# QCD CONDENSATES

## LECTURE IV

## CRYSTALLINE COLOR SUPERCONDUCTORS

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**GGI March 2014**

# Outline

- Realistic conditions in compact stars
- General properties of crystalline color superconductors (CCSC)
- One plane wave strustcure
- Crystalline structures

Review: 1302.4624

# MOTIVATIONS

# Compac star conditions

**sizable strange quark mass**
**+**
**weak equilibrium**
**+**
**electric neutrality**

$\longrightarrow$ **mismatch of Fermi momenta**

**No pairing case**

**Fermi momenta** $\quad p_u^F = \mu_u \qquad p_d^F = \mu_d \qquad p_s^F = \sqrt{\mu_s^2 - m_s^2}$

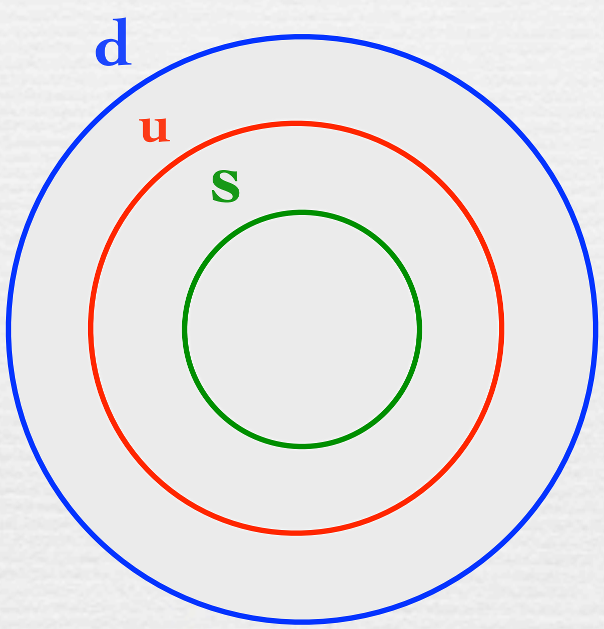
**weak decays**

$$u \to d + \bar{e} + \nu_e$$
$$u \to s + \bar{e} + \nu_e$$
$$u + d \leftrightarrow u + s$$

$\longrightarrow$

$$\mu_u = \mu_d - \mu_e$$
$$\mu_d = \mu_s$$

**electric neutrality** $\longrightarrow$ $\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$

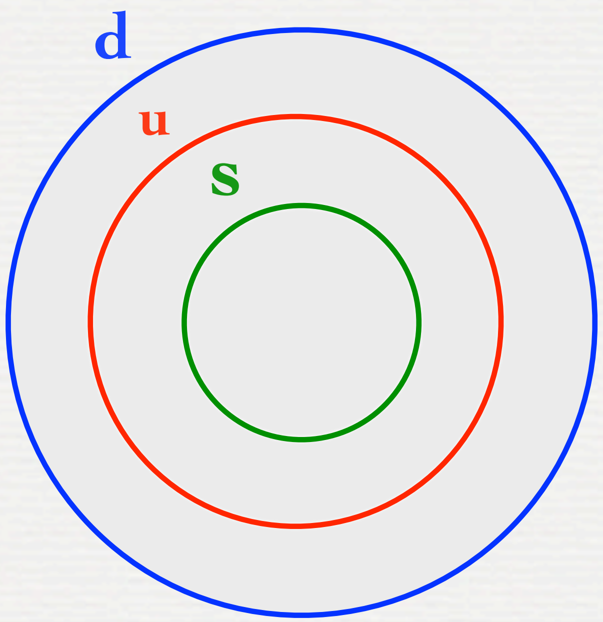



Fermi spheres of u,d, s quarks

$$\mu = \frac{\mu_u + \mu_d + \mu_s}{3} \qquad \mu_e \simeq \frac{m_s^2}{4\mu} \qquad p_d^F = \mu + \frac{1}{3}\mu_e \qquad p_u^F = \mu - \frac{2}{3}\mu_e \qquad p_s^F \simeq \mu - \frac{5}{3}\mu_e$$

Alford and Rajagopal, JHEP 0206 (2002) 031

# Mismatch vs Pairing




- Energy gained in pairing $\sim 2\Delta_{CFL}$
- Energy cost of pairing $\sim \delta\mu \sim \dfrac{m_s^2}{\mu}$

The CFL phase is favored for $\dfrac{m_s^2}{\mu} \lesssim 2\Delta_{CFL}$

Forcing the superconductor to a homogenous gapless phase $E(p) = -\delta\mu + \sqrt{(p-\mu)^2 + \Delta^2}$

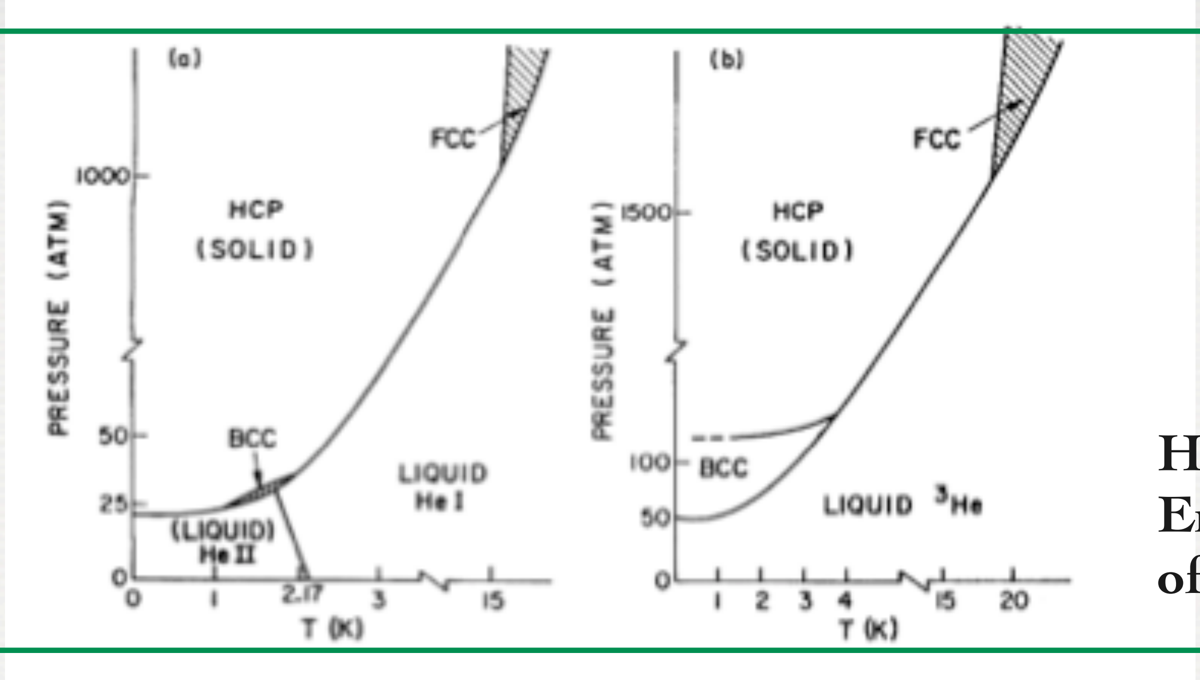
leads to the "chromomagnetic instability" $M^2_{\rm gluon} < 0$

Casalbuoni, MM et al. Phys.Lett. B605 (2005) 362

For $\dfrac{m_s^2}{\mu} \gtrsim 2\Delta_{CFL}$ some less symmetric CSC phase should be realized

# CAN A SUPERFLUID BE SOLID?

## Solid helium !!




H. R.Glyde, Encyclopedia of Physics (2005)

**Can a superfluid be solid?** **Can solids become superfluid?**

The naive answer seems NO WAY! (Penrose and Onsager, 1956)

Supersolid

- Solid
  - Long range order spontaneously produced
  - Difficult to deform (shear modulus)
- Superfluid
  - Highly delocalized system
  - Frictionless motion

It seems helium cannot become a supersolid.
Supersolid with ultracold trapped atoms?

Rev. Mod. Phys. 84, 759 (2012) and [arXiv:1110.1323v2](arXiv:1110.1323v2) [cond-mat.quant-gas]

# CRYSTALLINE COLOR SUPERCONDUCTORS

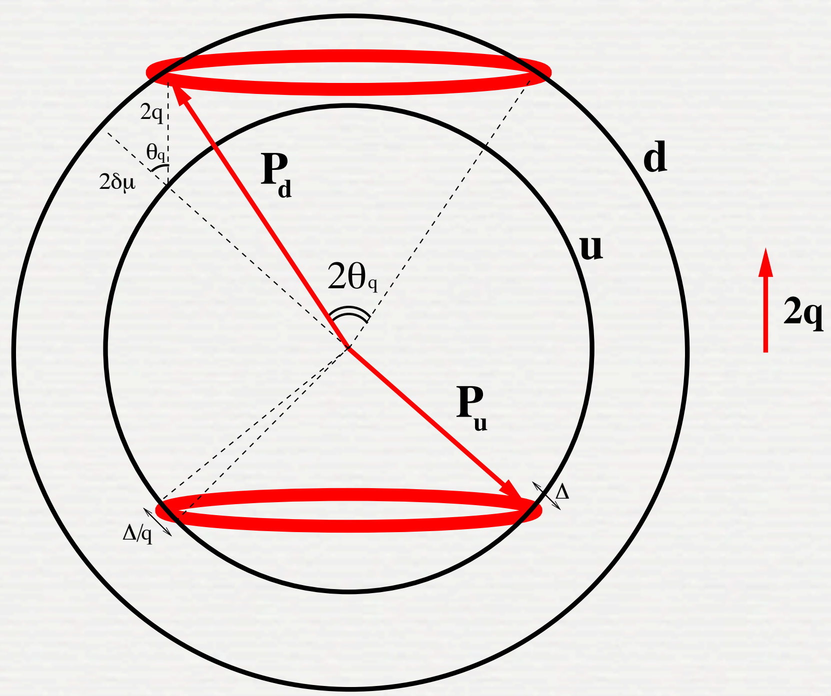
# LOFF (or FFLO)-phase

**LOFF: Larkin-Ovchinnikov and Fulde-Ferrel (1964)**

**For** $\delta\mu_1 < \delta\mu < \delta\mu_2$ **the superconducting phase named LOFF is favored with Cooper pairs of non-zero total momentum**

**For two flavors**

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$




- **In momentum space**

$$< \psi(\boldsymbol{p}_u)\psi(\boldsymbol{p}_d) > \sim \Delta\,\delta(\boldsymbol{p}_u + \boldsymbol{p}_d - 2\boldsymbol{q})$$

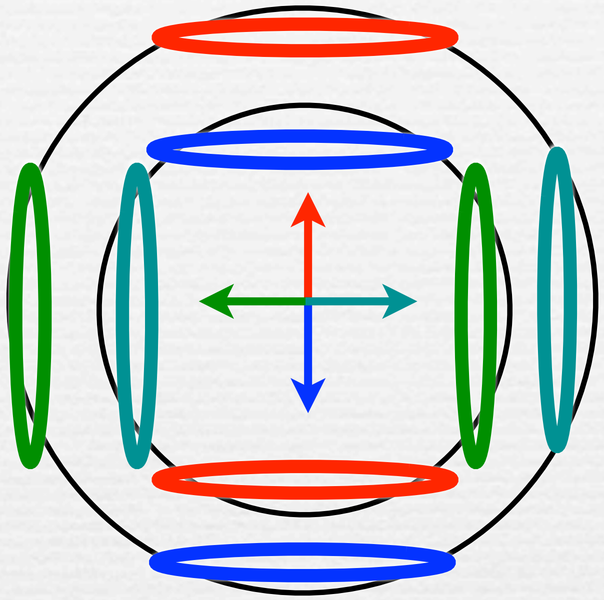
- **In coordinate space**

$$< \psi(\boldsymbol{x})\psi(\boldsymbol{x}) > \sim \Delta\, e^{i2\boldsymbol{q}\cdot\boldsymbol{x}}$$

**Inhomogeneous superconductor, with a spatially modulated condensate in the spin 0 channel**

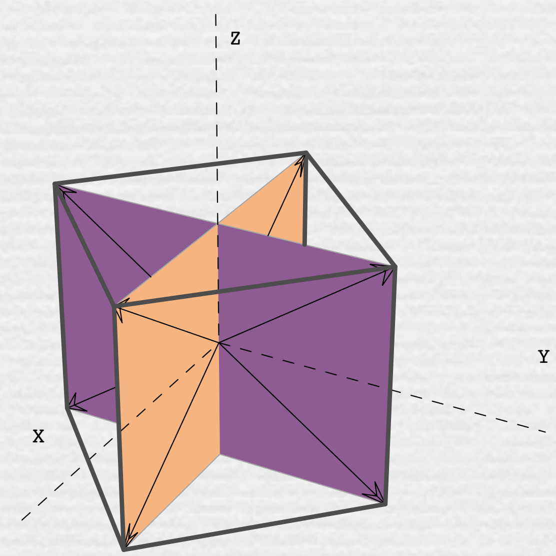
**The dispersion law of quasiparticles is gapless in some directions, but no instability**

# Crystalline structures: CCSC phase

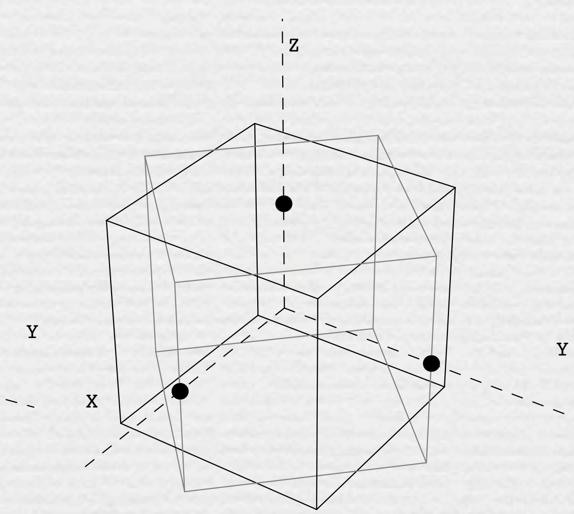


- Compliacted structures can be obtained combining more plane waves
- “no-overlap” condition between ribbons

CX




2cube45z




- Three flavors

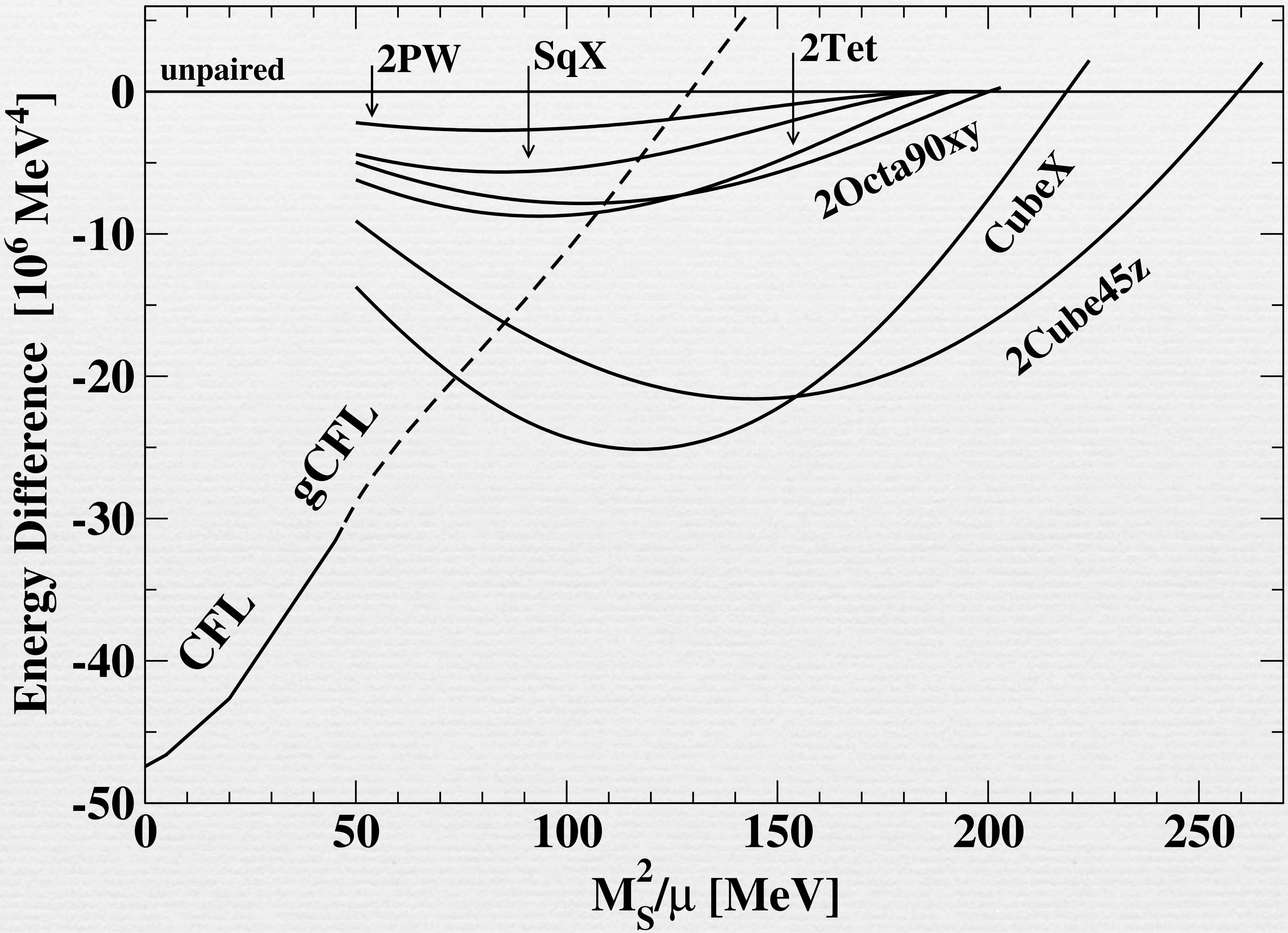
$$< \psi_{\alpha i} C \gamma_5 \psi_{\beta j} > \sim \sum_{I=2,3} \Delta_I \sum_{\mathbf{q}_I^m \in \{\mathbf{q}_I^m\}} e^{2i\mathbf{q}_I^m \cdot \mathbf{r}} \epsilon_{I\alpha\beta} \epsilon_{Iij}$$

simplifications

$$\mathbf{q}_I^m = q\, \mathbf{n}_I^m$$

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

# Free energy evaluation




Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

# Fermionic dispersion laws

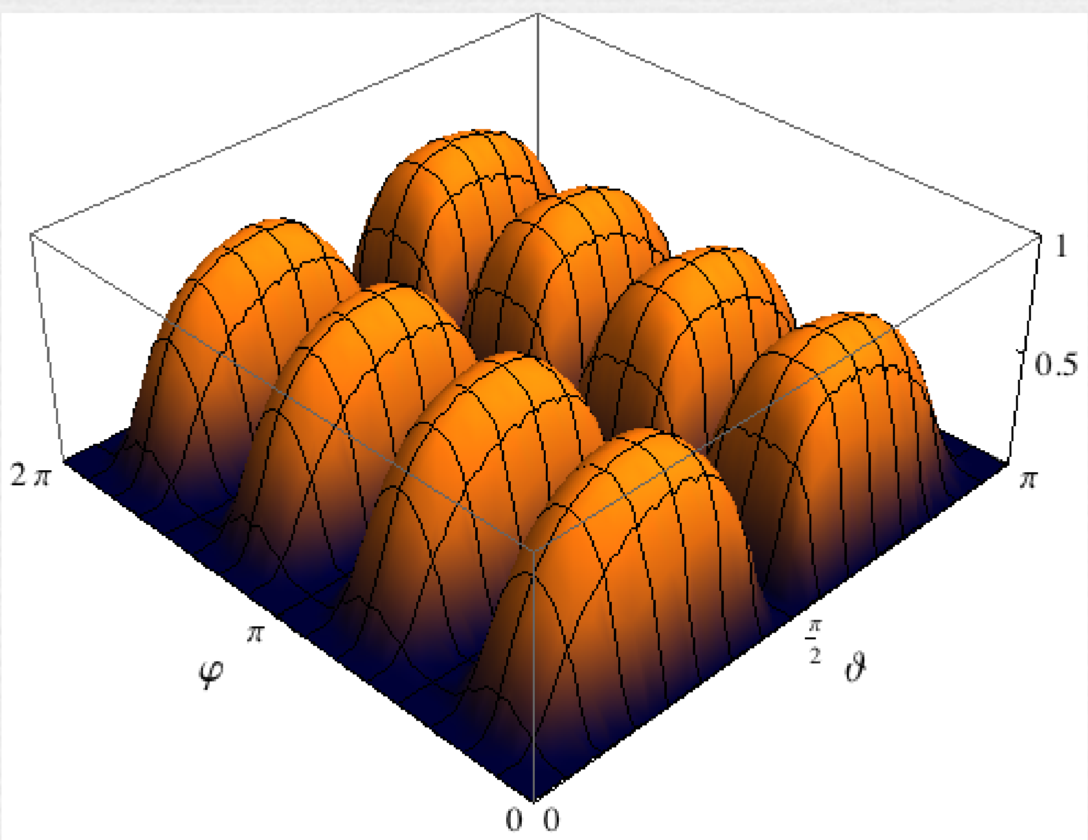
**Fermions have an unisotropic gapless dispersion law:**

**direction dependent velocity**

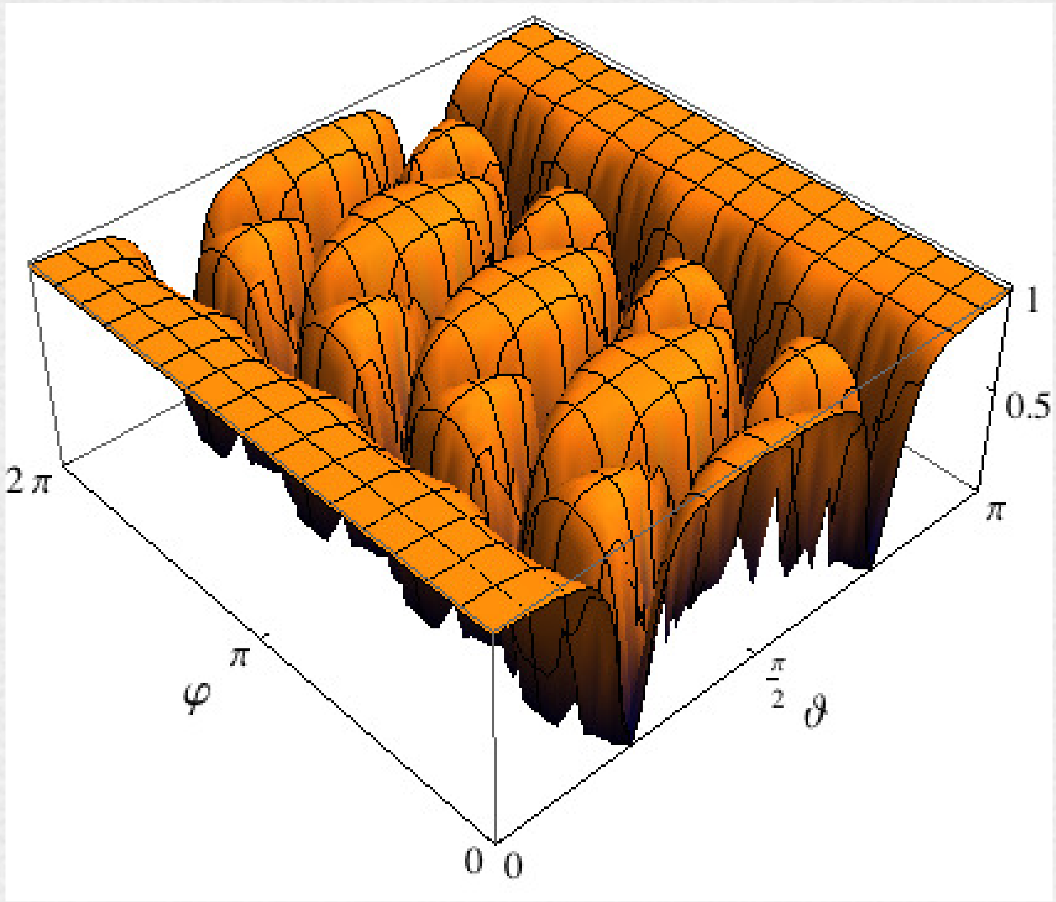
$$E = c(\theta, \phi)\,\xi$$

**Velocity of fermions in two different structures**

**BCC**




**FCC**






MM et al. 1302.4624

# SHAKING THE CRYSTALLINE STRUCTURE

# Phonons in the CCSC phase

Phonon fields, $\mathbf{u}_I$, describe the fluctuations of the condensate $\Delta_I(\mathbf{r}) \to \Delta_I(\mathbf{r} - \mathbf{u}_I)$

**Low-energy Lagrangian (from GL and momentum expansions)**

$$\mathcal{L}^{\Delta^2} = \frac{1}{2} \sum_I \kappa_I \sum_{\mathbf{n}_I^m} [\partial_0(\mathbf{n}_I^m \cdot \mathbf{u}_I)\partial_0(\mathbf{n}_I^m \cdot \mathbf{u}_I) - (\mathbf{n}_I^m \cdot \partial)(\mathbf{n}_I^m \cdot \mathbf{u}_I)(\mathbf{n}_I^m \cdot \partial)(\mathbf{n}_I^m \cdot \mathbf{u}_I)]$$

**interaction channel** **crystalline structure** **kinetic term** **potential term**

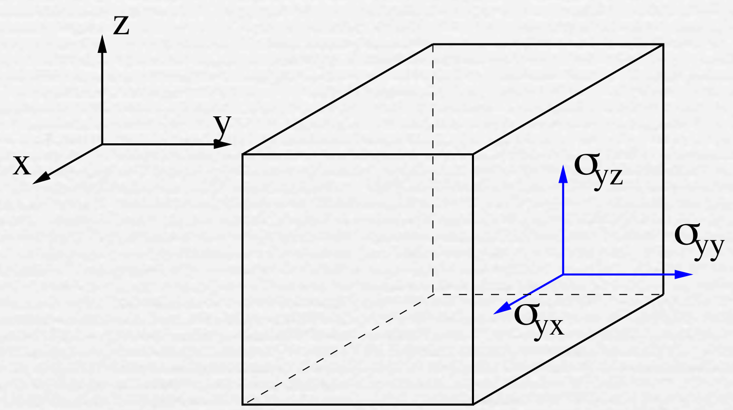
alternative description $\varphi_I^m = 2\mathbf{n}_I^m \cdot \mathbf{u}_I$

$$\kappa_I \equiv \frac{2\mu^2 |\Delta_I|^2}{\pi^2(1 - z_q^2)}$$

**This quantity multiplies also the "potential term": and is thus the energy price needed to produce these fluctuations**

**MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026**

# Shear modulus




**The shear modulus describes the response of a crystal to a shear stress**

$$\nu^{ij} = \frac{\sigma^{ij}}{2s^{ij}} \qquad \text{for} \quad i \neq j$$

$\sigma^{ij}$ **stress tensor acting on the crystal**

$s^{ij}$ **strain (deformation) matrix of the crystal**

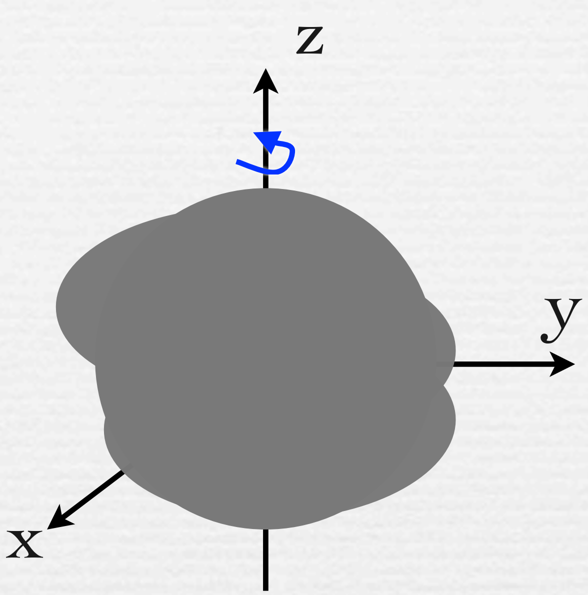
- **Crystalline structure given by the spatial modulation of the gap parameter**
- **It is this pattern of modulation that is rigid (and oscillates)**

$$\nu = 2.47 \frac{\text{MeV}}{\text{fm}^3} \left(\frac{\Delta}{10\text{MeV}}\right)^2 \left(\frac{\mu}{400\text{MeV}}\right)^2$$

**More rigid than diamond!!**
**20 to 1000 times more rigid than the crust of neutron stars**

**MM, Rajagopal and Sharma Phys.Rev. D76 (2007) 074026**

# Gravitational waves from “mountains”




**If the star has a non-axial symmetric deformation (mountain) it can emit gravitational waves**

**ellipticity**

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

**GW amplitude**

$$h = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{r}$$

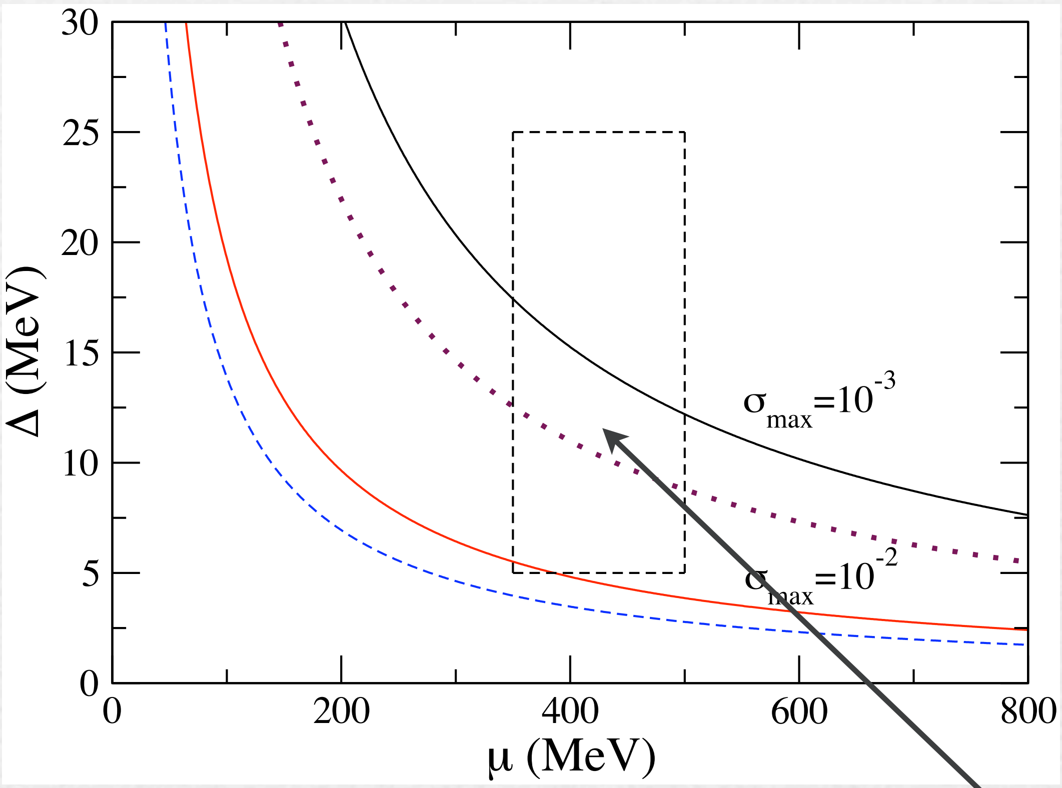
- **The deformation can arise in the crust or in the core**
- **Deformation depends on the breaking strain and the shear**

**To have a “large” GW amplitude**

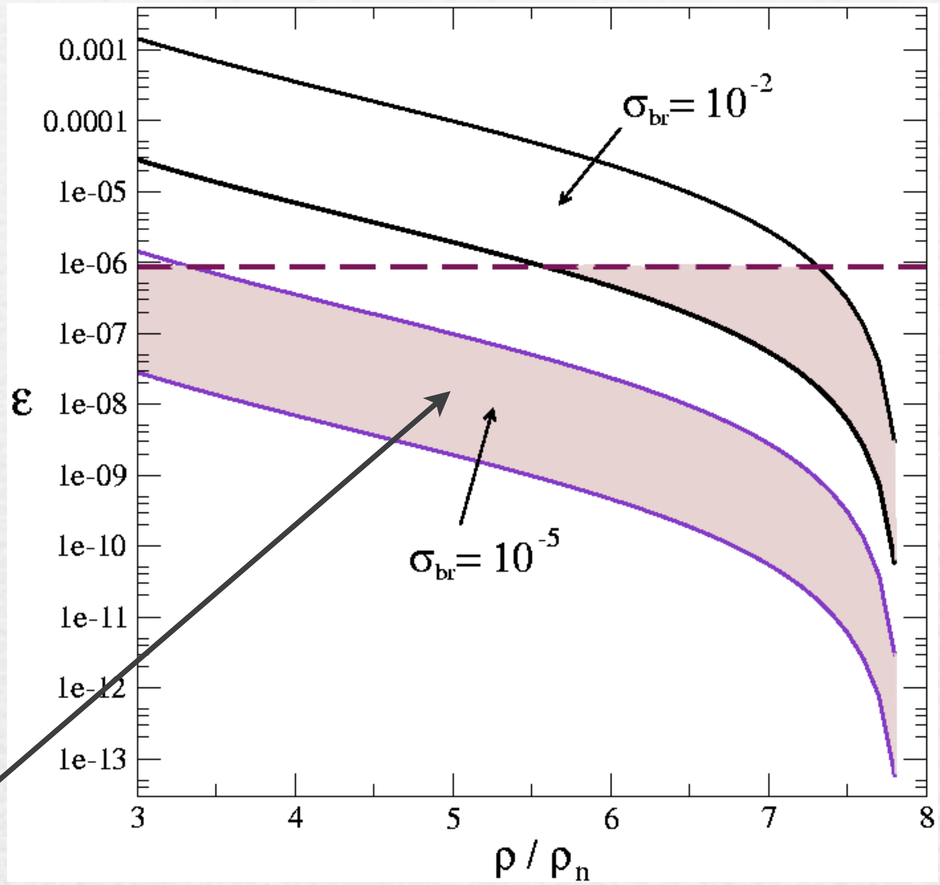
- **Large shear modulus**
- **Large breaking strain**

# Gravitational waves

**Using the non-observation of GW from the Crab by the LIGO experiment**




Lin, Phys.Rev. D76 (2007) 081502

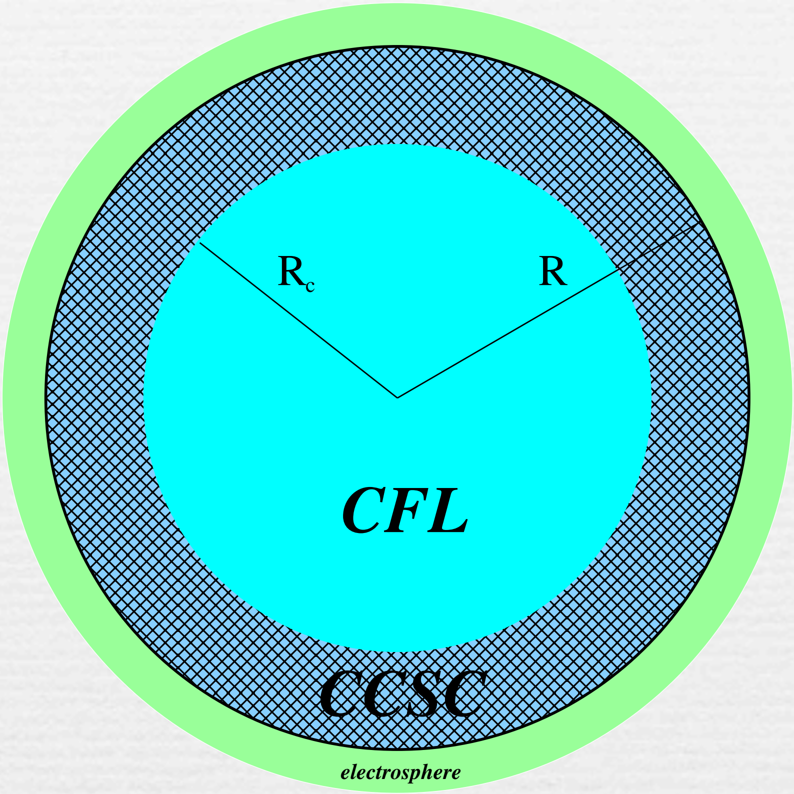



Andersson et al. Phys.Rev. Lett.99. 231101 (2007)

allowed regions

**...we can restrict the parameter space!**

# EM signals from bare strange stars




**Since the crust is rigid it can sustain large torsional oscillations**

**Since the star is bare, it has a large positive charge (compensated by the electrosphere)**

**Frequency of oscillation**

$$\omega \propto \frac{1}{R - R_c}\sqrt{\frac{\nu}{\rho}}$$

**about 1 GHz for a few centimeters thick crust**

**Estimated emitted power**

$$P \propto 10^{50} \text{ erg/s}$$

**assuming a giant Vela-like glitch as the trigger**

M.M., G.Pagliaroli, A. Parisi, L. Pilo, arXiv:1403.0128

**see the talk by Giulia Pagliaroli**

# Summary

- **Motivated by compact stellar observations, the study of matter in extreme conditions allows to shed light on some properties of QCD**

- **Color superconductivity is a phase of matter predicted by QCD at extreme densities**

- **We have developed various EFT for describing it**

- **Crystalline color superconductors are extremely rigid, more rigid than any known material.**

- **The study of various observables related to the large rigidity are under way**