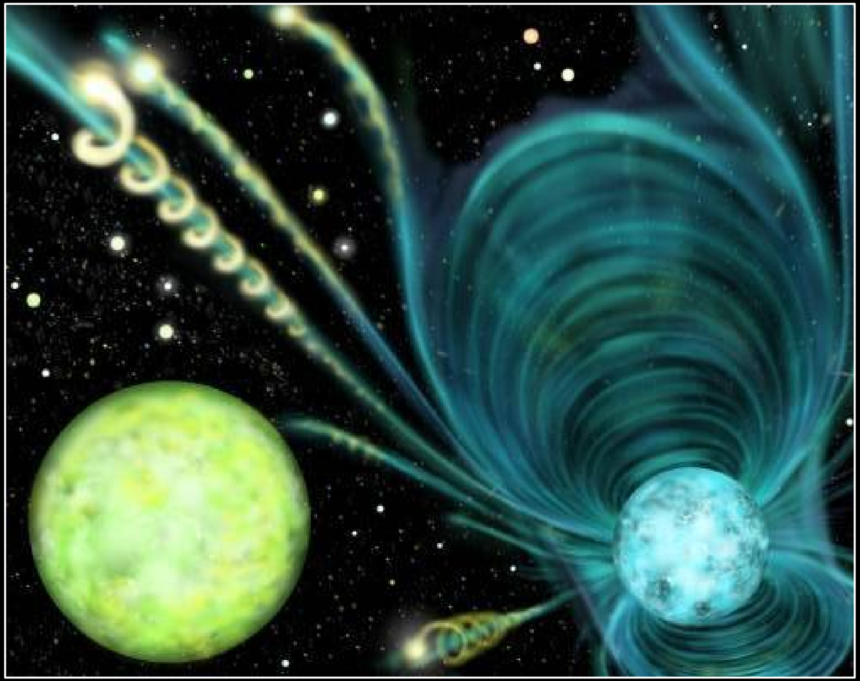
Neutron stars as superfluid laboratories



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Mathematics

the theory challenge

Want to use astronomical observations to probe matter at supranuclear densities.

A “minimal” theory model requires:

— supranuclear equation of state
(hyperons, quarks etc.)

— elastic crust
(neutron superfluid+vortices)

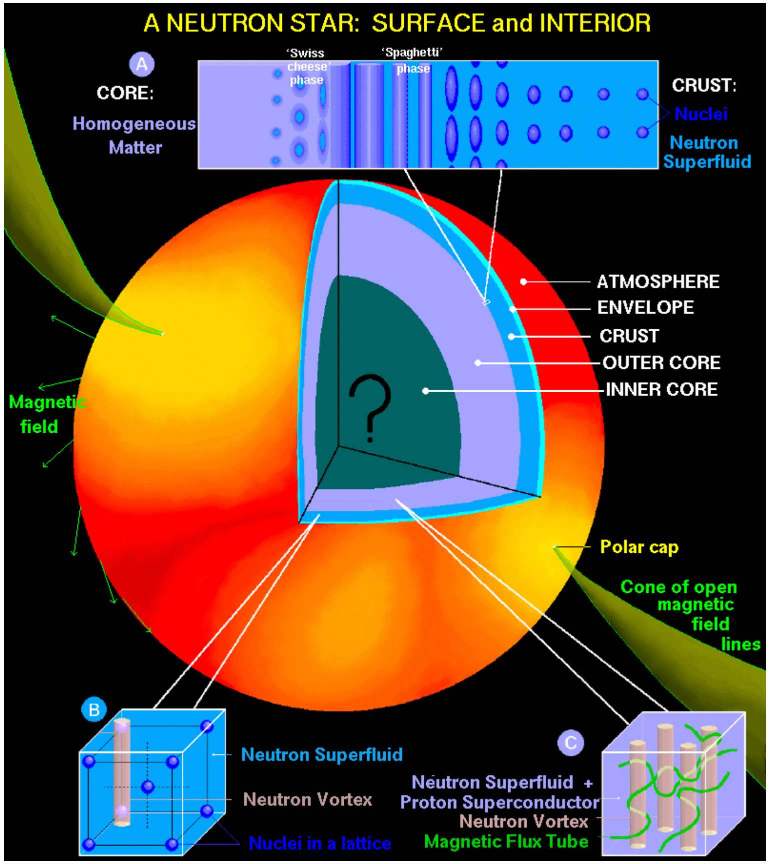
— magnetic field
(configuration, currents?)

— temperature profiles
(exotic cooling mechanisms?)

— superfluid/superconductors
(vortices vs flux tubes?)

— rotation (various instabilities)

— relativistic gravity



[courtesy Dany Page]

superfluidity

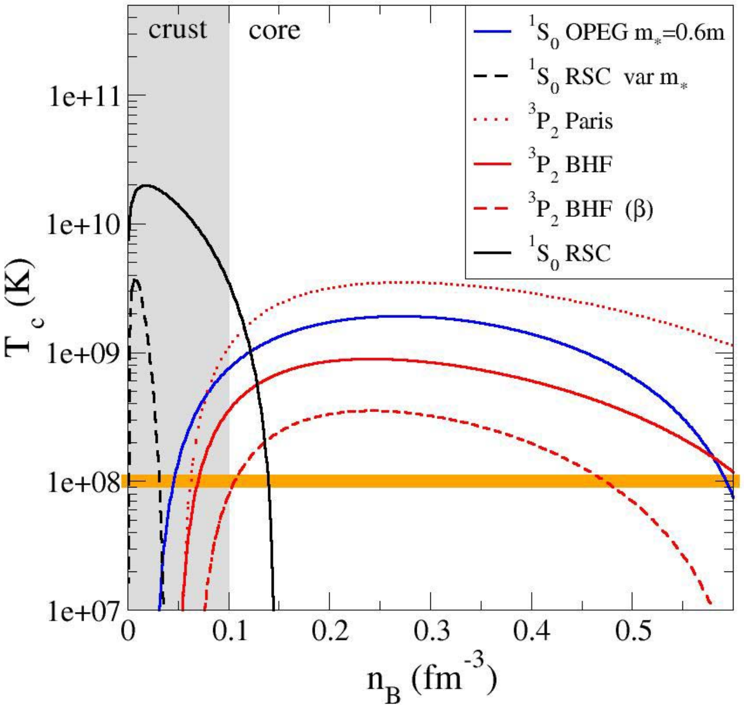
Since mature neutron stars are "cold" (10^8K<< T_{Fermi}=10^{12}K) they should be either solid or superfluid.

Theory:

Since late 1950's, nuclear physics calculations indicate "BCS-like" pairing gaps for neutrons and protons.

Exotica:

Deep core may contain superfluid hyperons and/or colour superconducting deconfined quarks; phases which allow exotic pairing states.



Remark: Must consider spatial phase transitions (boundary layers etcetera).

[NA, Comer & Glampedakis Nucl Phys A 763, 212 (2005)]

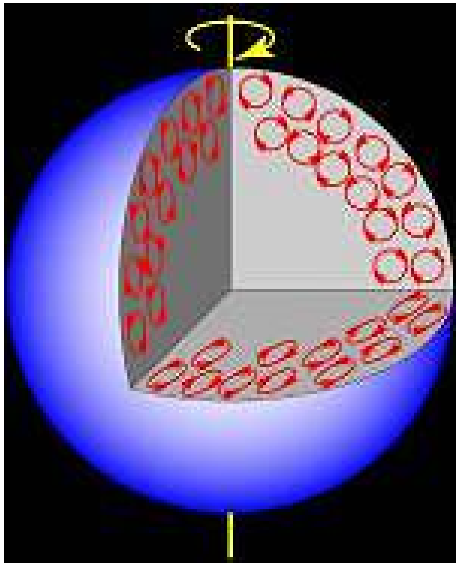
two "fluids"

Assume that:

— Electrons/muons in the core are coupled to the protons on very short timescales.

— Vortices and fluxtubes are sufficiently dense that a smooth-averaging can be performed.

The system is reduced to a <u>two-fluid model</u>.



One fluid is the superfluid neutrons in the inner crust and core, and the other fluid is a conglomerate of all charged constituents.

The equations of motion (analogous to Landau's model for superfluid Helium) can be derived from a variational principle. The model can be generalized to even more complex/exotic settings.

[NA & Comer Living Reviews 2007]

laboratory systems

Useful to compare and contrast to Helium:

For ^{4}He the two fluids represent the condensate and excitations.

In the (simplest) neutron star case we have zero temperature and a mixture of

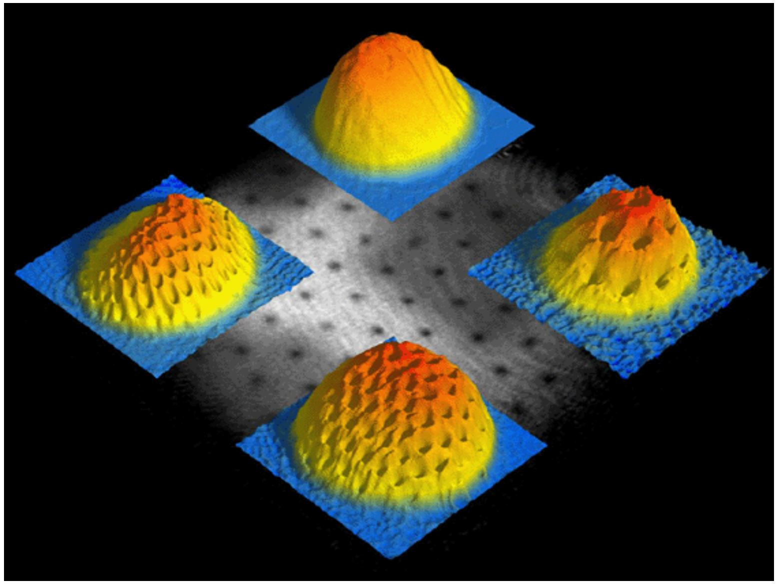
— superfluid neutrons

— superconducting proton (+ electrons)

At finite temperature we must also account for excitations and the associated entropy.

This problem is far from easy…

Note: May be relevant to compare to mixtures of ^{4}He and ^{3}He.



Some good news:

The basic multi-fluid aspects and the vortex dynamics are likely to be similar in the two cases. Can also learn from atomic condensates.

equations of motion

The equations describing a superfluid neutron star take the form (x & y are n or p):

$$\partial_t n_{\mathrm{x}} + \nabla_i (n_{\mathrm{x}} v_{\mathrm{x}}^i) = 0$$

$$(\partial_t + v_{\mathrm{x}}^j \nabla_j) p_i^{\mathrm{x}} + \nabla_i (\Phi + \tilde{\mu}_{\mathrm{x}}) + \varepsilon_{\mathrm{x}} w_j^{\mathrm{yx}} \nabla_i v_{\mathrm{x}}^j = f_i^{\mathrm{x}}$$

Here we have defined the relative velocity $w_i^{\mathrm{yx}} = v_i^{\mathrm{y}} - v_i^{\mathrm{x}}$ and the momenta are given by

$$p_i^{\mathrm{x}} = v_i^{\mathrm{x}} + \varepsilon_{\mathrm{x}} w_i^{\mathrm{yx}}$$

This encodes the "entrainment effect", due to which the velocity of each fluid does not have to be parallel to its momentum.

Can be thought of in terms of an "effective mass"; $\rho_{\mathrm{p}} \varepsilon_{\mathrm{p}} = n_{\mathrm{p}} (m_{\mathrm{p}} - m_{\mathrm{p}}^*)$

Remark: Recent calculations suggest that the effective neutron mass may be very large in the inner crust of the star.

[NA & Comer CQG 23, 5505 (2006)]

mutual friction

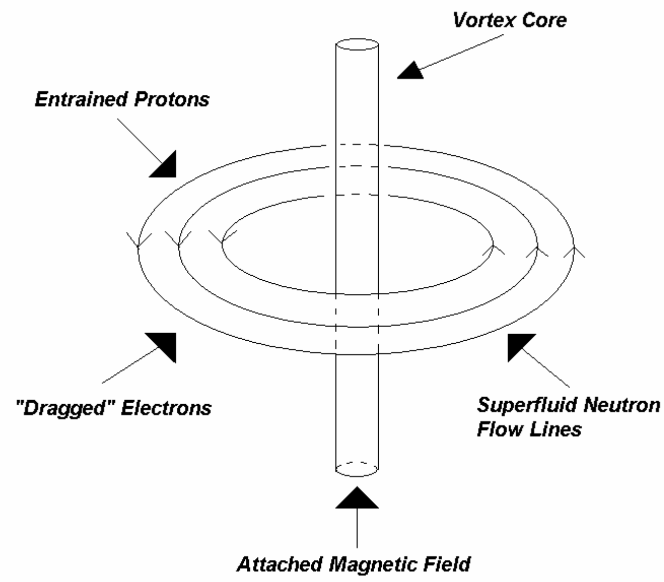
The presence of vortices leads to "mutual friction".

Standard form (for a straight vortex array);

$$f_i^{\mathrm{mf}} = \frac{R}{1+R^2}\varepsilon_{ijk}\hat{\omega}_{\mathrm{n}}^j \varepsilon^{klm}\omega_l^{\mathrm{n}} w_m^{\mathrm{np}} + \frac{R^2}{1+R^2}\varepsilon_{ijk}\omega_{\mathrm{n}}^j w_{\mathrm{np}}^k$$

where

$$\omega_{\mathrm{n}}^i = \varepsilon^{ijk}\nabla_j p_k^{\mathrm{n}}$$



Possibilities:

– electron scattering off vortices leads to $R<<1$
– vortex clusters lead to $R>>1$
– vortex/fluxtube interaction?

The, usually neglected, strong drag regime leads to interesting dynamics.

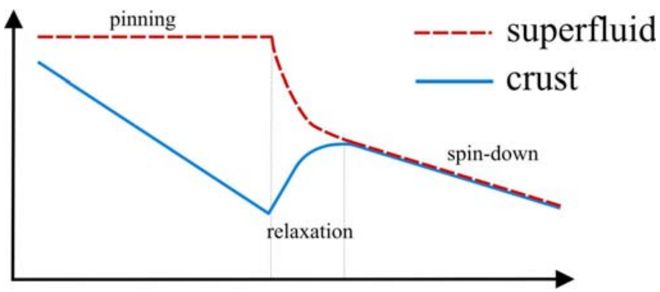
[NA, Sidery & Comer MNRAS 368, 162 (2006)]

observations

Observational support from “glitches” in many young neutron stars.

The standard model for these events is based on transfer of angular momentum from a superfluid component to the crust.

– the crust slows down due to magnetic braking



– the superfluid can only spin down if vortices move outwards

– if the vortices are pinned to the crust, the superfluid lags behind

– at some critical level, a large number of vortices are released. As a result the crust is spun up.

Observations suggest unpinning of vortices at relative rotation

$$\Delta\Omega/\Omega_p \approx 5\times10^{-4}$$

Our understanding is, however, far from satisfactory.

In particular, we do not know what <u>triggers</u> a glitch.

relaxation

Standard form for mutual friction assumes a straight vortex array.

Leads to a model that predicts that the system evolves according to

$$\begin{aligned} n_{\rm n}\partial_t p_i^{\rm n} + \ldots &= f_i \\ n_{\rm p}\partial_t p_i^{\rm p} + \ldots &= -f_i \end{aligned} \quad \to \quad \frac{m_{\rm p}^*}{m_{\rm p}}\partial_t w_i^{\rm np} + \ldots \approx -\frac{B\kappa n_{\rm v}}{x_{\rm p}} w_i^{\rm np}$$

following a glitch event. This corresponds to a typical coupling timescale

$$t_d \approx 10P(s)\left(\frac{m_{\rm p}^*}{m_{\rm p}-m_{\rm p}^*}\right)^2\left(\frac{x_{\rm p}}{0.05}\right)^{-1/6}\left(\frac{\rho}{10^{14}\ {\rm g/cm^3}}\right)^{-1/6}$$

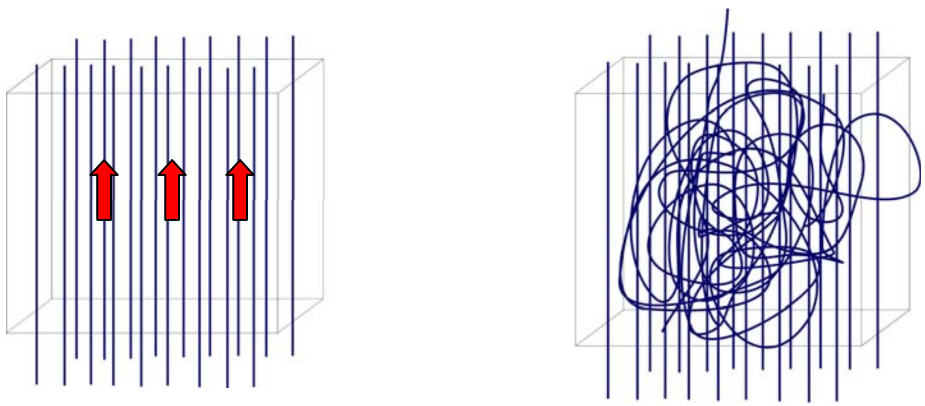
Since this is much faster than the observed relaxation time in, for example, the Vela pulsar (weeks/months), glitches may not be associated with the star's core…

Note: The mutual friction is much weaker in the crust, so the relaxation time would be longer.

turbulence

This simple model may, however, be "wrong"...

If there is a large scale flow along the vortex array, then short wavelength inertial modes become unstable.



Then the system is turbulent (overwhelming evidence from lab experiments), and the mutual friction may have a different form;

$$f_i = \frac{8\pi^2 \rho_{\rm n}}{3\kappa} \left(\frac{\chi_1}{\chi_2} \right)^2 B^3 w_{\rm pn}^2 w_i^{\rm pn}$$

This leads to non-exponential relaxation (locally), so we need simulations...

[NA, Sidery & Comer MNRAS 381, 747 (2007)]

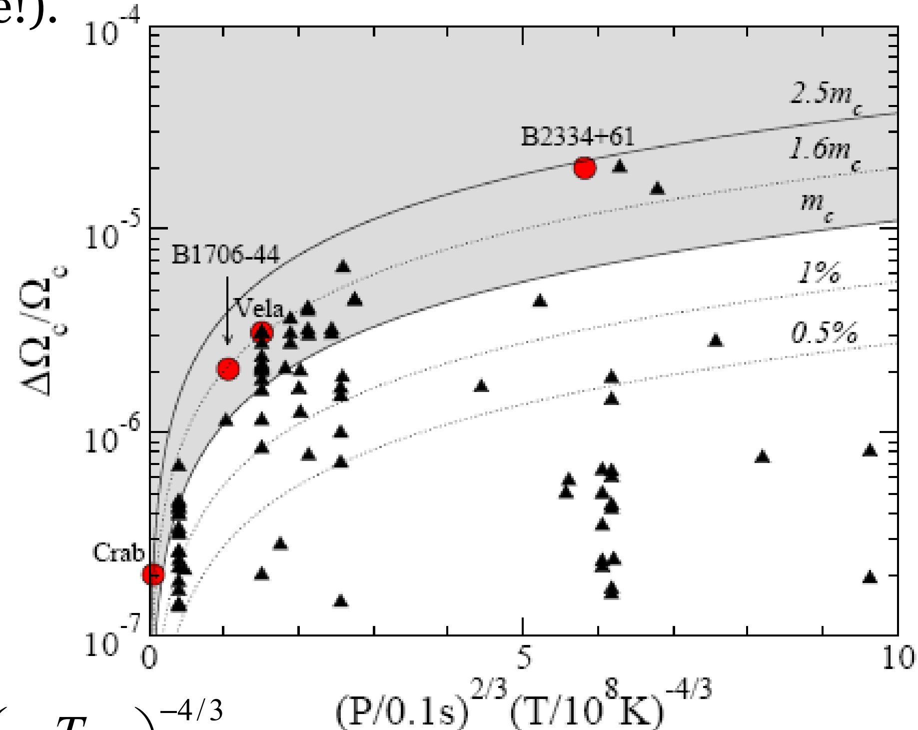
superfluid instability

Global r-mode calculation for model with mutual friction and different background rotation rates (first time!).

Large *m* modes (short wave-length) become <u>dynamically</u> unstable beyond critical rotational lag in system with <u>strong</u> coupling.

Could this be the mechanism that triggers large glitches?

Balance mode growth and shear viscosity damping to get;



$$\left.\frac{\Omega_n - \Omega_p}{\Omega_p}\right|_{\rm critical} \approx 6\times10^{-5}\left(\frac{P}{0.1\rm s}\right)^{2/3}\left(\frac{T}{10^8\rm K}\right)^{-4/3}$$

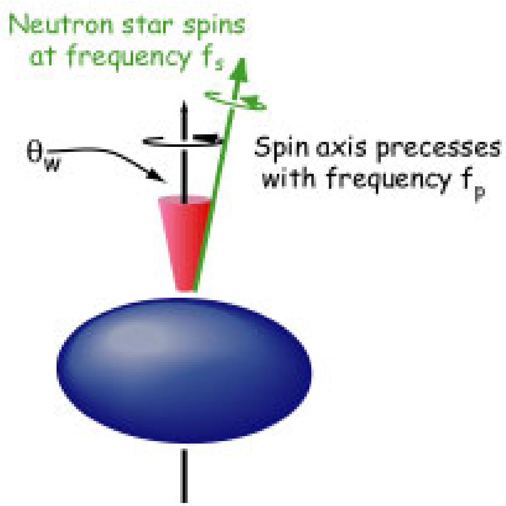
Note: Possible link between oscillation modes and vortex turbulence.

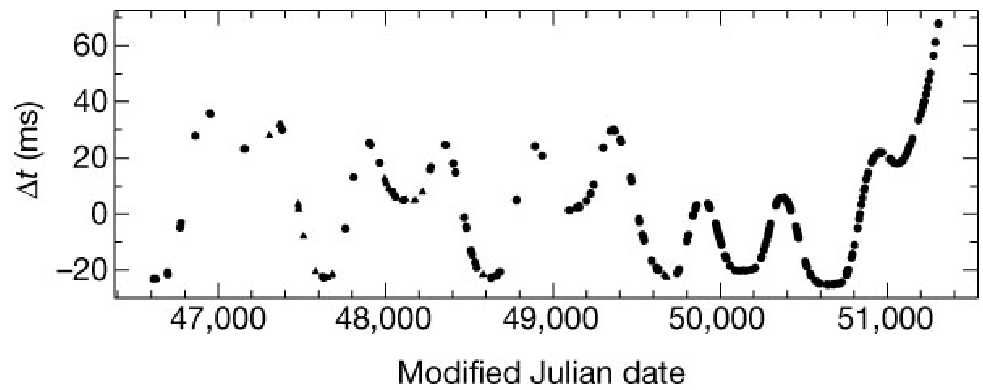
[Glampedakis & NA PRL 102, 141101 (2009)]

free precession

Free precession is the most general motion of a solid body. (“Chandler wobble”)

Neutron star will precess if the crust is deformed in some way. Expect small deformations and <u>long period</u> precession.





Strongest observational evidence (?):
1009d (or 500d) periodicity in PSR B1828-11

Since the precession motion is a normal mode of the coupled core-crust system it depends on the interior dynamics.

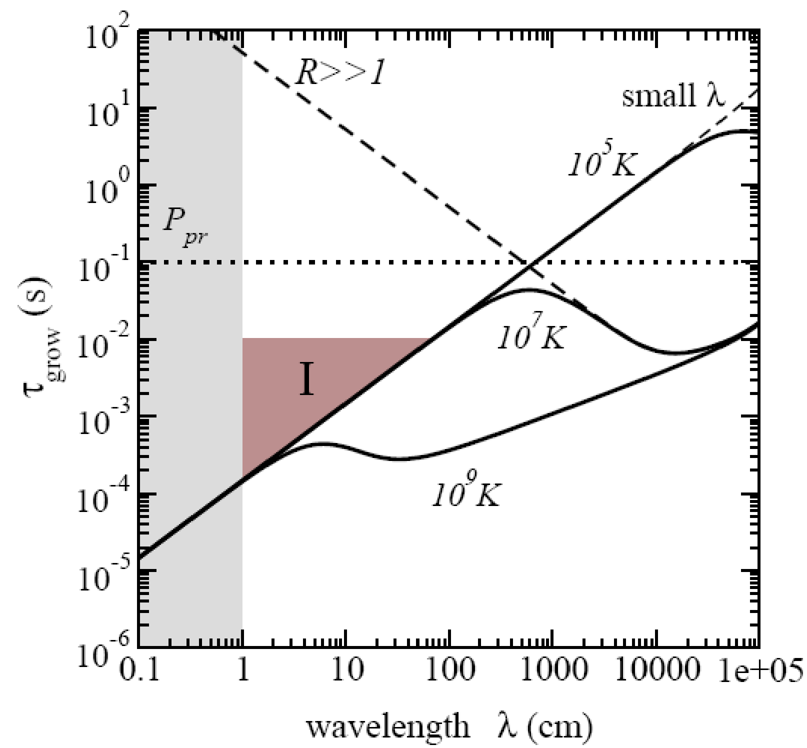
[Jones & NA MNRAS 324, 811 (2001)]

fast precession?

Long period precession is not possible if there is significant pinning between vortices and magnetic fluxtubes in the star's core.

Observations indicate that the core is not a type II superconductor?

Perhaps, but…



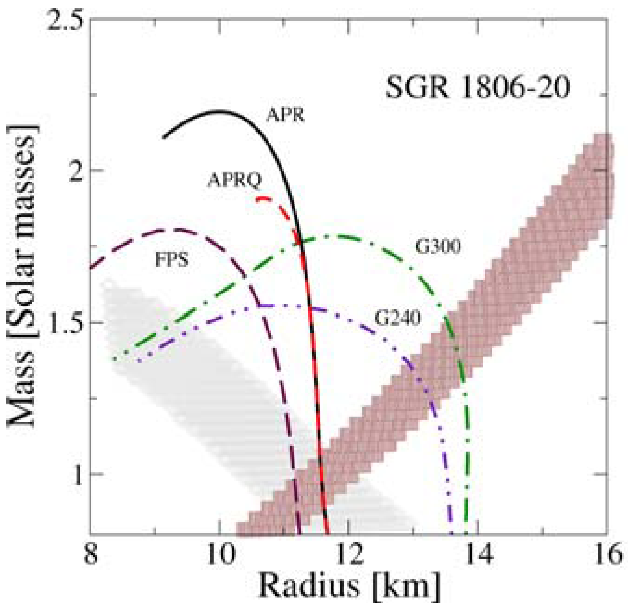
A local analysis shows that short wavelength waves may be unstable in a precessing star. Strong coupling/fast precession motion is generically unstable.

May explain why precessing neutron stars are rare.

Note: We need to consider the hydrodynamics asssociated with precession. This is a very hard problem given the range of timescales involved.

[Glampedakis, NA & Jones PRL 100, 081101 (2008)]

magnetar seismology



Oscillations seen in the tails of SGR flares may be associated with crust modes.

If so, observed spectrum constrains the equation of state (at least in principle).

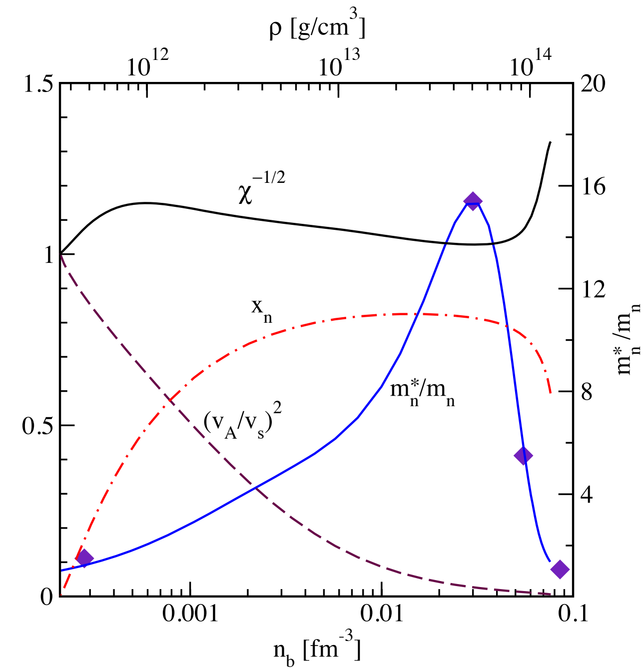
Fundamental modes (different multipoles) and a single overtone provide a relatively strong constraint.

The superfluid in the crust affects the oscillations;

$$\omega^2 \to \tilde{\omega}^2 \approx \frac{x_c}{\chi} \omega^2$$

χ encodes the <u>effective mass</u> of the free neutrons.

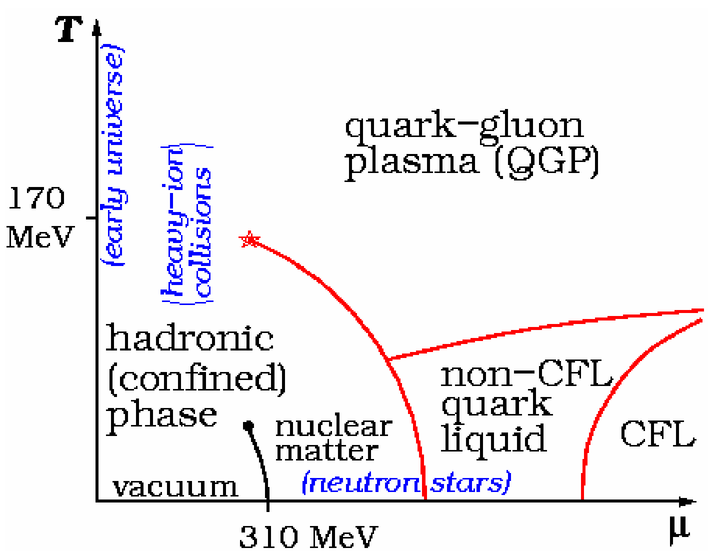
This may be a 20% effect.



[NA, Glampedakis & Samuelsson MNRAS 396, 894 (2009)]

quarks

Various Colour-Flavour-Locked quark phases provide interesting challenges.



Hierarchy of models with increasing complexity;

1. Condensate + massless excitations (phonons) (Essentially ^{4}He…)
2. "Thermal kaons"
   Need additional, massive, fluid component (coupling to phonons?)
3. Kaon condensate (CFL-K^o)
   Requires a second condensate and another set of thermal excitations (vortices?)

A "natural" variational model for ^{4}He would distinguish the <u>atoms</u> from the massless <u>entropy</u> (the excitations). Then we have

$$\pi_i^{\rm n} = mn\tilde{p}_i^{\rm n} = mn(v_i^{\rm n} + \varepsilon_{\rm n} w_i^{\rm sn})$$

$$\pi_i^{\rm s} = mn\varepsilon_{\rm n} w_i^{\rm ns}$$

$$\boxed{\varepsilon^{ijk}\nabla_j \tilde{p}_k^{\rm n} = 0}$$

<u>Entropy entrainment</u> encodes "normal fluid" density.

summary

Neutron star superfluidity impacts on key astrophysics.

Pulsar glitches, free precession and oscillations (gravitational waves/magnetar flares).

Ongoing effort to understand the dynamics of multi-fluid neutron stars.

A number of "interesting" results;

– relevance of entrainment and the mutual friction damping

– two-stream instability (turbulence?), with impact on precession and glitches;

– role of pinning and core superconductivity.

Long to-do list:

– consider finite temperature effects (<u>always</u> important near critical density);

– model crust penetrated by neutron superfluid in detail (<u>pinning</u>);

– analyse vortex behaviour at crust-core interface (viscous boundary layer?).

– understand role of vortex turbulence (<u>polarisation</u>)