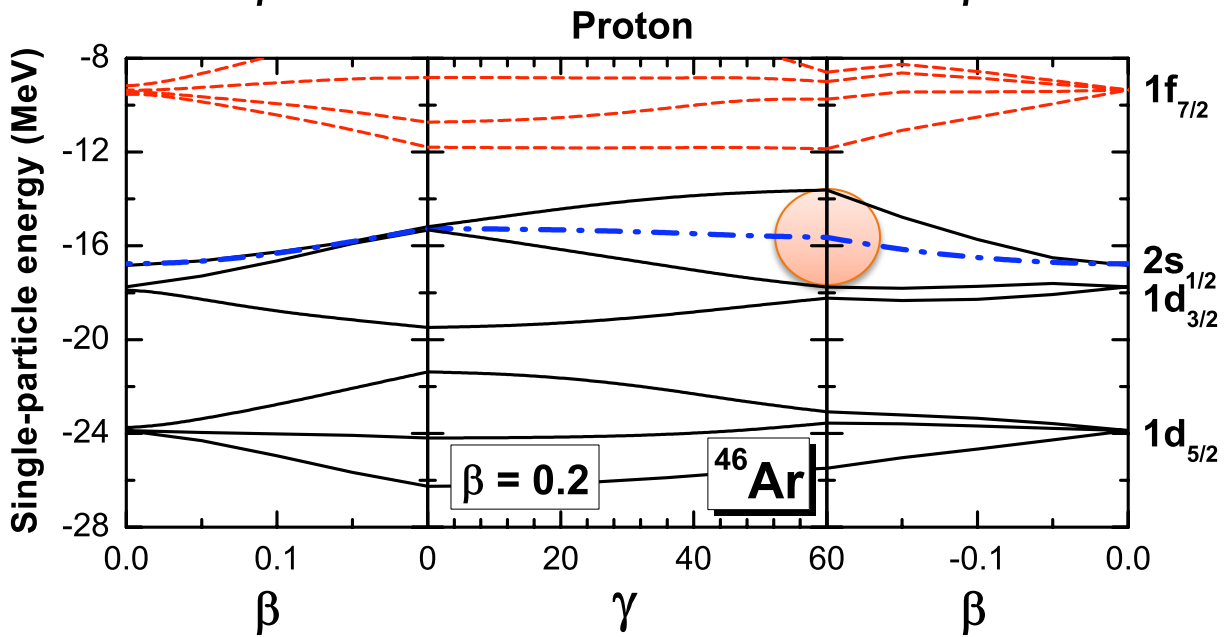
Exp. values		$\Delta^{\text{sph.}}_{N=28}$	β_{min}
4.80 MeV	^{48}Ca	4.73	0.00
4.47 MeV	^{46}Ar	4.48	-0.19
	^{44}S	3.86	0.34
	^{42}Si	3.13	-0.35
	^{40}Mg	2.03	0.45

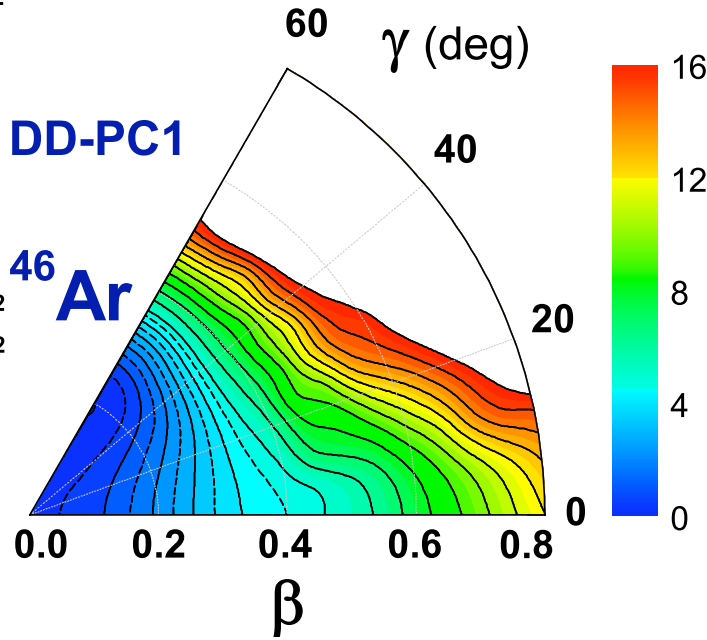




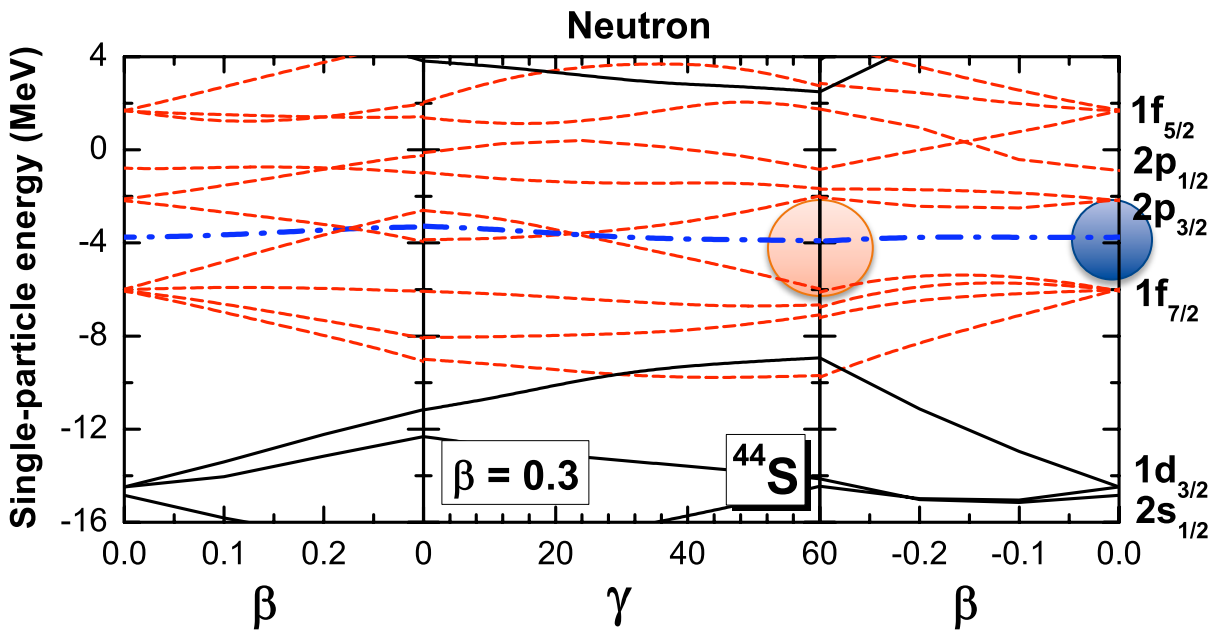
^{46}Ar: single-particle levels

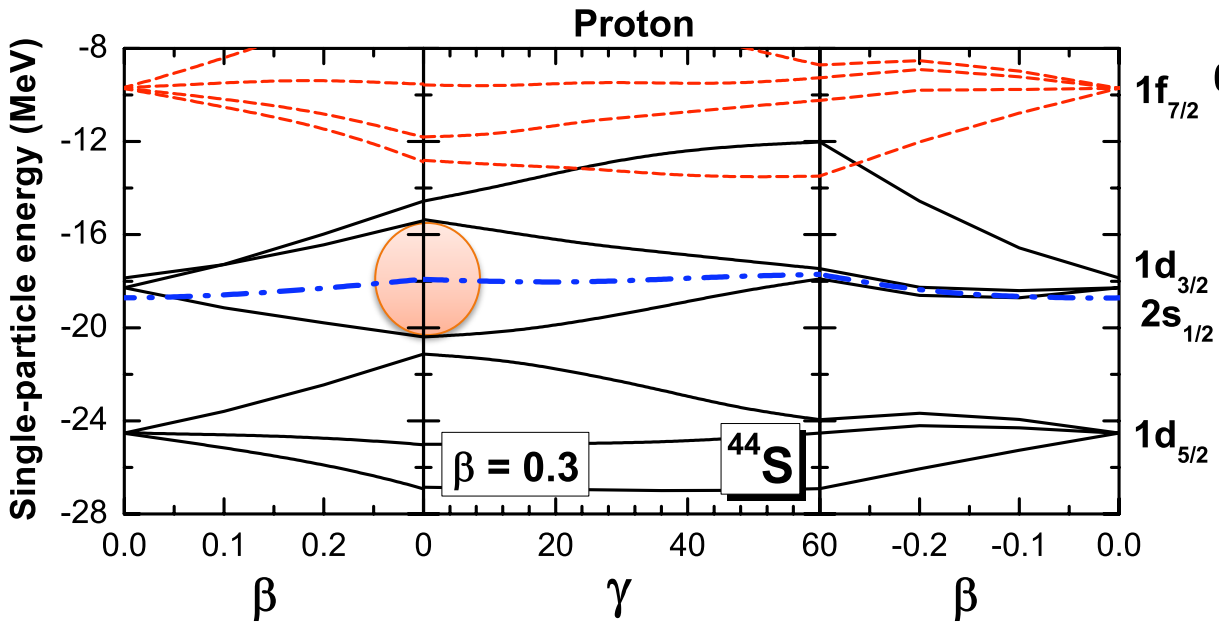


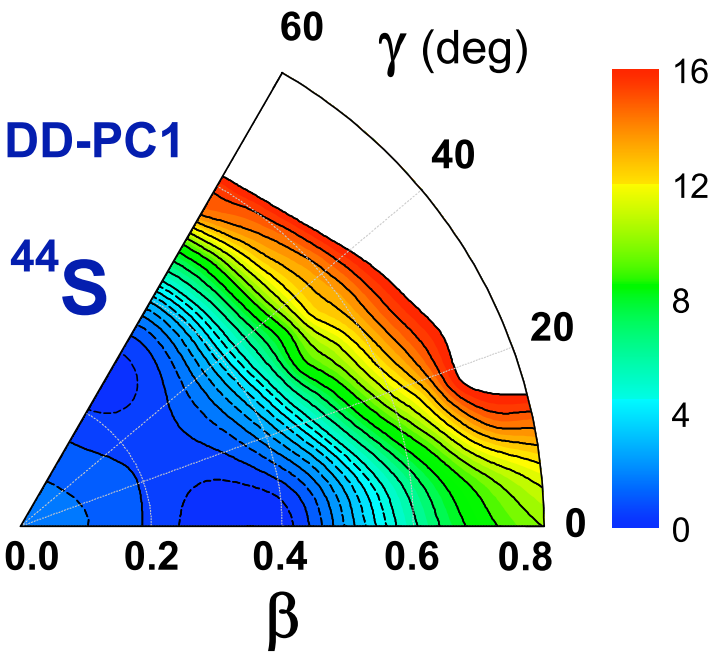




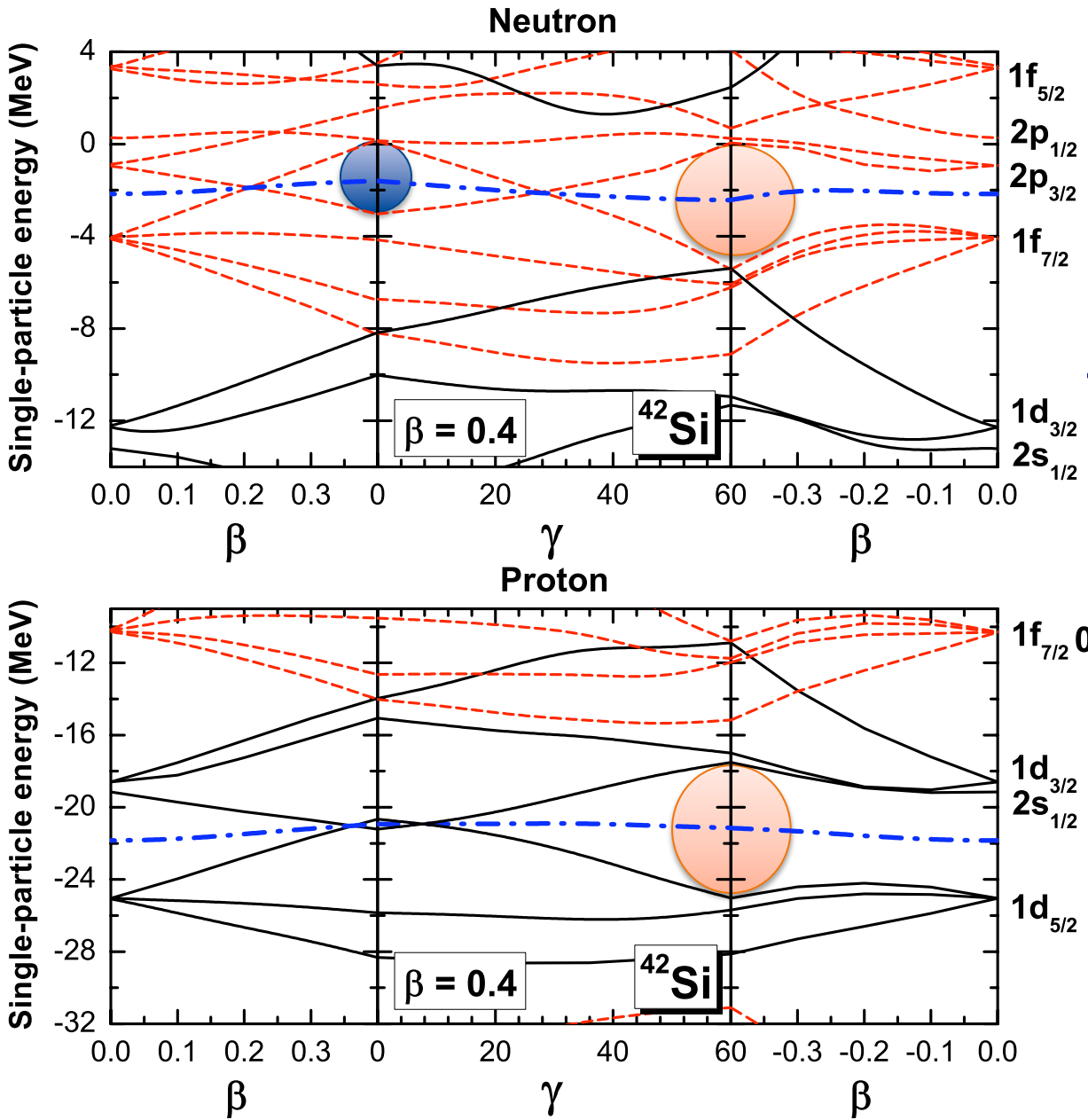
^{44}S: single-particle levels

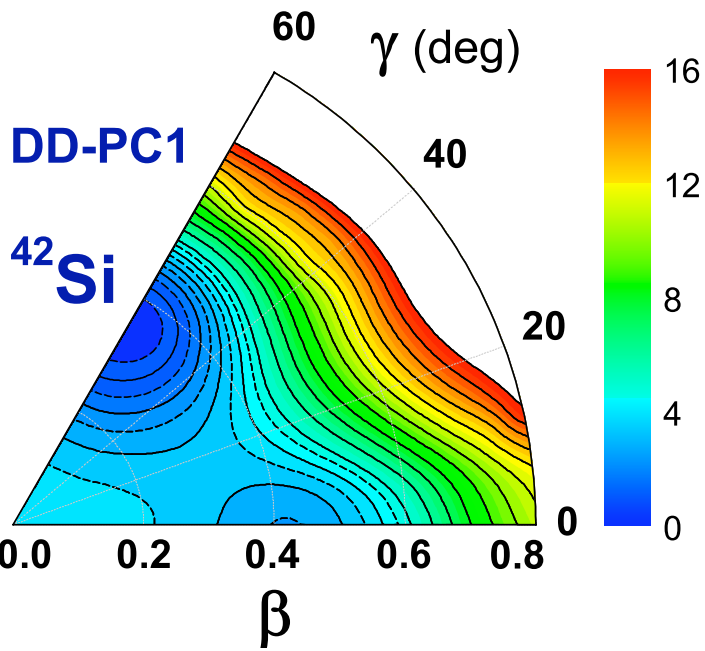


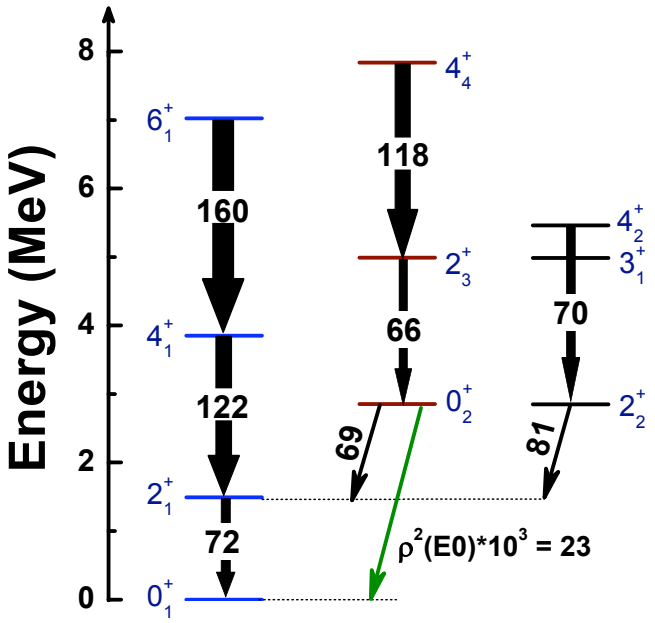




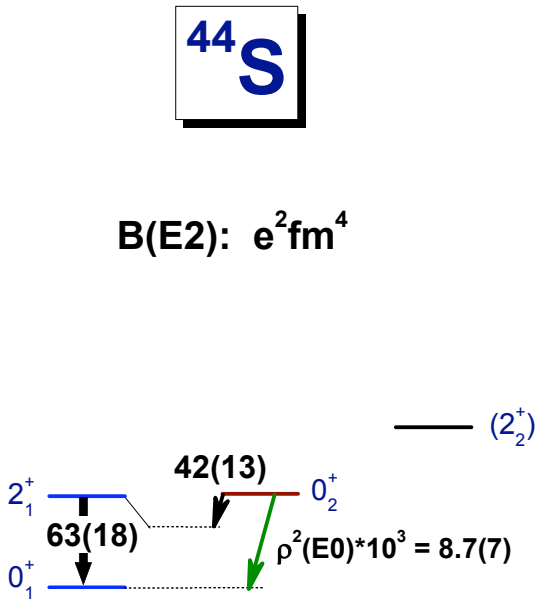
^{42}Si: single-particle levels



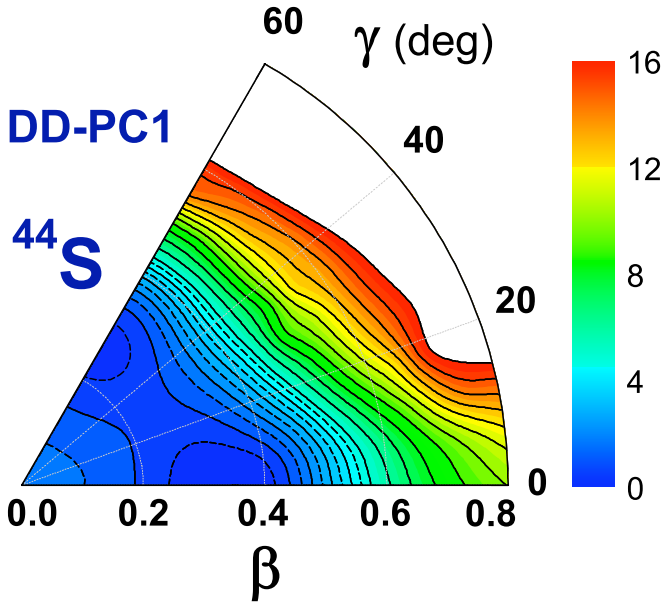




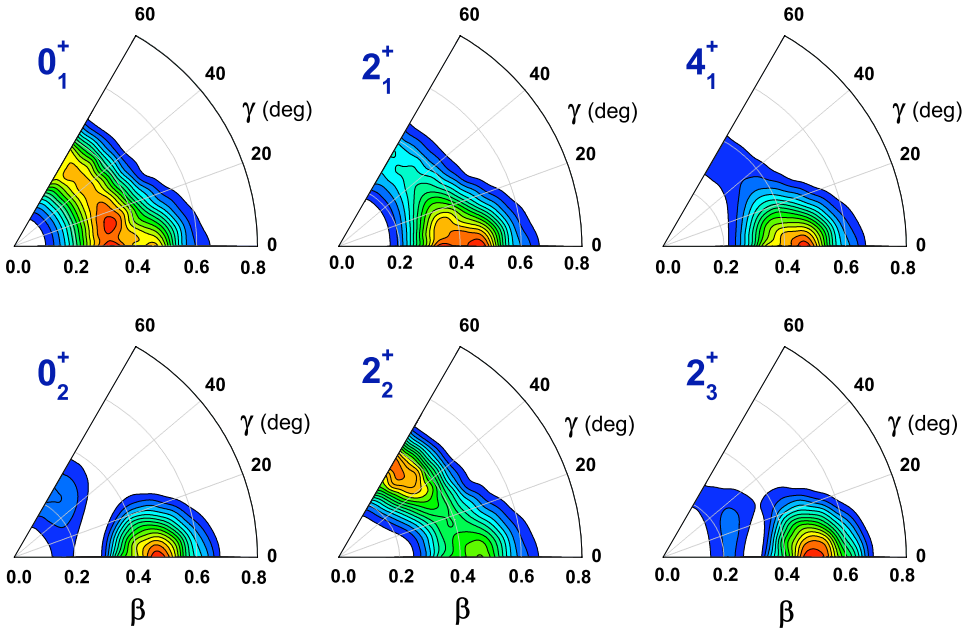
DD-PC1



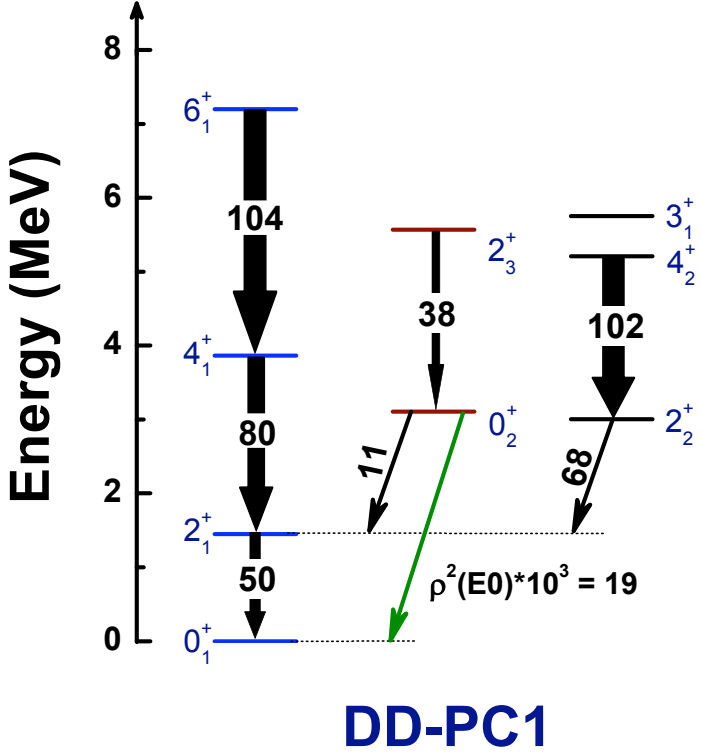
Exp.



Probability density distributions:



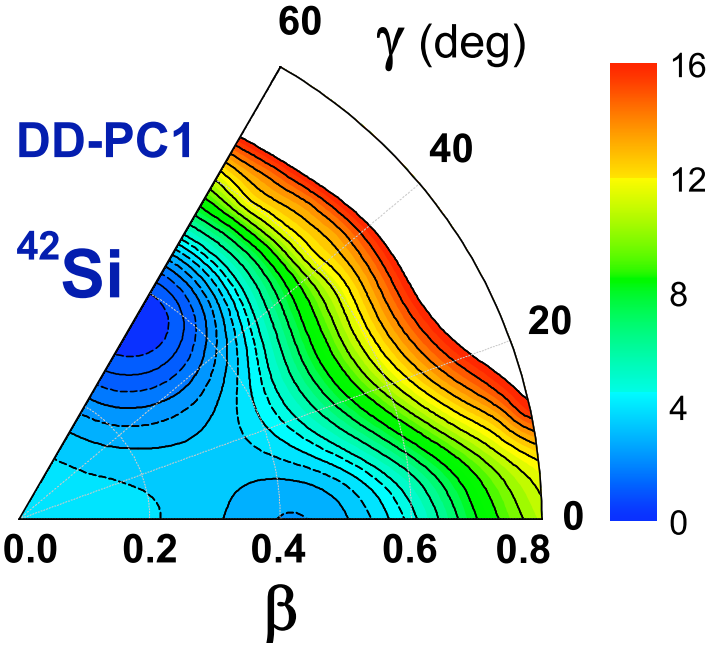
	$K=0$	$K=2$	$Q_{\rm spec.}$
2_1^+	88.4	11.6	-10.9
2_2^+	21.5	78.5	7.8
2_3^+	80.0	20.0	-9.6



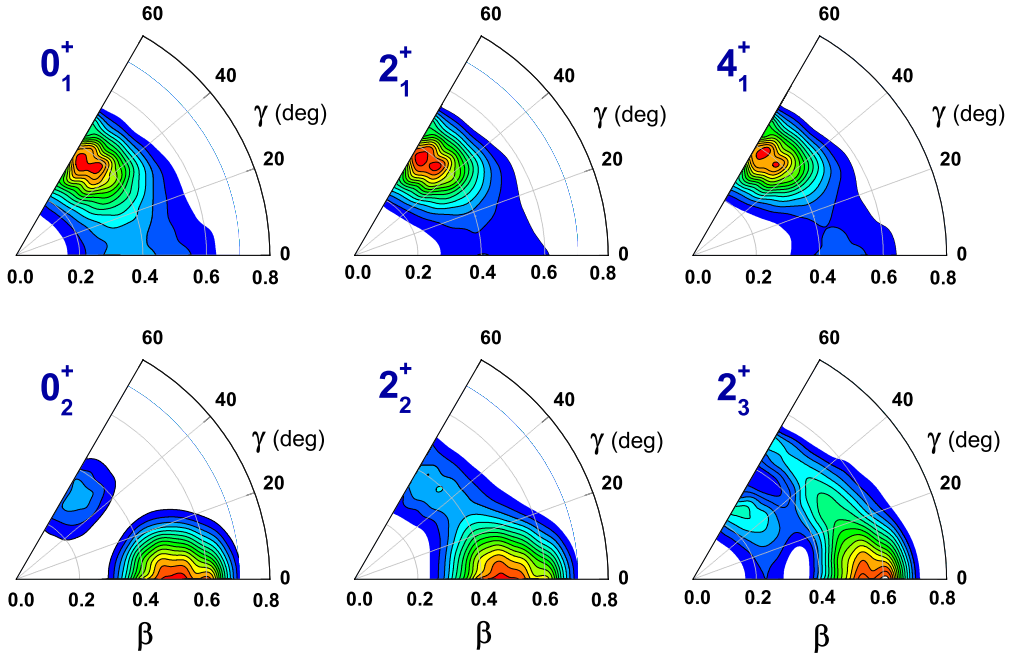
DD-PC1

^{42}Si

Exp.



Probability density distributions:



E. Global study of quadrupole correlation effects

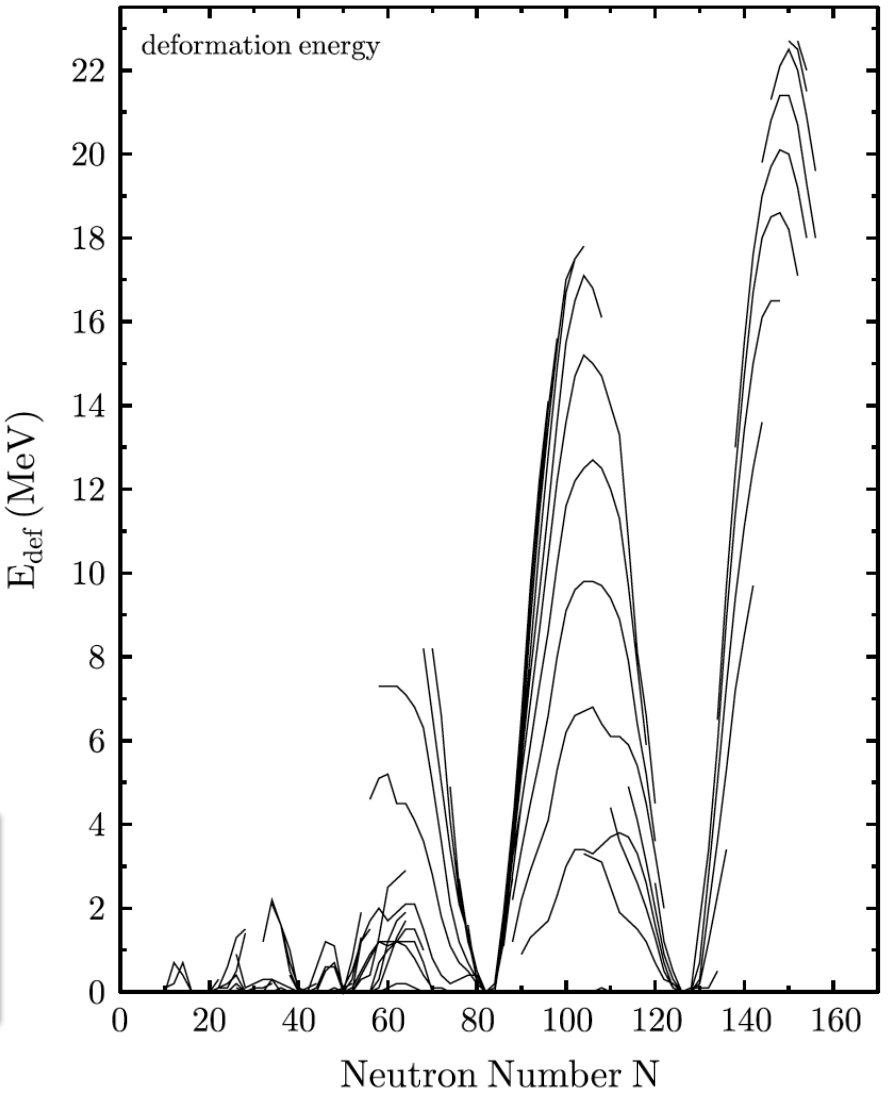
M. Bender, G. F. Bertsch, and P.-H. Heenen

Phys. Rev. C 73, 034322

Definition of correlation energies

1) The ***static deformation energy*** is the energy difference between a mean-field configuration q and the corresponding spherical state:

$$E_{\rm def}(q) = E(Q_2 = 0) - E(q)$$



Static deformation energy as a function of neutron number N. Isotopic chains are connected by lines.

2) The energy gained by the projection of a deformed mean-field state |q > (on angular momentum I=0) is its ***rotational energy***:

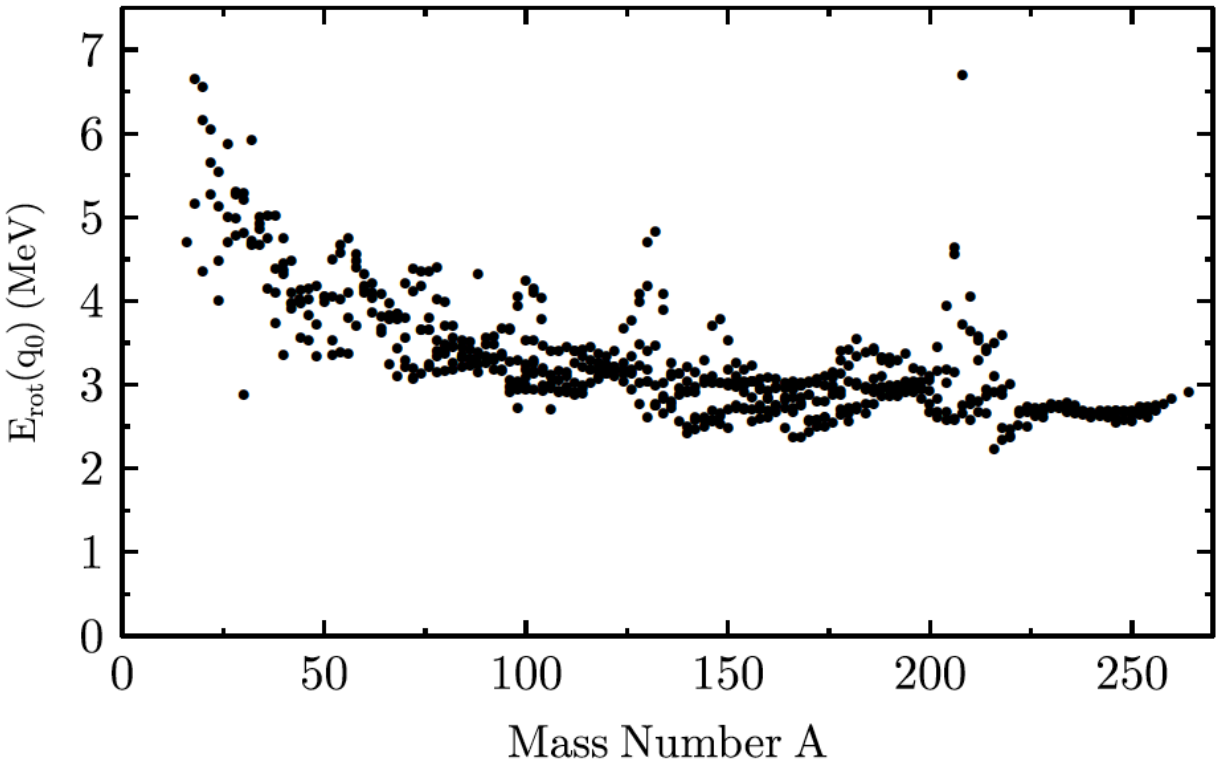
$$E_{\rm rot}(q) = E(q) - E_0(q)$$

3) The ***rotational energy correction***:

$$E_{I=0} = E(q_{\rm mf}) - E_0(q_0)$$

mean-field minimun

minimum after projection



Rotational energy $E_{rot}(q_0)$ at the minimum of the J = 0 projected energy curve.

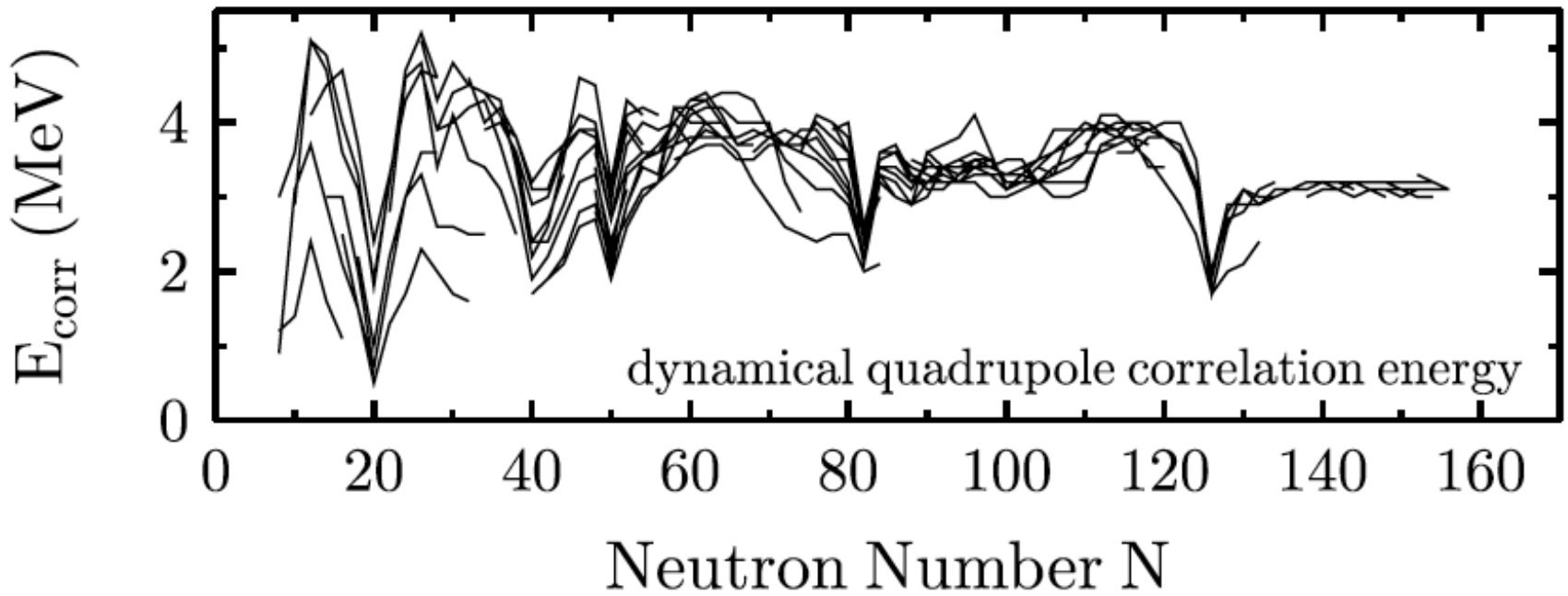
4) The correlation energy gained by configuration mixing:

$$E_{\mathrm{GCM}} = E_0(q_0) - E_{k=0}$$

GCM ground state

The total ***dynamical correlation energy*** is the energy difference between the mean-field ground state and the projected GCM ground state:

$$\begin{aligned} E_{\mathrm{corr}} &= E(q_{\mathrm{mf}}) - E_{k=0} \\ &= E_{I=0} + E_{\mathrm{GCM}} \end{aligned}$$



(i) The quadrupole correlation energy varies between a few 100 keV and about 5.5 MeV.

(ii) Projection on angular momentum J = 0 provides the major part of the energy gain of up to about 4 MeV; all nuclei gain energy by deformation.

(iii) the mixing of projected states with different intrinsic axial deformation adds a few 100 keV up to 1.5 MeV to the correlation energy.

(iv) Typically nuclei below mass A ≤ 60 have a larger correlation energy than static deformation energy, whereas the heavier deformed nuclei have larger static deformation energy than correlation energy.