Overview of nuclear reaction theory for rare isotopes

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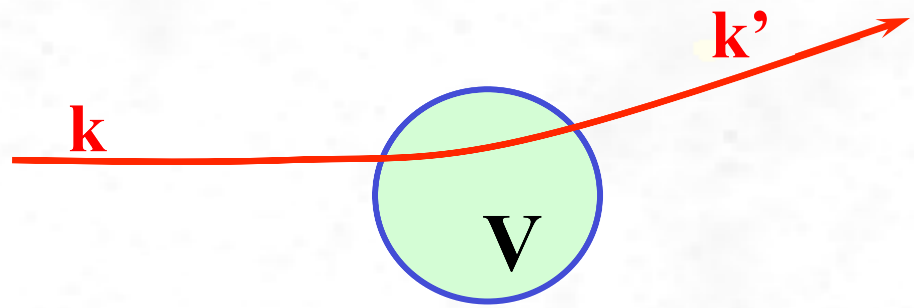
Lecture 2

Direct Reactions

Low energy scattering

Scattering Problem

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$



$$\left. \begin{array}{l} \left[H_0 + V\right]\Psi = E\Psi \\ H_0\phi = E\phi \end{array} \right\}$$

$$\Psi = \phi + G_0(E)V\Psi$$

Lippmann-Schwinger eq.

Green's function

$$G_0(E) = \frac{1}{E - H_0}$$

Coordinate space:

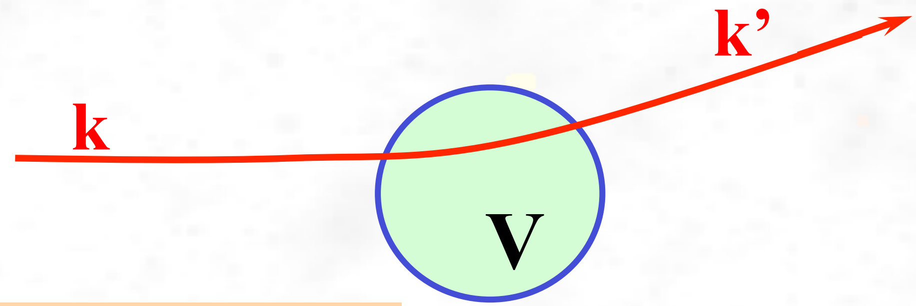
$$G_0^{\pm}(E,\mathbf{r},\mathbf{r}') = -\frac{2\mu}{\hbar^2}\frac{\mathrm{e}^{\pm ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \xrightarrow[r\to\infty]{} -\frac{2\mu}{\hbar^2}\frac{\exp\left[\pm ik\left(r - \hat{\mathbf{r}}\cdot\mathbf{r}'\right)\right]}{r}$$

$$\Psi_{\mathbf{k}}^{\pm}(\mathbf{r}) \xrightarrow[r\to\infty]{} \phi_{\mathbf{k}}(\mathbf{r}) + \frac{e^{\pm ikr}}{(2\pi)^{3/2}r}\underline{\left[-\frac{4\pi^2\mu}{\hbar^2}\left\langle\phi_{\pm\mathbf{k}'}\middle|V\middle|\Psi_{\mathbf{k}}^{\pm}\right\rangle\right]}$$

$$f(\theta)$$

Transition matrix

$$T_{\mathbf{k},\mathbf{k}'} = \left\langle \phi_{\mathbf{k}'} \middle| T \middle| \phi_{\mathbf{k}} \right\rangle = \left\langle \phi_{\mathbf{k}'} \middle| V \middle| \Psi_{\mathbf{k}}^{(+)} \right\rangle$$



$$\Longrightarrow \quad f(\theta) = -\frac{4\pi^2\mu}{\hbar^2} T_{\mathbf{k},\mathbf{k}'}$$

$$T = V + VG_0(E)T$$

Lippmann-Schwinger

Scattering matrix

$$S_{\mathbf{k},\mathbf{k}'} = \left\langle \Psi_{\mathbf{k}'}^{(-)} \middle| \Psi_{\mathbf{k}}^{(+)} \right\rangle = \left\langle \phi_{\mathbf{k}'} \middle| S \middle| \phi_{\mathbf{k}} \right\rangle$$

$$S_{\mathbf{k},\mathbf{k}'} = \delta(\mathbf{k} - \mathbf{k}') - 2\pi i\, \delta\left(E_k - E_{k'}\right) T_{\mathbf{k},\mathbf{k}'}$$

Two potential formula

$$V = V_0 + U$$

$$\chi^{\pm} = \phi + G_0^{\pm}(E) V_0 \chi^{\pm}$$

$$\Psi^{\pm} = \chi^{\pm} + G^{\pm}(E) U \Psi^{\pm}$$

$$G_0 = \frac{1}{E - H_0}, \quad G = \frac{1}{E - H_0 - V_0}$$

$$T_{\mathbf{k}',\mathbf{k}} = \left\langle \phi_{\mathbf{k}'} \middle| V_0 \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle + \left\langle \chi_{\mathbf{k}'}^{(-)} \middle| U \middle| \Psi_{\mathbf{k}}^{(+)} \right\rangle$$

Gellmann-Goldberger relation

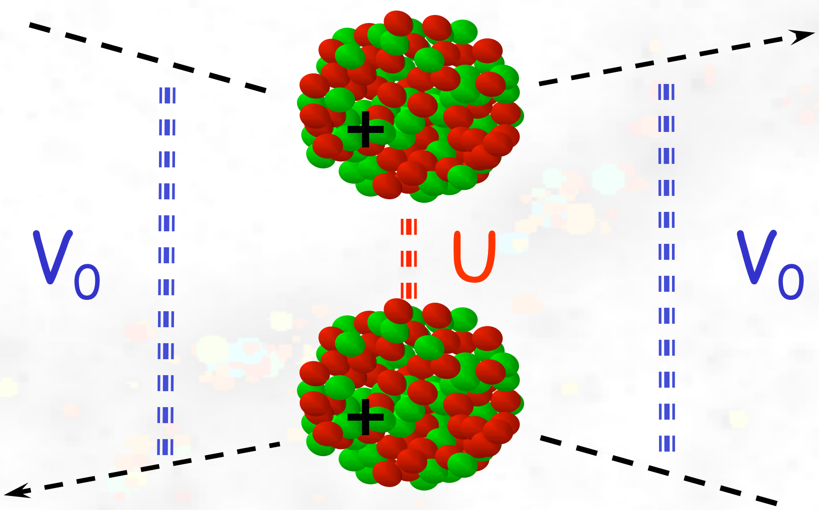
(Basis of DW calculations)

$$\Longrightarrow \qquad f_{inel}(\theta) = -\frac{4\pi^2\mu}{\hbar^2}\int d^3r\,\chi_{\mathbf{k'}}^{(-)*}(\mathbf{r})\,U(\mathbf{r})\,\Psi_{\mathbf{k}}^{(+)}(\mathbf{r})$$

Iterations: DWBA

$$\Psi^{\pm} \sim \chi^{\pm} \qquad \Longrightarrow \qquad f_{DWBA}(\mathbf{k'},\mathbf{k}) = -\frac{4\pi^2\mu}{\hbar^2}\left\langle \chi_{\mathbf{k'}}^{(-)}\middle|U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$

$$T_{DWBA}(\mathbf{k'},\mathbf{k}) = \left\langle \chi_{\mathbf{k'}}^{(-)}\middle|U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$



Distorted: all orders in V_0

Born: only first order in U

DWBA: another way to interpret it

$$d\sigma = \frac{\mu}{\hbar k}\frac{2\pi}{\hbar}\left|U_{fi}\right|^2 dn(E_f)$$

$$dn(E) \sim d^3k = k^2 dk\, d\Omega$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \left|f_{DWBA}(\mathbf{k'},\mathbf{k})\right|^2$$

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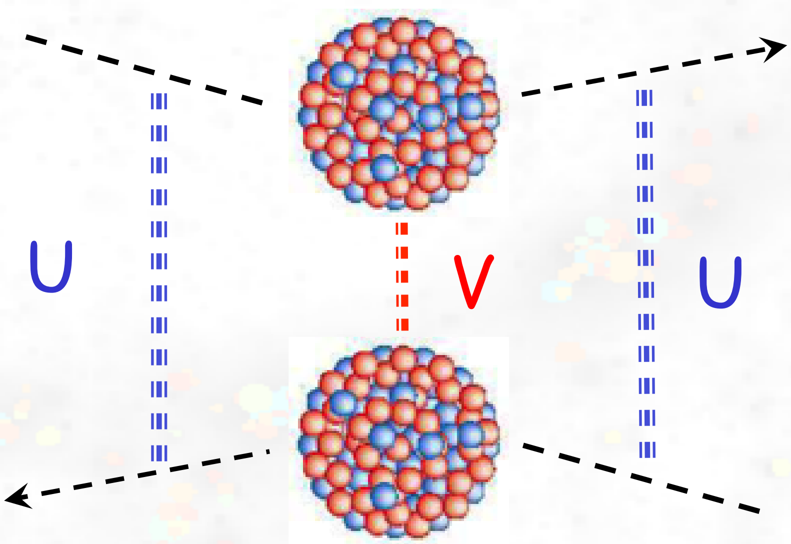
$$f_{DWBA}(\mathbf{k'},\mathbf{k}) = -\frac{4\pi^2\mu}{\hbar^2}\left\langle \chi_{\mathbf{k'}}^{(-)}\middle|U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$



$$T_{DWBA}(\mathbf{k'},\mathbf{k}) = \left\langle \chi_{\mathbf{k'}}^{(-)}\middle|U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$

Distorted: all orders in V

Born: only first order in U

Application: Coulomb + Nuclear excitation

$$f_{inel}^{C}(\theta) \approx \int d^3r\, d^3r'\, \chi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, \varphi_f(\mathbf{r}')\, \boxed{V_C(\mathbf{r},\mathbf{r}')}\, \chi_{\mathbf{k}}^{(+)}(\mathbf{r})\, \varphi_i(\mathbf{r}')$$

nice, well known,
angel

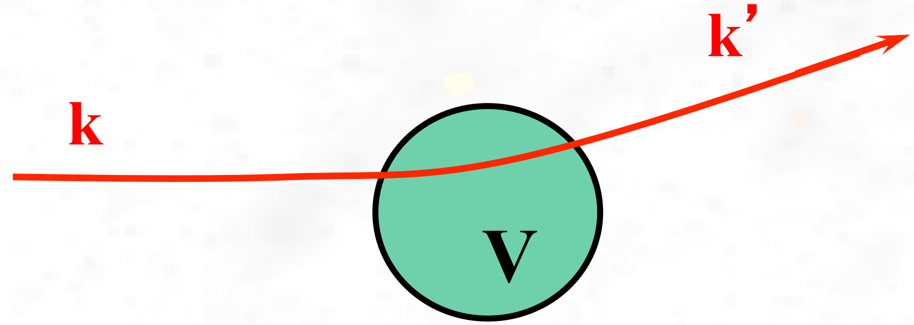
$$f_{inel}^{N}(\theta) \approx \int d^3r\, d^3r'\, \chi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, \phi_f(\mathbf{r}')\, \boxed{V_N(\mathbf{r},\mathbf{r}')}\, \chi_{\mathbf{k}}^{(+)}(\mathbf{r})\, \phi_i(\mathbf{r}')$$

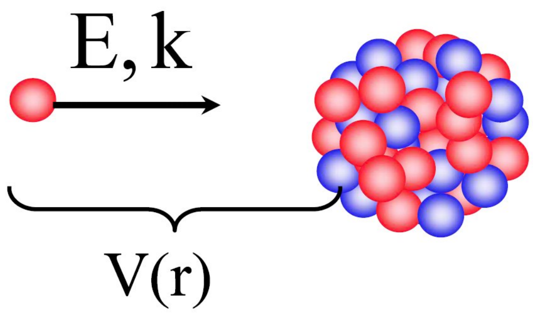
bad, not well known, a
true monster

$$\frac{d\sigma}{d\Omega} = \left| f_{inel}^{N}(\theta) + f_{inel}^{C}(\theta) \right|^2$$

Partial waves

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$





$$\Psi = \sum_l \frac{u_l(r)}{r} P_l(\cos\theta)$$

Partial wave expansion

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u_l(r) + \left[V(r) + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]u_l(r) = Eu_l(r)$$

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

Incoming wave

Outgoing wave

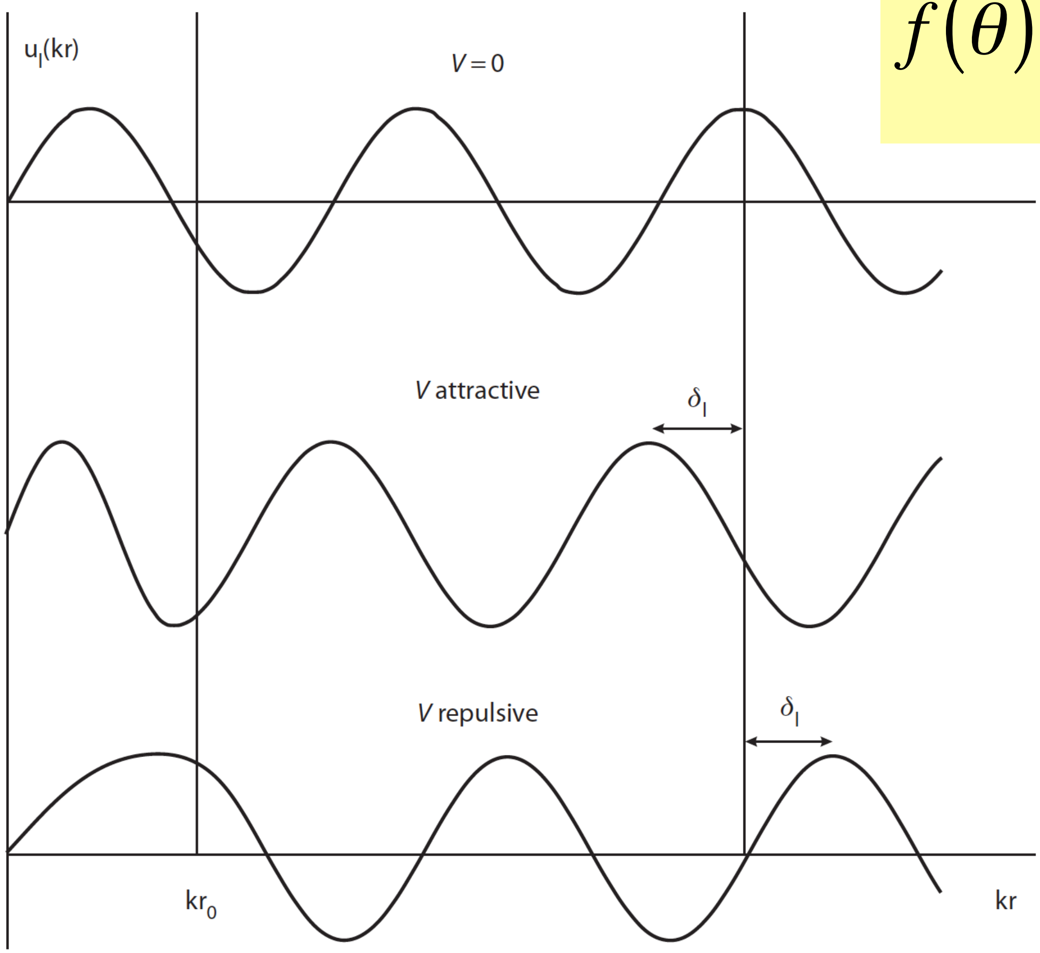
"Survival" amplitude
(S-matrix)

Low energy Scattering

$$S_l = e^{2i\delta_l}$$

(δ_l = Phase shift)

$|S_l|^2$ = "Survival" probability ≤ 1



$$f(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$\sigma = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l$$

$\Rightarrow$ δ_l is everything!

to get δ_l solve S.E. numerically and match $u_l(r)$ to asymptotic $u_l(r)$, and their derivatives at a large r (outside range of V(r)).

Application:

Elastic Scattering

$$f(\theta) = \frac{i}{k}\sum_l (l+\tfrac{1}{2})(1-S_l)P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = \left|f(\theta)\right|^2$$

Inelastic Scattering

$$\chi^{(\pm)}_{\mathbf{k}(\mathbf{k}')} \longrightarrow \chi^{(\pm)}_{\mathbf{k}(\mathbf{k}');lm}$$

$$U \rightarrow \left\langle \alpha_f \middle| U \middle| \alpha_i \right\rangle$$

$$\alpha = JMTN, etc$$

$$f_{DWBA}(\theta) = -\frac{4\pi^2\mu}{\hbar^2}\left(\frac{k_f}{k_i}\right)^{1/2}\sum_{lm}\left\langle \chi^{(-)}_{\mathbf{k}';lm};\alpha_f \middle| U \middle| \chi^{(+)}_{\mathbf{k};lm};\alpha_i \right\rangle$$

U_{fi} from microscopic NN interactions

$$U = \sum_{i=1}^{A}\sum_{j=1}^{B} V_{ij} \xrightarrow{\quad Pauli \quad} U = \sum_{i=1}^{A}\sum_{j=1}^{B} V_{ij}\left(1-P_{ij}\right)$$

Neglect it for now

Point-like projectiles: Reaction x Structure factorization

Assume
$$U_{fi} = \sum_{k=1}^{B} \left\langle \alpha_f \left| \int d^3r\, V(\mathbf{r}_k - \mathbf{r}) \sum_{j=1}^{A} \delta(\mathbf{r} - \mathbf{r}_j) \right| \alpha_i \right\rangle = \sum_{k=1}^{B} \int d^3r\, V(\mathbf{r}_k - \mathbf{r})\, \rho_{fi}(\mathbf{r})$$

$$\rho_{fi}(\mathbf{r}) = \left\langle \alpha_f \left| \int d^3r\, \sum_{j=1}^{A} \delta(\mathbf{r} - \mathbf{r}_j) \right| \alpha_i \right\rangle$$

One-body transition density

Use
$$V_{ji}(|\mathbf{r}_j - \mathbf{r}_i|) = \frac{1}{(2\pi)^3} \int d^3q\, e^{i\mathbf{q}\cdot(\mathbf{r}_j - \mathbf{r}_i)}\, V(\mathbf{q})$$

$$\Longrightarrow \quad T_{DWBA}^{fi}(\mathbf{k_f}, \mathbf{k_i}) = \int d^3q\, R(\mathbf{k_f}, \mathbf{k_i}; \mathbf{q})\, V(\mathbf{q})\, \rho_{fi}(\mathbf{q})$$

Structure

$$\rho_{fi}(\mathbf{q}) = \left\langle \alpha_f \left| \int d^3r\, \sum_{j=1}^{A} \exp(i\mathbf{q}\cdot \mathbf{r}_j) \right| \alpha_i \right\rangle$$

Reaction

$$R(\mathbf{k_f}, \mathbf{k_i}; \mathbf{q}) = \frac{1}{(2\pi)^3} \sum_{k=1}^{B} \int d^3r_k\, \left\langle \chi_{\mathbf{k}_f}^{(-)} \middle| \exp(-i\mathbf{q}\cdot \mathbf{r}_k) \middle| \chi_{\mathbf{k}_i}^{(+)} \right\rangle$$

Projectile nuclei are structureless plane waves

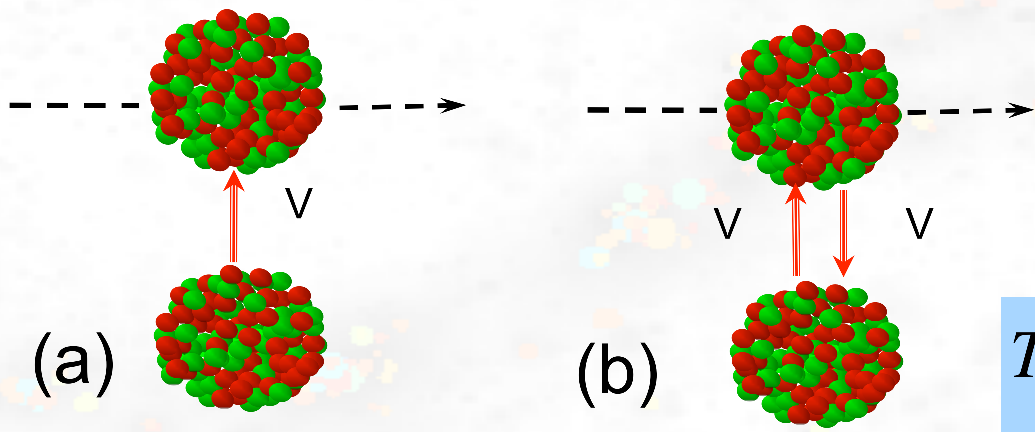
$$\frac{1}{(2\pi)^3}\sum_{k=1}^{B}\int d^3 r_k \left\langle \chi_{\mathbf{k}_f}^{(-)}\middle|\exp\left(-i\mathbf{q}\cdot\mathbf{r}_k\right)\middle|\chi_{\mathbf{k}_i}^{(+)}\right\rangle = B\delta\left(\mathbf{q}-\mathbf{k}_i+\mathbf{k}_f\right)$$

$$\Rightarrow \quad T_{PWIA}^{fi}\left(\mathbf{k}_f,\mathbf{k}_i\right) = B\,V(\mathbf{q})\,\rho_{fi}(\mathbf{q}), \qquad \mathbf{q}=\mathbf{k}_f-\mathbf{k}_i$$

Perfect factorization: "reasonable" for nucleon-nucleus reactions

"Gentle" ($k_f \sim k_i$) reactions probe $\rho_{fi}(\mathbf{q})$ with $\mathbf{q}\sim\mathbf{k}_f-\mathbf{k}_i$

One step in U (or higher?)



One step

Two step

(a)

$$T_{DWBA}(\mathbf{k}',\mathbf{k}) = \left\langle \chi_{\mathbf{k}'}^{(-)}\middle|U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$

(b)

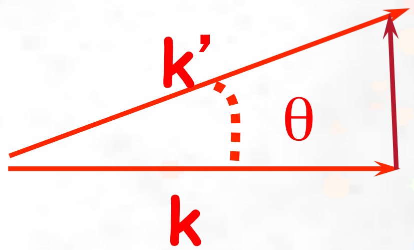
$$T_{DWBA2}(\mathbf{k}',\mathbf{k}) = \sum_{\gamma} C_{\gamma}\left\langle \chi_{\mathbf{k}'}^{(-)}\middle|UG_{\gamma}^{(+)}U\middle|\chi_{\mathbf{k}}^{(+)}\right\rangle$$

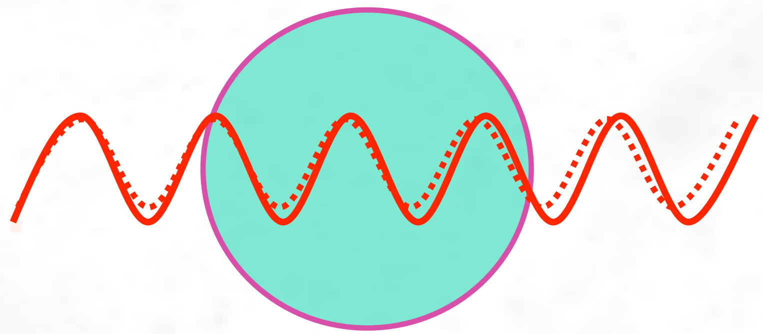
propagation through intermediate states

High energy scattering

Eikonal Waves

$\Delta E << E$, $\theta << 1$ radian , $|\Delta\psi/\psi|_{\Delta r=\lambda} << 1$





$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$

Assume ψ is almost like a plane wave

$$\Psi(\mathbf{r}) = S(\mathbf{b},z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

where S(b,z) is a slowly varying function of b and z.

$$\left|\nabla^2 S\right| << k\left|\nabla S\right|$$

$$2ik\,e^{ikz}\frac{\partial S}{\partial z} + \cancel{e^{ikz}\frac{\partial^2 S}{\partial z^2}} + \cancel{e^{ikz}\nabla_b^2 S} - \frac{2\mu}{\hbar^2}Ve^{ikz}S = 0$$

$$\frac{\partial S}{\partial z} = -\frac{i}{\hbar v}V(\mathbf{r})S \Rightarrow S(\mathbf{b},z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\mathbf{r}')\,dz'\right\}$$

$$T_{DWBA}(\mathbf{k}',\mathbf{k}) = \left\langle \chi_{\mathbf{k}'}^{(-)};\alpha_f \middle| U \middle| \chi_{\mathbf{k}}^{(+)};\alpha_i \right\rangle$$

Using eikonal,

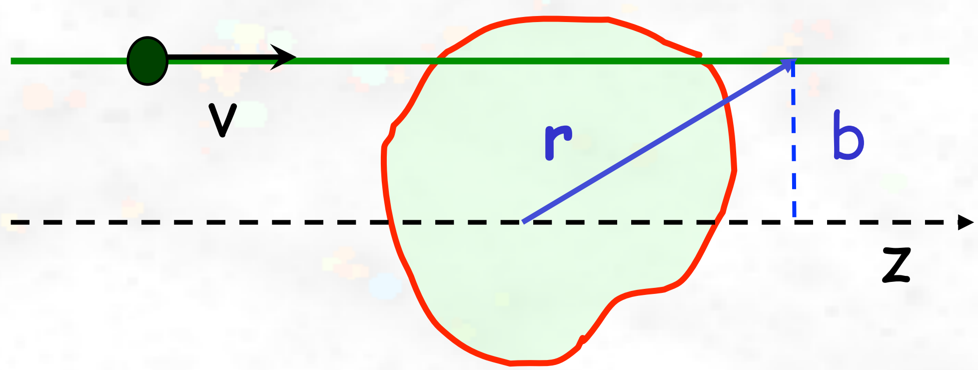
$$\chi_{\mathbf{k}'}^{(-)^*}\chi_{\mathbf{k}}^{(+)} = \mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}\exp\left\{-\frac{i\mu}{\hbar^2 k}\int_{-\infty}^{z} V(r')\,dz' - \frac{i\mu}{\hbar^2 k}\int_{z}^{\infty} V(r')\,dz'\right\}$$

$$= \exp\left\{ i\mathbf{q}\cdot\mathbf{r} + i\delta(b)\right\}$$

$$\mathbf{q} = \mathbf{k}' - \mathbf{k}, \qquad \delta(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} V(r')\,dz'$$

(eikonal phase)

$$v = \hbar k/\mu$$



$$\chi_{\mathbf{k}'}^{(-)^*}\chi_{\mathbf{k}}^{(+)} = S(b)\,\mathrm{e}^{i\mathbf{q}\cdot\mathbf{r}}$$

$$S(b) = e^{i\delta(b)}$$

(eikonal S-matrix)

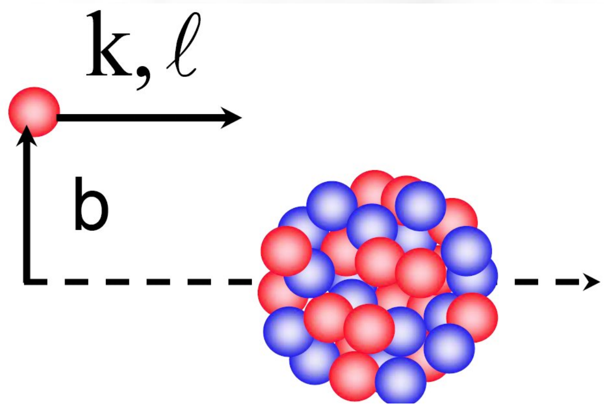
Eikonal waves (reactions)

Harmonic oscillator (structure)

Pearls of quantum mechanics

$$\Rightarrow \quad T_{DWBA}(\mathbf{k}',\mathbf{k}) = \int d^3r\, S(b)\, \exp[i\mathbf{q}\cdot\mathbf{r}]\, \langle \alpha_f | U(\mathbf{r}) | \alpha_i \rangle$$

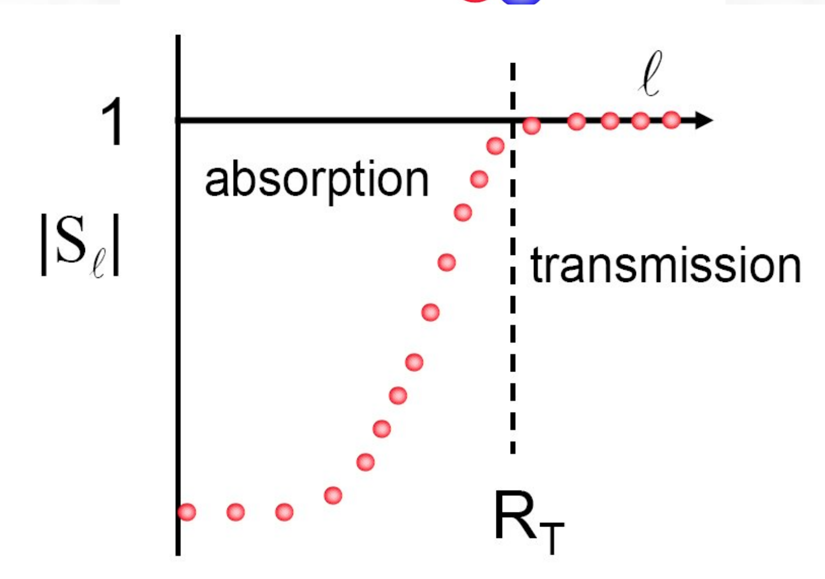
Partial wave expansion ⟶ simple integral



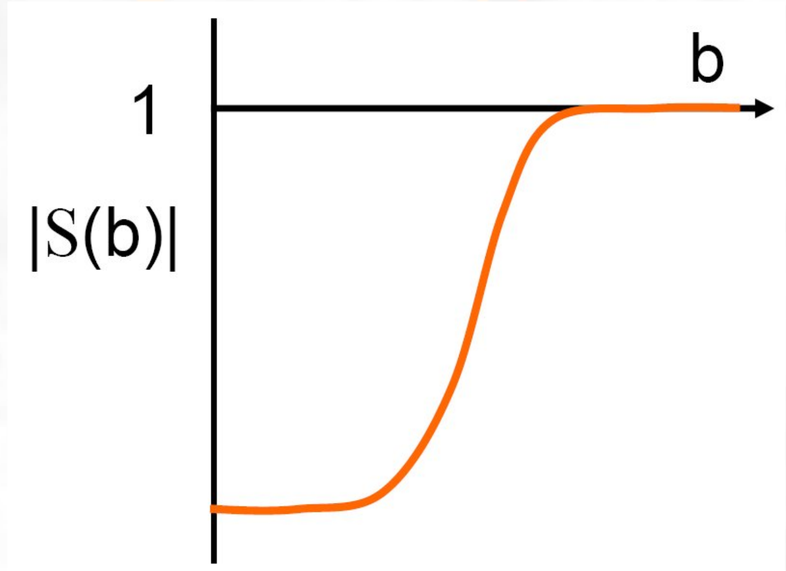
interpretation:

b = impact parameter

l = kb (actually l + 1/2 = kb)

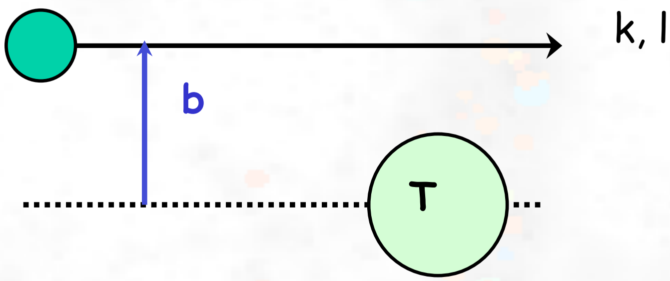


l (discrete values)



b (continuous)

S-matrices ("Survival" Amplitudes)



b = impact parameter

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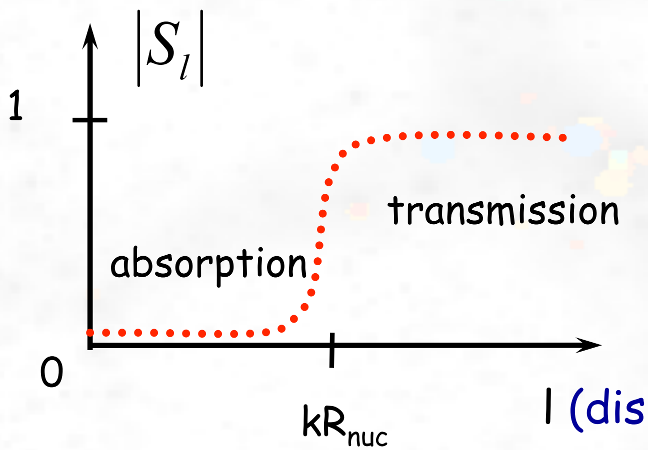
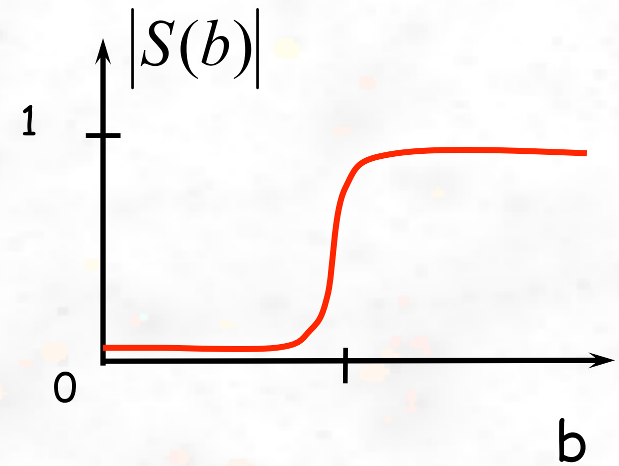


$\Longrightarrow$

eikonal



Ex: Elastic Scattering

$$f(\theta) = \frac{i}{k}\sum_l (l+\tfrac{1}{2})(1-S_l)P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$f(\theta) = ik\int db\, b\, J_0(kb)\{1-S(b)\}$$

Coulomb phase

$$V_C = \frac{Z_1 Z_2 e^2}{r}$$

$$\delta_C(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} V_C(r')\,dz' \quad \rightarrow \quad \infty$$

But $$\delta_C(b) = \frac{2Z_1 Z_2 e^2}{\hbar v}\ln(kb)$$ yields

$$f_{eikonal}(\theta) = ik\int db\, b\, J_0(kb)\left\{1 - \exp\left[iX_C(b)\right]\right\}$$

$$= \frac{Z_1 Z_2 e^2}{2\mu v^2 \sin^2(\theta/2)}\exp\left[-i\eta\ln\left(\sin^2(\theta/2) + 2\sigma_0\right)\right]$$

(correct)

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

- Also used for distortion in inelastic channels
- Finite nuclear size effects on δ_C easily included

Connection to NN interaction: tρρ approximation

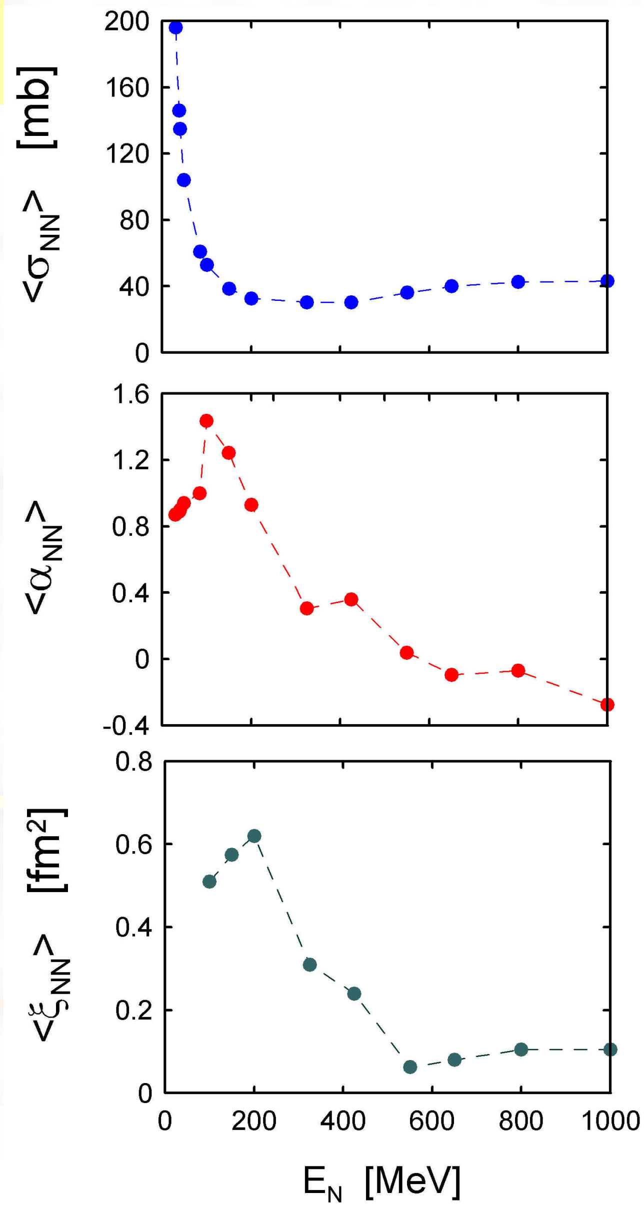
$$V_{PT} \sim t_{NN}\left(\theta \sim 0^o, E\right)\int d\mathbf{r}'\rho_P(\mathbf{r}')\rho_T(\mathbf{r}-\mathbf{r}')$$
$$= -4\pi\frac{E}{k^2} f_{NN}\left(\theta \sim 0^o, E\right)\int d\mathbf{r}'\rho_P(\mathbf{r}')\rho_T(\mathbf{r}-\mathbf{r}')$$

Hussein, Rego, CB, Phys. Rep. 5 (1991) 279

$$f_{NN}\left(\theta \sim 0^o, E\right) = f_{NN}(q) = \frac{k_{NN}}{4\pi}\sigma_{NN}\left(i+\alpha_{NN}\right)e^{-\xi_{NN}\, q^2}$$

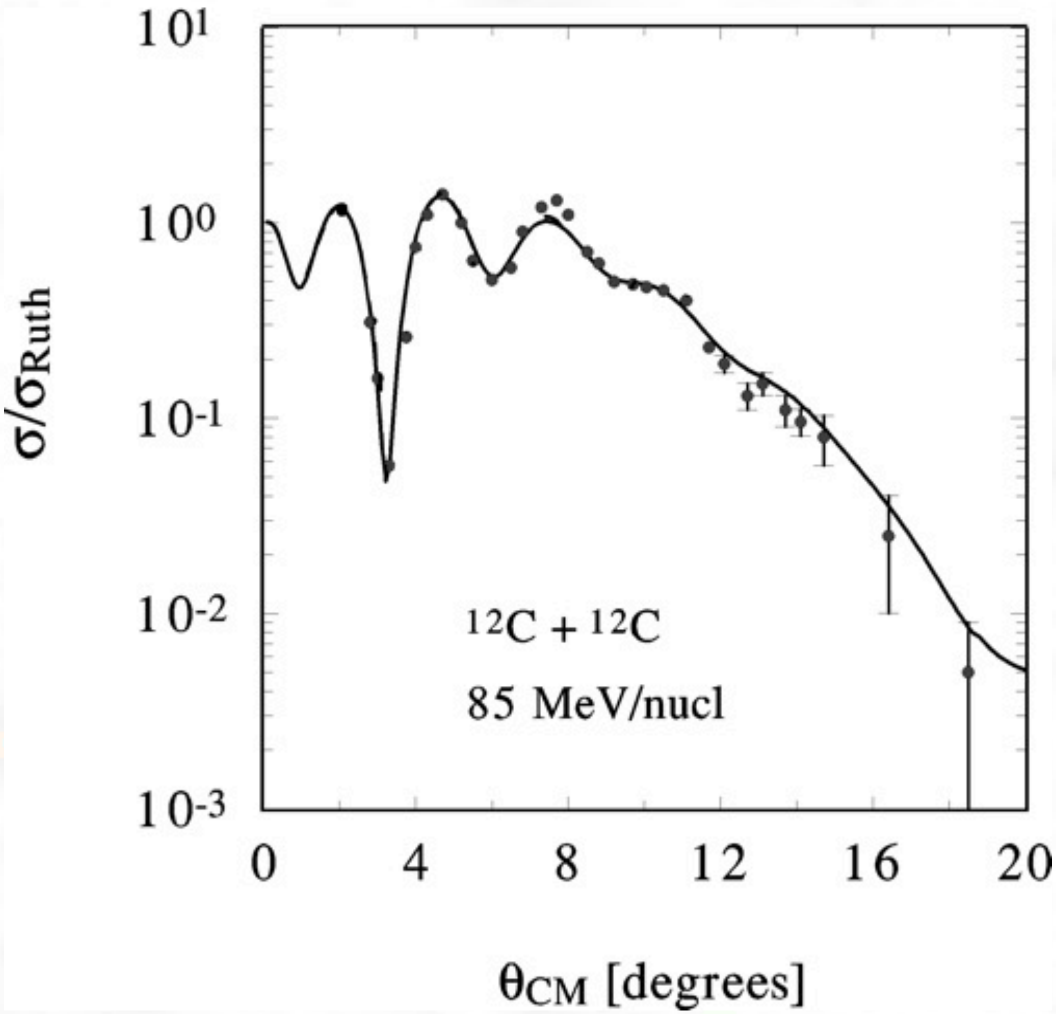
$$\delta_{PT}^{(N)}(b) = \frac{\sigma_{NN}}{4\pi}\left(i+\alpha_{NN}\right)\int_0^\infty dq\, q\, \tilde{\rho}_P(q)\tilde{\rho}_T(q)\, e^{-\xi_{NN}\, q^2} J_0(qb)$$

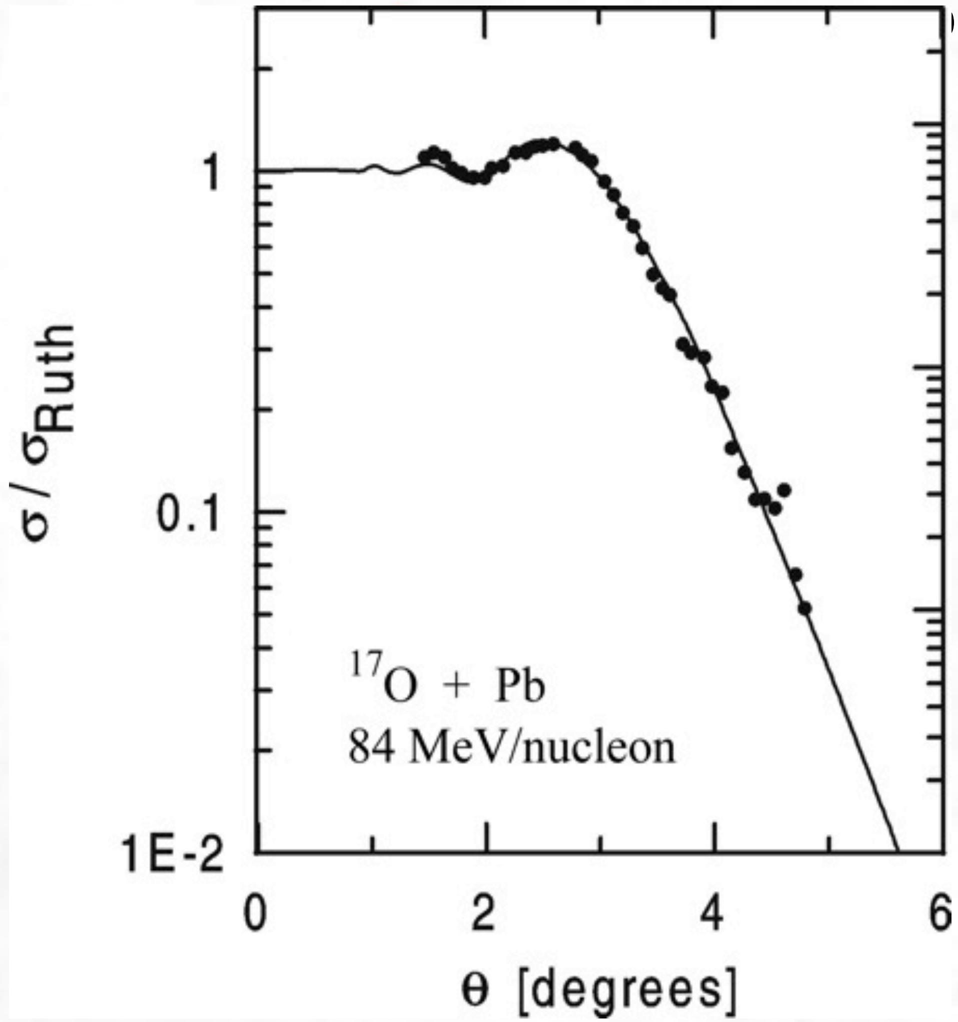
$$S(b) = \exp\left\{ i\delta_{PT}^{N}(b) + i + \frac{2Z_1Z_2e^2}{\hbar v}\ln(kb)\right\}$$



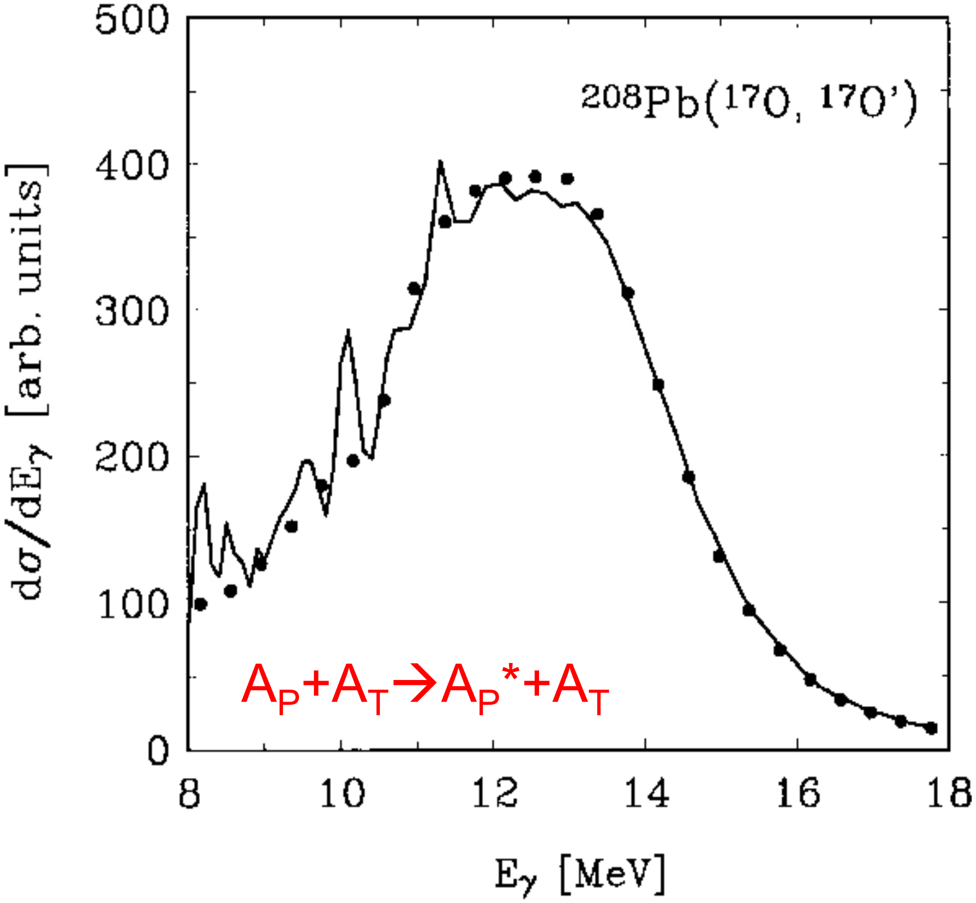
Probing nuclear densities

Same optical potential as used in partial wave analysis



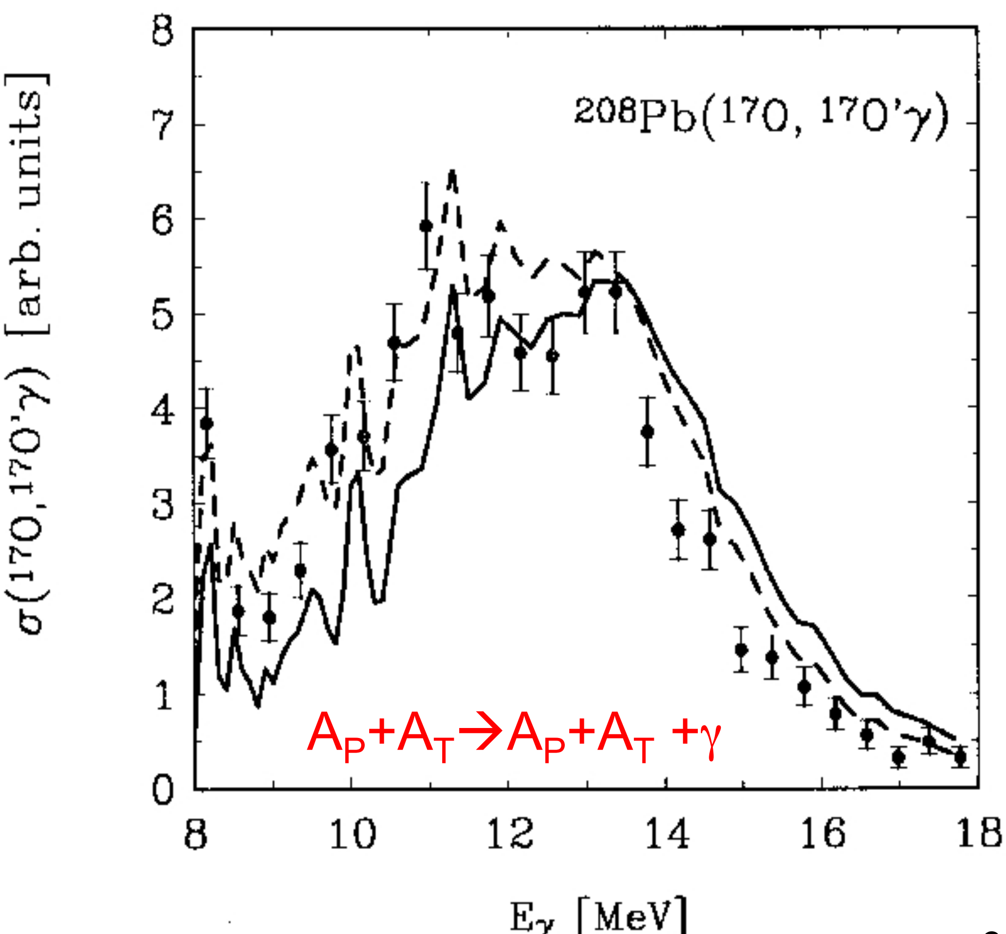


Another application:
Probing collective states



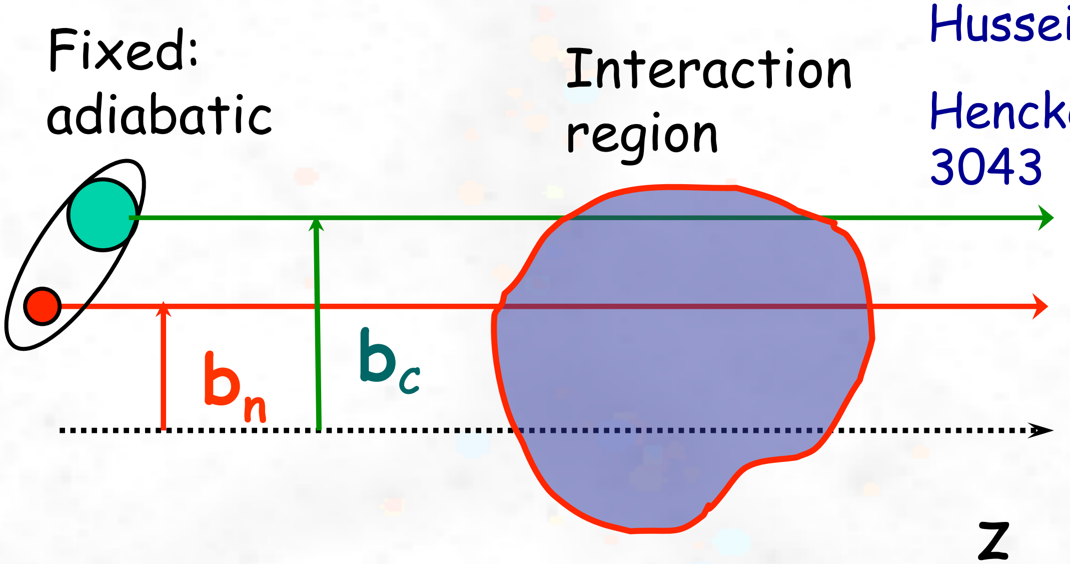
GANIL data, PRC 41 (1990) 920

CB, Nathan, NPA 554 (1993) 158



Knockout Reactions

Breakup Reactions (two-body): (a) elastic



Hussein, McVoy, NPA 445, 124 (1985)

Hencken, Bertsch, Esbensen, PRC 54 (1996) 3043

Elastic:

including breakup effects

$$\Psi^{eik}(\mathbf{r}) = S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \phi_0$$

Best possible wfs:

(Spectroscopy)

$$S_{elast}(\mathbf{b}) = \left\langle \phi_0 \middle| S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n) \middle| \phi_0 \right\rangle$$

Survival amplitude

for projectile at impact parameter b

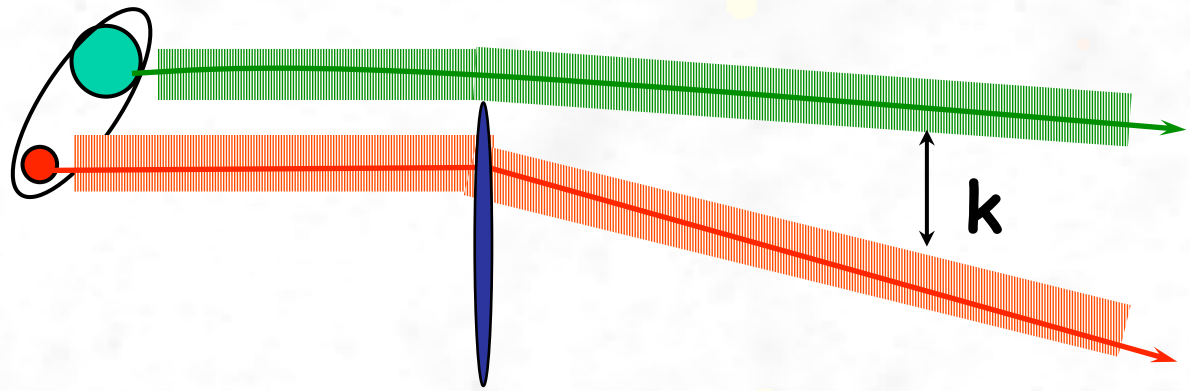
Survival amplitudes

for particles C and n at impact parameters b_C and b_n

(Dynamics)

(b) Diffraction Dissociation

Breakup amplitude:
to state $\phi_{\mathbf{k}}$



$$1 - S_{dif.dis.}(\mathbf{b}) = 1 - S_C(\mathbf{b}_C) + 1 - S_n(\mathbf{b}_n) - \left[1 - S_C(\mathbf{b}_C)\right]\left[1 - S_n(\mathbf{b}_n)\right]$$

$$\varphi_{\mathbf{k}} \perp \phi_0 \quad \Longrightarrow \quad S_{dif.dis.}(\mathbf{b}) = \left\langle \phi_{\mathbf{k}} \middle| S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n) \middle| \phi_0 \right\rangle$$

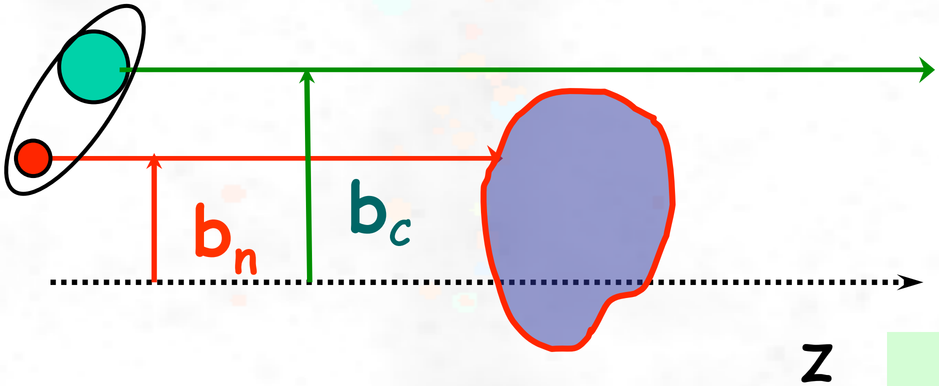
Closure:

$$\int d\mathbf{k} \left|\phi_{\mathbf{k}}\right\rangle\left\langle\phi_{\mathbf{k}}\right| = 1 - \left|\phi_0\right\rangle\left\langle\phi_0\right| - \left|\phi_1\right\rangle\left\langle\phi_1\right| - \cdots$$

$\Rightarrow$ Breakup X-section (only one bound state):

$$\sigma_{dif.dis.}(\mathbf{b}) = \int d\mathbf{b} \left[\left\langle \phi_0 \middle| \left|S_C\, S_n\right|^2 \middle| \phi_0 \right\rangle - \left|\left\langle \phi_0 \middle| S_C\, S_n \middle| \phi_0 \right\rangle\right|^2\right]$$

(c) Stripping:

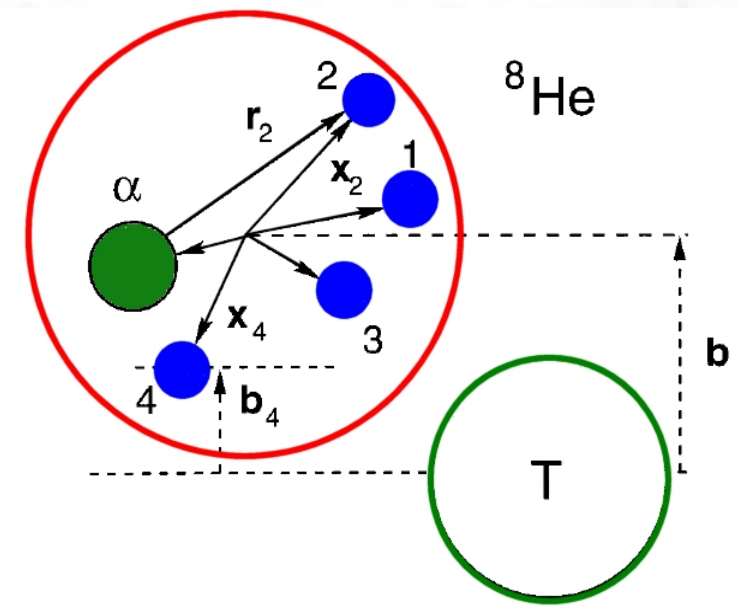


$$\left|S_C(\mathbf{b}_C)\right|^2\left(1-\left|S_n(\mathbf{b}_n)\right|^2\right)$$

C survives, n absorbed

$$\sigma_{strip}(\mathbf{b}) = \int d\mathbf{b}\left\langle\phi_0\middle|\,\left|S_C\right|^2\left(1-\left|S_n\right|^2\right)\middle|\,\phi_0\right\rangle^2$$

(d) Composite particles:



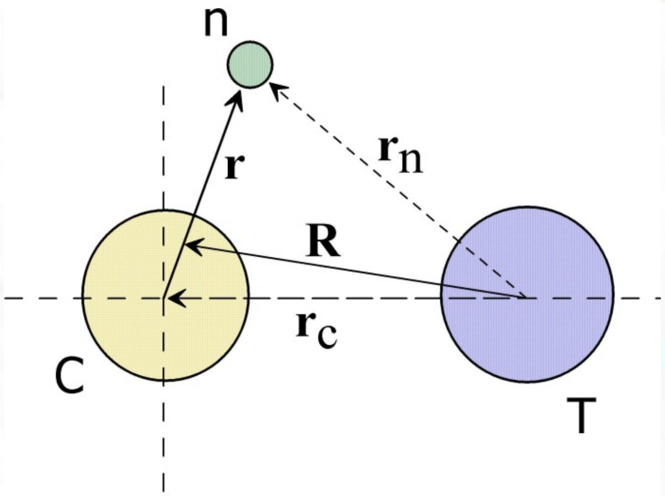
$$S_{dif.dis.}(\mathbf{b}) = \left\langle\phi_8\middle|S_\alpha(\mathbf{b}_\alpha)\prod_{i=1}^{4}S_i(\mathbf{b}_i)\middle|\,\phi_8\right\rangle$$

$$\prod_{j\ survive}\left|S_j(\mathbf{b}_j)\right|^2\prod_{k\ \ absorbed}\left(1-\left|S_k(\mathbf{b}_k)\right|^2\right)$$

Momentum Distributions: (a) Stripping

C scatters elastically and C + n breaks up:

$$\left| \left\langle \phi_{Continuum}(\mathbf{r}) \middle| S_C(\mathbf{b}_C)\, \phi_{l_0,m_0}(\mathbf{r}) \right\rangle \right|^2$$

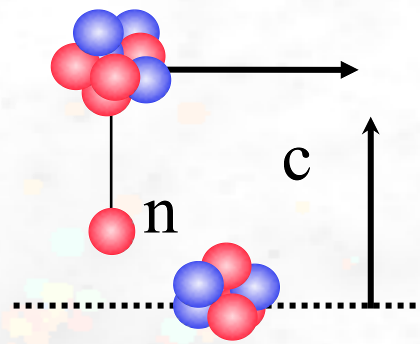


$$\mathbf{r}_n = \mathbf{R} + \frac{m_C}{m}\mathbf{r}, \qquad \mathbf{r}_C = \mathbf{R} - \frac{m_n}{m}\mathbf{r}$$

$$\mathbf{K} = \mathbf{k}_C - \mathbf{k}_n, \qquad \mathbf{k} = \frac{m_n}{m}\mathbf{k}_C - \frac{m_C}{m}\mathbf{k}_n$$

n is absorbed:

$$1 - \left| S_n(\mathbf{b}_n) \right|^2$$



$$\phi_{Continuum}(\mathbf{r}) \sim e^{i\mathbf{k}.\mathbf{r}}$$

$$\frac{d\sigma_{strip}}{d^3k_C} = \frac{1}{(2\pi)^3}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n \left[1 - \left|S_n(\mathbf{b}_n)\right|^2\right] \left| \int d^3r\, e^{-i\mathbf{k}_C\cdot\mathbf{r}}\; S_C(\mathbf{b}_C)\, \phi_{l_0,m_0}(\mathbf{r}) \right|^2$$

Momentum Distributions

Serber model:
PR 72 (1947) 1008

$$S_C(\mathbf{b}_C) \approx 1 \quad \Longrightarrow \quad \frac{d\sigma_{strip}}{d^3k_C} = C_{geometry} \left| \tilde{\phi}_{l_0 m_0}(\mathbf{k}_c) \right|^2$$

(b) Diffraction dissociation

C and n scatters elastically:

$$S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r})$$

Project onto continuum CM and relative coordinates:

$$\left\langle e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \phi^*_{\mathbf{k}}(\mathbf{r}) \right| S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r}) \rangle$$

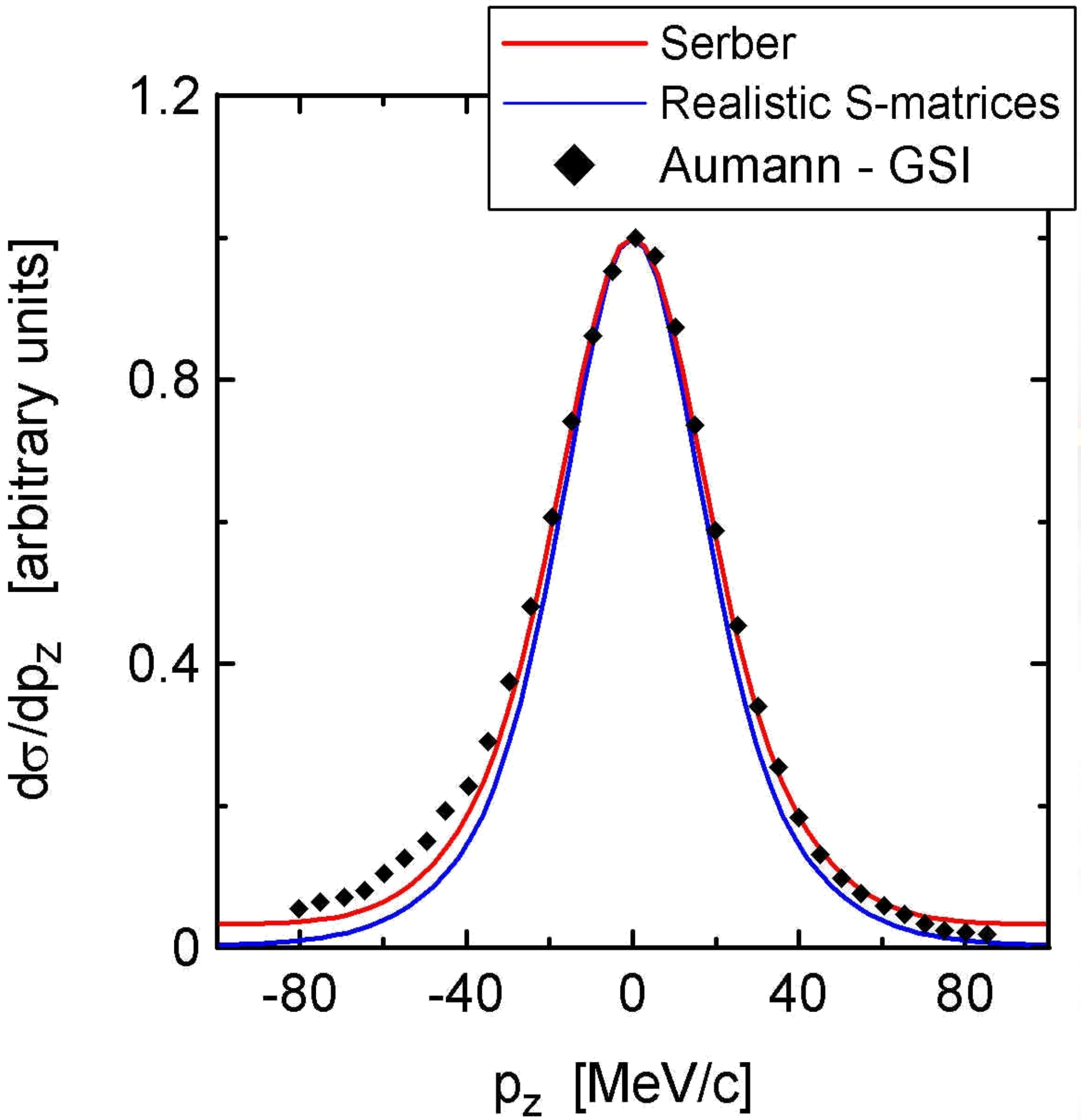
CM

Relative motion

$$\frac{d\sigma_{dif.dis.}}{d^2K_\perp d^3k} = \frac{1}{(2\pi)^5} \frac{1}{(2l_0+1)} \sum_{m_0} \left| \int d^3r\, d^2b\, e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \phi^*_{\mathbf{k}}(\mathbf{r})\, S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r}) \right|^2$$

Applications: (a) Longitudinal Momentum Dist. (LMD)

$$\frac{d\sigma_{strip}}{dk_C^z} = \frac{1}{2\pi}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n\left[1-\left|S_n(\mathbf{b}_n)\right|^2\right]\int d^2b_C\left|S_C(\mathbf{b}_C)\right|^2\left|\int dz\, e^{-ik_C^z\cdot z}\quad \phi_{l_0,m_0}(\mathbf{r})\right|^2$$



$^{9}\mathrm{Be}\left(^{11}\mathrm{Be}\,,^{10}\mathrm{Be}+\gamma\right)\mathrm{X}$

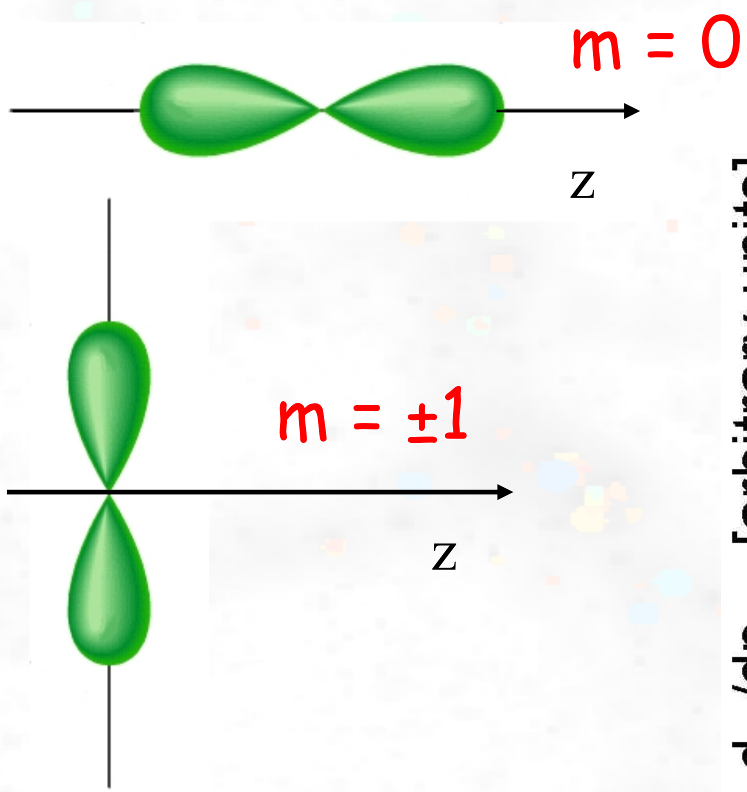
One neutron-removal

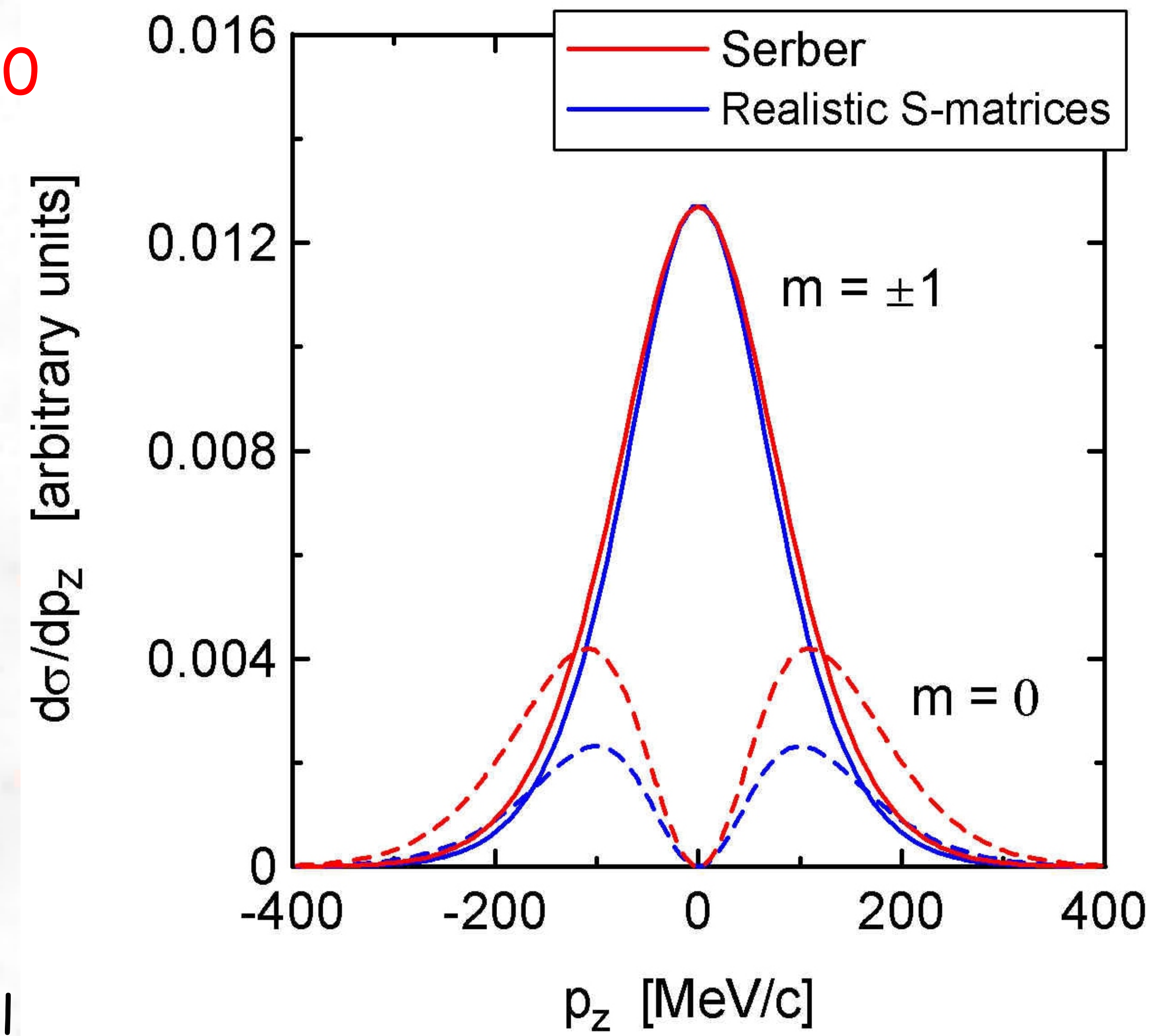
60 MeV/nucleon

$1s_{\frac{1}{2}}$ neutron, S_n = 0.503 MeV

Tails & asymmetry:
higher order
corrections

LMD: Orbital Alignment





$^{9}Be\left(^{7}Li,{}^{6}He\right)X$

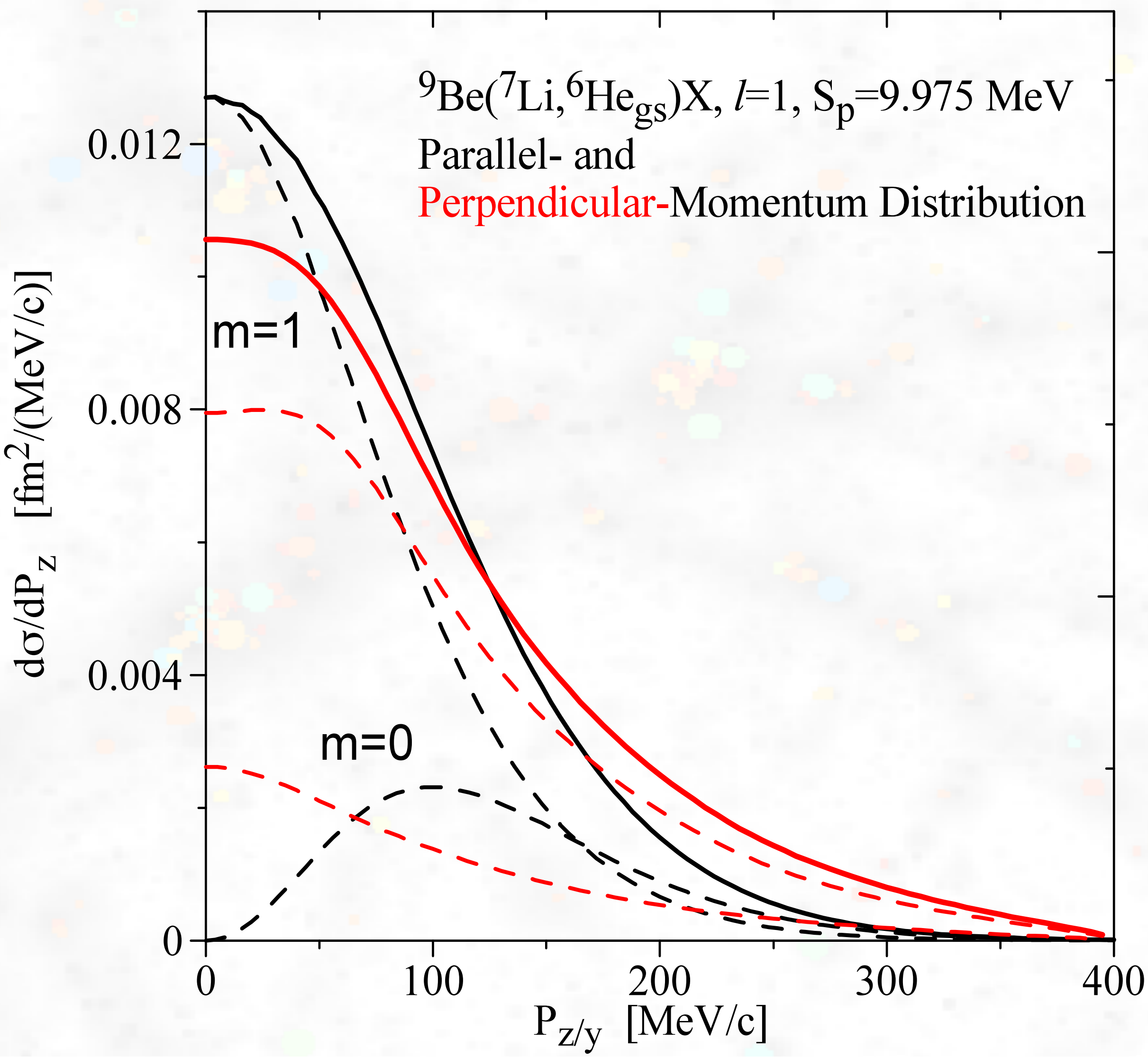
One proton-removal

80 MeV/nucleon

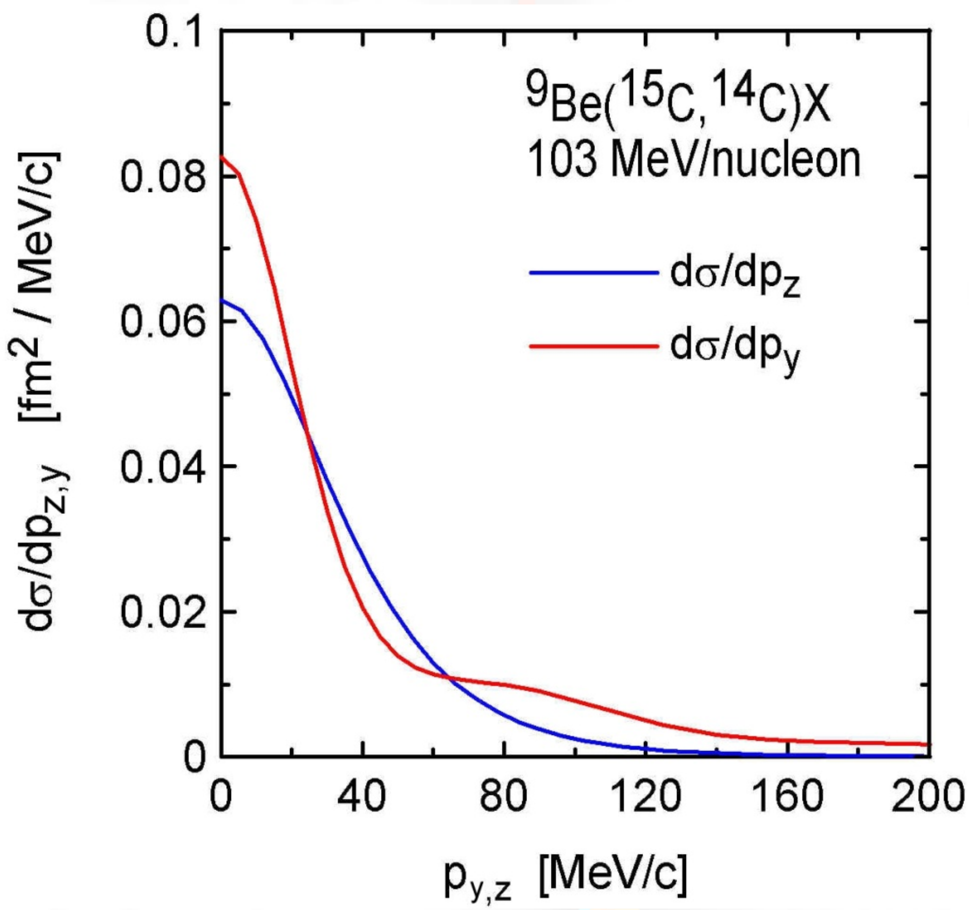
0p3/2 proton, S_p = 9.98 MeV

(b) Transverse Momentum Distribution (TMD)

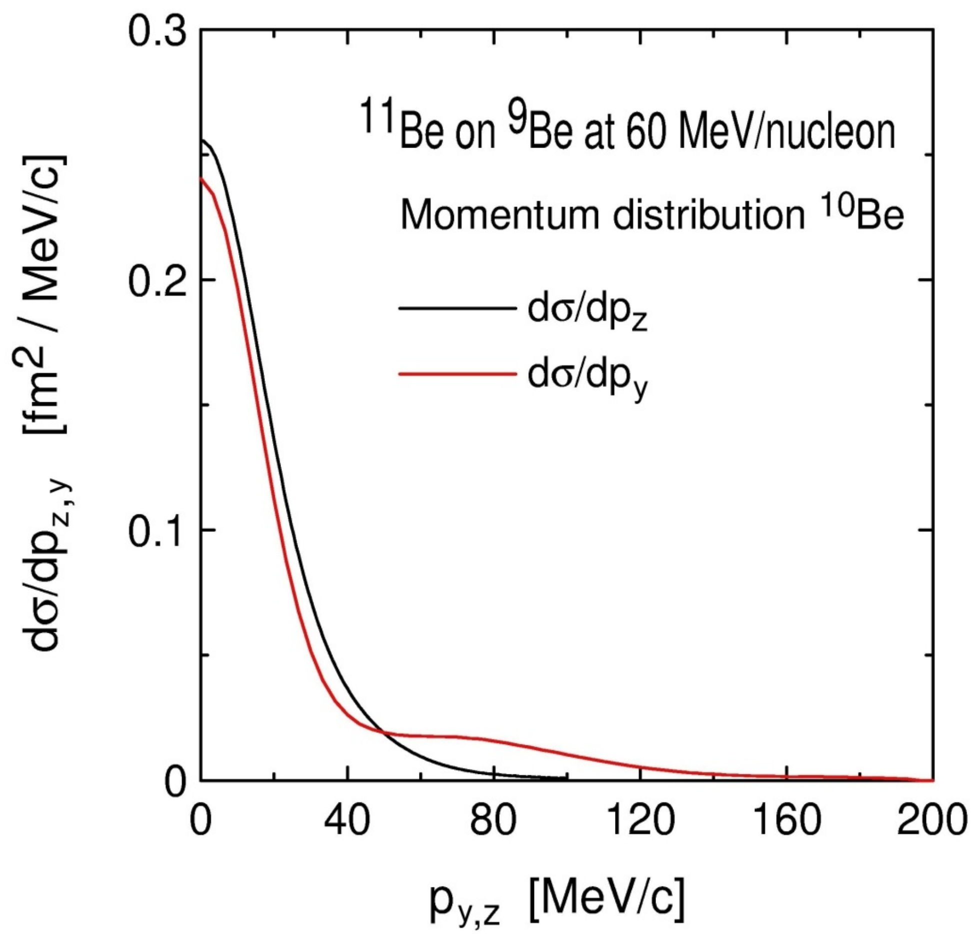
$$\frac{d\sigma_{strip}}{d^2k_C^{\perp}} = \frac{1}{2\pi}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n\left[1-\left|S_n(\mathbf{b}_n)\right|^2\right]\int dz\,\left|\int d^2b_C\, e^{-i\mathbf{k}_C^{\perp}\cdot\mathbf{r}}\, S_C(\mathbf{b}_C)\,\phi_{l_0,m_0}(\mathbf{r})\right|^2$$



TMD x LMD:

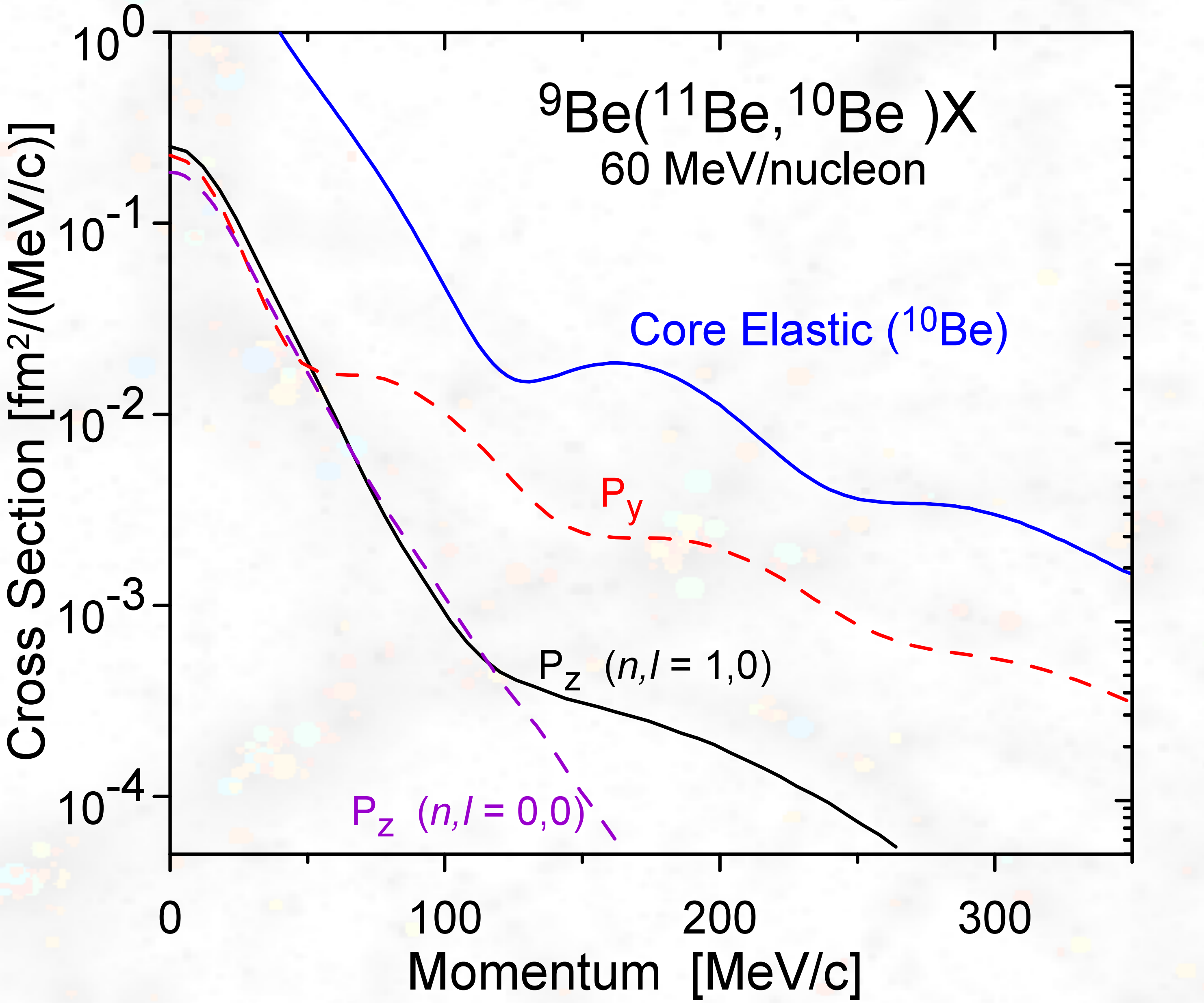


$^{9}Be(^{15}C,^{14}C)X$
103 MeV/nucleon
$d\sigma/dp_z$
$d\sigma/dp_y$
$d\sigma/dp_{z,y}$ [fm^2 / MeV/c]
$p_{y,z}$ [MeV/c]
0.1
0.08
0.06
0.04
0.02
0
0
40
80
120
160
200



^{11}Be on ^{9}Be at 60 MeV/nucleon
Momentum distribution ^{10}Be
$d\sigma/dp_z$
$d\sigma/dp_y$
$d\sigma/dp_{z,y}$ [fm^2 / MeV/c]
$p_{y,z}$ [MeV/c]
0.3
0.2
0.1
0
0
40
80
120
160
200

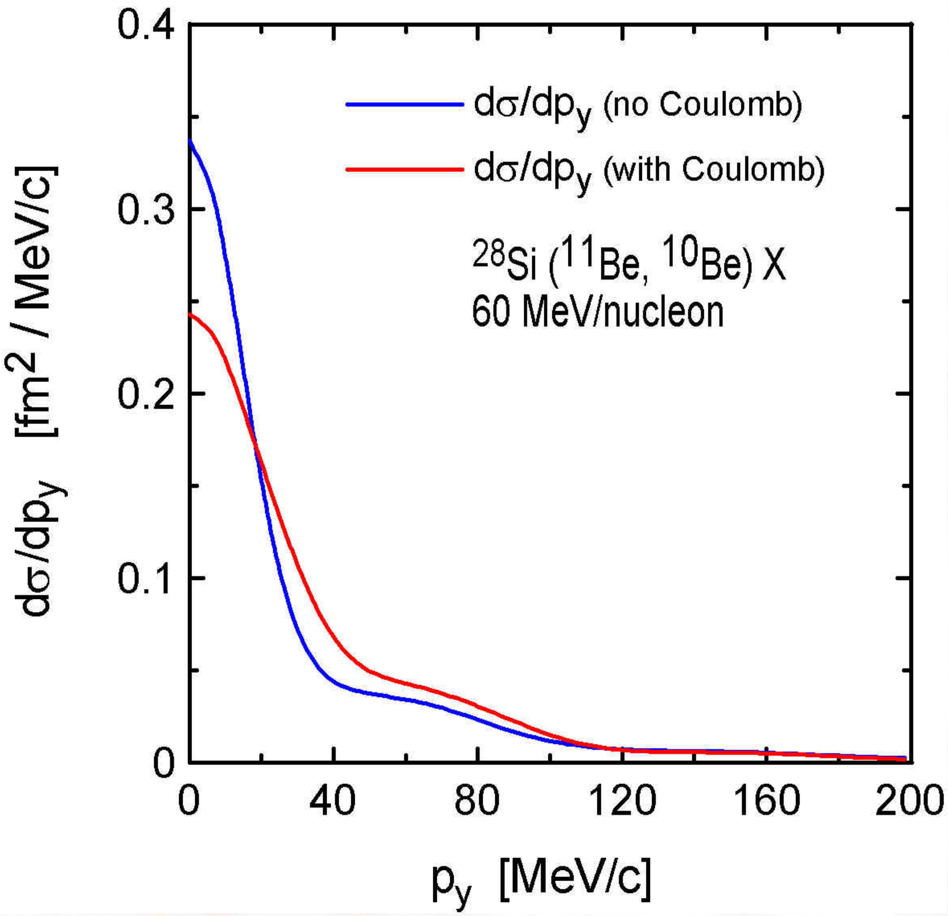
TMD: final state elastic core-target scattering

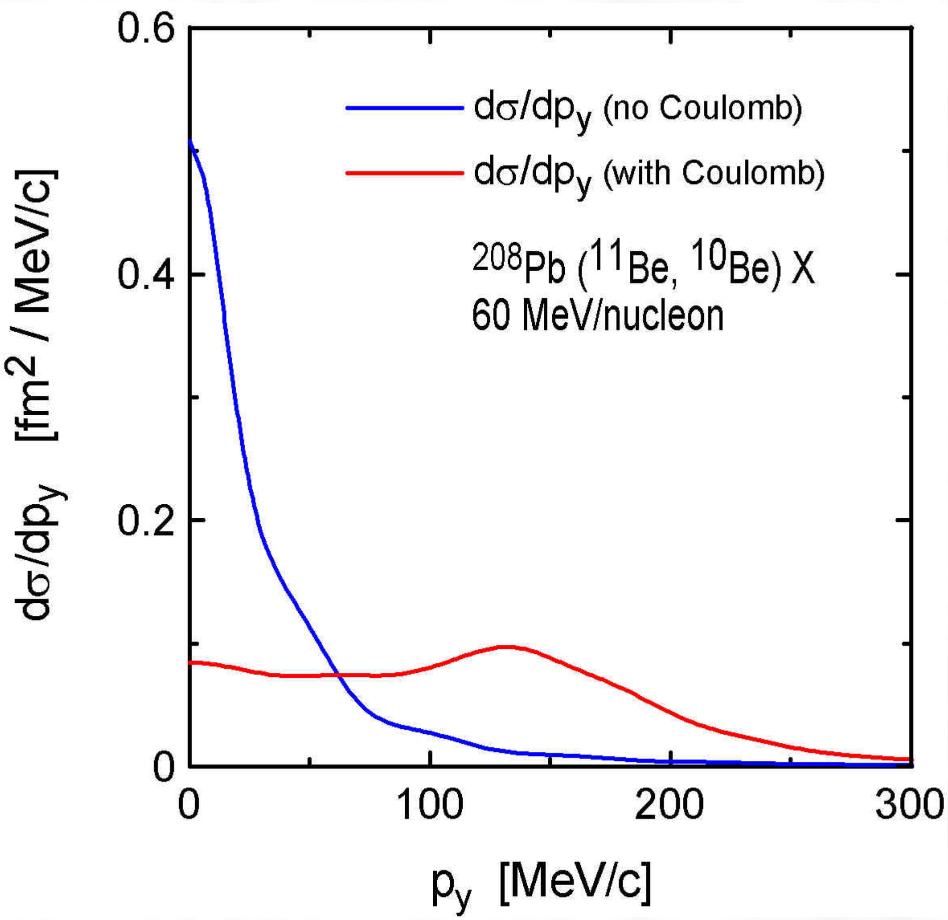


CB, Hansen, PRC 70, 034609 (2004)

Coulomb and medium effects in knockout reactions

TMD: final state core-target Coulomb scattering





Medium effects in σ_{NN}

$$\langle \mathbf{k}|G|\mathbf{k}_0\rangle = \langle \mathbf{k}|V_{NN}|\mathbf{k}_0\rangle - \int \frac{d^3k'}{(2\pi)^3} \frac{\langle \mathbf{k}|V_{NN}|\mathbf{k'}\rangle Q(\mathbf{k'})\langle \mathbf{k'}|G|\mathbf{k}_0\rangle}{E(\mathbf{k'}) - E_0 - i\varepsilon}$$

$$E(\mathbf{P},\mathbf{k}) = e(\mathbf{P}+\mathbf{k}) + e(\mathbf{P}-\mathbf{k})$$

e = single-particle energies

E_0 = E on-shell

$$Q(\mathbf{P},\mathbf{k}) = 1, \quad \text{if} \quad k_{1,2} > k_F$$
$$= 0, \quad \text{otherwise}$$

$$\mathbf{k}_{1,2} = \mathbf{P} \pm \mathbf{k}$$

In real calculations:

$$\overline{Q}(P,k) = \frac{\int d\Omega\, Q(\mathbf{P},\mathbf{k})}{\int d\Omega}$$

$$e(p) = T(p) + v(p)$$

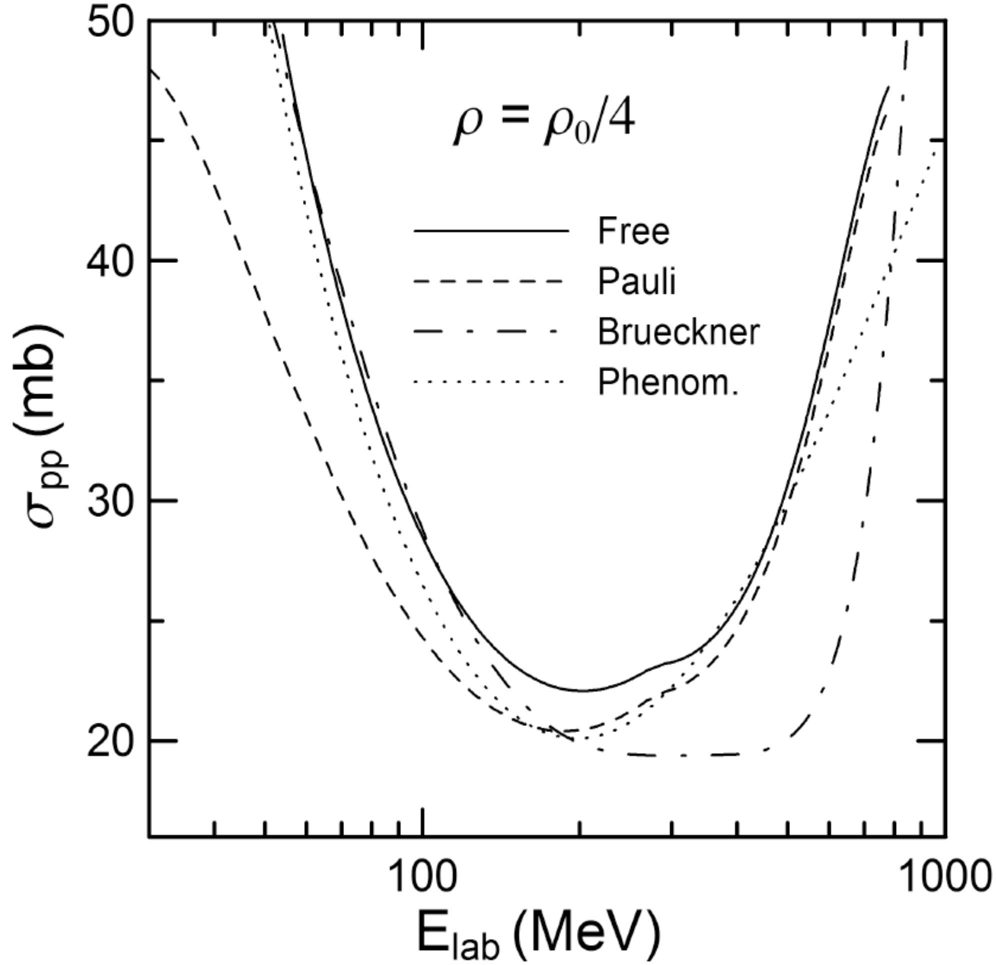
$$v(p) = \langle p|v|p\rangle = \mathrm{Re} \sum_{q \le k_F} \langle pq|G|pq - qp\rangle$$

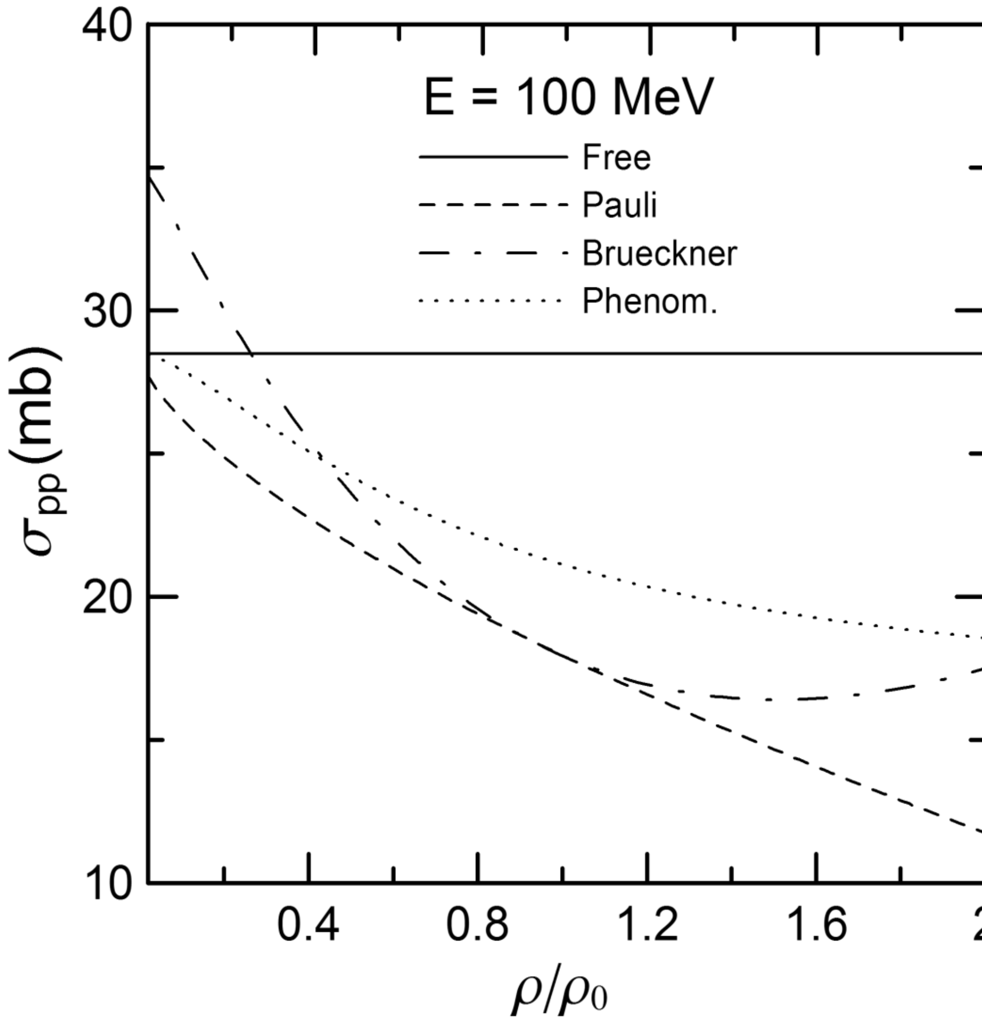
- e depends on v
- v depends on G
- G depends on v

$\Longrightarrow$ Solve self-consistently

(Brueckner theory)

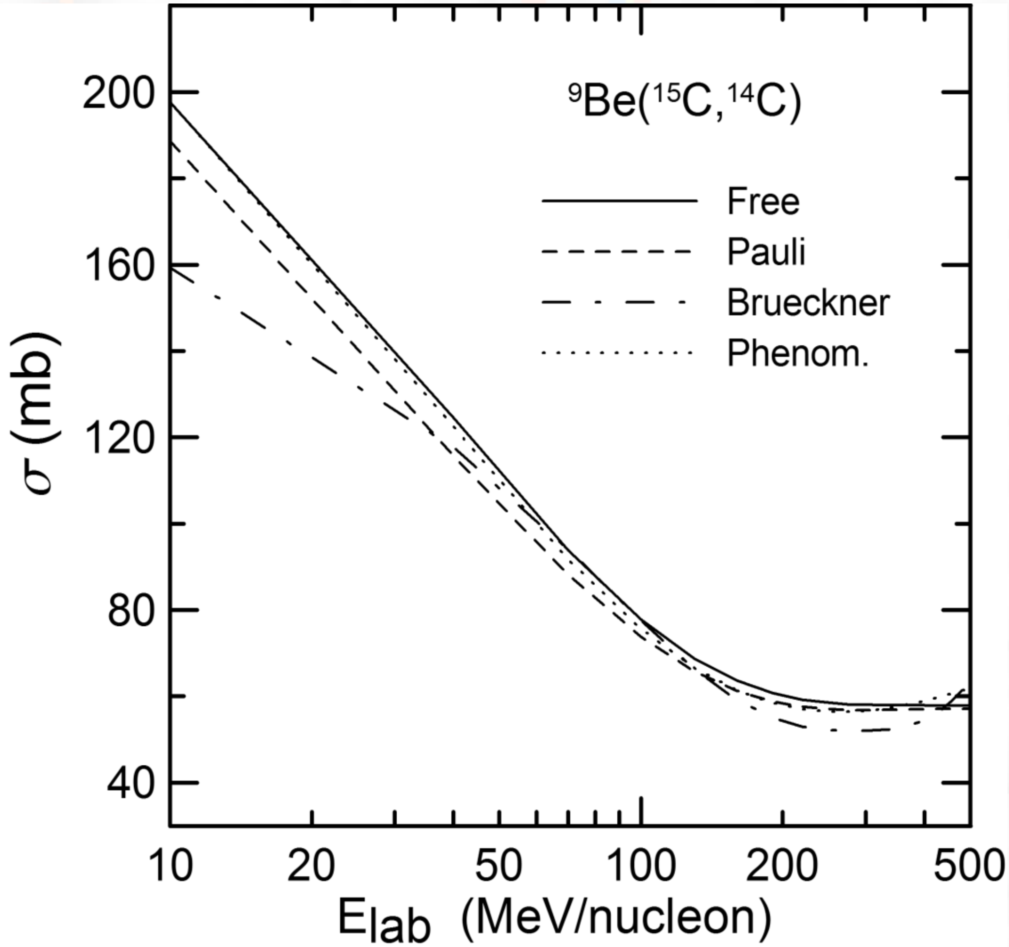
Medium effects in σ_{NN}





CB, De Conti,
PRC 81, 064603 (2010)

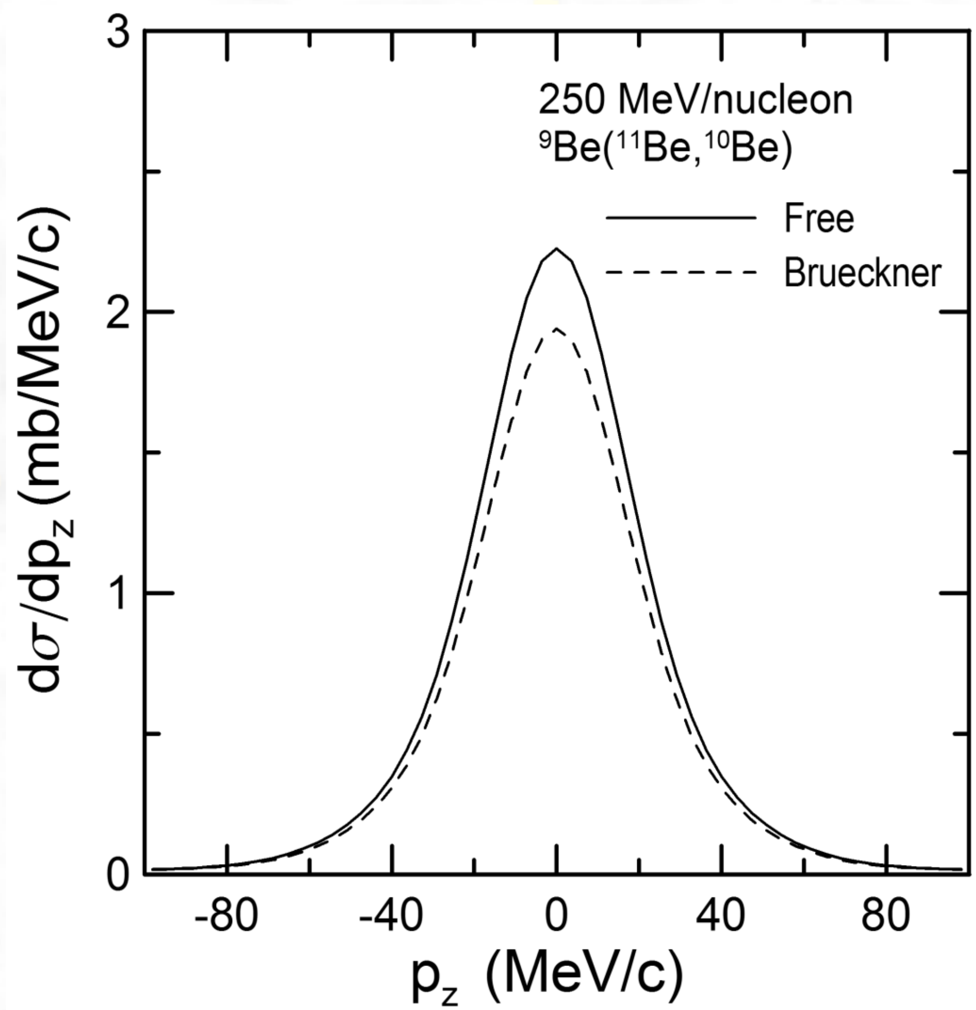
Chen, Sammarruca, CB,
PRC 87, 054616 (2013)



Medium effects in knockout reactions

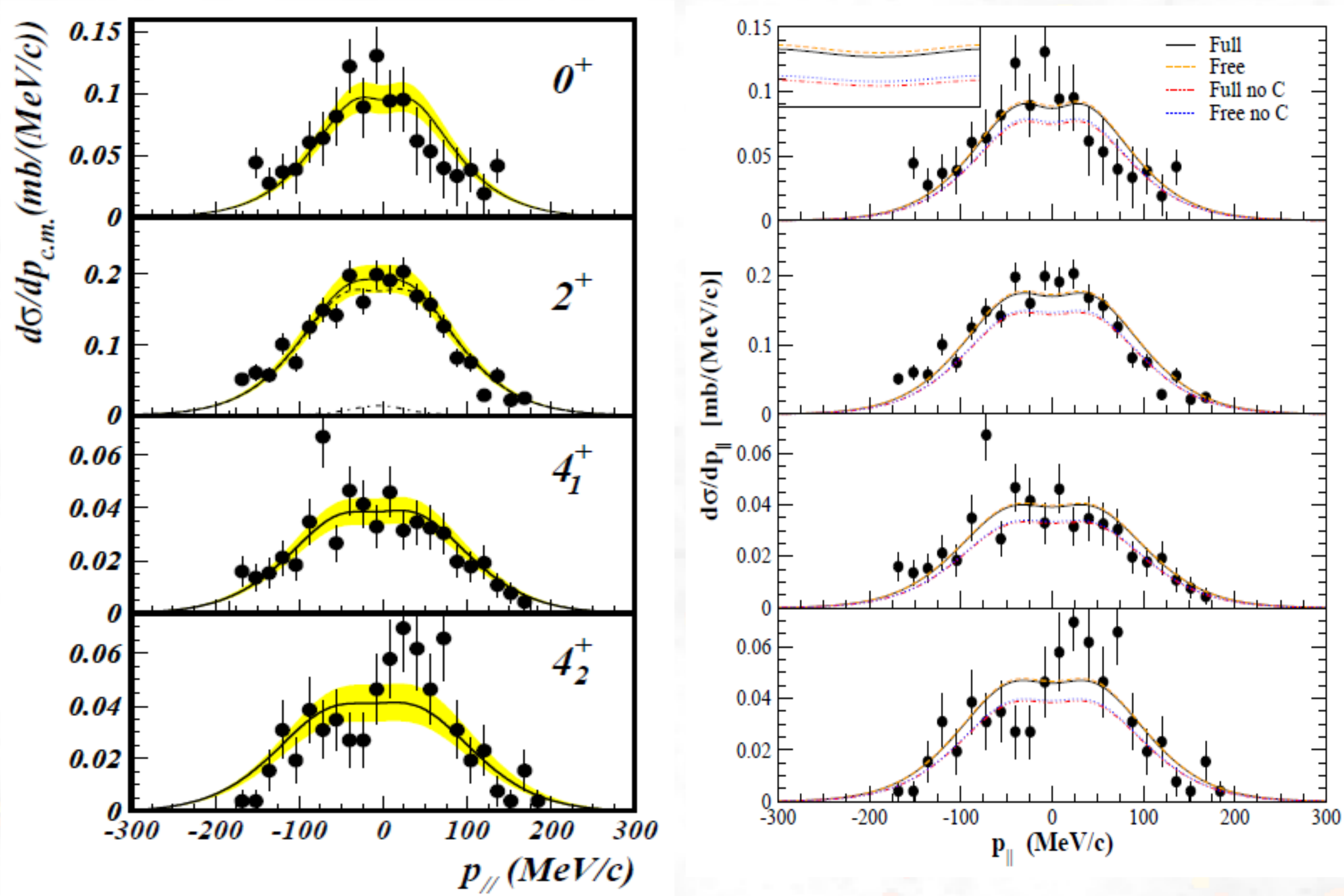
and

momentum distributions

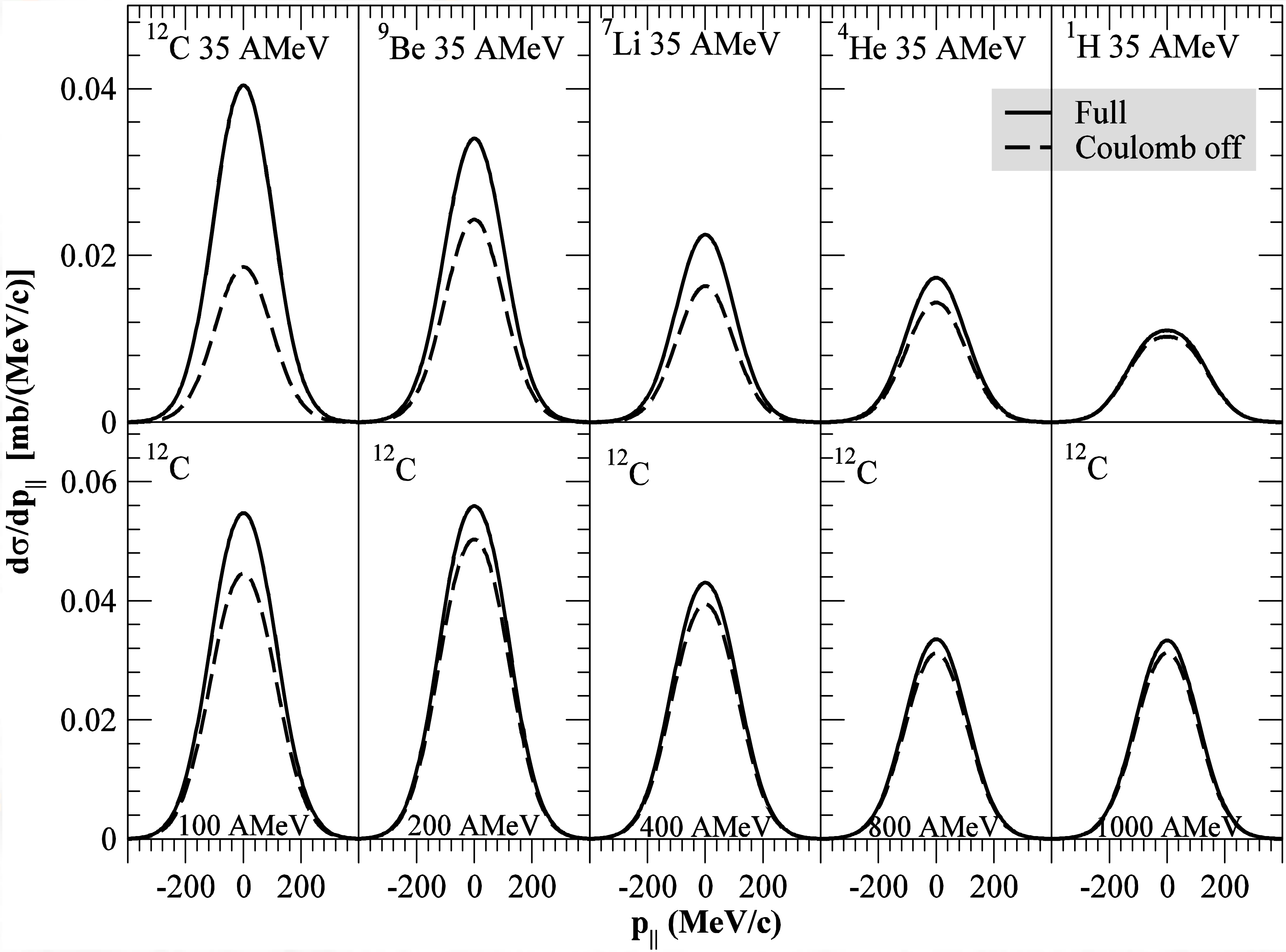


CB, De Conti,
PRC 81, 064603 (2010)

$^{12}C(^{23}Al,^{22}Mg)X$ @ 50 MeV/u



Karakoc, Banu, CB, Trache, PRC 87 024607 (2013)

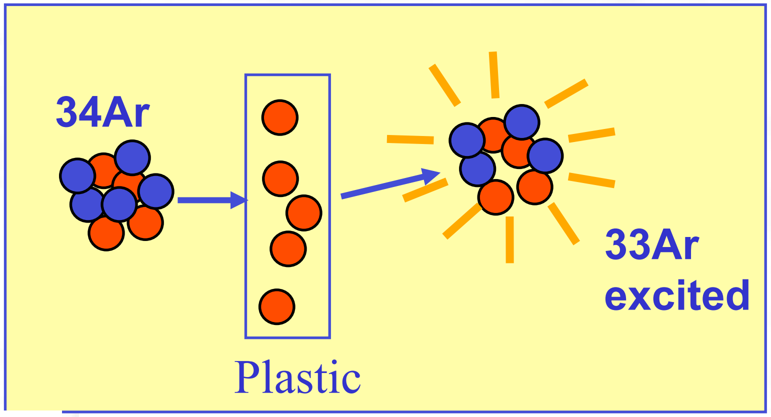


^{12}C 35 AMeV
^{9}Be 35 AMeV
^{7}Li 35 AMeV
^{4}He 35 AMeV
^{1}H 35 AMeV
Full
Coulomb off
^{12}C
100 AMeV
200 AMeV
400 AMeV
800 AMeV
1000 AMeV
dσ/dp$_{||}$ [mb/(MeV/c)]
p$_{||}$ (MeV/c)
0
0.02
0.04
0.06
-200
200

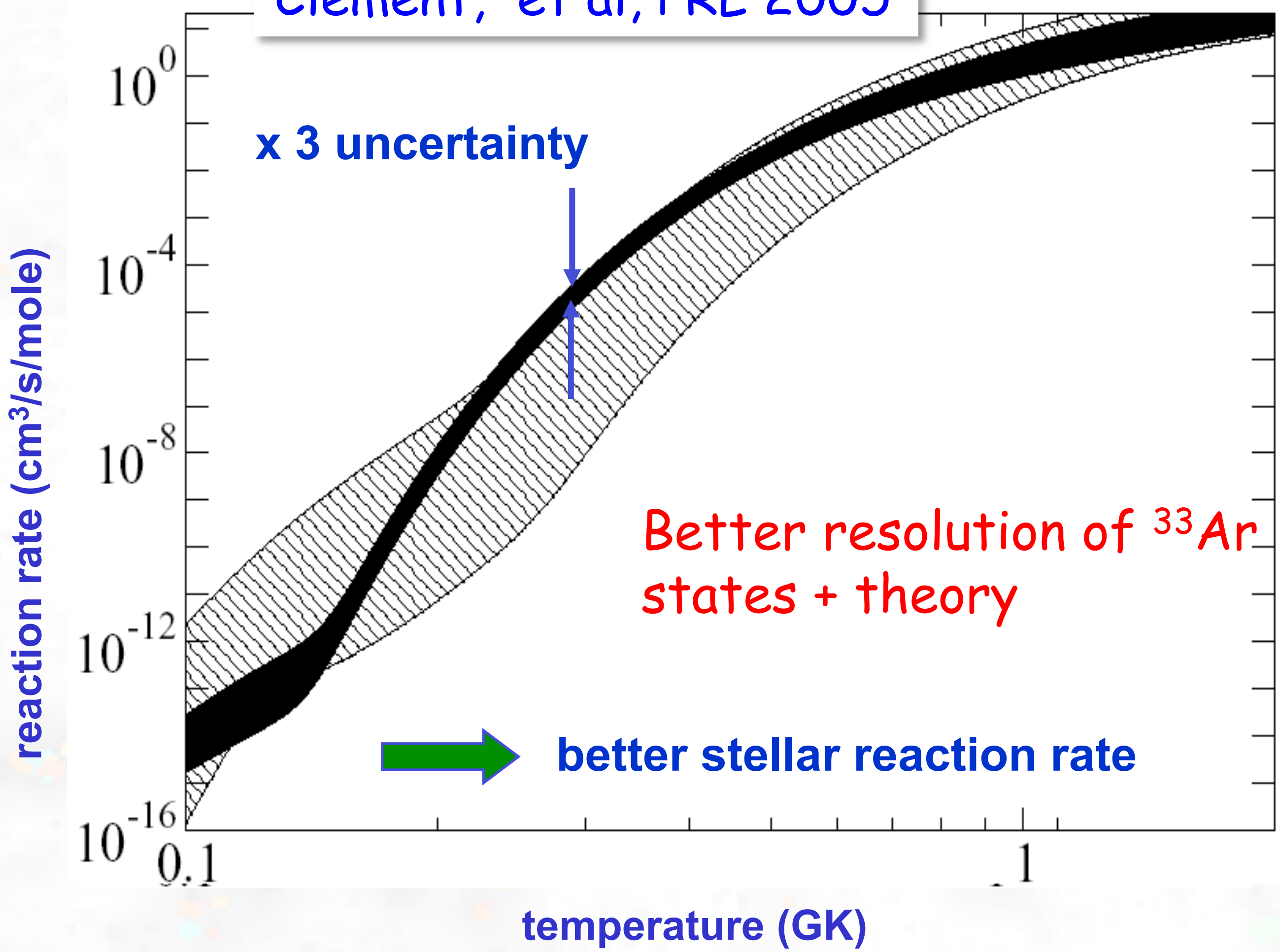
Astrophysics Application: Capture on Excited States in Stars

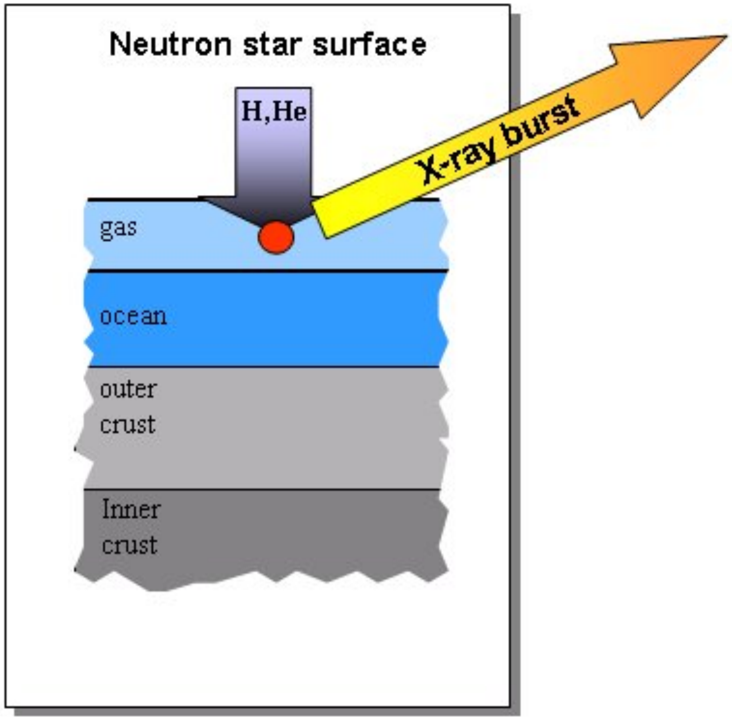
$^{32}Cl^{*} + p \rightarrow {}^{33}Ar^{*} \rightarrow {}^{33}Ar + \gamma$

Enhancement through capture on 89 keV state in ^{32}Cl



Clement, et al, PRL 2005





Capture on excited state of ^{32}Cl 4 times larger!

End Lecture 2