Nuclear Structure Information from Peripheral Nuclear Reactions

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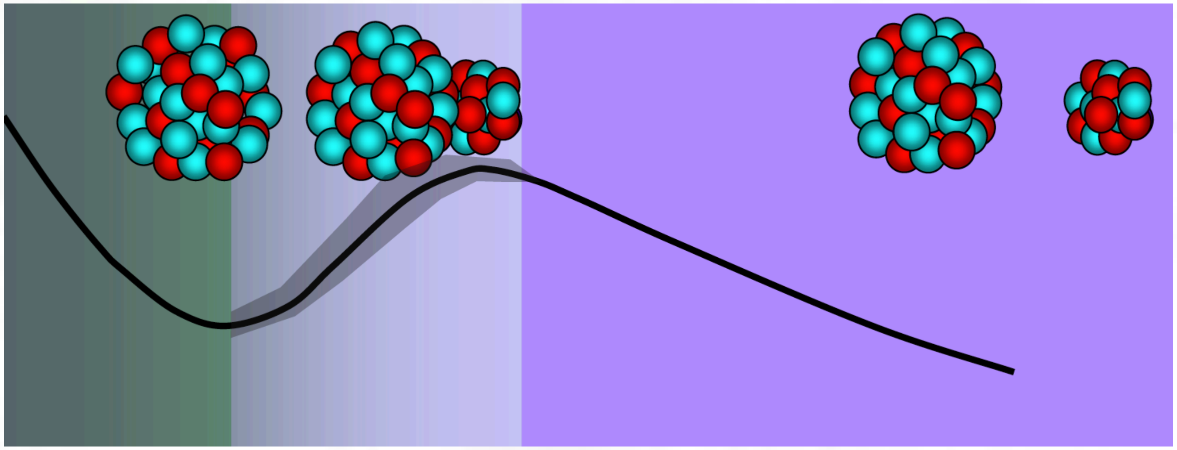
Lecture 4

Fusion & Transfer

Fusion

Fusion of heavy nuclei



Classical area: $2\pi\int b\,db$

Quantum area. Since $l+\frac{1}{2}=kb$ → $b\,db=\frac{l+1/2}{k^2}\Delta l$

$\Delta l = 1$ ⟶ $2\pi\int b\,db=\frac{\pi}{k^2}\sum_{l}(2l+1)$

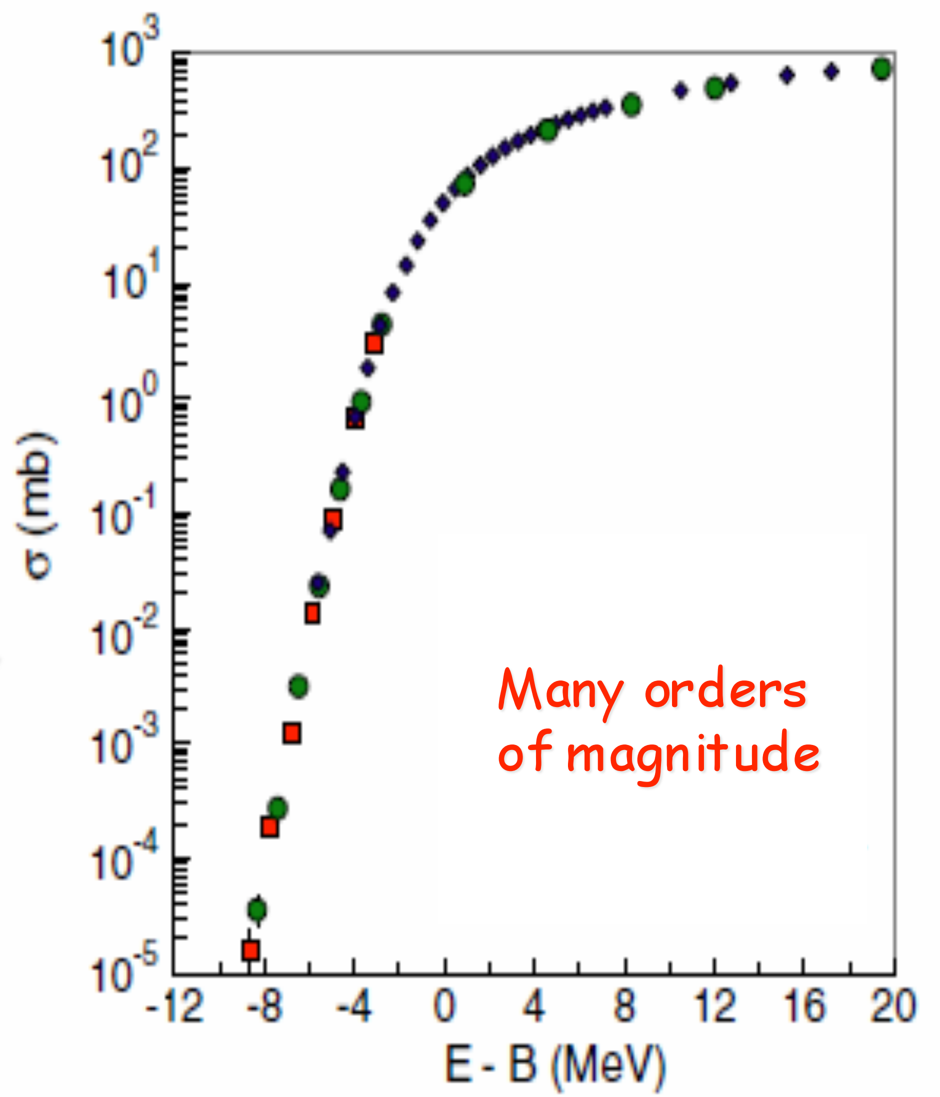
Fusion of heavy nuclei

Fusion cross section

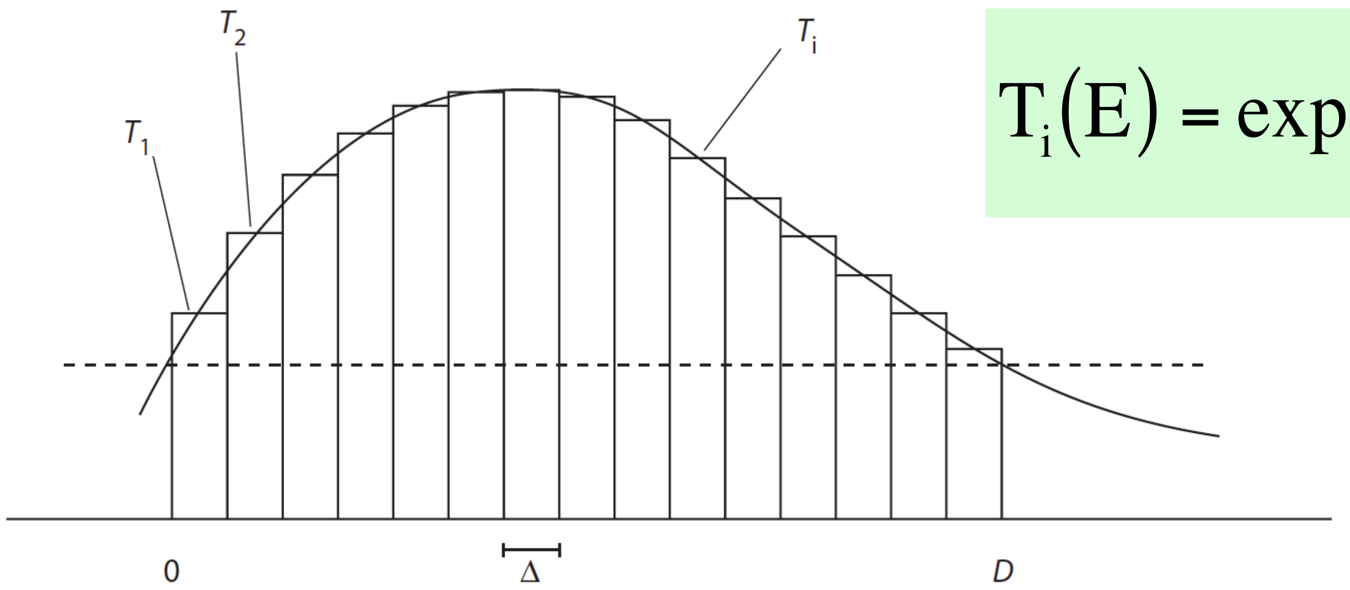
$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_l (2l+1)T_l(E)$$

Fusion or transmission probability
or
"Penetrability factor"

Decreases exponentially with barrier width



Barrier penetration model



$$T_i(E) = \exp\left[-\frac{2}{\hbar}\Delta\sqrt{2\mu(U-E)}\right]$$

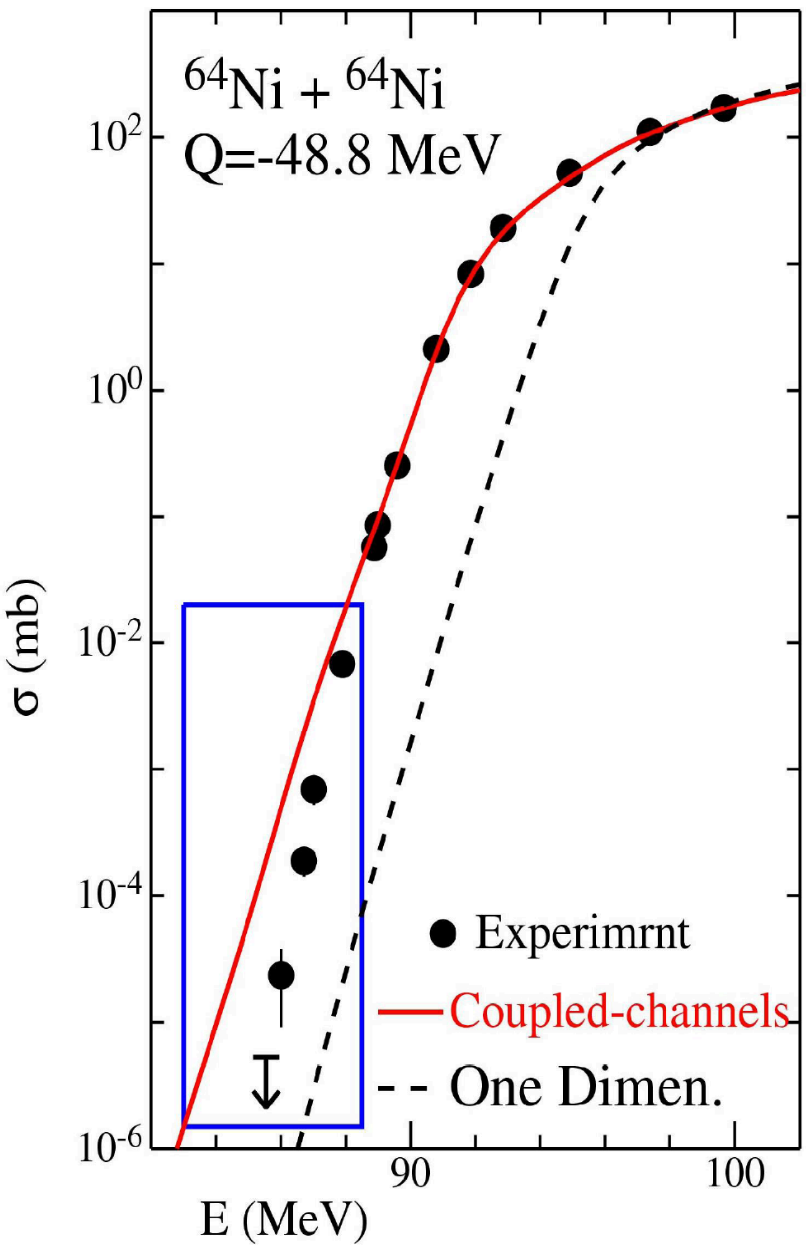
$$T(E) = \prod_i T_i(E)$$

(e.g., WKB approximation)

$$T_l(E) = \exp\left[-\frac{2}{\hbar}\int_{R_1}^{R_2} dr\sqrt{2\mu\left[U_l(r)-E\right]}\right]$$

Barrier Penetration Model (BPM)

Fusion

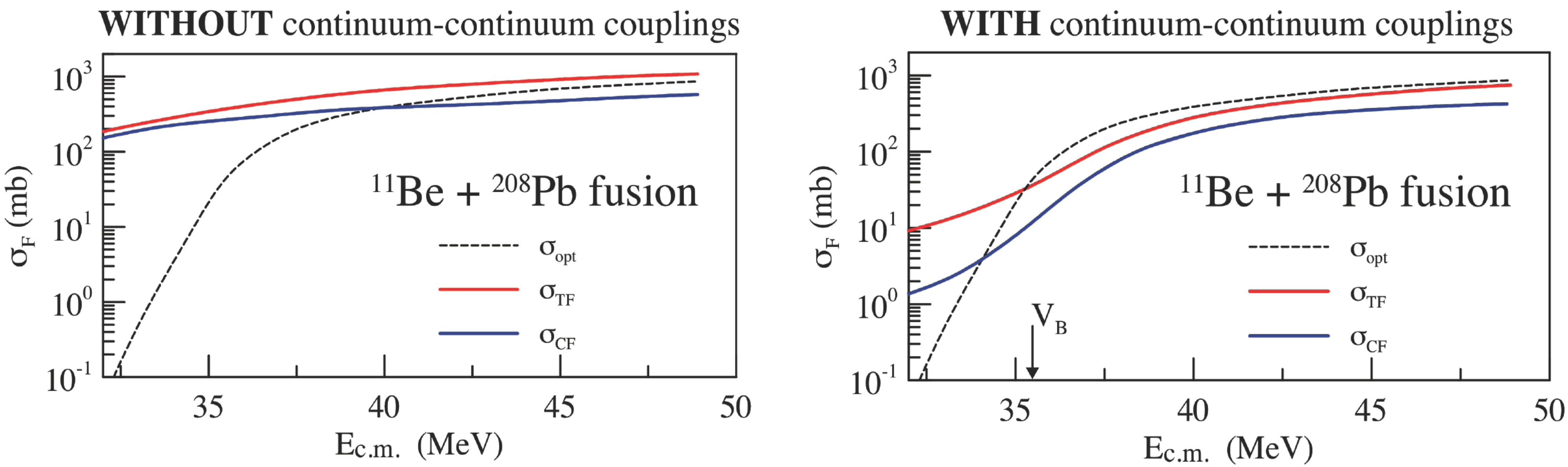


Jiang et al, PRL 93, 012701 (2004)

Often BPM does not work

Coupled-channels and/or
microscopic models often necessary.

CDCC fusion calculations



Canto, Fusion 2011

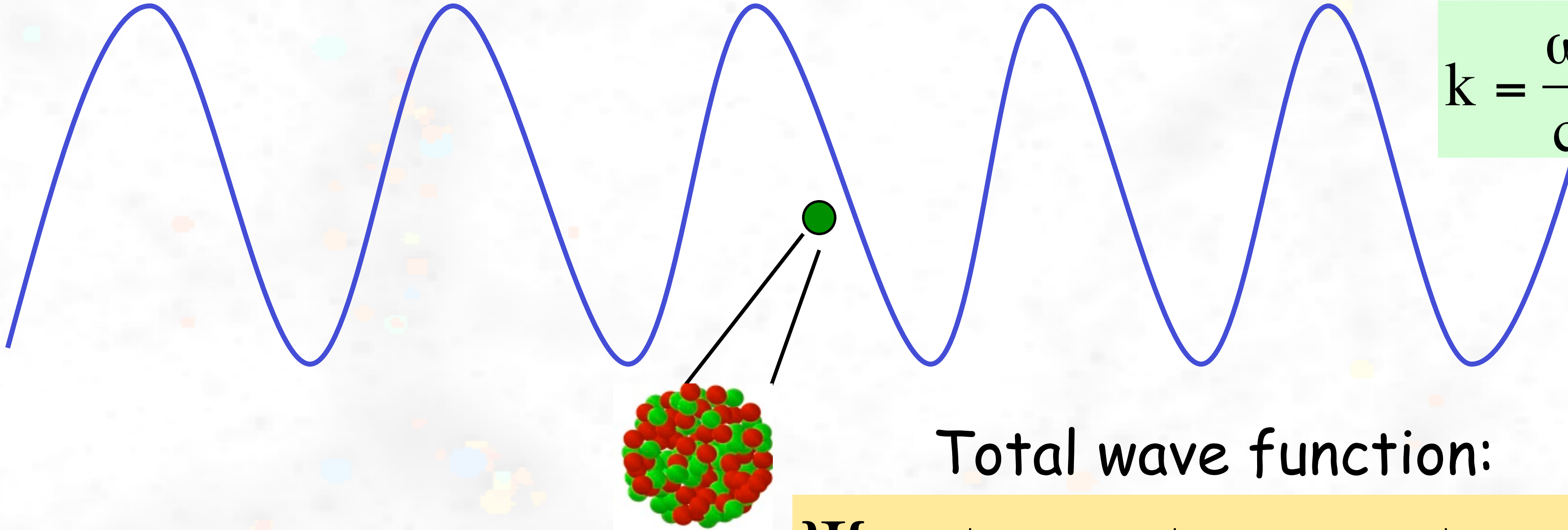
Continuum-continuum couplings hinders fusion …. but what is the mechanism?

Coupled channels one of the least controllable calculations: couplings can add as +-+--++-+ or as +++-+++ or ----+----, depending on the system

→ Suppression or enhancements are difficult to understand.

Fusion + Photon Emission
(radiative capture)

Nucleus + photon



$$k = \frac{\omega}{c} = \frac{E_\gamma}{\hbar c}$$

Total wave function:

$$\Psi \sim \psi_{\mathrm{nucleus}}\,\psi_{\mathrm{phtoon}} \sim \psi_{\mathrm{nucleus}}\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$e^{i\mathbf{k}\cdot\mathbf{r}} = 4\pi \sum_{l} i^{l} j_{l}(kr)\, Y^{*}_{lm}(\vec{\mathbf{r}})\, Y_{lm}(\hat{\mathbf{k}})$$

Long wavelength, $r \ll 1/k = \lambda_\gamma$:

$$j_l(kr) \sim (kr)^l$$

➡ Transition amplitude for radiative capture:

$$\left\langle \psi^{(f)}_{\mathrm{nucleus}}\,\psi_{\mathrm{phtoon}} \middle| \psi^{(i)}_{\mathrm{nucleus}} \right\rangle \sim \left\langle \psi^{(f)} \middle| r^{l}\, Y_{lm}(\hat{\mathbf{r}}) \middle| \psi^{(i)} \right\rangle$$

Potential model

Tombrello, NP 71, 459 (1965)
Robertson, PRC 7 (1973) 543

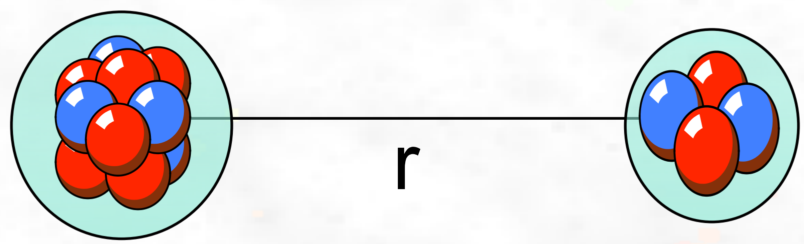
- Internal structure neglected
- Schrödinger equation:

Potential
Ex: Gauss,
Woods-Saxon

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi_l(r)+\left[V(r)+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]\psi_l(r)=E\psi_l(r)$$

Very simple to solve numerically for E < 0 or E > 0

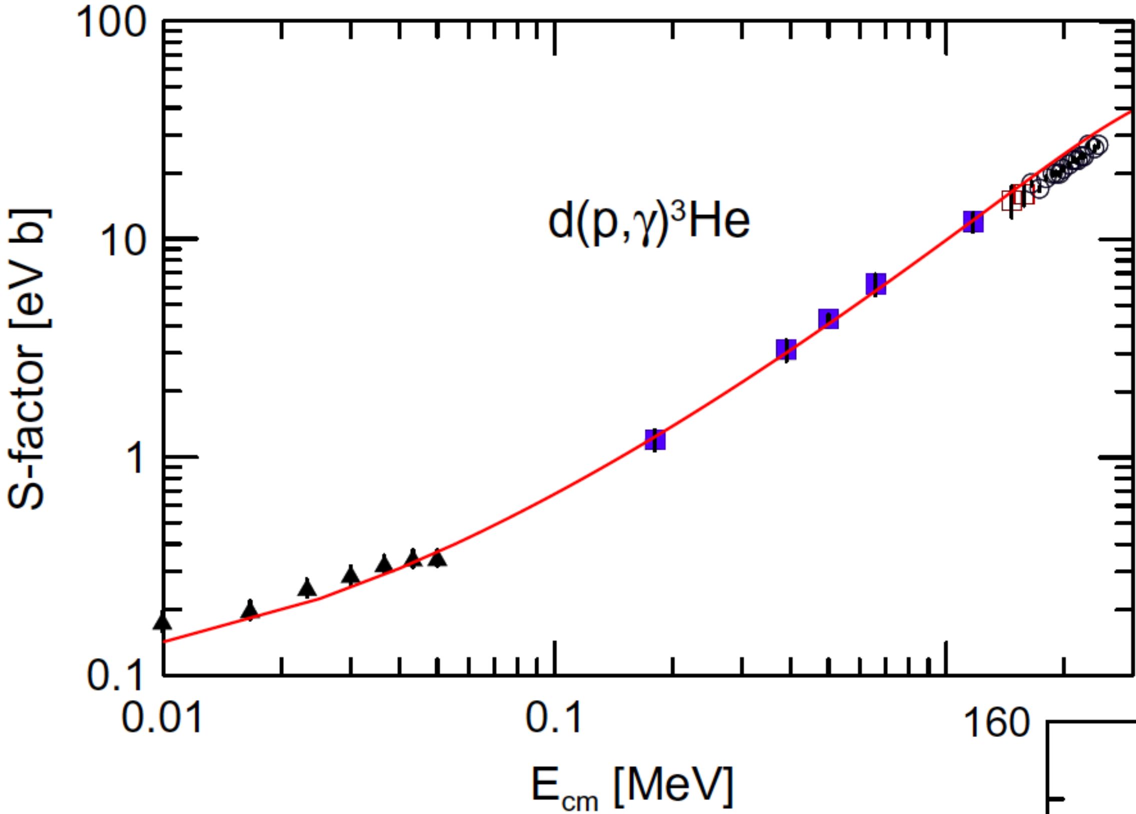
- initial state: scattering $E_i > 0$
  final state: bound $E_f < 0$



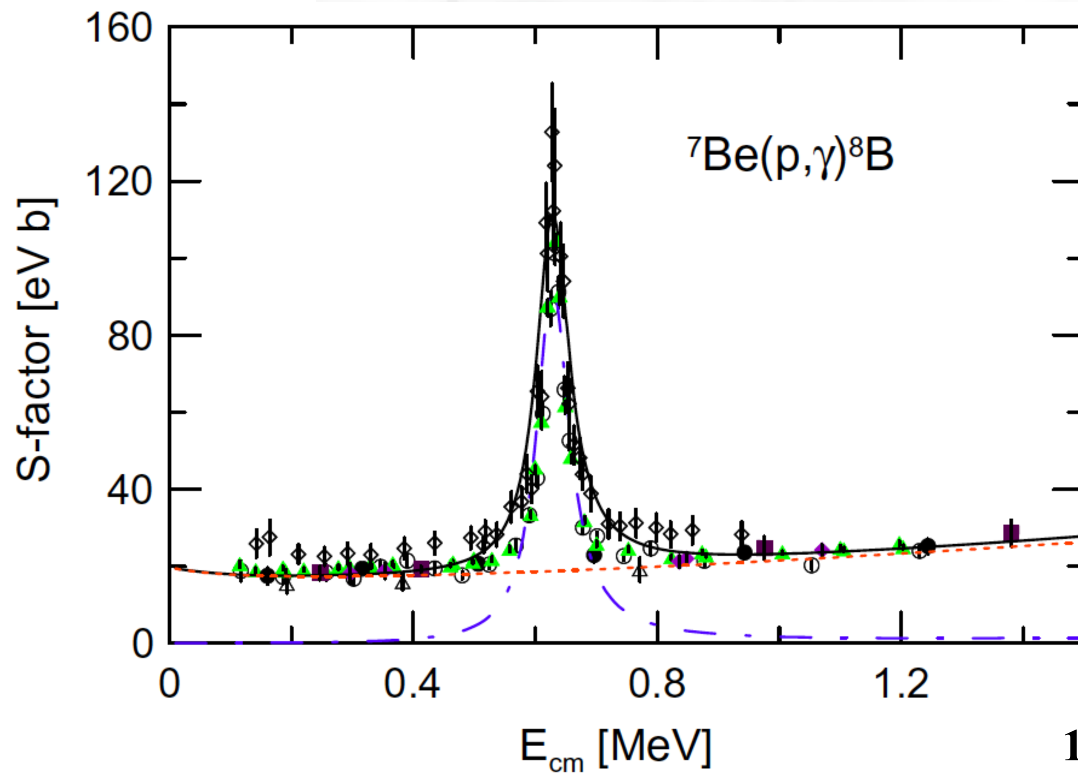
- Capture cross section (electric)

$$\sigma_{E\lambda} \sim \left|\int_0^\infty r^\lambda \psi^l(E_i,r)\psi^{l'}(E_f,r)r^2dr\right|^2$$

Potential model

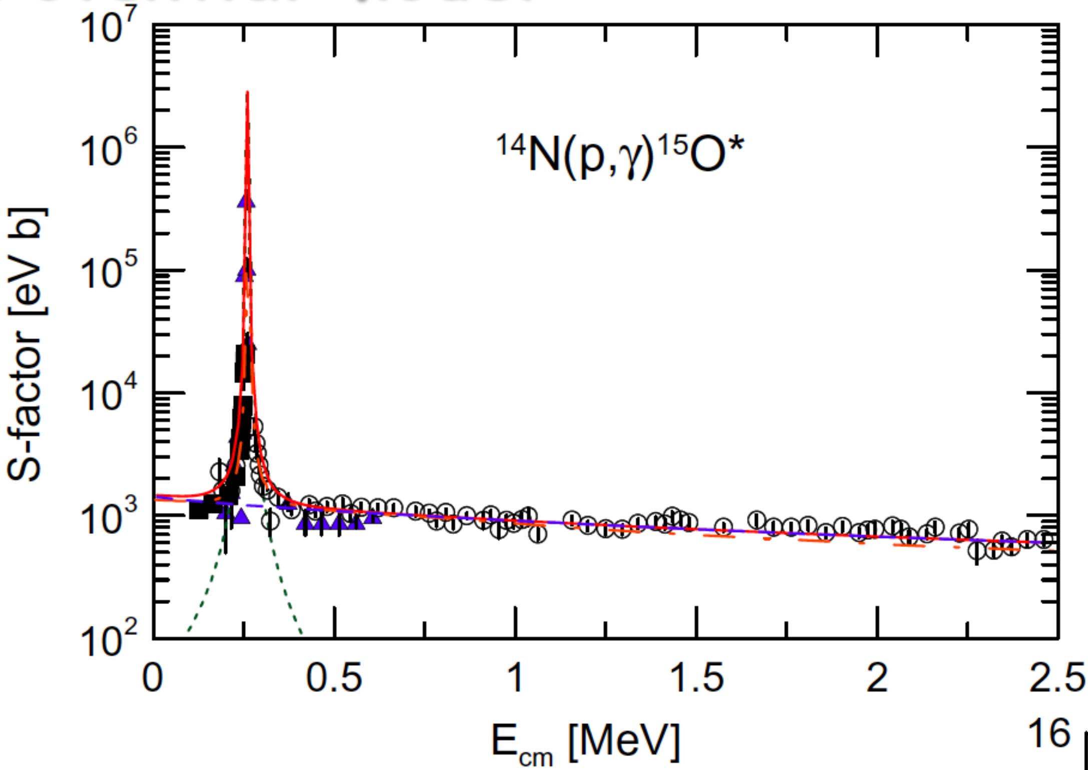


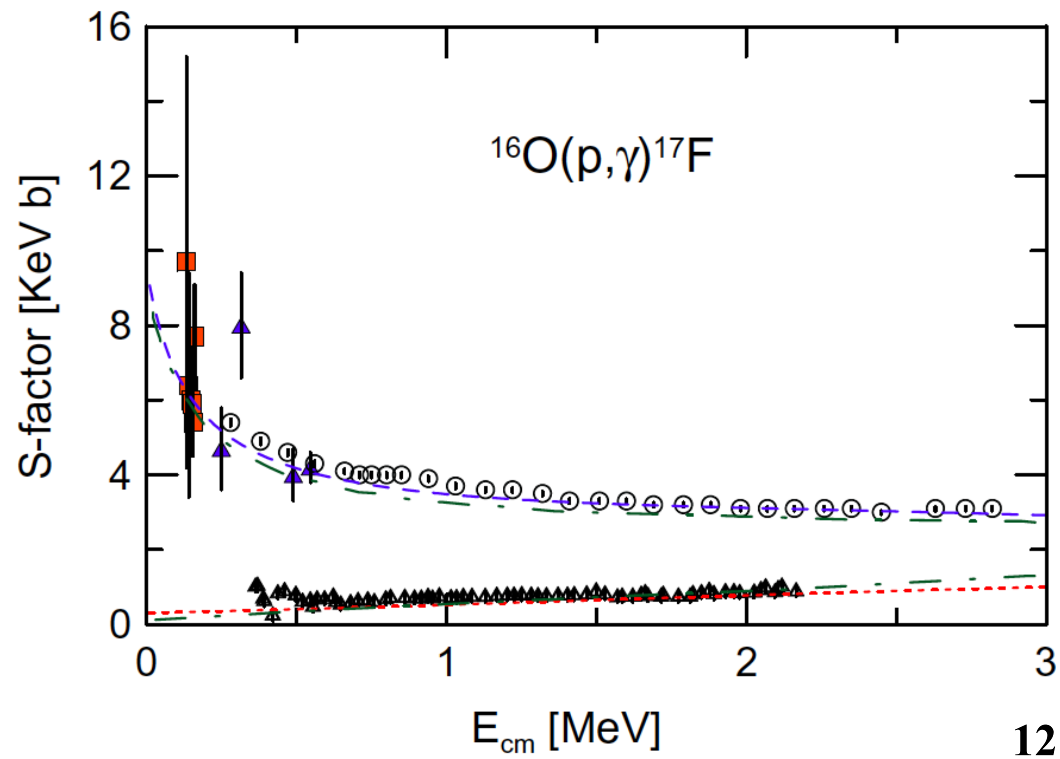
Huang, CB, Guimaraes, ADNDT 96, 824 (2010)



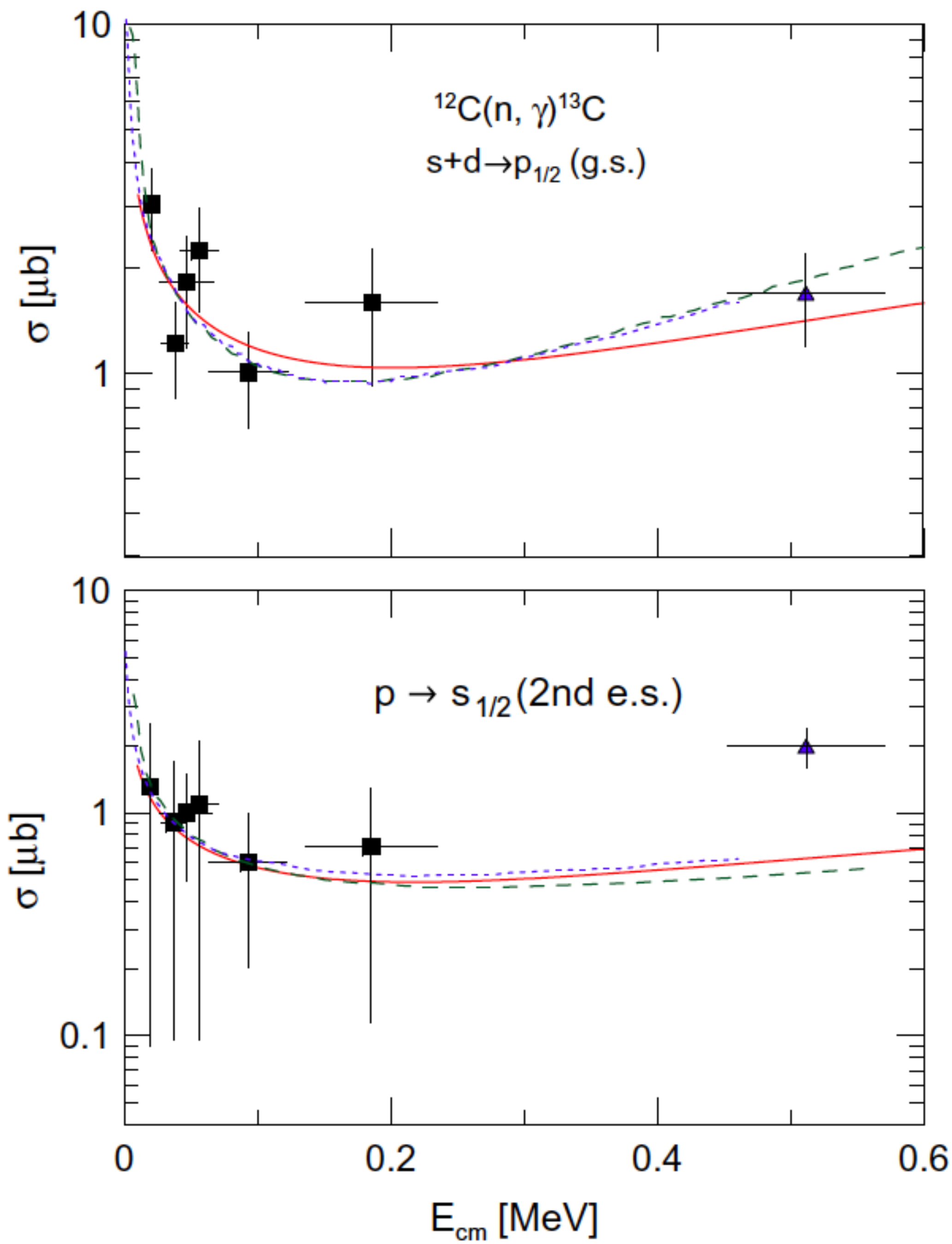
RADCAP, CB, Comp. Phys. Comm. 156 (2003) 123

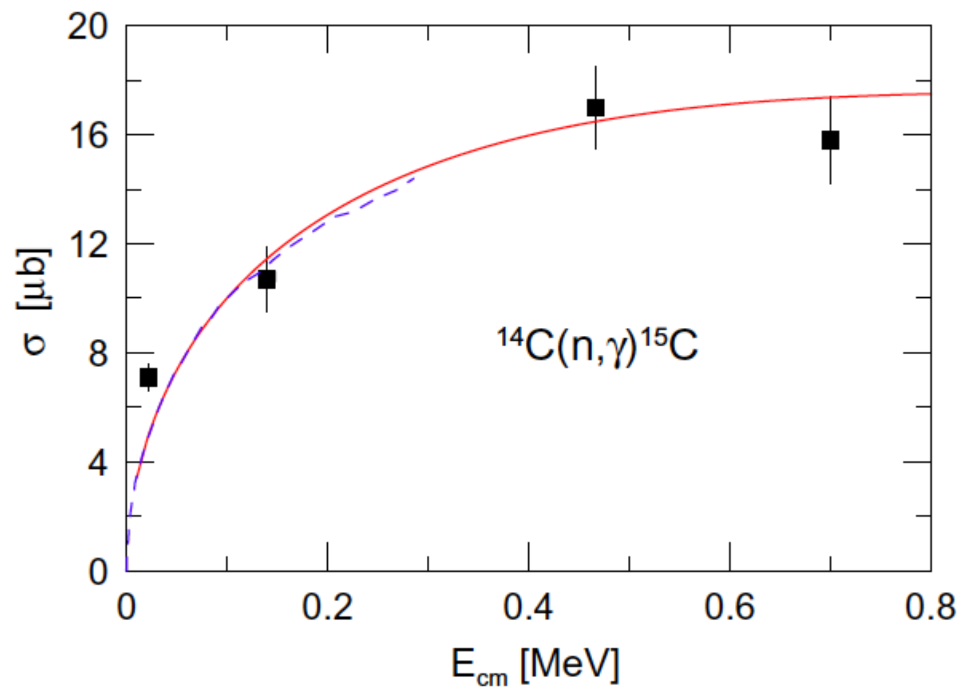
Potential model





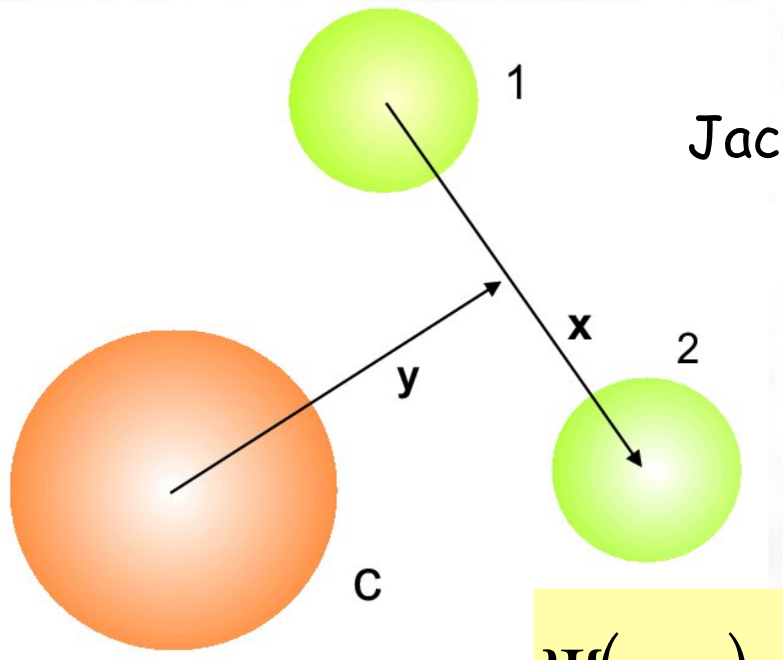
Potential model





Three-body potential models

3-body potential model - ex: hyperspherical harmonics



Jacobi coordinates (x, y)

Hyperspherical harmonics

$$\Psi(\mathbf{x},\mathbf{y}) = \frac{1}{\rho^{5/2}} \sum_{KLSl_xl_y} \Phi_{KLS}^{l_xl_y}(\rho)\left[\Gamma_{KL}^{l_xl_y}(\Omega_5) \otimes \chi_S\right]_{JM}$$

Hyperangle

$$\Omega_5 = (\theta_x, \phi_x, \theta_y, \phi_y, \theta)$$

$$y = \rho\sin\theta, \qquad x = \rho\cos\theta$$

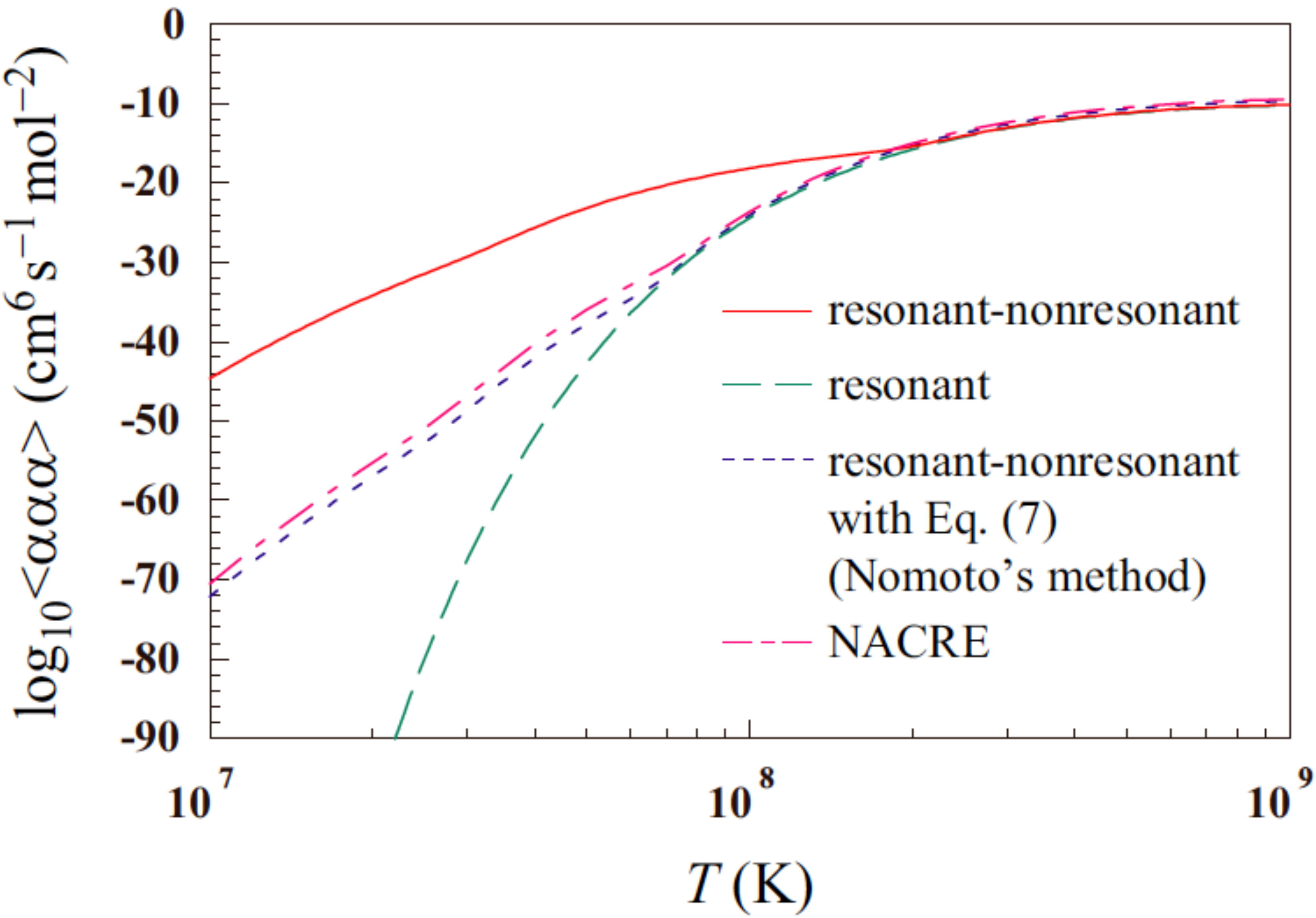
$$\sigma_{E1} \propto \left| \int dxdy \frac{\Phi_\alpha(\rho)}{\rho^{5/2}} y^2 x\, u_p(x) u_q(y) \right|^2$$

$$E_r = \frac{\hbar^2}{2m_N}(q^2 + p^2)$$

Triple-alpha reaction rate

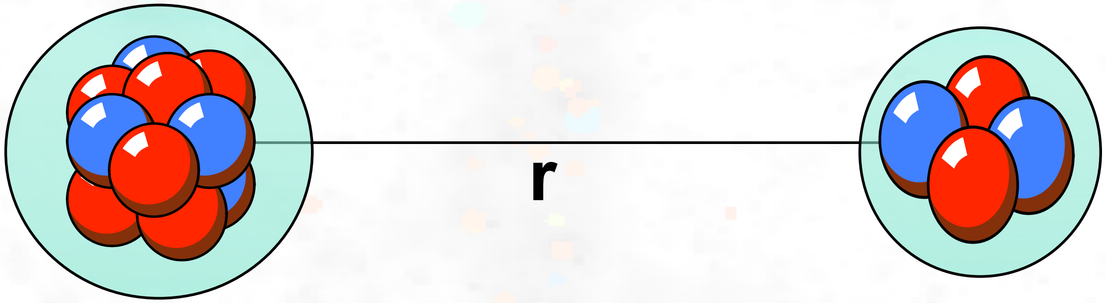
Ogata et al, Prog.Theor. Phys. 122, 1055 (2010)
finds 10^{20} factor larger than expected at low temperatures

See also, Suzuki, Descouvemont, arXiv:1308.4021



Microscopic cluster models

Resonating Group Method



Includes the internal structure of the nuclei: $\Phi_1\Phi_2$

antisymmerization

$$\Psi = \mathcal{A}\ \Phi_1\Phi_2\, g(r)$$

Hamiltonian:

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk}$$

T_i = kinetic energy of nucleon **i**

V_{ij} = nucleon-nucleon **effective** interaction

C_{lj} = ANC

$$g(r) = \left\langle \chi^{(12)} \middle| \hat{A}\Phi_1\ \Phi_2\,\delta(r - r_{1,2}) \right\rangle$$

$$g_{bound}(r) \to C_{lj}\,\frac{W_{-\eta,l+1/2}(r)}{r}$$

$$g_{scat}(r \to \infty) \sim I_l(r) - S_l O_l(r)$$

Resonating Group Method

Variational principle:

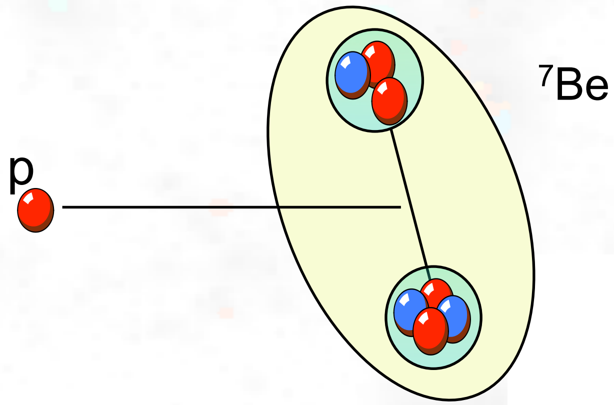
$$\delta\left[\frac{\langle\Psi_f|H|\Psi_i\rangle}{\langle\Psi_f|\Psi_i\rangle}\right]=0$$

$$\Rightarrow \quad \int dr'\left[H(r,r')-EN(r,r')\right]g(r')=0 \quad \text{all } r$$

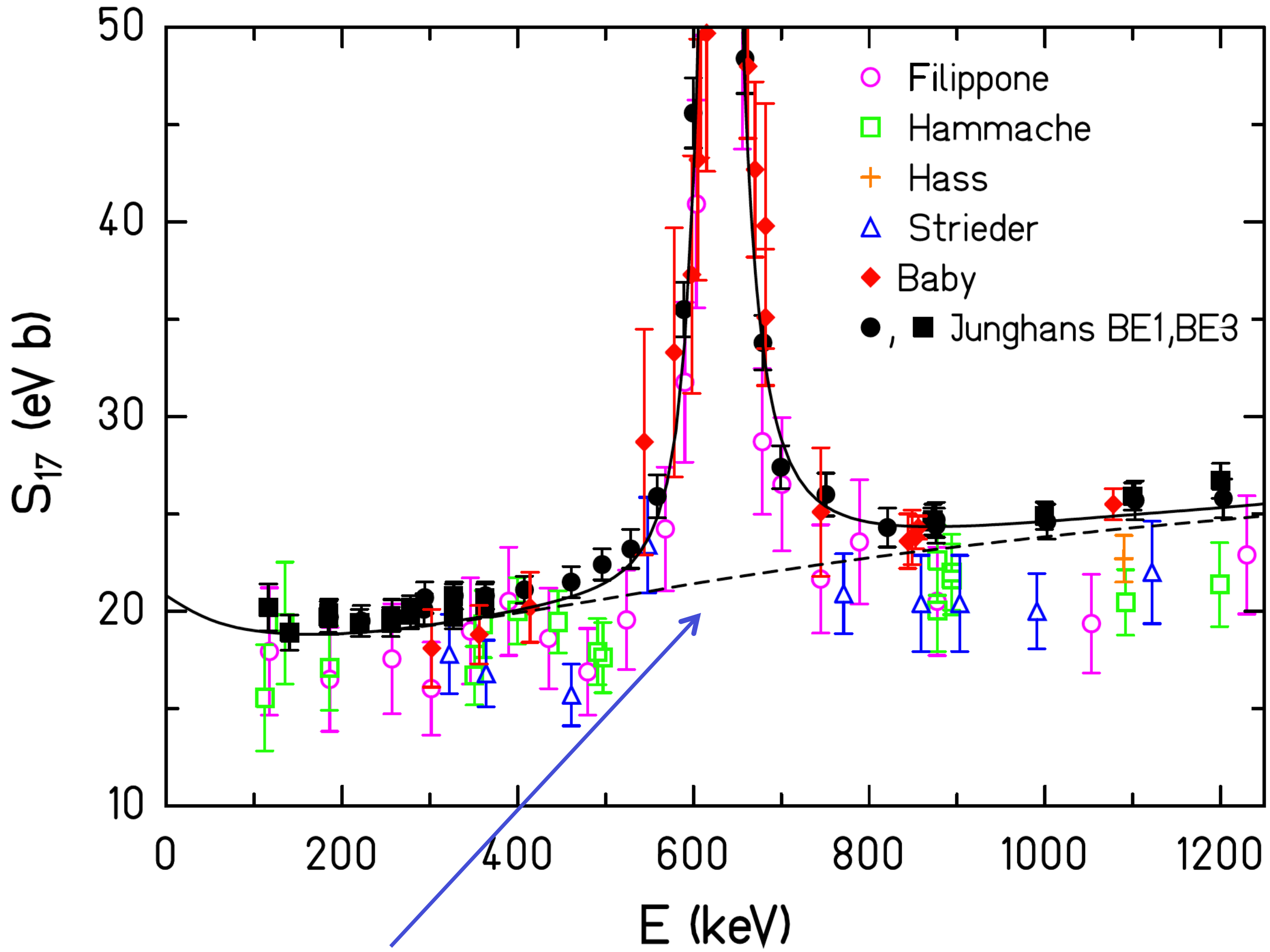
Hill-Wheeler (1955)

$$\begin{Bmatrix}H\\N\end{Bmatrix}(r,r')=\left\langle\left[\hat{\mathcal{A}}\,\Phi_1(r_{1i})\Phi_2(r_{2i})\right](r')\left|\begin{Bmatrix}\widehat{H}\\1\end{Bmatrix}\right|\left[\hat{\mathcal{A}}\,\Phi_1(r_{1i})\Phi_2(r_{2i})\right](r)\right\rangle$$

Resonating Group Method



- Volkov (gaussians) effective interactions



Descouvemont, Baye, NPA 567, 341 (1994)

"Ab-initio" models

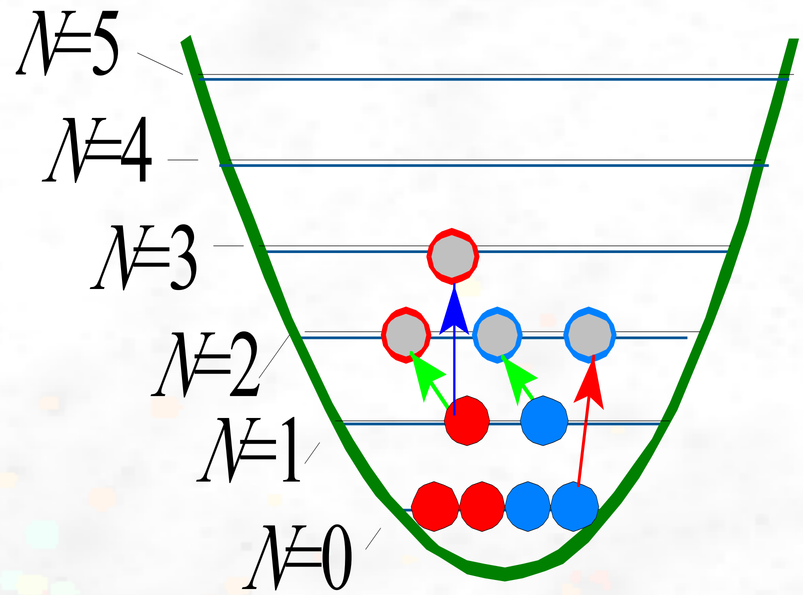
The ab-initio No-Core Shell Model

$$H = T_{rel} + V_{NN} + V_{3N} + \bullet\bullet\bullet$$

$$H|\Psi_i\rangle = E_i|\Psi_i\rangle$$

$$|\Psi_i\rangle = \sum_{n=0}^{\infty} A_n^i |\Phi_n\rangle$$

$$\text{Diagonalize } \{\langle\Phi_m|H|\Phi_n\rangle\}$$



- φ_n = finite harmonic oscillator (HO) basis
    - Jacobi relative coordinates
    - $N_{max}\hbar\Omega$ model space
    - Translational invariance preserved
    - Effective interaction tailored to model-space truncation

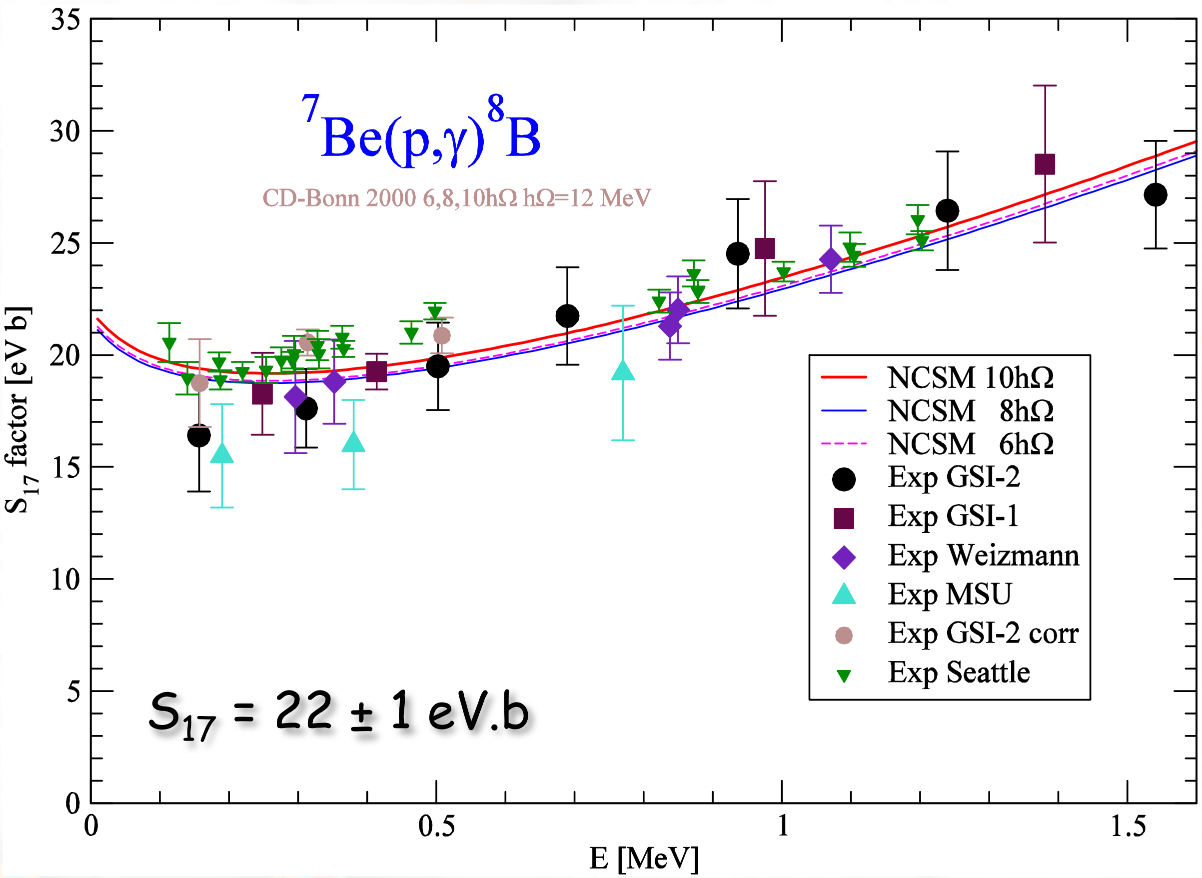
Navrátil, Vary, Barrett, PRC 62, 054311 (2000)

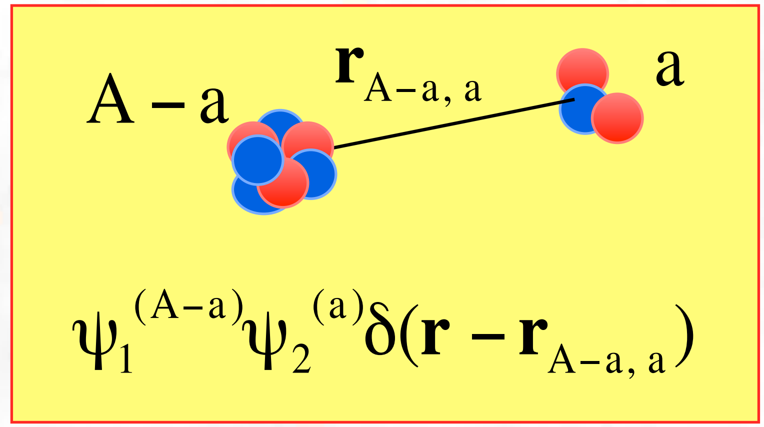
- **Convergence to exact solution with increasing N_{max} for bound states.** **No coupling to continuum**.

"Ab-initio" model for LE reactions

- Accurate wave functions of ^{7}Be – Ab-initio calculations

Navratil, CB, Caurier, PLB 2006
Navratil, CB, Caurier, PRC 2006





Eigenstates of $H_{(A-a)}$ and $H_{(a)}$ in the **ab initio** NCSM basis

Continuum = potential model

"Ab-initio" model with continuum

What one needs

$$g(r) = \left\langle \chi^{(A)} \middle| \hat{\mathcal{A}} \Phi^{(A-a)} \Phi^{(a)} \delta(r - r_{A-a,a}) \right\rangle$$

$$g_{\text{bound}}(r) \to C_{lj} \frac{W_{-\eta, l+1/2}(r)}{r}$$

$$g_{\text{scat}}(r \to \infty) \sim I_l(r) - S_l O_l(r)$$

What one does   Quaglioni, Navratil, PRL 2007, PRL 2009

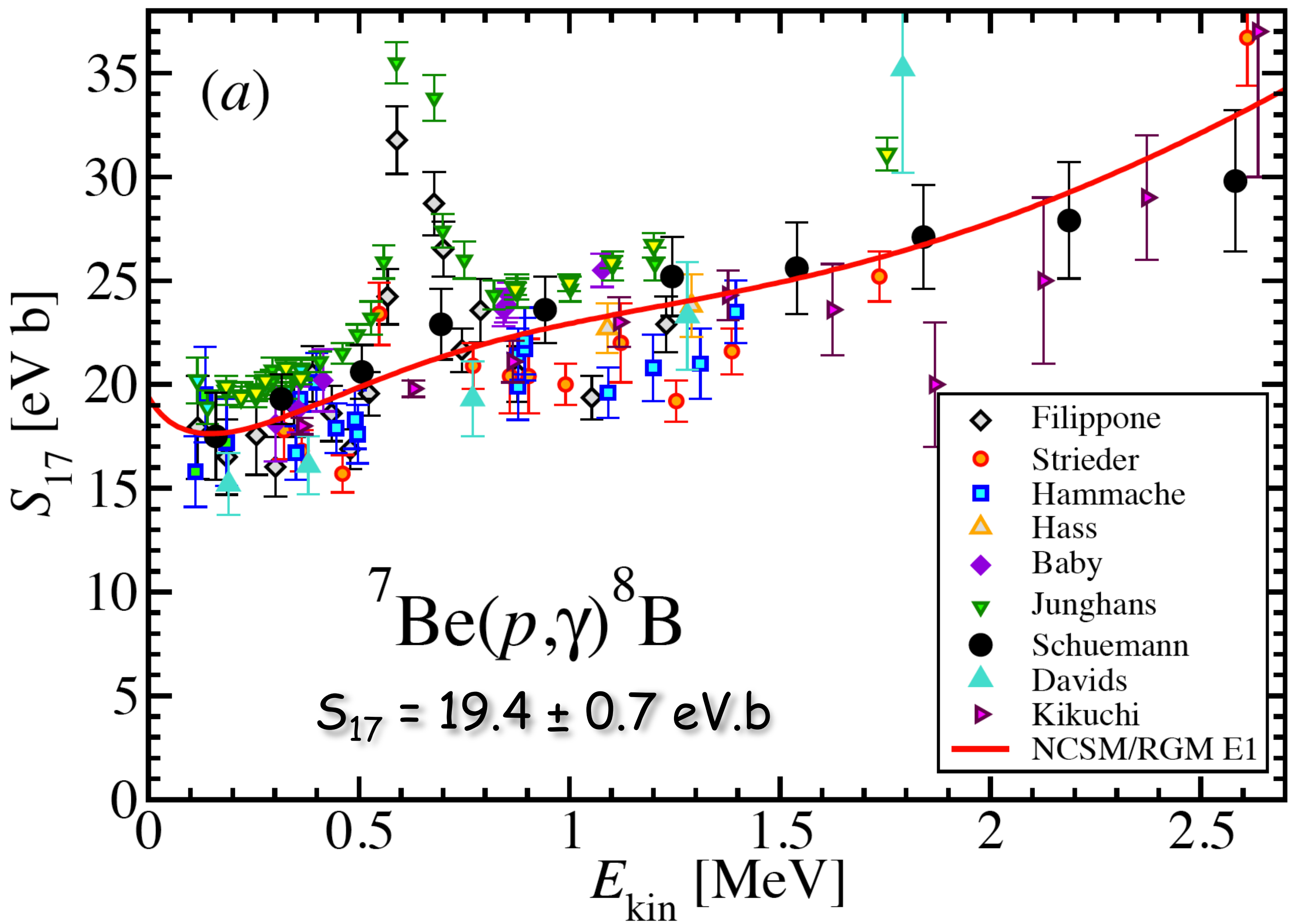
$$\int dr' \left[H(r,r') - EN(r,r') \right] g(r') = 0 \quad \text{all } r$$

Hill-Wheeler (1955)

$$\begin{Bmatrix} H \\ N \end{Bmatrix}(r,r') = \left\langle \hat{\mathcal{A}} \Phi^{(A-a)} \Phi^{(a)}(r') \middle| \begin{Bmatrix} H \\ N \end{Bmatrix} \middle| \hat{\mathcal{A}} \Phi^{(A-a)} \Phi^{(a)}(r) \right\rangle$$

e.g. NCSM

No core shell model + Hill-Wheeler



Quaglioni, Navratil, Roth, PLB 704 (2011) 379

Resonance manipulation models

(R-matrix)

R-matrix theory

Instead of

$$-\frac{\hbar^2}{2\mu}\frac{d^2\Psi}{dr^2} + V\Psi = E\Psi$$

solve

$$-\frac{\hbar^2}{2\mu}\frac{d^2\varphi_\lambda}{dr^2} + V\varphi_\lambda = E_\lambda\varphi_\lambda$$

with boundary conditions

$$r\left.\frac{d\varphi_\lambda/dr}{\varphi_\lambda}\right|_a = -b$$

a = channel radius
b = positive constant

ϕ_λ a complete basis: $\Psi = \sum_\lambda A_\lambda \varphi_\lambda$

$$\Rightarrow \quad \left.\frac{r\,d\Psi/dr}{\Psi}\right|_a = \frac{1-b\mathcal{R}}{\mathcal{R}}$$

$$\mathcal{R} = \sum_\lambda \frac{\gamma_\lambda^{\;2}}{E_\lambda - E}$$

R-matrix
(here one channel)

$$\gamma_\lambda^2 = \frac{\hbar^2\varphi_\lambda^2(a)}{2\mu a}$$

reduced width
(does not depend on E)

One-channel R-Matrix theory

Outside the channel radius a: $\Psi \sim \underbrace{I}_{e^{-ikr}} + \underbrace{\mathcal{S}}_{\text{S-matrix}} \underbrace{O}_{e^{ikr}}$

Assume E near one of the E_λ → neglect all but channel α

Channel radius matching condition $\left.\dfrac{r\,d\Psi/dr}{\Psi}\right|_a = \dfrac{1 - b\mathcal{R}}{\mathcal{R}}$

$$\Gamma_\alpha = \frac{\hbar^2 k \varphi_\alpha^2(a)}{\mu}$$

resonance width

$$\Rightarrow \quad \mathcal{S} = \left[1 + \frac{i\Gamma_\alpha}{\left(E_\alpha + \Delta_\alpha - E\right) - i\Gamma_\alpha/2}\right] e^{-2ika}$$

$$\Delta_\alpha = -\frac{b\Gamma_\alpha}{2ka}$$

resonance shift

$$\sigma = \frac{\pi}{k^2}\left|1 - \mathcal{S}\right|^2 = \frac{\pi}{k^2}\left|\underbrace{e^{2ika} - 1}_{\text{non-resonant}} + \underbrace{\frac{i\Gamma_\alpha}{\left(E_\alpha + \Delta_\alpha - E\right) - i\Gamma_\alpha/2}}_{\text{Breit-Wigner}}\right|^2$$

Multi-channel R-Matrix theory

Generalize to possible many channels also using many a's

$$\mathcal{R}_{\alpha\alpha'} = \sum_{\lambda} \frac{\gamma_{\lambda\alpha}\gamma_{\lambda\alpha'}}{E_{\lambda} - E}$$

$$\mathcal{S}_{\alpha\alpha'} = \frac{I(a)}{O(a)}\left[\frac{1 - L^{*}\mathcal{R}}{1 - L\mathcal{R}}\right]$$

$$L = \left.\frac{r\,d\Psi/dr}{\Psi}\right|_{a}$$

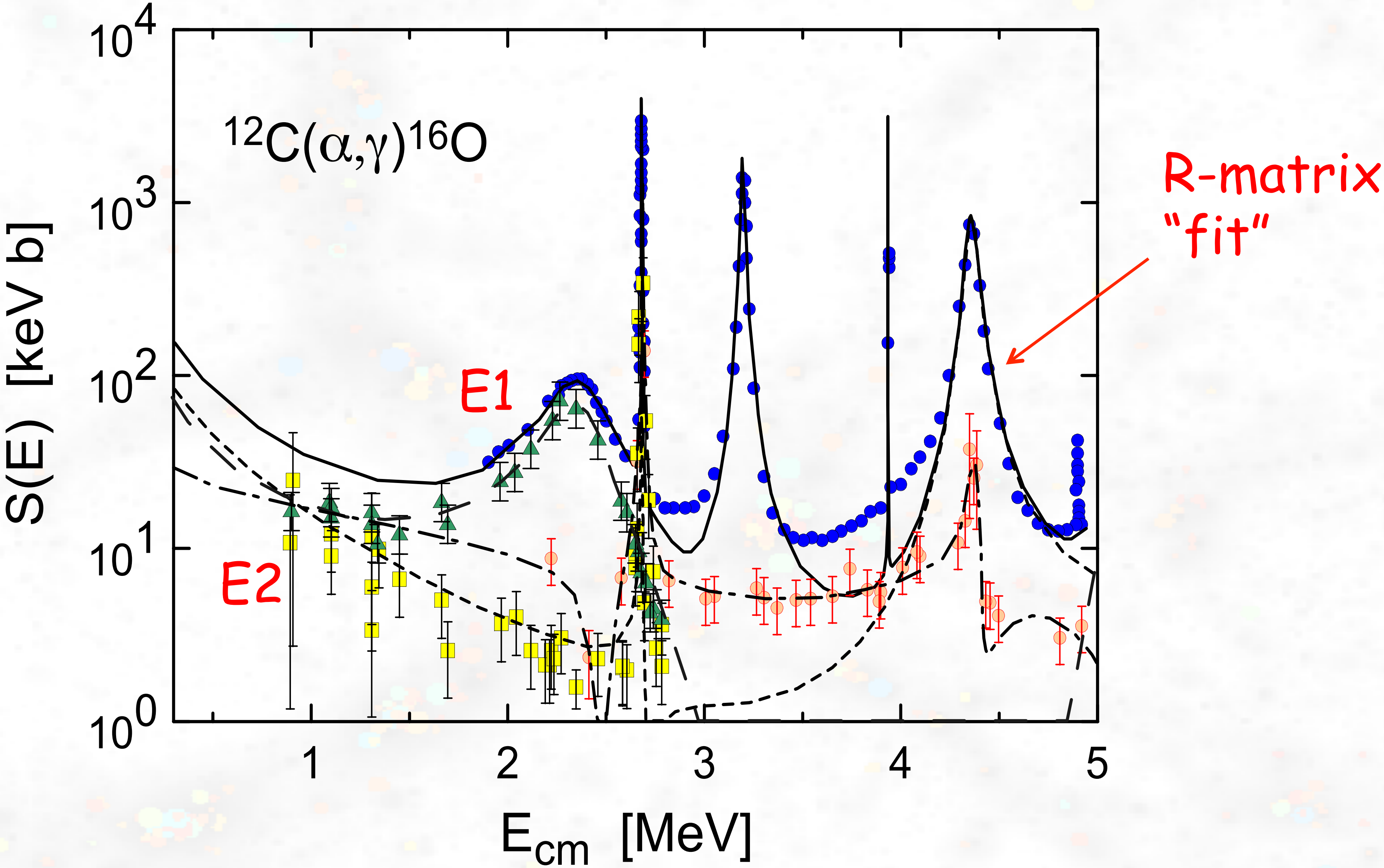
For each channel transition $\alpha \rightarrow \alpha'$

Finally:

$$\sigma_{\alpha\alpha'} = \frac{\pi}{k_{\alpha}^{2}}\left|\delta_{\alpha\alpha'} - \mathcal{S}_{\alpha\alpha'}\right|^{2^{2}} = \frac{\pi}{k_{\alpha}^{2}} T_{\alpha\alpha'}$$

transition (transmission) probability (matrix)

R-Matrix theory - example



Review: Descouvemont, Baye Rep. Prog. Phys. **73,** 036301 (2010)

Compound Nuclei

Compound nucleus theory

Heisenberg relation: $\Delta E \Delta t \sim \hbar$ → for a state with width Γ

→ decay time: $\Delta t \sim \dfrac{\hbar}{\Gamma_\alpha}$

If many decay channels → decay probability = $\dfrac{\Gamma_\alpha}{\sum_\alpha \Gamma_\alpha} = \dfrac{\Gamma_\alpha}{\Gamma}$

Bohr hypothesis: formation independent of decay

$$\sigma_{\alpha\alpha'} = \sigma_{CN}(\alpha)\frac{\Gamma_{\alpha'}}{\Gamma}$$

$$\left.\begin{array}{l} a + b \\ \text{or} \\ c + d \\ \text{or} \ldots \end{array}\right. \xrightarrow[\text{formation}]{\alpha} X \xrightarrow[\text{decay}]{\alpha'}$$

Ewing-Weisskopf (1940)

detailed balance: $g_\alpha k_\alpha^2 \sigma_{\alpha\alpha'} = g_{\alpha'} k_{\alpha'}^2 \sigma_{\alpha'\alpha}$

spin counting

for CN:

$$g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)\Gamma_{\alpha'} = g_{\alpha'} k_{\alpha'}^2 \sigma_{CN}(\alpha')\Gamma_\alpha$$

$$\frac{\Gamma_{\alpha'}}{g_{\alpha'} k_{\alpha'}^2 \sigma_{CN}(\alpha')} = \frac{\Gamma_\alpha}{g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)} \Rightarrow \Gamma_\alpha = g_\alpha k_\alpha^2 \sigma_{CN}(\alpha)$$

introducing density of levels ρ of final states:

$$\sigma_{\alpha\alpha'} = \sigma_{CN}(\alpha)\frac{(2J_{\alpha'}+1)\mu_{\alpha'} E_{\alpha'} \sigma_{CN}(\alpha')\rho(E_{\alpha'})}{\sum_\alpha \int (2J_\alpha+1)\mu_\alpha E_\alpha \sigma_{CN}(\alpha)\rho(E_\alpha)dE_\alpha}$$

Hauser-Feshbach (1952)

Herman Feshbach: include angular momentum in Ewing-Weisskopf theory

CN ang. mom.

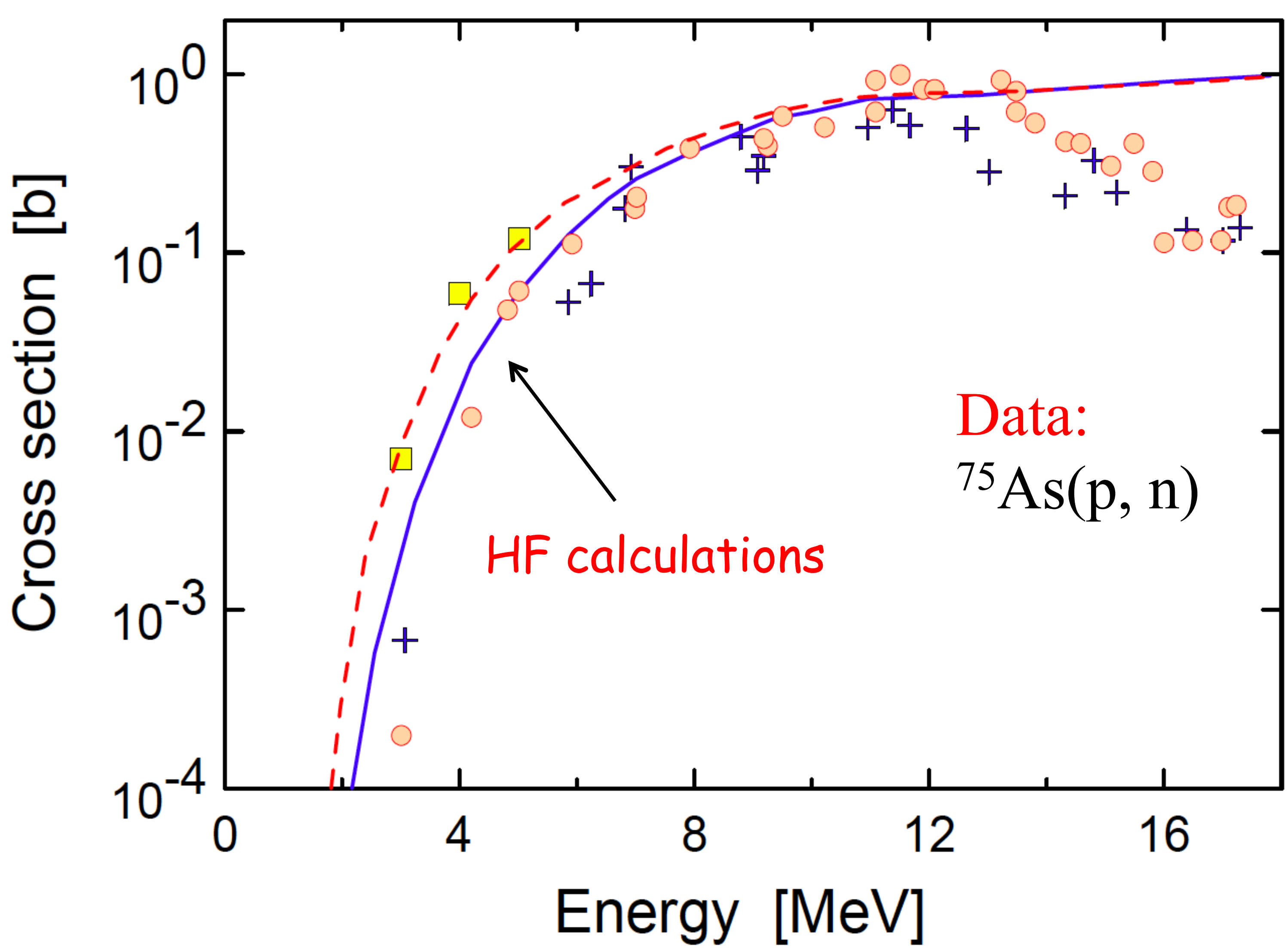
$$\sigma_{\alpha\alpha'} = \frac{\pi}{k^2} \sum_J \frac{2J+1}{(2i_c+1)(2I_c+1)} \frac{\sum_{s,l} T_{l,s}(c) \sum_{s',l'} T_{l',s'}(c')}{\sum_c \sum_{s,l} T_{l,s}(c)}$$

projectile spin

target spin

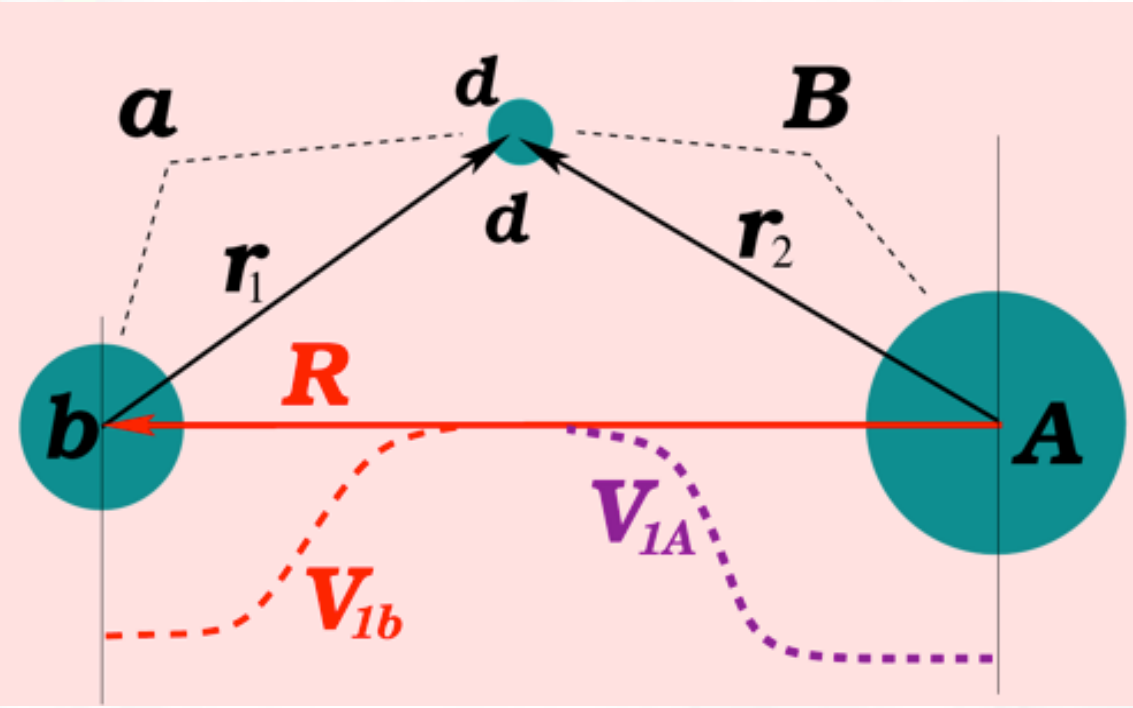
transmission probability

Hauser-Feshbach



Transfer Reactions

One-nucleon transfer (Born approximation)



$\mathcal{Q}$ = momentum transfer
V_{1A} transfer interaction

POST or PRIOR representation

transfer matrix element

$$M_{\beta\alpha}(\mathbf{R}) \sim \int d^3\mathbf{r}_1\, e^{i\mathcal{Q}\bullet\mathbf{r}_1} \varphi^{(A)}_{a_n}(\mathbf{R}+\mathbf{r}_1)\left(V_{1A} - \langle U\rangle\right)\varphi^{(b)}_{a'_n}(\mathbf{r}_1)$$

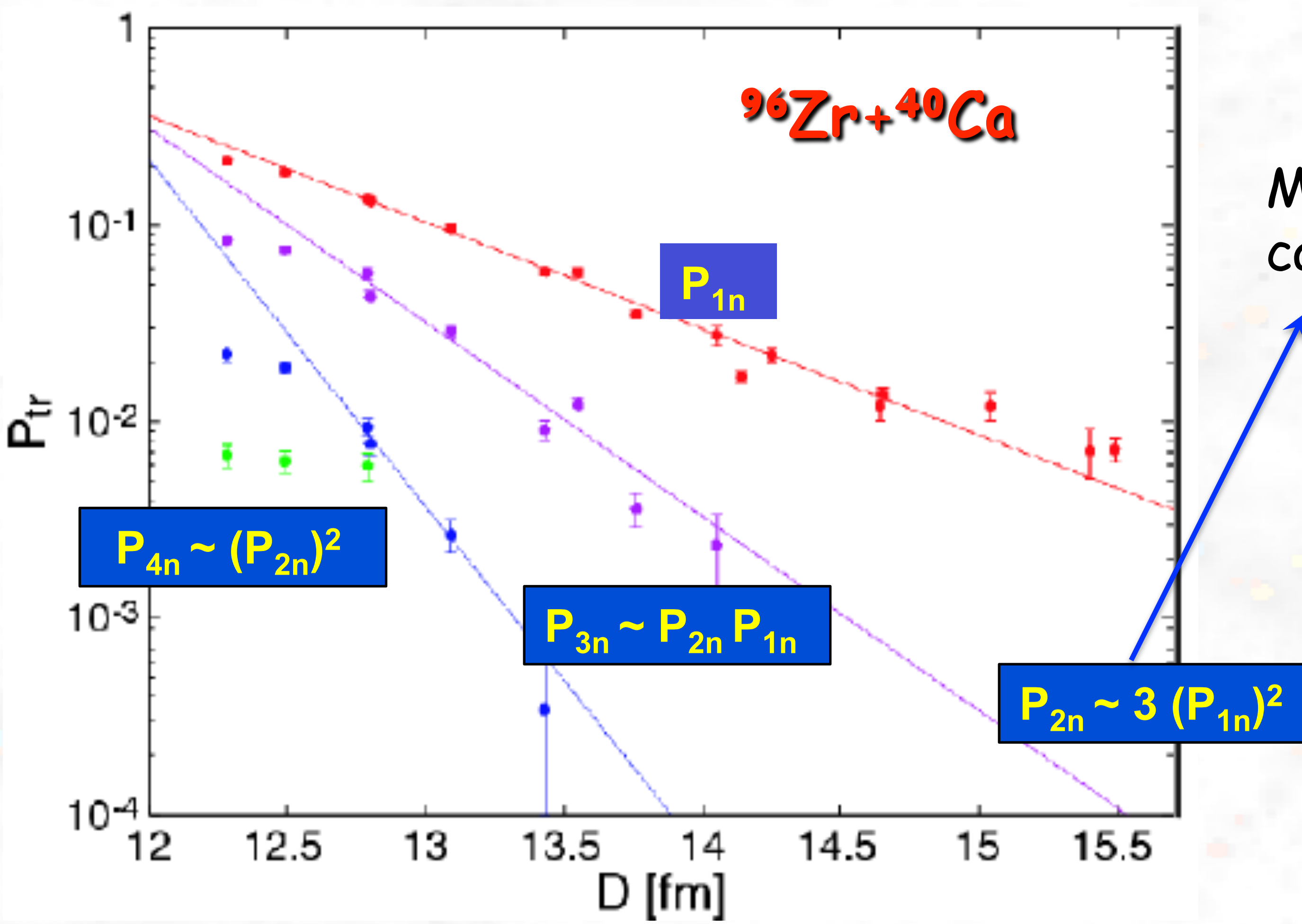
Transfer probability

$$P_\beta = \left|\frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, M_{\beta\alpha}(\mathbf{R})\, e^{i(E_\beta - E_\alpha)t/\hbar + (\ldots)}\right|^2 \sim \tau_{coll}\left|M_{\beta\alpha}(D)\right|^2 g(Q_{\beta\alpha})$$

Multi-nucleon transfer (Born approximation)

$$\frac{P_{tr}}{\sin(\theta_{cm}/2)} \sim \exp(-2\alpha D)$$

$$D = \frac{Z_1 Z_2 e^2}{2E_{cm}}\left(1 + \frac{1}{\sin(\theta_{cm}/2)}\right)$$



More sophisticated calculations necessary

Asymptotic Normalization Coefficients

Spectroscopic factors

- What is the amplitude for ^{12}C + n in ^{13}C?
- Define overlap function:

$$\mathrm{I}(\mathbf{r}) = \left\langle \phi_A(\zeta_A)\phi_n(\zeta_n) \middle| \phi_B(\zeta_A;\zeta_n;\mathbf{r}) \right\rangle$$

And the spectroscopic factor is

$$\int d^3r \; |\mathrm{I}^c_{\ell j}(\mathbf{r})|^2 = S(\ell j)$$

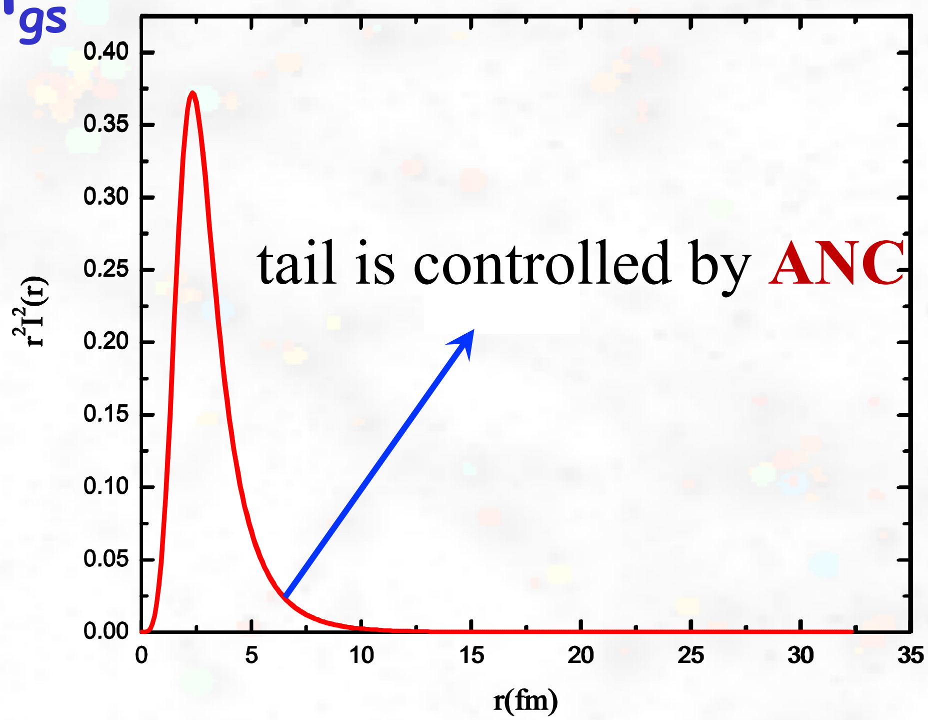
An example: $(^7Li_{gs} + n)_{2+} \leftrightarrow {}^8Li_{gs}$

Single particle approach:

$$\mathrm{I}^c_{\ell j}(\mathbf{r}) = \sqrt{S(\ell j)}\;\phi(\mathbf{r})$$

$$0 \le r \le 6\,\mathrm{fm}$$

96%



Asymptotic Region

- Single particle overlap function for **r > R$_N$**

$$\phi_{(lj)}(r) \underset{r>R_N}{\longrightarrow} b_{(lj)} i\kappa h^{(1)}_{(lj)}(i\kappa r)$$

$$\kappa = \sqrt{2\mu\varepsilon^B_n}$$

$$\varepsilon^B_n = m_A + m_n - m_B$$

- Model independent definition:

$$I_{(lj)}(r) \underset{r>R_N}{\longrightarrow} K_{(lj)}\phi_{(lj)}(r) = C_{(lj)} b_{(lj)} i\kappa h^{(1)}_{(lj)}(i\kappa r)$$

- Asymptotic Normalization Coefficient

$$C_{(lj)} = K_{(lj)} b_{(lj)}$$

Contains information about the many-body physics at the tail of the w.f.

$$\Rightarrow \quad S_{(lj)}(r) = \int_0^\infty dr\, r^2 I^2_{(lj)}(r) = K^2_{(lj)} \int_0^\infty dr\, r^2 \phi^2_{(lj)}(r) = K^2_{(lj)}$$

Transfer reaction

Cross section for **A(d,p)B**

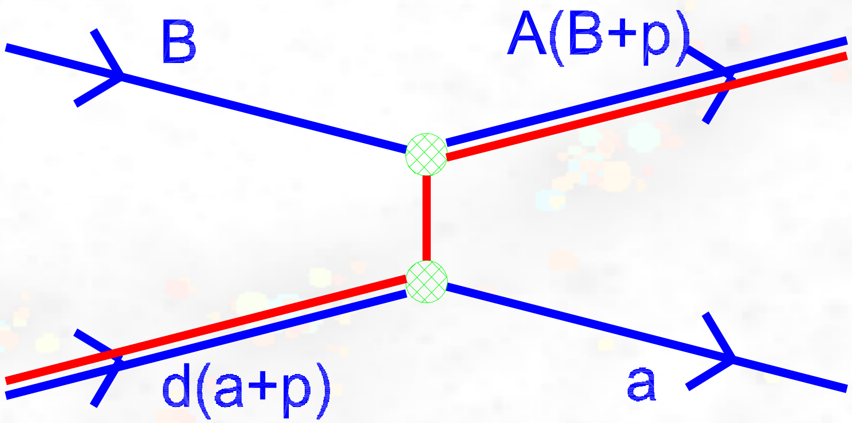
$$\sigma^{DW} = |M|^2 = \left|\langle \psi_f^{(-)} I_{An}^{B} | V | \phi_{pn} \psi_i^{(+)} \rangle\right|^2$$

With the single particle approximation

$$\sigma^{DW} = S\left|\langle \psi_f^{(-)} \phi_{An}(n_r lj) | V | \phi_{pn} \psi_i^{(+)} \rangle\right|^2$$

$[S = C^2/b^2]$

S is the normalization (i.e. 'spectroscopic') factor



$$I_{Bp}^{A} \overset{r_{Bp} > R_N}{\approx} C_{Bp}^{A} \frac{W_{-\eta, l+\frac{1}{2}}\left(2\kappa_{Bp} r_{Bp}\right)}{r_{Bp}}$$

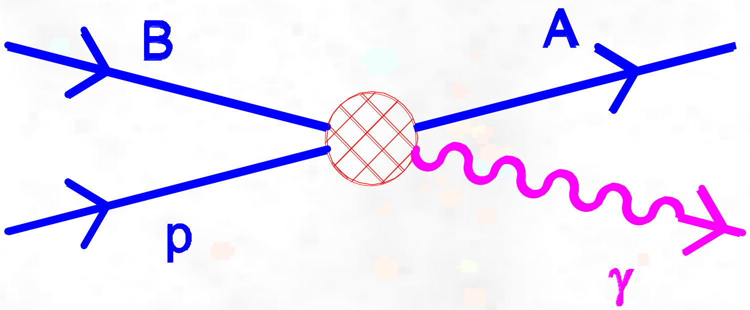
$$\frac{d\sigma}{d\Omega} = \left(C_{Bpl_A j_A}^{A}\right)^2 \left(C_{Bpl_d j_d}^{A}\right)^2 \frac{\sigma_{l_A j_A l_d j_d}^{DW}}{b_{Bpl_A j_A}^2 b_{Bpl_d j_d}^2}$$

Use of ANCs

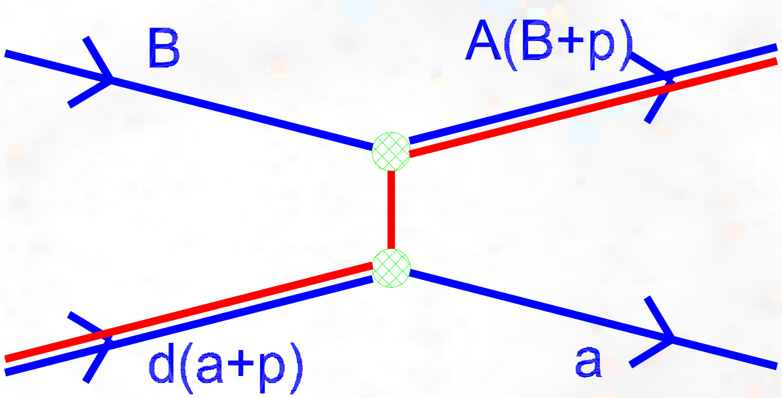
- Find a peripheral transfer reaction
- Measure angular distribution (abs. c.s.)
- DWBA calculation (optical model parameters)
- Determine single particle ANCs

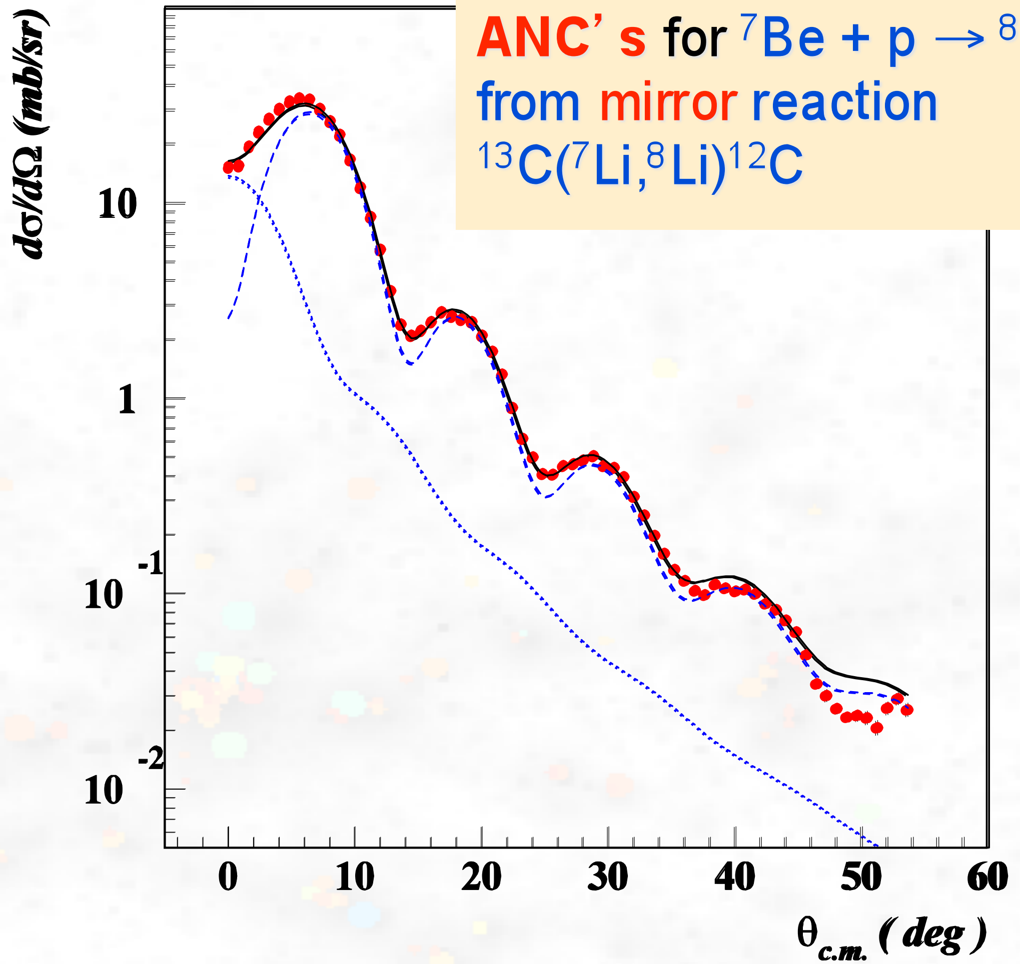
Use the information (ANCs) obtained for the wavefunctions to calculate matrix elements of astrophysical interest.

Asymptotic normalization coefficients



$$\sigma_{capture} \approx \left(C^{A}_{Bp}\right)^2$$





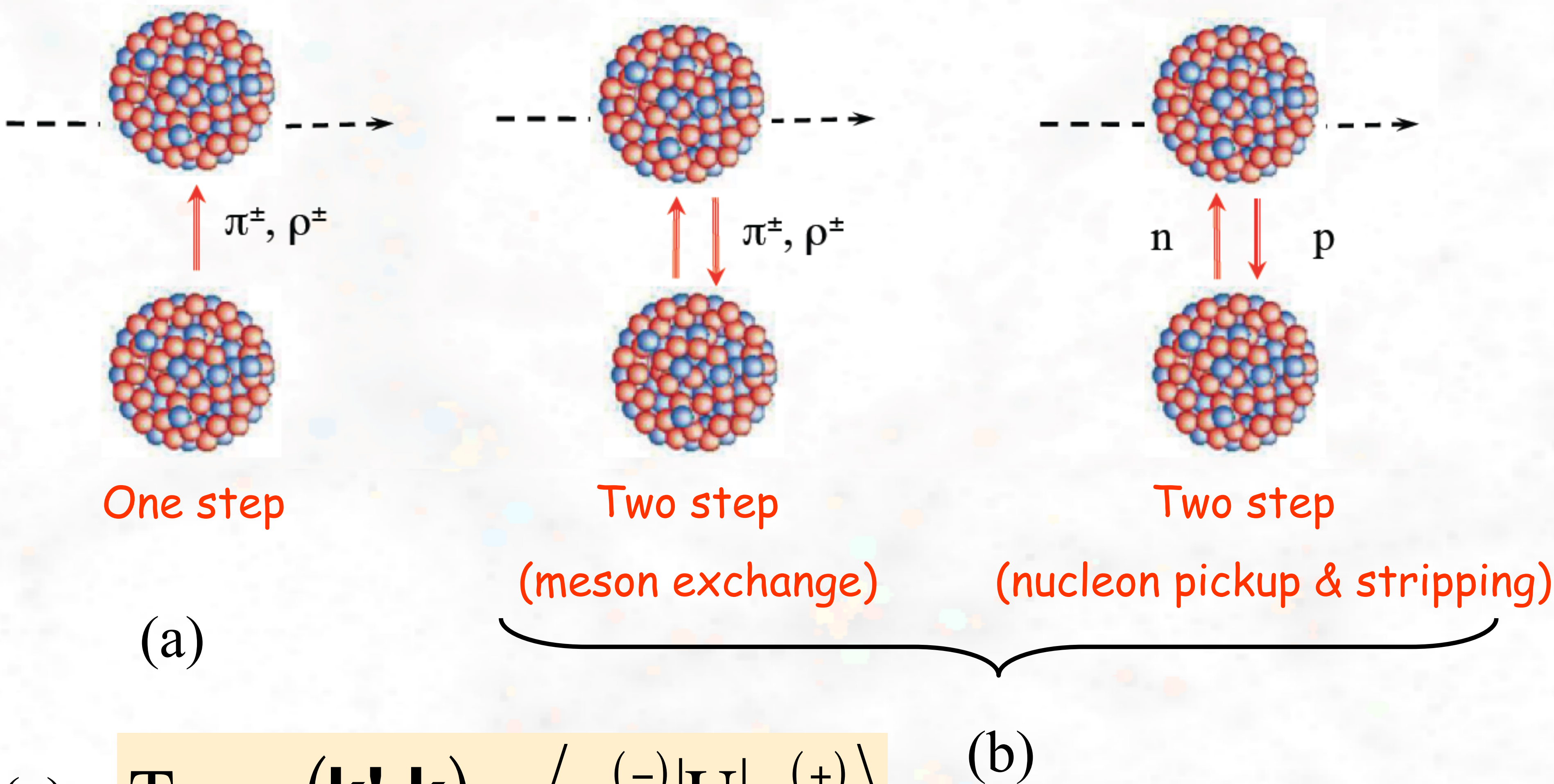
ANC's for $^7Be + p \to {}^8B$ from mirror reaction $^{13}C(^7Li,^8Li)^{12}C$

$$\frac{d\sigma}{d\Omega} = \left(C^{A}_{Bpl_A j_A}\right)^2 \left(C^{A}_{Bpl_d j_d}\right)^2 \frac{\sigma^{DW}_{l_A j_A l_d j_d}}{b^2_{Bpl_A j_A}\, b^2_{Bpl_d j_d}}$$

$S_{17}(0)$ = 17.6 ± 1.7 eV.b

A.M. Mukhamedzhanov et al., PRC 56, 1302 (1997)

Charge-exchange reactions



(a) $$T_{\mathrm{DWBA}}(\mathbf{k}',\mathbf{k}) = \left\langle \chi_{\mathbf{k}'}^{(-)} \middle| U \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

(b) $$T_{\mathrm{DWA}}(\mathbf{k}',\mathbf{k}) = \sum_{\gamma=0} C_\gamma \left\langle \chi_{\mathbf{k}'}^{(-)} \middle| U\left(G^{(+)}U\right)^{\gamma} \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

Effective interaction V_{NN}

$$V_{NN}(\mathbf{r}) = V^{C}(r) + V_{\sigma}^{C}(r)\left(\sigma_1 \cdot \sigma_2\right) + \left[V_{\tau}^{C}(r) + V_{\sigma\tau}^{C}(r)\left(\sigma_1 \cdot \sigma_2\right)\right]\left(\tau_1 \cdot \tau_2\right)$$
$$+\left[V^{T}(r) + V_{\tau}^{T}(r)\left(\tau_1 \cdot \tau_2\right)\right] S_{12}(\hat{\mathbf{r}}) + V^{LS}(r)\ \boldsymbol{l} \cdot \left(\sigma_1 + \sigma_2\right)$$

Antisimetrization: $V_{NN}(\mathbf{r}) = \left[1-(-)^{l}P_{x}\right] V_{12}(\mathbf{r})$ $\quad P_x : \mathbf{r} \rightarrow -\mathbf{r}$

$V^{LS}(r)\ \boldsymbol{l} \cdot \left(\sigma_1 + \sigma_2\right)$ small and usually neglected

Notation:

$$V^{C}(r) = V_{00}^{0}(r), \quad V_{\sigma}^{C}(r) = V_{10}^{0}(r), \quad V_{\tau}^{C}(r) = V_{01}^{0}(r)$$
$$V_{\sigma\tau}^{C}(r) = V_{11}^{0}(r), \quad V^{T}(r) = V_{10}^{2}(r), \quad V_{\tau}^{T}(r) = V_{01}^{2}(r)$$

$$V_{12}(\mathbf{r}) = \sum_{\substack{K=0,2 \\ ST}} V_{ST}^{K}(r) C_{S}^{K}\ Y_{K}(\hat{\mathbf{r}}) \left[\sigma_1 \otimes \sigma_2\right]^{K} \left[\tau_1 \cdot \tau_2\right]^{T}$$

K = 0 : central force

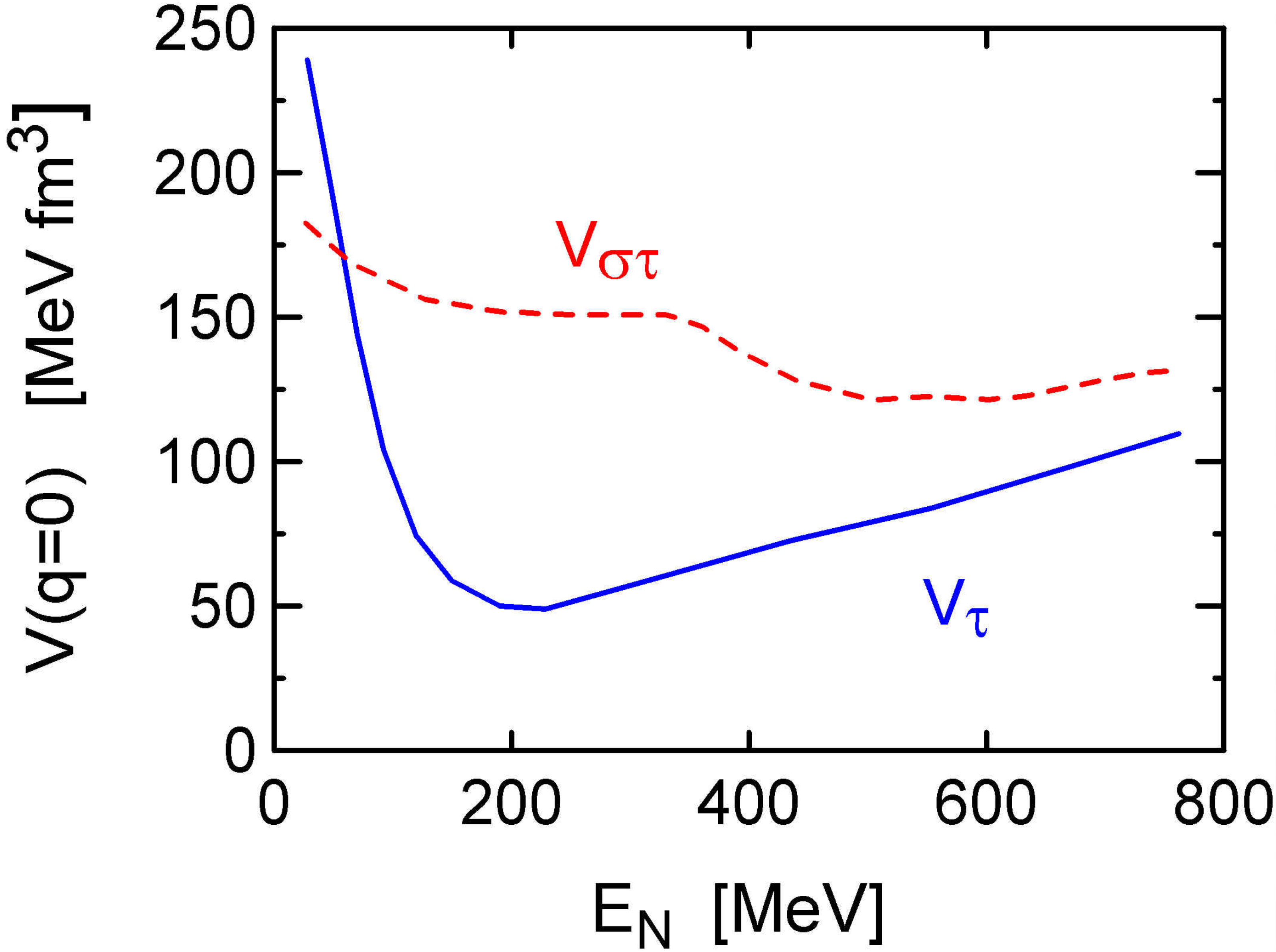
K = 2 : tensor force

$$\sigma^{S=0} = 1, \quad \sigma^{S=1} = \sigma$$
$$\tau^{T=0} = 1, \quad \tau^{T=1} = \tau$$

$$C_0^0 = \sqrt{4\pi}, \quad C_1^0 = -\sqrt{12\pi}$$
$$C_0^2 = 0, \qquad C_1^2 = \sqrt{25\pi/5}$$

Effective interaction V_{NN}

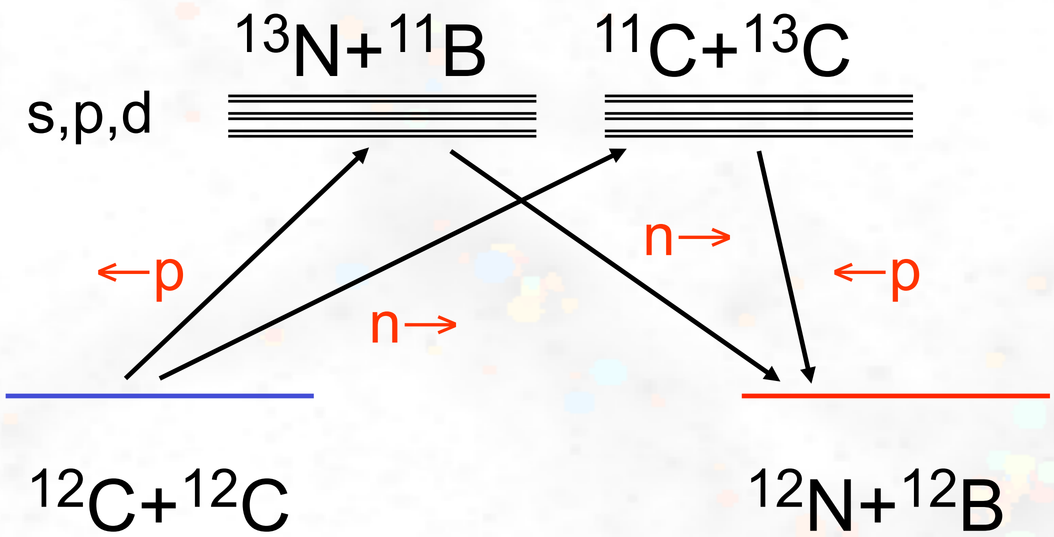
Love, Franey, NPA 1981, 1985

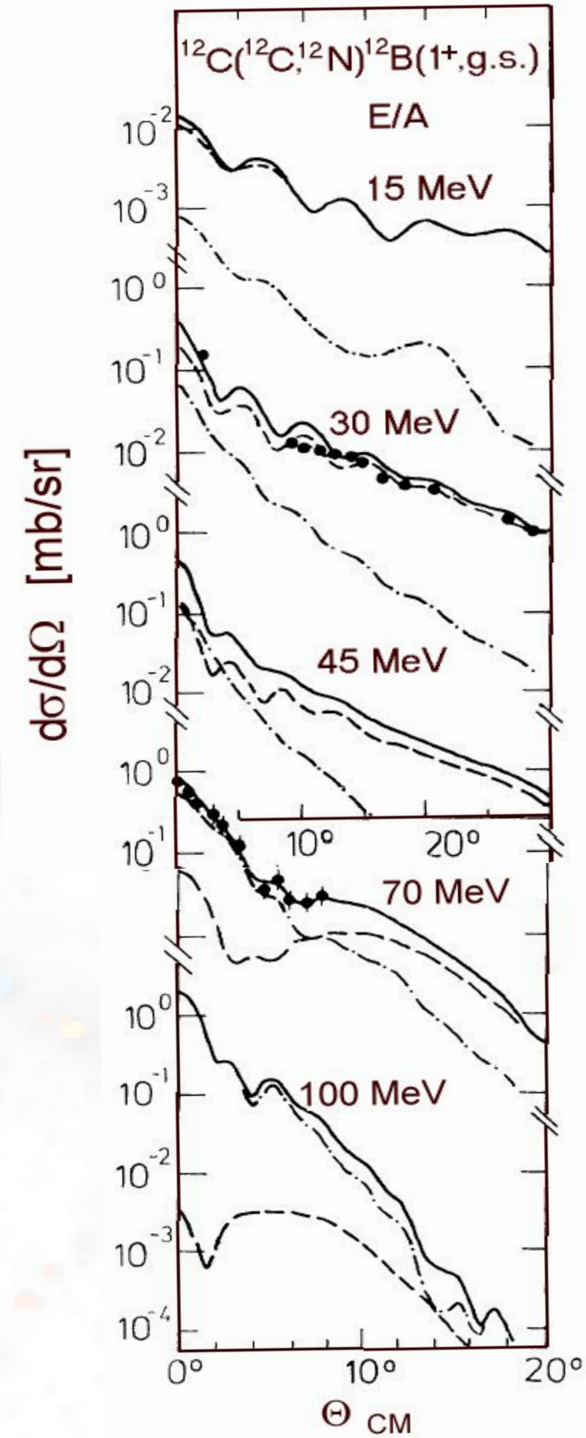


Two step (proton pickup & neutron-stripping)

Lenske, Wolter, Bohlen, PRL 1989

$^{12}C(^{12}C,^{12}N)^{12}B(1^+,gs)$





Two step (double $\pi+\rho$ exchange)

Bertulani, NPA 1993

$^{13}C(^{13}N,^{13}C)^{13}N$ 70 MeV/nuc.

$\sigma_{2nd} \sim 10^{-4} \times \sigma_{1st}$

DBWA

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \int d^3r\, S(b)\, \exp[i\mathbf{q}\cdot\mathbf{r}]\, \langle bB | U(\mathbf{r}) | aA \rangle$$

$$| aA\rangle = | aA; J_a M_a T_a N_a ; J_A M_A T_A N_A \rangle$$

eikonal + few pages of algebra

CB, NPA 554, 493 (1993)

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \sum_{\substack{K=0,2 \\ ST}} \sum_{\substack{LL'JJ' \\ MM'\mu}} C(KS;LL'JJ'MM'\mu) \int db\, b\, S(b)\, J_0(qb) \times \int dp\, p\, J_{M'-M-\mu}(pb)\, \underbrace{\tilde{V}^K_{ST}(p)\, \tilde{\rho}^{aA}_{LJST}(p)\, \tilde{\rho}^{bB}_{L'J'ST}(p)}$$

$$\tilde{\rho}^{aA}_{LJST}(p) = \int dr\, r^2\, j_L(pr) \left\langle J_a T_a \left\| \sum_i \frac{\delta(r-r_i)}{r_i^2} \mathfrak{I}^{LSJ}_M \tau^T \right\| J_b T_b \right\rangle$$

STRUCTURE INPUT factorized

$$\mathfrak{I}^{LSJ}_M = \sum_{\mu M_L} \langle L M_L S\mu | JM \rangle i^L\, Y_{LM_L}(\hat{\mathbf{r}})\, \sigma^{S\mu}$$

Charge exchange at forward angles

$$T_{aA\to bB}(\mathbf{k}',\mathbf{k}) = \sum_{\cdots}\sum_{\cdots}\cdots\int db\, b\, S(b)\, J_0(qb)\int dp\, p\, J_{\cdots}(pb)\, \tilde{\rho}^{aA}_{\cdots}(p)\, \tilde{\rho}^{bB}_{\cdots}(p)$$

$$S(b)\sim 1 \longrightarrow p\sim q$$

- $S(b)\neq 1$ but largest value of $T_{aA\to bB}$ occurs when

$J_0(qb)$ oscillates in phase with $J_{\cdots}(pb)$

$\Longrightarrow$ $p\sim q$

Forward scattering: $q\sim 0$

CB, NPA 554, 493 (1993)

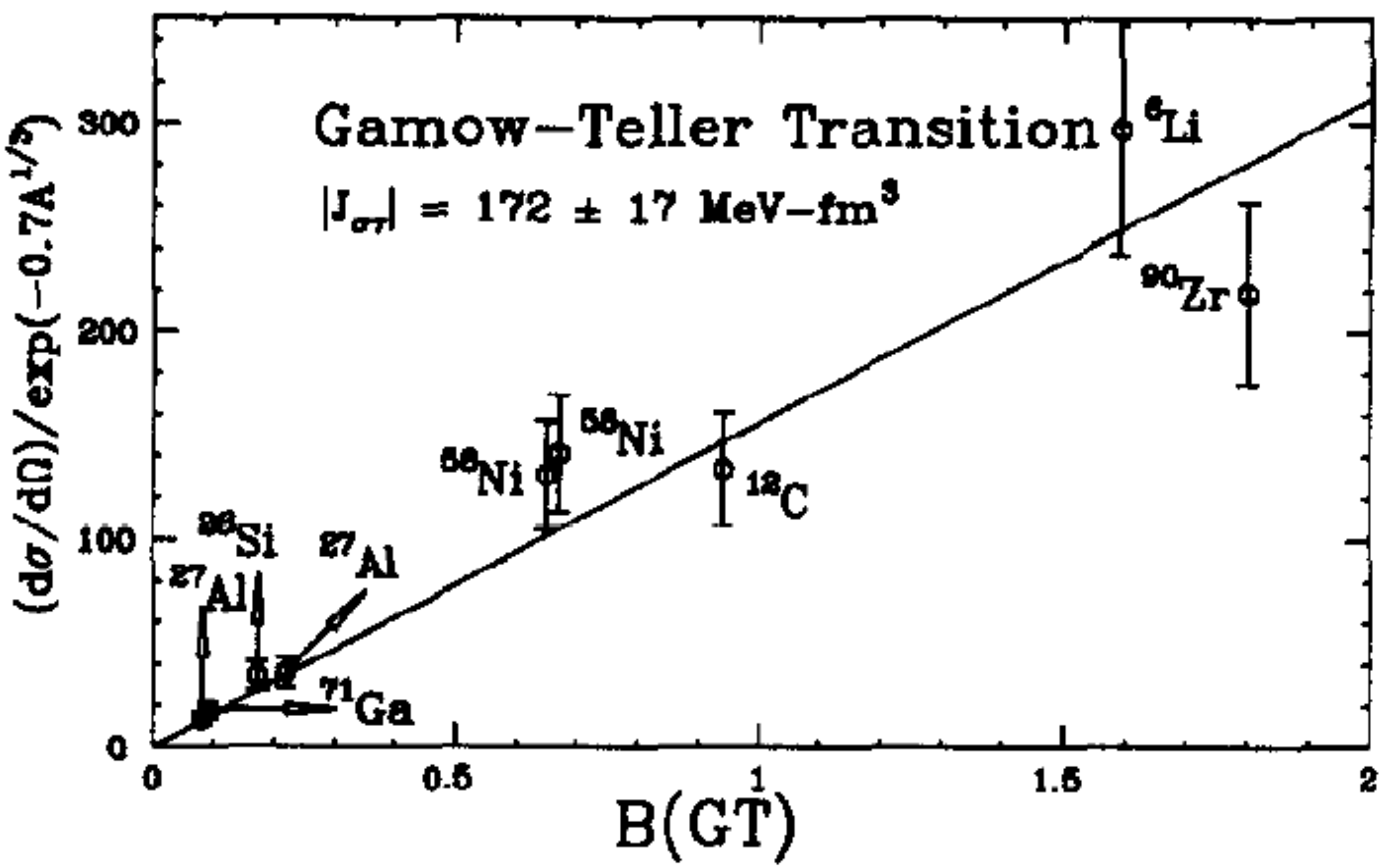
$$f_{aA\to bB}(\theta\sim 0) = \ldots\tilde{\rho}^{aA}_{\cdots}(0)\,\tilde{\rho}^{bB}_{\cdots}(0)\times\int dp\, p\, V^{K}_{ST}(p)\times\int db\, b\, J_0(qb)\, e^{iX(b)}$$

$$\tilde{\rho}^{aA}_{\cdots}(0) = \cdots\langle A\|\sigma^S\tau\|a\rangle \qquad \frac{d\sigma}{d\Omega}(\theta\sim 0^0) = \cdots\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2\left|\langle B\|\sigma^S\tau\|b\rangle\right|^2$$

$\Longrightarrow$ • If $\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2$ well known. E.g. (a,A) = (n,p) then

Fermi and Gamow-Teller m.e. READ DIRECTLY from $\frac{d\sigma}{d\Omega}(\theta\sim 0^0)$

Charge exchange at forward angles



End Lecture 4