Aim of the lectures: qualitative and semiquantitative description of some phases of cold and dense hadronic matter

Topics

Lecture I, Overview

Motivations, Compact objects, General introduction to superfluids and superconductors, Introduction to color superconductors

Lecture II, Toy Model

Remarks on CFL, Discussion of a toy model, Stability criteria

Lecture III, Tools

Some tools, NJL model, High Density Effective Theory, Low energy effective theories

Lecture IV, Crystalline Color Superconductors

Astrophysical relevance, Main properties, Applications

Remarks

- There are several exercises/question which you should try to do/answer. They appear with a red cornice. They should help to better understand the discussed topic.

- Most of the discussions are rigorous, but not all of them!

- If something is unclear in the first two lectures, do not worry, it should become more clear in the next lecture. I will indeed discuss some topics in various different ways.

- The last lecture is about crystalline color superconductors, it will not help you to clarify any previous topic.

- You are very welcome to ask questions. Although I may not have the answer for all of them.

Reviews: Wilczek and Rajagopal hep-ph/0011333
Alford et al. arXiv:0709.4635,
MM et al. arXiv:1302.4264

QCD CONDENSATES

LECTURE I
OVERVIEW

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GGI March 2014

OUTLINE

- *General motivations*
- *What are compact stars made of?*
- *Superfluids and Superconductors*
- *Color Superconductors*

GENERAL MOTIVATIONS

Two main motivations

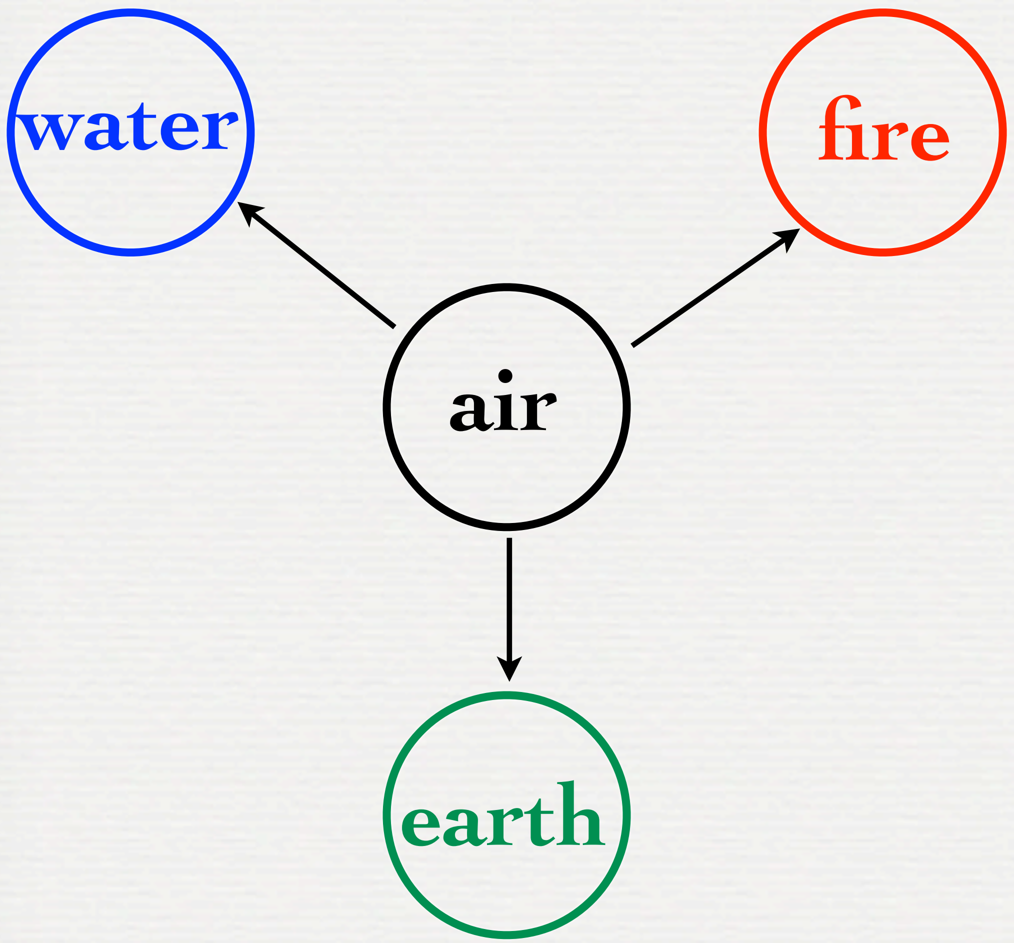
Understanding the properties of hadronic matter in the deconfined phase, in which the correct degrees of freedom are quarks and gluons

Find astrophysical observables related with deconfined quark matter

What is matter made of ?

Reductionism: complex objects are made by simple objects

For ***Anaximenes of Miletus***, **air** is the elementary object, the archè

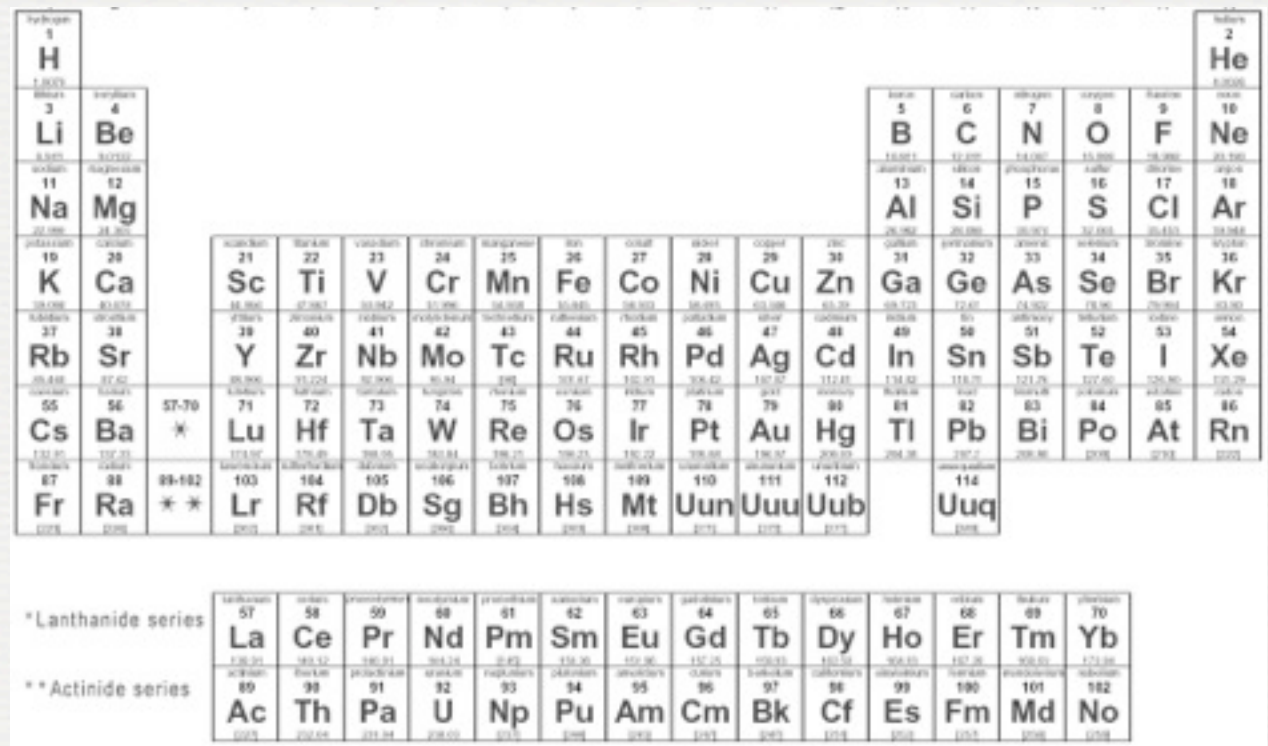


water, fire and earth
are condensed states of air

Matter is made by water, earth and fire in different states of aggregation

What is matter made of ?

A chemist answer:



Systematics suggests substructure

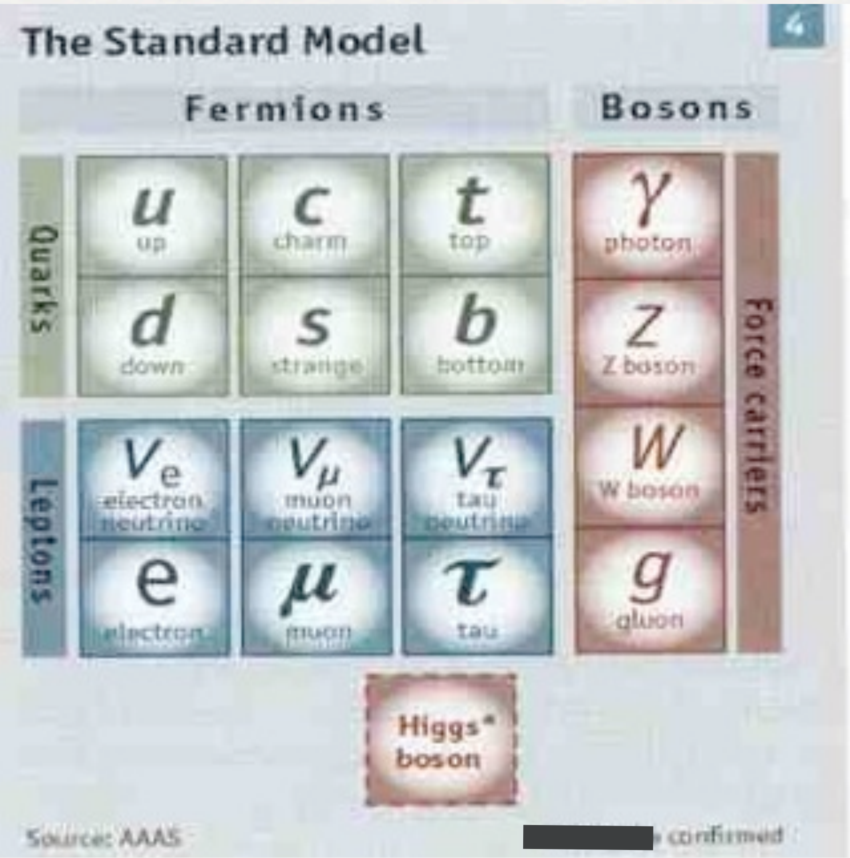
protons, neutrons and electrons

Proliferation of particles

	J^P	Particles (mass in MeV)
mesons	0^-	π^0 (135), $\pi^\pm$ (140), η (547), η' (958), $K^\pm$ (494), $K^0, \bar{K}^0$ (498)
	1^-	$\rho^{\pm,0}$ (771), ω (783), $K^{*\pm}, K^{*0}, \bar{K}^{*0}$ (892), Φ (1020)
baryons	$\frac{1}{2}^+$	p (938), n (939), Λ (1116), $\Sigma^{\pm,0}$ (1193), $\Xi^{0,-}$ (1318)
	$\frac{3}{2}^+$	Δ^{++}, Δ^+, Δ^-, Δ^0 (1232), $\Sigma^{*\pm,0}$ (1385), $\Xi^{*\pm,0}$ (1318), Ω^- (1672)

systematics suggests substructure

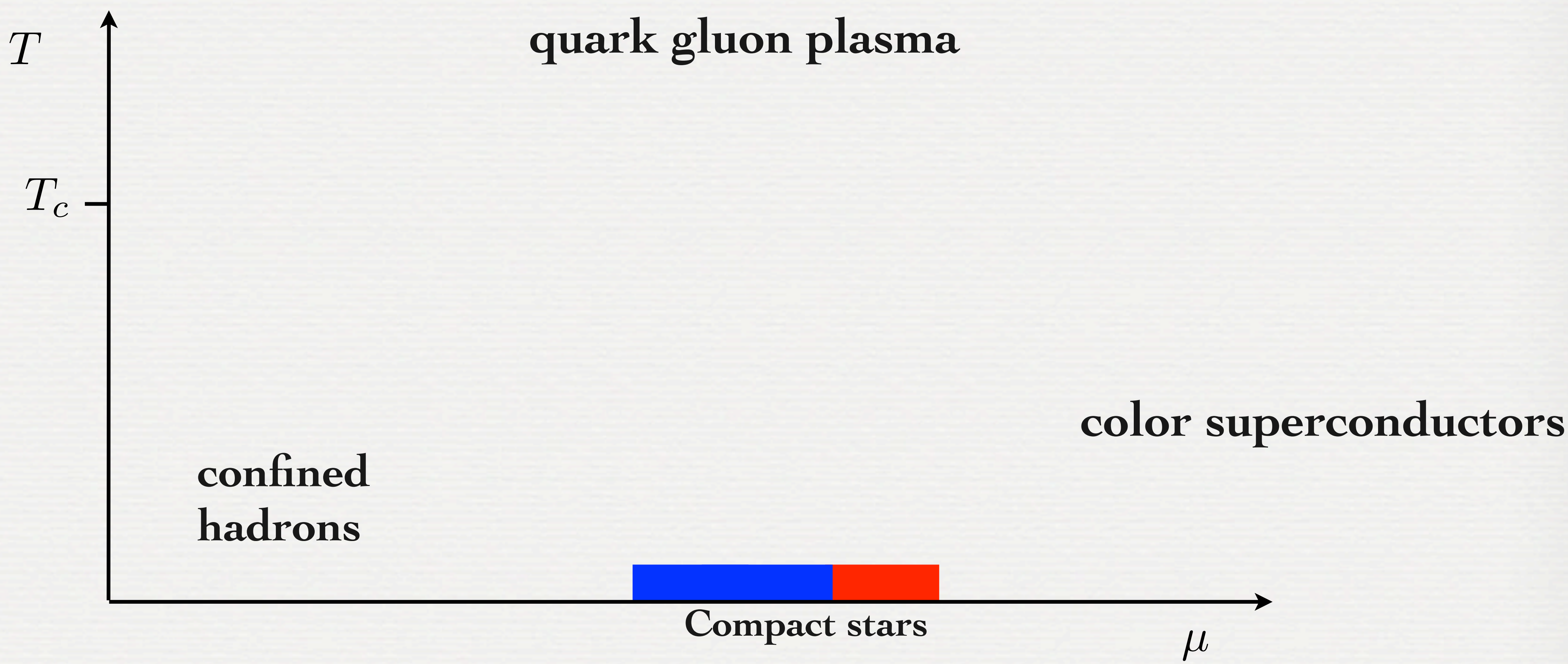
Particle physicist:
Matter is made by elementary particles in different states of aggregation



A modern vision: it does not exist one single description of matter

The description depends on the energy of the probe, on the "thermodynamic conditions", on the kinematic state of the observer

QCD phase diagram

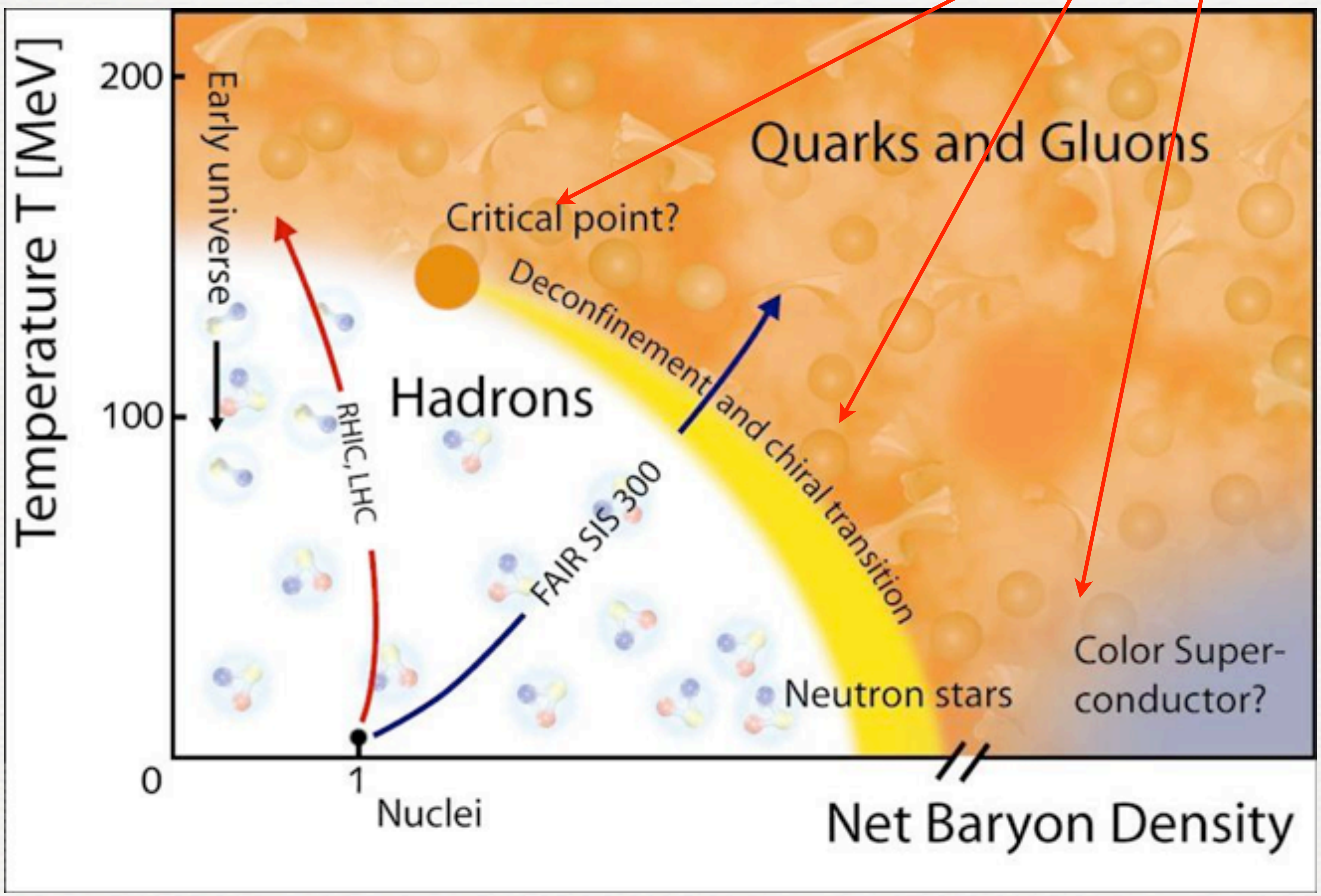


Few pieces of information about the phase diagram

HOT MATTER	ENERGY-SCAN	LATTICE QCD	EMULATION
RHIC LHC	RHIC NA61/SHINE@CERN-SPS CBM@FAIR/GSI MPD@NICA/JINR	Different approaches. See M.P. lecture Sign problem.	Ultracold fermi atoms

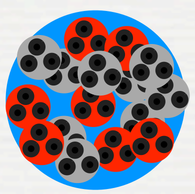
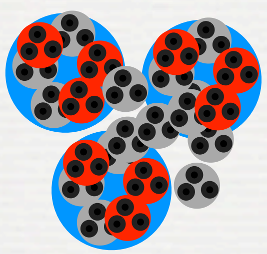
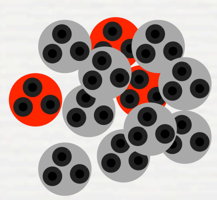
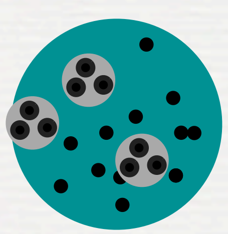
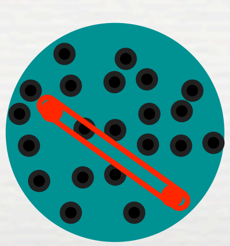
How we think it should be

what we know we don't know



http://homepages.uni-regensburg.de/~sow28704/ftd_lqcd_ss2012/ftd_lqcd_ss2012.html

Increasing baryonic density

		Density	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
	H			
	He	low		light nuclei
.....		......		.....
	Ni		confining	heavy nuclei
	neutron drip			
		high		neutrons and protons
	neutron proton soup			
	quark drip	very large	strong coupling	quarks and gluons Cooper pairs of quarks? quarkyonic phase?....
	quark soup	extreme	weak coupling	Cooper pairs of quarks NGBs

ρ

The physics at high baryonic density is difficult to handle

1. **QCD is nonperturbative**
2. **Lattice simulations do not work** **(Barbour et al. 1986 Nucl.Phys. B275 296)**
3. **No experimental facility (so far) can reproduce the correct conditions**

Let me rephrase it:

1. **We do not know how to do computations**
2. **We do not have numerical methods for doing tests**
3. **We have no terrestrial lab for validating the theoretical results**

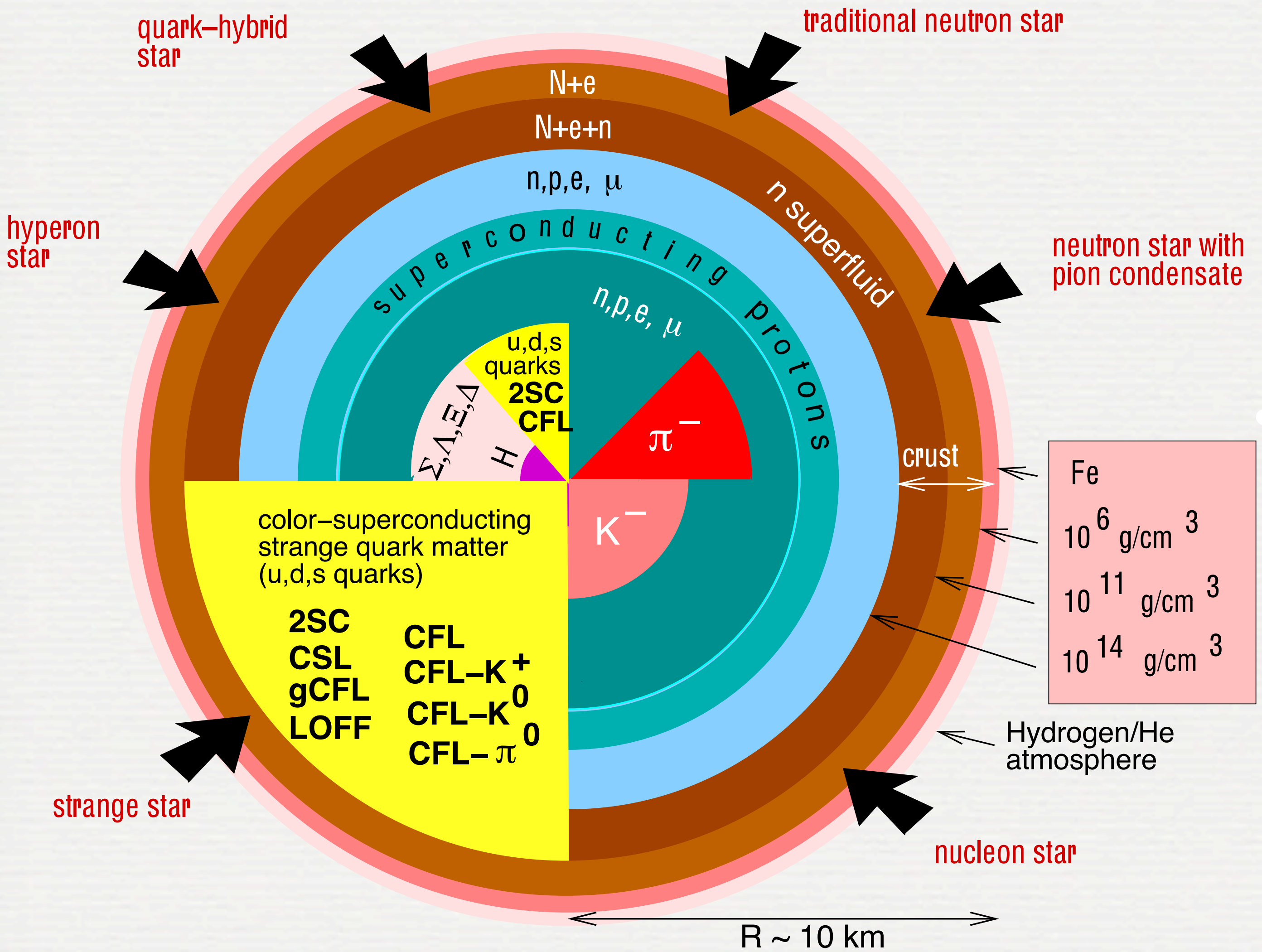
The way out:

We can use symmetries and analogies for obtaining qualitative and semiquantitative results

We can use compact stars as the "lab"

What are compact stars made of ?

Compact stellar objects



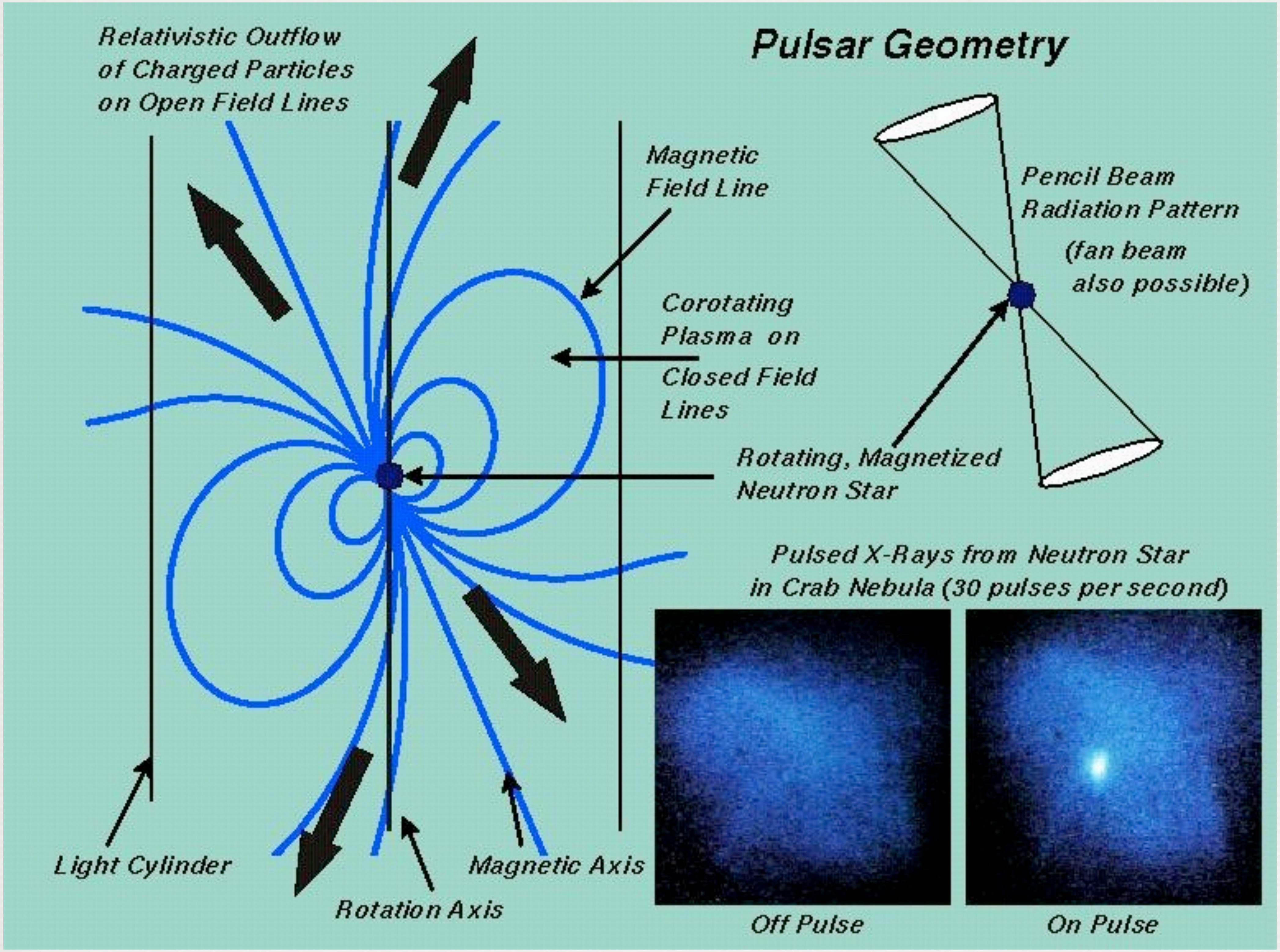
"Probes"
cooling
glitches
instabilities
mass-radius
magnetic field
GW
……

F. Weber, Prog.Part.Nucl.Phys. 54 (2005) 193

Very massive compact stars?
PSR J1614-2230 mass M ~ 2 M⊙ Demorest et al Nature 467, (2010) 1081
Tension with quark matter models Bombaci et al. Phys. Rev. C 85, (2012) 55807
Unlikely one single model can explain everything

Pulsars

Compact stars are typically observed as pulsars



EPN arxiv

Crab 33ms

Vela 89ms

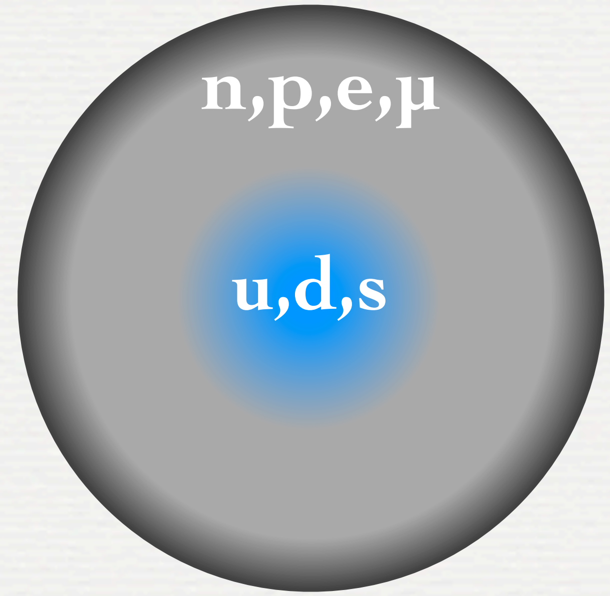
Classification of CSOs

Neutron star

$R \sim 10 \text{ km} \quad M = 1.4\, M_\odot$

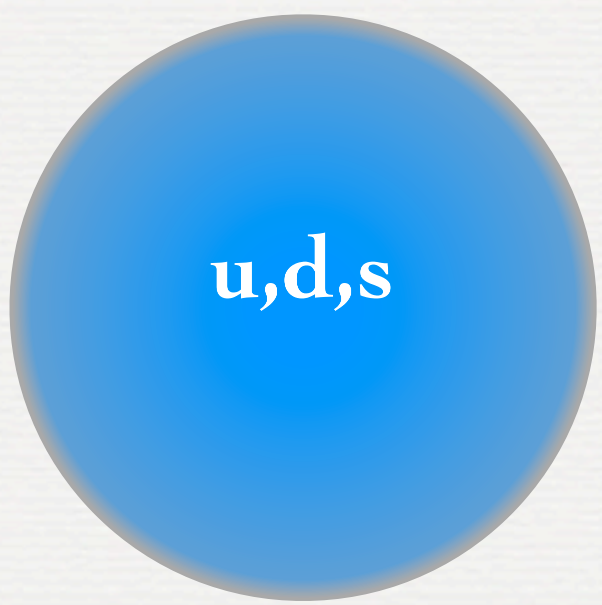
See I.V. Lecture

Hybrid star



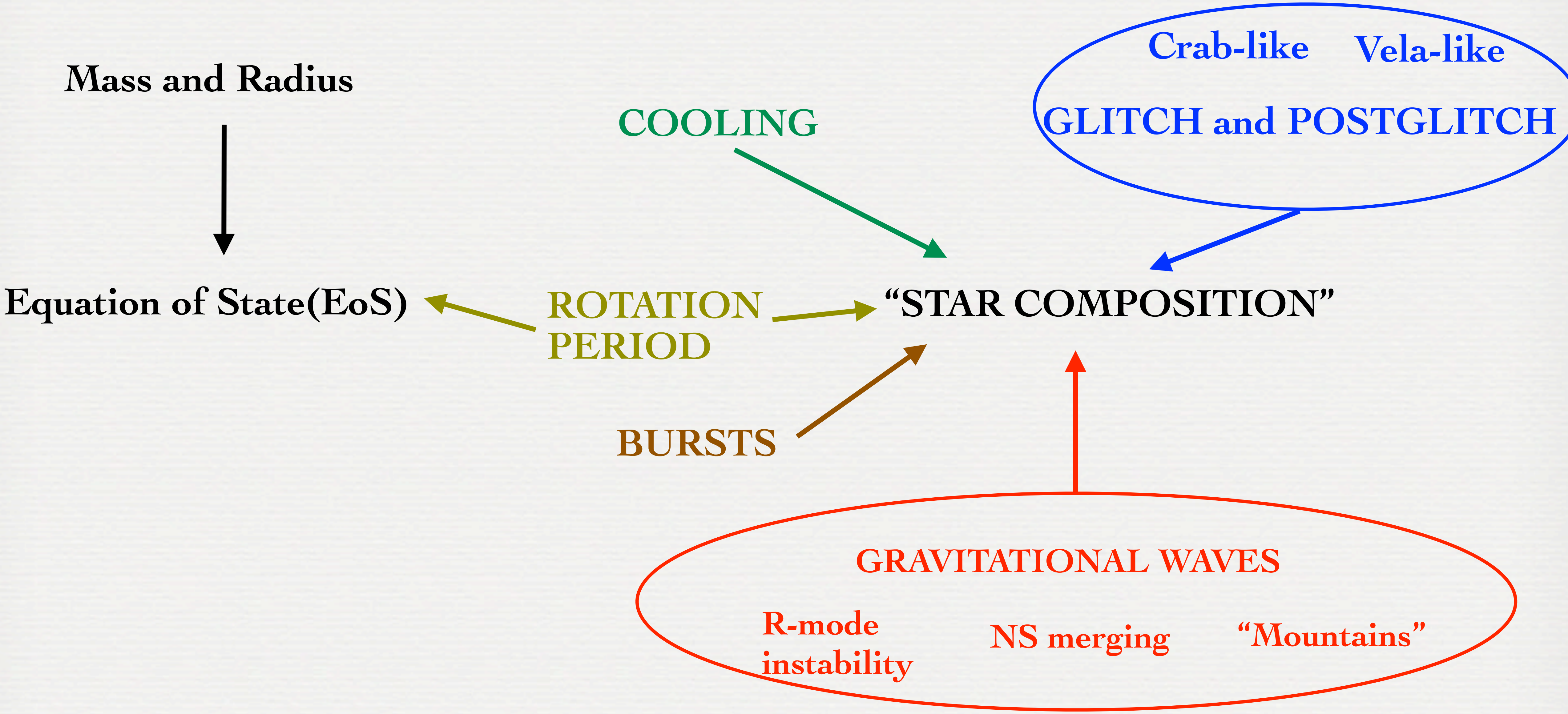
$R \sim 10 \text{ km} \quad M = 1.4\, M_\odot$

Strange star



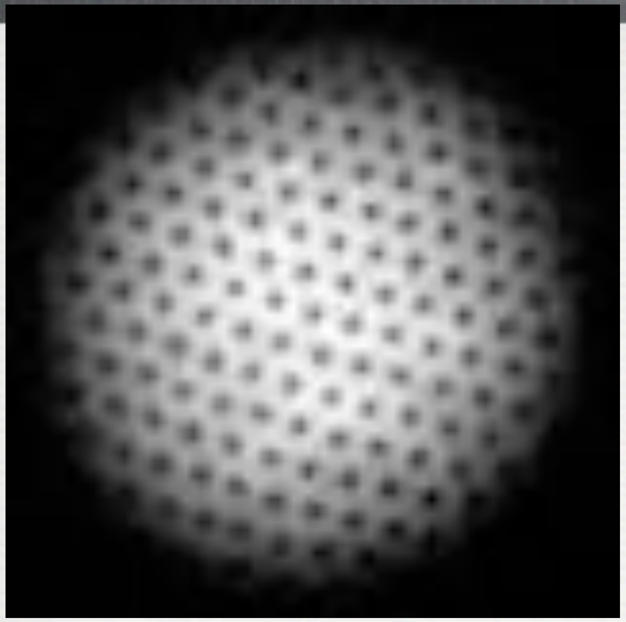
$R \sim 0 - 10 \text{ km} \quad M < 2.5\, M_\odot$

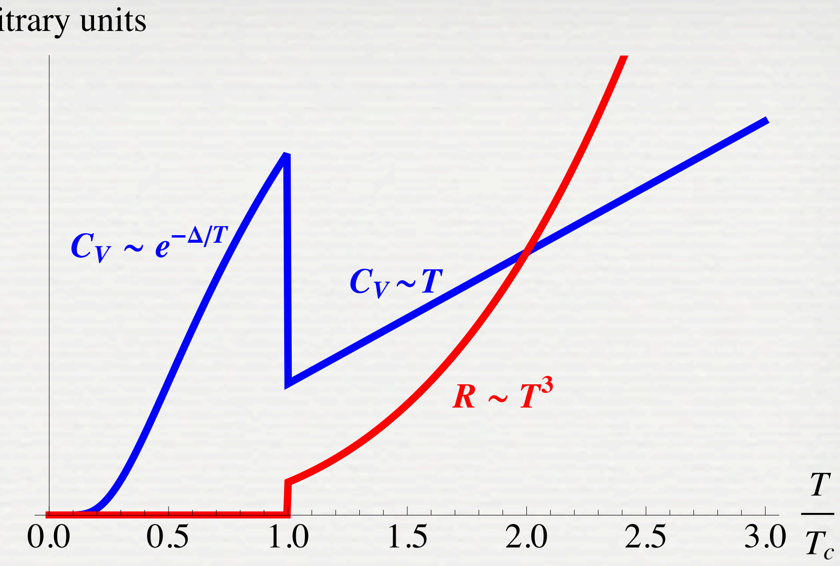
Information from CSOs



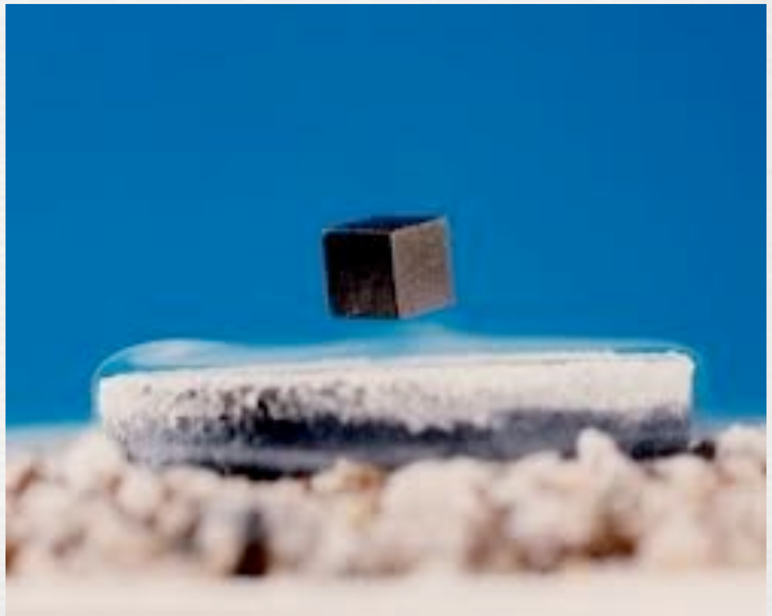
The quality of the information is not the same in all the observables.
For example, we now VERY well the rotation period of pulsars.
We have NOT DETECTED gravitational waves

	Neutron and hybrid stars	Strange stars
holding force	gravity	strong interactions and gravity
mass-radius	constrained	$M \propto R^3$ no lower bound
top spinning period	$P \sim 1$ ms	$P < 1$ ms
cooling	might be the same	
r-modes	gravitational waves emission	model dependent
glitches	more or less explained	qualitative explanation



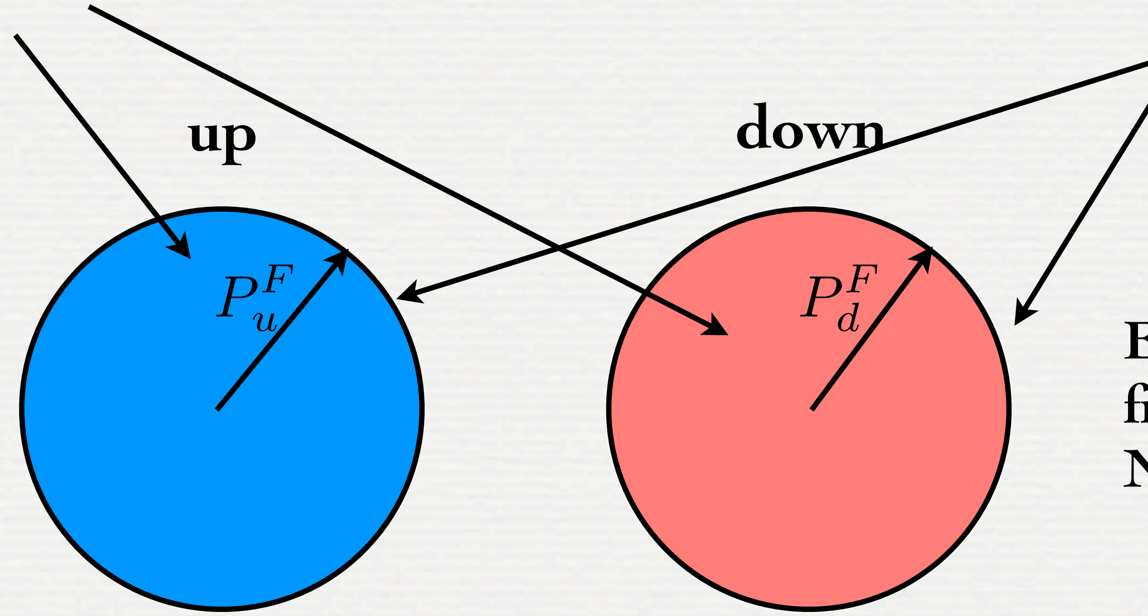


SUPERFLUIDS AND SUPERCONDUCTORS



BCS qualitative description

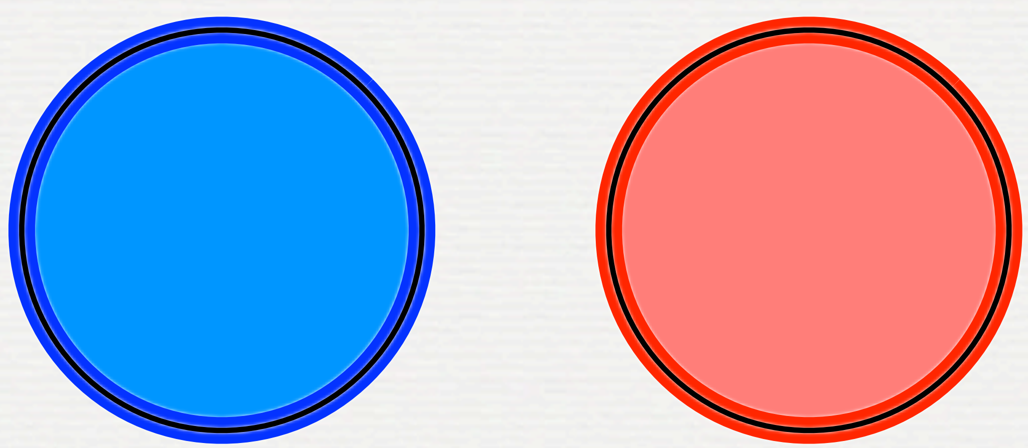
Degenerate system of spin up and spin down fermions



No interaction

Exclusion principle rules: fermions fill the Fermi sphere.
Naive noninteracting ground state.

Attractive interaction



Active fermions form correlated pairs.
BCS state

The naive Fermi sphere ground state is unstable: the BCS state has minimum energy.
We can integrate out the Pauli blocked fermions

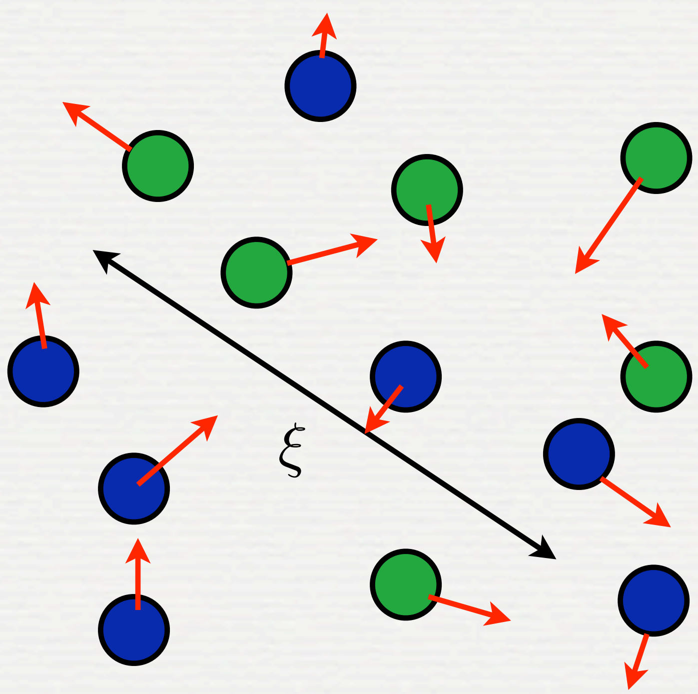
BCS qualitative description

fermions

spin up

spin down

momentum



Cooper pairs: "difermions" with total spin 0 and total momentum 0

Free energy gain $\sim \Delta$

$$\xi \sim \frac{v_F}{\Delta}$$

BCS: loosely bound pairs $\xi \gtrsim n^{-1/3}$

BEC: tightly bound pairs $\xi \lesssim n^{-1/3}$

Type I (Pippard): $\lambda \ll \xi$ first order phase transition to the normal phase

Type II (London): $\lambda \gg \xi$ richer behavior, 2nd order phase transition

Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with $v = \nabla\varphi \Rightarrow \nabla \times v = 0$ (irrotational or quantized vorticity)

Superconductor: "screening" of the magnetic field: Meissner effect (almost perfect diamagnet)

Superfluid

Broken global symmetry

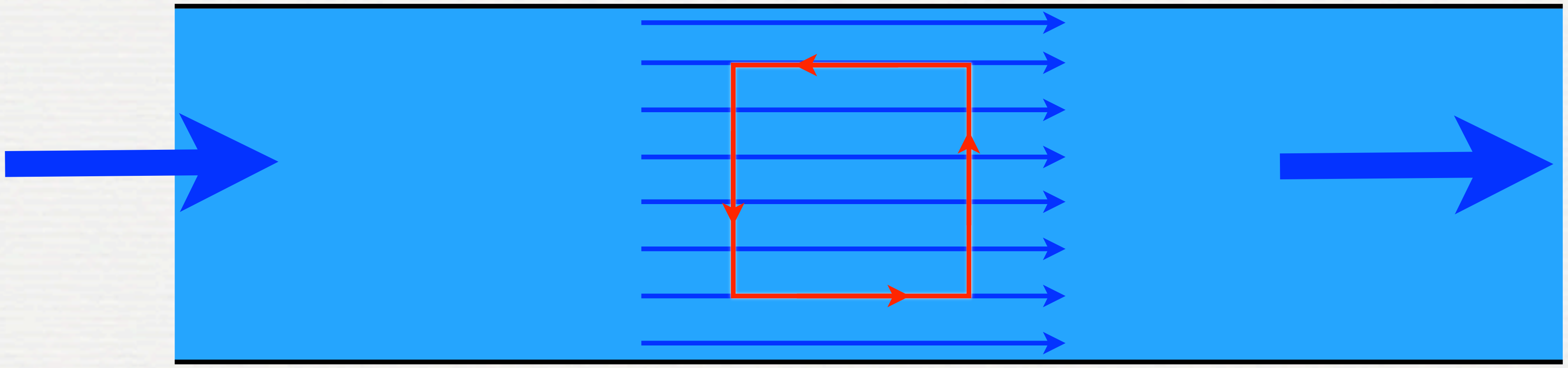
Goldstone theorem $\longrightarrow$ Transport of the quantum numbers of the broken group with almost no dissipation

Superconductor

"Broken gauge symmetry"

Higgs mechanism $\longrightarrow$ **Broken gauge fields with mass, M, penetrate for a length** $\lambda \propto 1/M$

Unviscid (dry) fluid



Continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$

Euler equation $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla \mathbf{p}}{\rho} - \nabla \phi$

Vorticity $\mathbf{\Omega} = \nabla \times \mathbf{v}$ **Unviscid fluid** $\Omega = 0$

The flow is permanently irrotational $\mathbf{v} = \nabla \varphi$

Viscous fluid

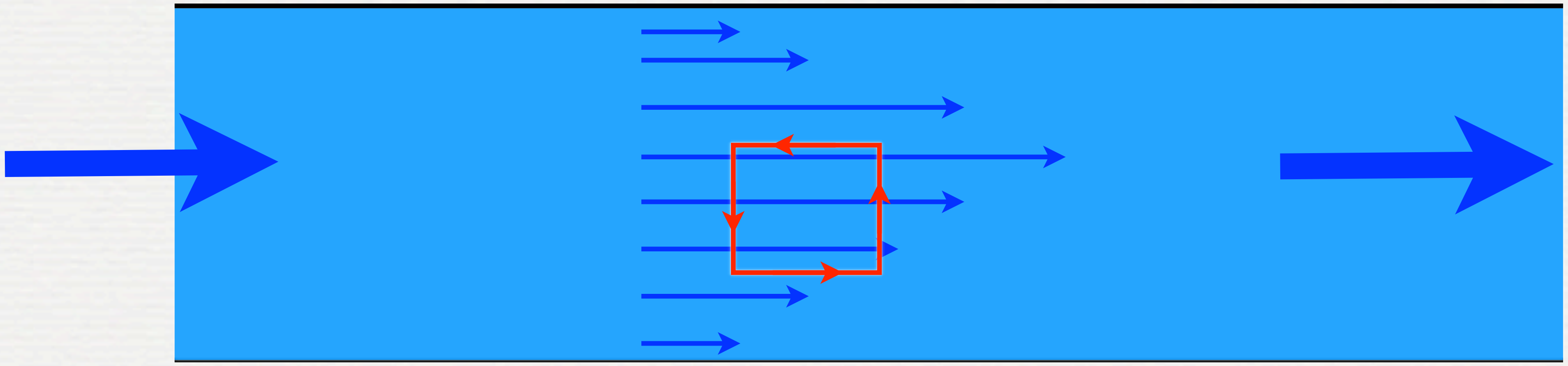
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p - \rho\nabla\phi + \eta\nabla^2\mathbf{v} + \zeta\nabla(\nabla\cdot\mathbf{v})$$

shear viscosity **bulk viscosity**

Using the vorticity

$$\frac{\partial \mathbf{\Omega}}{\partial t} + \nabla\times(\mathbf{\Omega}\times\mathbf{v}) = \frac{\eta}{\rho}\nabla^2\mathbf{\Omega}$$

Vorticity is generated by the shear viscosity



Superfluid

Superfluidity is due to the absence of low energy excitations which dissipate energy

When does an excitation appear?

Suppose the superfluid moves with respect to a a boundary

Low energy excitation in the comoving frame $\epsilon(p)$

Low energy excitation in the the lab frame $\epsilon(p) + \mathbf{p}\cdot\mathbf{v}$

$\epsilon(p) + \mathbf{p}\cdot\mathbf{v} < 0$ transition to the excited state

Landau's criterion:

Superfluidity occurs when $v < v_{cr} = \mathrm{Min}\dfrac{\epsilon(p)}{p}$ $\qquad \mathrm{Min}\dfrac{\epsilon(p)}{p} \neq 0$

Example

Consider a system with a global $U(1)$ symmetry

Spontaneous symmetry breaking: Goldstone boson $\epsilon(p) = c\,p$

Question: Does the Goldstone theorem imply a linear dispersion law?

Univiscid flow is not the characterizing property of a superfluid.

Consider a ballistic gas. It "flows" without dissipation. But it is certainly not a superfluid.

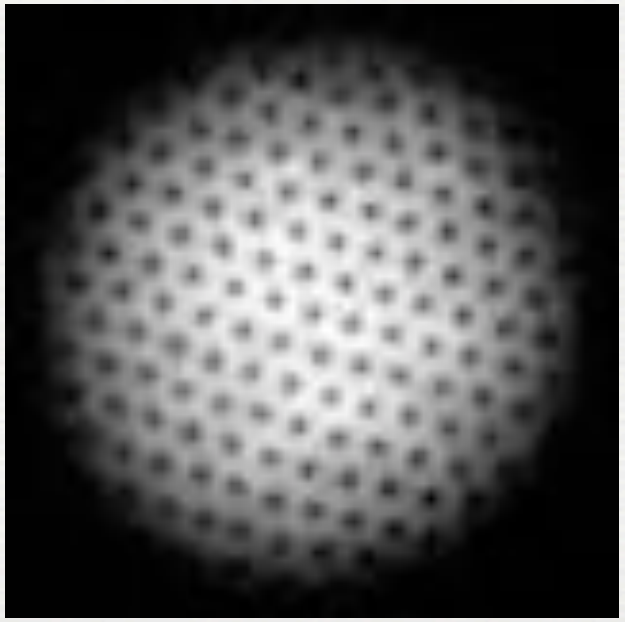
The characterizing property of a superfluid is potential flow

with quantized vortices when in rotation

$$\boldsymbol{v} = \nabla\varphi \;\rightarrow\; \nabla \times \boldsymbol{v} = 0 \textbf{ almost everywhere}$$

$$\oint_C d\boldsymbol{l} \cdot \boldsymbol{v} = \Delta\varphi = n\pi \;\; \textbf{because } \varphi \textbf{ is a phase}$$

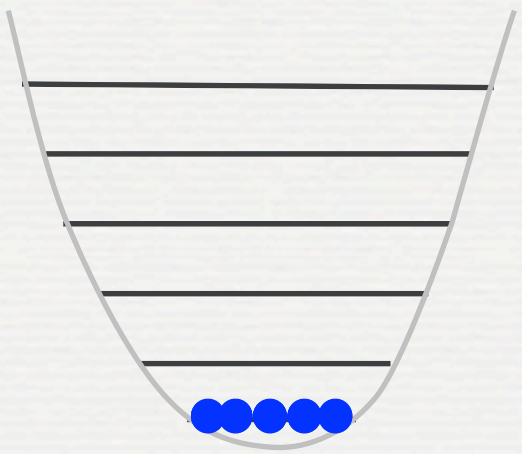
Since vortices repel they
form an Abrikosov lattice



Fermionic and bosonic superfluids at T=0

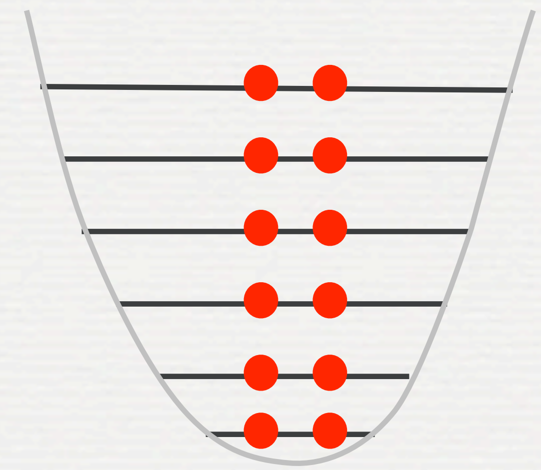
^{4}He

BOSONS



^{3}He

FERMIONS



Bosons “like” to move together, no dissipation

^{4}He becomes superfluid at
$T_c \simeq 2.17$ K, Kapitsa et al (1938)

An arbitrary weak interaction leads to the formation of Cooper pairs

^{3}He becomes superfluid at
$T_c \simeq 0.0025$ K, Osheroff (1971)

BEC

BCS

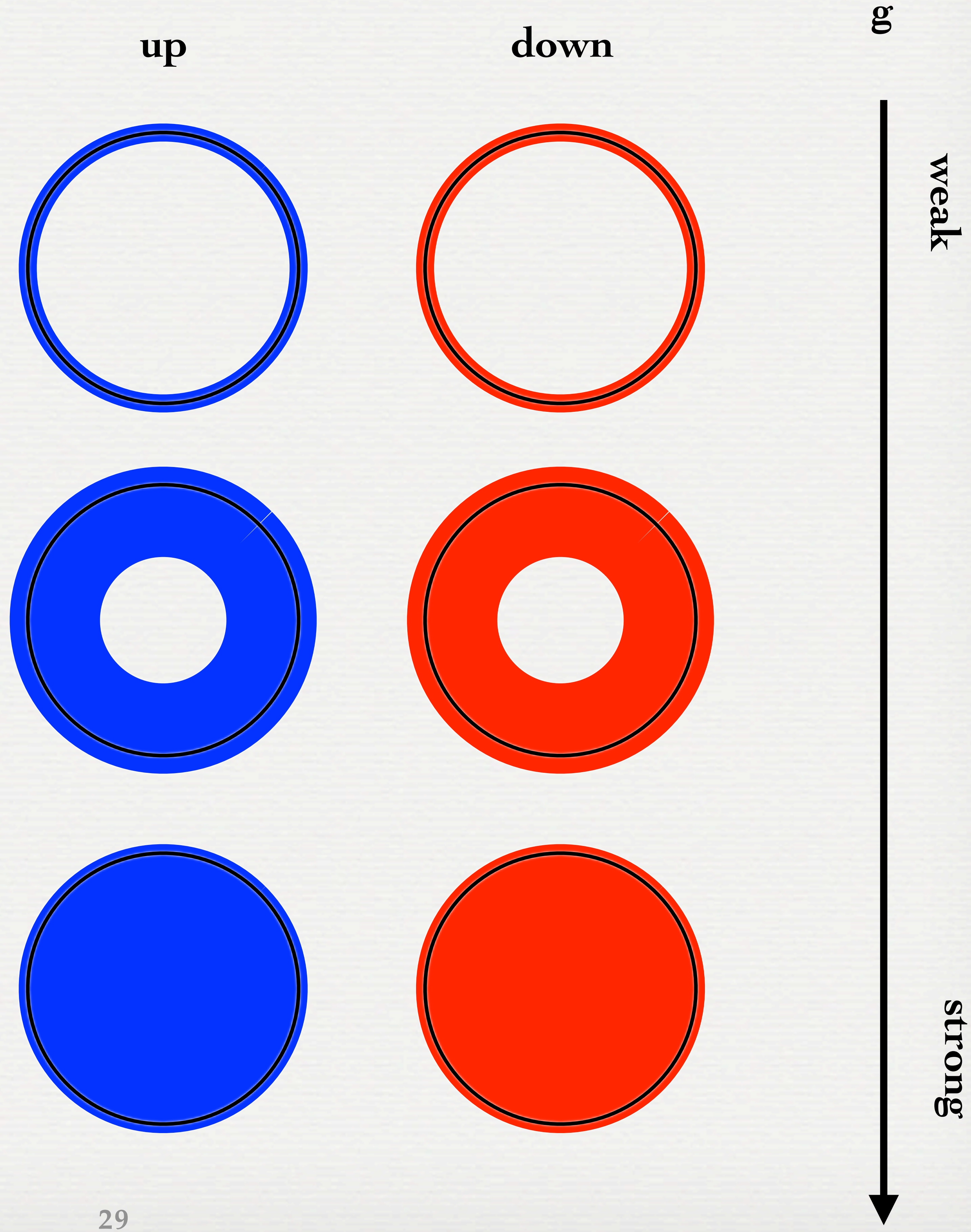
increasing the attractive interaction between fermions

BCS-BEC crossover

BCS
fermi surface phenomenon

BCS-BEC crossover
start to form bound pairs

BEC
tightly bound pairs



Main concepts so far:

- We don't know what is inside compact stars. Some exotic phase might be there
- Superfluid, spontaneous breaking of a global symmetry
- Superconductor, "spontaneous breaking" of a local symmetry
- Superfluids are weird systems: vanishing viscosity, quantized vorticity...
- Both fermions (like ^3He) and bosons (like ^4He) can become superfluid

Next:

What about quark matter? Can it be superfluid? Does it happen? Which are the consequences? ....

COLOR SUPERCONDUCTORS

A bit of history

- Quark matter inside compact stars, Ivanenko and Kurdgelaidze (1965), Paccini (1966) ...

- Quark Cooper pairing was proposed by Ivanenko and Kurdgelaidze (1969)

- With asymptotic freedom (1973) more robust results by Collins and Perry (1975), Baym and Chin (1976)

- Classification of some color superconducting phases: Bailin and Love (1984)

Interesting studies but predicted small energy gaps ~ 10 ÷100 keV
small/negligible phenomenological impact for compact stars

- A large gap with instanton models by Alford et al. (1998) and by Rapp et al. (1998)

- The color flavor locked (CFL) phase was proposed by Alford et al. (1999)

Do we have the ingredients?

Recipe for superconductivity

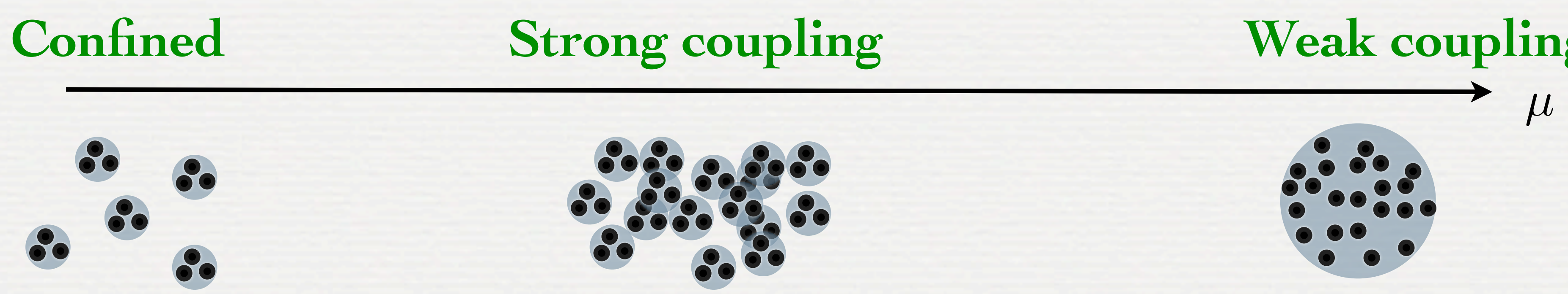
- Degenerate system of fermions
- Attractive interaction (in some channel)
- $T < T_c$

Color superconductivity

- At large μ, degenerate system of quarks
- Attractive interaction between quarks in the antitriplet color channel
- We expect $T_c \sim (10 - 100)$ MeV >> $T_{star\text{-}core} \sim 10 \div 100$ keV

THE SYSTEM IS EFFECTIVELY ULTRACOLD

Color superconductors

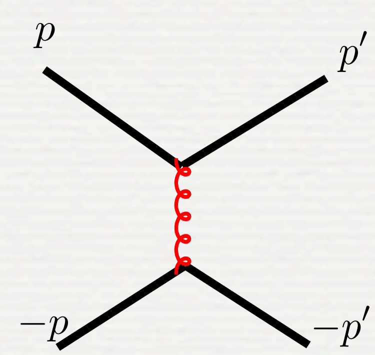


Degenerate system of quarks

Attractive interaction between quarks

$$3 \times 3 = \bar{3}_A + 6_S$$

attractive channel



$$p, p' \simeq p_f$$

Using quarks as building blocks, one has color, flavor as well as spin degrees of freedom: the game is more complicated

QCD, allows for a zoo of colored phases and one has to single out the one with the smallest free-energy

Gap parameter

Any superconducting phase is characterized by a particular difermion condensate

The condensate express the fact that the ground state contains a sizable fraction of particles

The quantum numbers of the condensate allow to determine the residual symmetries of the theory

If the condensate is "charged" with respect to a particular quantum symmetry, then that symmetry is spontaneously broken.

Example

"Standard BCS superconductors"

Consider a degenerate system of fermions with charge Q
Suppose there is an attractive S-wave interaction channel, say because of a phonon exchange

If the temperature is sufficiently low, there will be a condensate made by two fermions

$$\langle \psi \psi \rangle$$

The gauge transformation $\psi(x) \to e^{iQ\alpha(x)}\psi(x)$ **does not leave the condensate invariant**

Indeed the condensate has electric charge 2 Q and it "brakes" $U(1)_{\rm em}$

By the Higgs-Anderson mechanism, the photon gets mass: the Meissner mass and the magnetic component of the gauge field is screened.

Important points:
1) we have only used symmetries (no specific description of the interaction channel)
2) we have not used an external Higgs field, it has been the dynamics of the fermions to induce the symmetry breaking.

General expression of the gap parameter

Quite general color superconducting condensate:

$$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \propto \sum_{I=1}^{3} \Delta_{\mathrm{I}} \varepsilon^{\alpha\beta I} \epsilon_{ijI}$$

gap parameters

$\alpha, \beta = 1, 2, 3$ **color indices**

$i, j = 1, 2, 3$ **flavor indices**

It has a flavor charge **It has a color charge** **It has a baryonic charge**

Δ_I **is the gap parameter between quarks whose flavor and color is not *I***

Δ_3 **is the gap parameter between** $\langle u_b d_r \rangle$ **and** $\langle u_r d_b \rangle$

Obs. the condensate has been written in a gauge variant way

2 Flavor color superconductor (2SC)

Suppose the strange quark mass is "large".
Strange quarks decouple and we effectively have only up and down quarks

$$\Delta_3 > 0\,,\ \Delta_2 = \Delta_1 = 0$$

The breaking pattern:

$$SU(3)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_S}_{\supset U(1)_{\mathrm{Q}}} \to SU(2)_c \times \underbrace{SU(2)_L \times SU(2)_R \times U(1)_{\tilde{B}} \times U(1)_S}_{\supset U(1)_{\tilde{\mathrm{Q}}}}$$

- **5 gauge bosons acquire a mass: the system IS A COLOR SUPERCONDUCTOR**
- **No global symmetry is broken: the system IS NOT A SUPERFLUID**
- **No chiral symmetry breaking**
- **The photon is rotated (mixed with gauge and global symmetries).**

The system is an "electrical" conductor

Color Flavor Locking (CFL)

Suppose that the strange quark mass is "small".
All quarks should be treated on an equal footing:

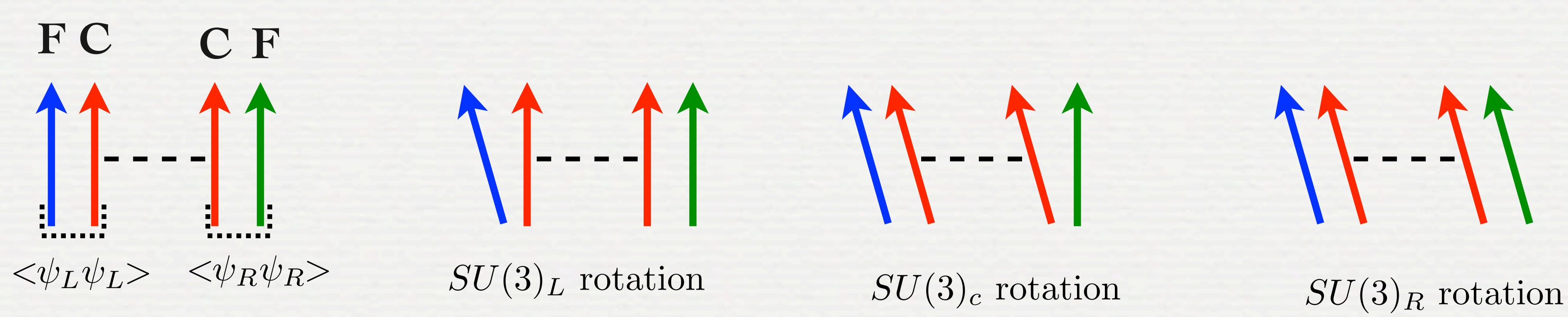
$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_{\rm CFL}$$

$$\langle \psi_{\alpha i} C\gamma_5 \psi_{\beta j}\rangle \propto \Delta_{\rm CFL} \sum_{I=1}^{3} \varepsilon^{\alpha\beta I}\epsilon_{ijI}$$

Alford, Rajagopal, Wilczek hep-ph/9804403

All quarks, contribute coherently to pairing. This phase maximizes the pairing.
It is very robust. It is expected to be the ground state of quark matter at very large densities

Locking color and flavor rotations:



This is "like" in the SU(2) x U(1) -> U(1) locking of the standard model

CFL breaking pattern

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_{\mathrm{Q}}} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{\mathrm{Q}}}} \times Z_2$$

- 8 gluons acquire "magnetic mass" CFL IS A COLOR SUPERCONDUCTOR
- χSB: 8 (pseudo) Nambu-Goldstone bosons
- $\mathrm{U(1)_B}$ breaking CFL IS A SUPERFLUID
- "Rotated" photon, mixing angle $cos\,\theta = \dfrac{g}{\sqrt{g^2 + 4e^2/3}}$

The system is an electromagnetic transparent insulator