The r-mode instability

what we (think) we know...

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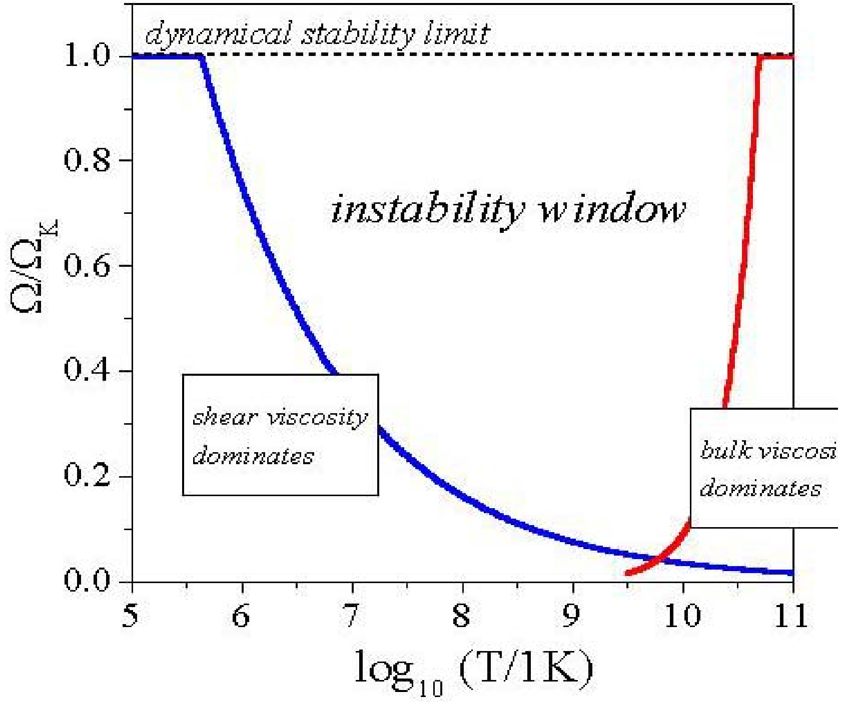
The r-modes belong to a large class of "inertial" modes, which may be driven unstable by the emission of GWs at all rates of rotation.

The *l=m=2* r-mode grows due to current-multipole radiation on a timescale of a few tens of seconds.

Instability window depends on core-physics.



The simplest model accounts for damping due to shear- and bulk viscosity.

Leads to a very large instability window.

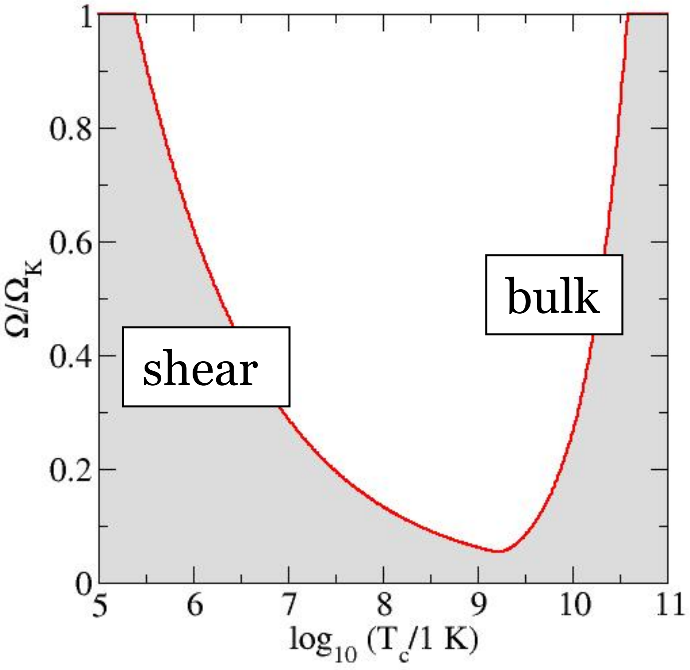
In principle, we should not observe any "usual" pulsar inside the instability region. Can we use this to "rule out" theoretical models?

“constraints”

Young radio pulsars: Original r-mode window consistent with the inferred birth spin of the Crab PSR (19 ms), but not with the 16 ms X-ray PSR J0537-6910.

Recycled pulsars: Need to allow the formation of a cold 716 Hz PSR (presumably after recycling). This constrains the instability window at low temperatures.

LMXBs: Nuclear burning of accreted material provides a thermostat that sets the core temperature to 10^8 K. Fastest systems (around 640 Hz) require smaller instability region.



Consider temperature limits for systems in quiescence (Brown+Ushomirsky).

If a system is r-mode unstable, how would we know?

Are there systems that behave (in some sense) “funny”?

Ekman layer

The story is, however, more complex.

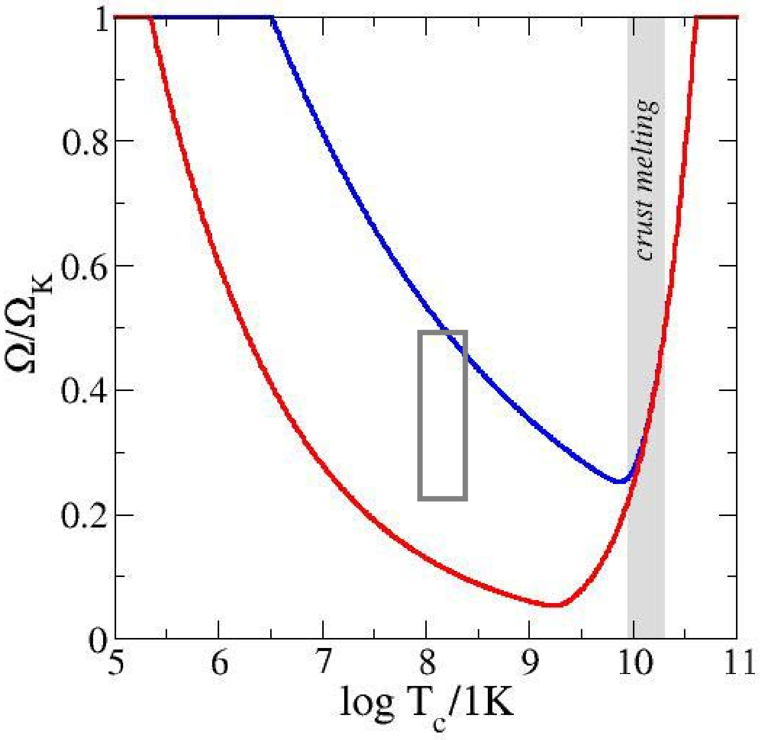
There are a number of additional damping mechanisms to consider.

Current results suggest that viscous “rubbing” at the base of the crust may dominate the damping.

For a solid crust an Ekman (boundary) layer forms at the core-crust interface, due to the required non-slip condition.

The typical length-scale of this layer is

$$\delta_E \sim \sqrt{\frac{\nu}{\Omega}} \sim 1 \text{ cm}$$



The short length-scale means that the Ekman layer leads to efficient damping.

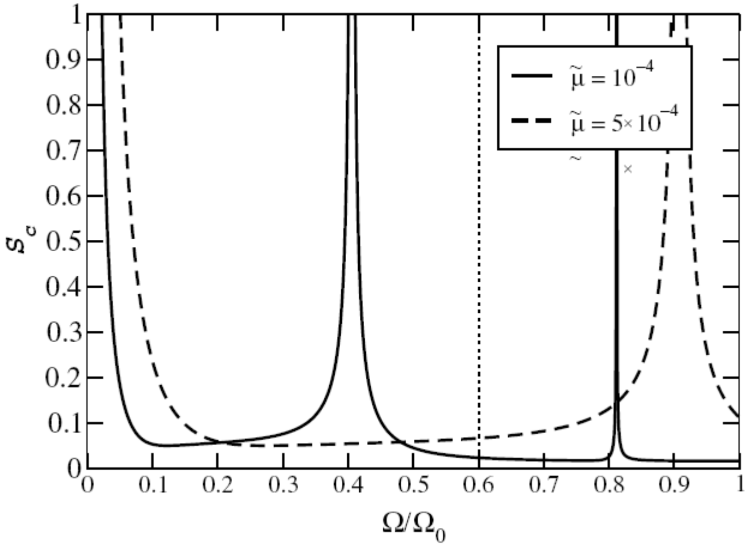
This is, perhaps, good news since the accreting LMXBs would otherwise be unstable.

crust-core slippage

However… the crust will take part in the r-mode motion.

For typical parameters, the crust is more like a “liquid” than a solid; the Coriolis force dominates over the elasticity;

$$S^2 = \left(\frac{\delta v}{v}\right)^2 \approx \frac{\mu}{\rho\Omega^2 R^2} \sim 10^{-4}\left(\frac{\Omega_K}{\Omega}\right)^2$$



[Glampedakis & Andersson 2006]

As a result, there may only be a slight “slippage” between the core and the crust.

The upshot is that the Ekman layer damping is weaker by a significant factor, perhaps as large as 1,000.

The picture is further complicated by;

– resonances with the “elastic” modes in the crust,

– compressibility in the fluid and internal stratification (factor of a few?)

– the magnetic field, which may make the boundary layer much more efficient.

superfluidity

Another factor, that could be highly relevant, is <u>superfluidity</u>.

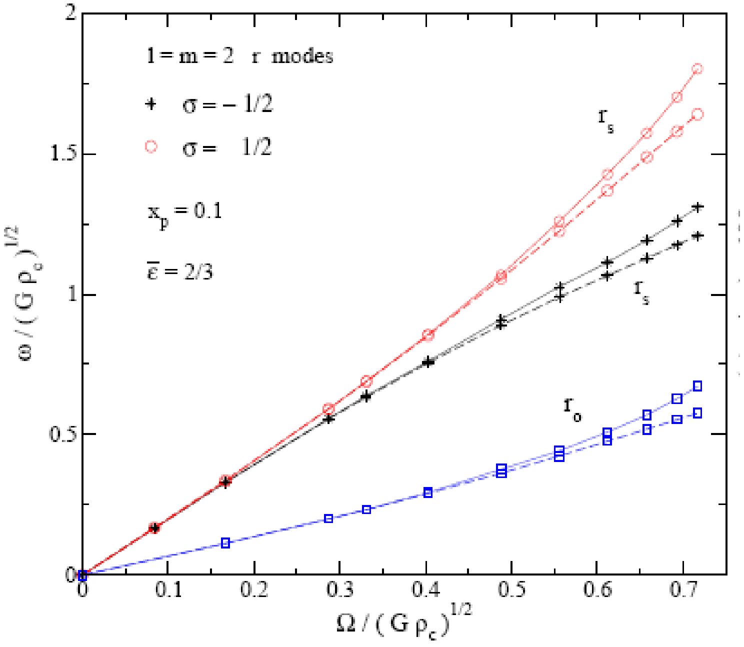
Mature neutron stars will contain <u>several</u> superfluid/superconducting components.

Such systems require a multi-fluid approach, accounting for entrainment and unique dissipation mechanisms like the vortex mediated mutual friction.

Recent work improves our understanding of both the r-modes in fast spinning systems and the mutual friction damping.

A key conclusion is that mutual friction is <u>unlikely to have a significant effect</u> on the r-mode instability.

Warning: The picture may change once we account for the proton superconductivity, and the associated magnetic fluxtubes…

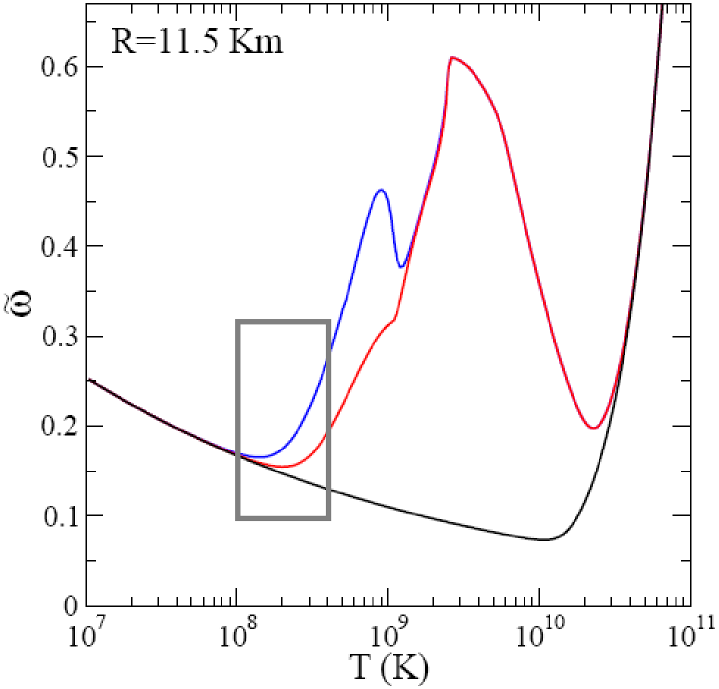


[Passamonti et al 2009]

hyperons

The core composition could also be highly significant.

In particular, non-leptonic reactions involving hyperons may lead to a much stronger bulk viscosity.



[Haskell & Andersson 2010]

This may suppress the r-mode, but… in order not to be in conflict with cooling data, the hyperons must be superfluid.

Then the bulk viscosity is also suppressed.

Need to account for "realistic" pairing gaps (multi-layered star) and multi-fluid aspects (extra bulk viscosity terms).

Recent work in this direction, but we need to do better…

Note: Similar issues arise for a star with deconfined quarks in the core.

magnetic field

The magnetic field is relevant for a number of reasons. Most importantly;

– the r-mode may change nature due to the magnetic forces (continuum?)

– the field may itself be affected by the mode (secular drift leads to wind-up?)

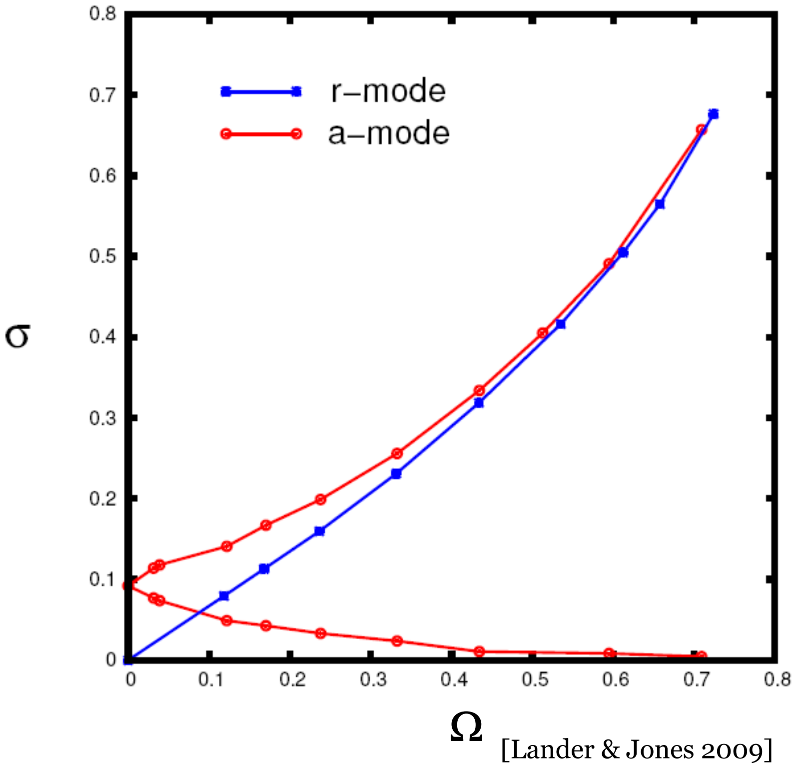
Recent work sheds some light on the nature of the modes.

At low rotation rates the r-modes are dominated by the magnetic forces (“replaced” by toroidal Alfven modes which may even be stable).

Unlikely to be relevant for realistic magnetic field strengths, but…

This problems needs <u>a lot</u> more attention.

Technically complicated… and we don’t “know” the interior field configuration.



[Lander & Jones 2009]

saturation

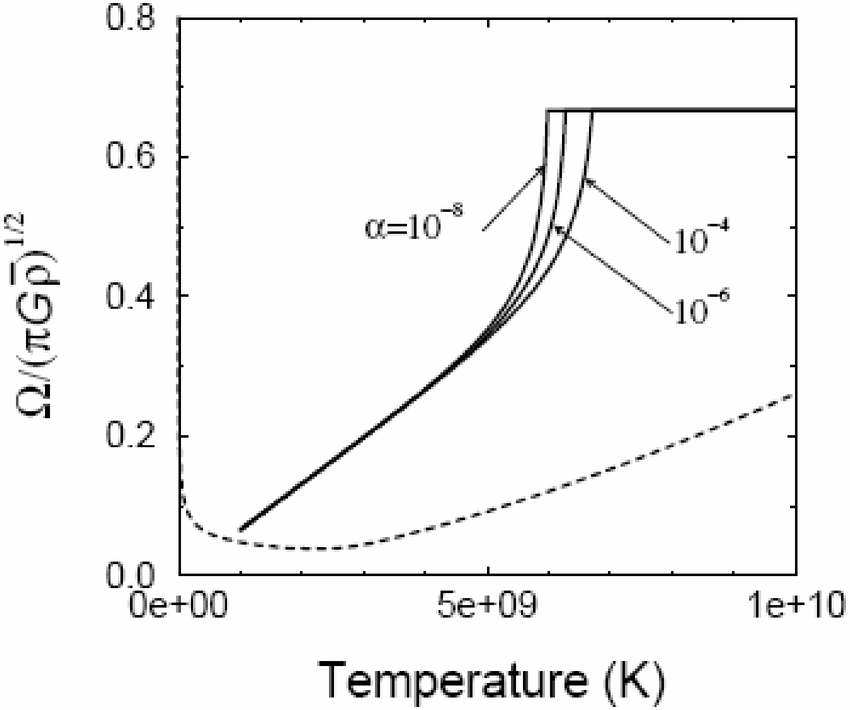
The r-mode growth phase is adequately described by linear theory, but nonlinear effects soon become important.

Phenomenology:
At some point the mode "saturates" and, as excess angular momentum is radiated away, the star spins down.

Model using Ω and α and evolution equations from the conservation of energy and angular momentum:

$$\delta v \propto \alpha \Omega R$$

Initial models used α=1 which is large!



[Owen et al 1998]

Now think that nonlinear mode-coupling limits the r-mode amplitude to $\alpha < 10^{-3}$

“realistic” evolutions

The non-linear evolution of the instability depends on many different parameters.

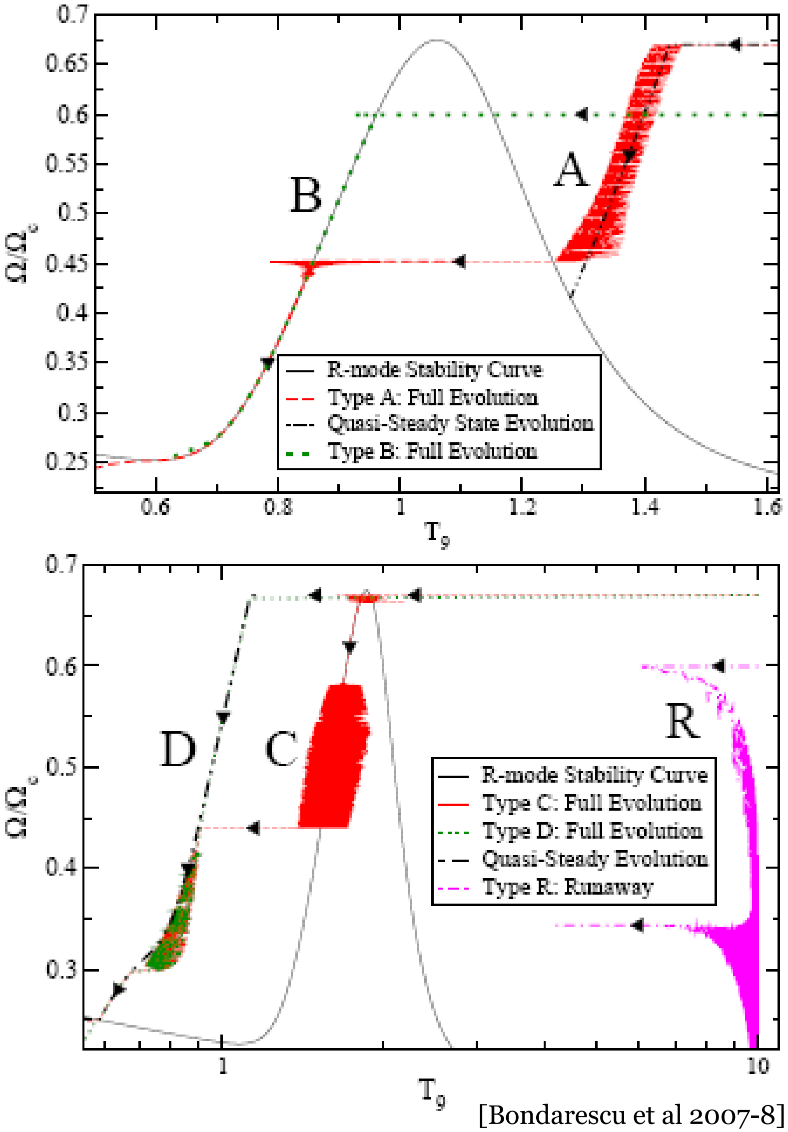
Good news: In many cases, a relatively simple “three-mode network” provides a good description.

Moreover, the general evolution can be understood from stationary solutions (cooling balance heating etc).

Bad news: The r-mode amplitude may not be slowly varying, meaning that it could be difficult to detect the GWs.

Estimates suggest that the signal from newly born neutron stars may be detectable out to 1kpc-1Mpc, meaning that the event rate will be low.

Moreover… if the system is turbulent…



[Bondarescu et al 2007-8]

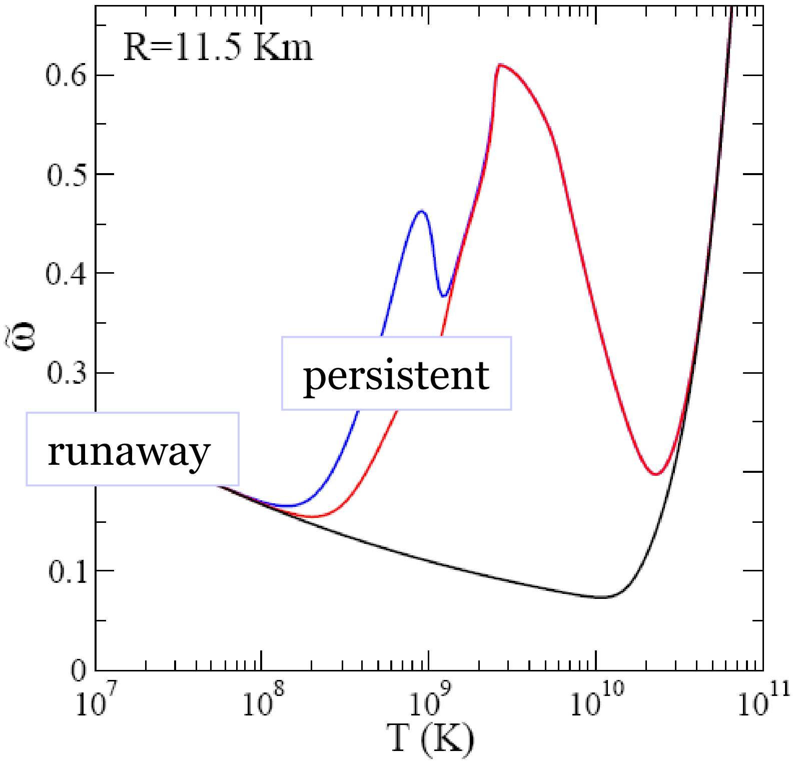
LMXBs

For accreting systems there are (essentially) two relevant scenarios:

- thermal “runaway”
- “steady state”

Outcome depends on a numbers of issues:

- the shape of instability window
- does the mode survive saturation?
- backreaction on the star (differential rotation?)



Stars with sizeable (superfluid) hyperon/quark cores seem the most promising for persistent emission.

With the GW torque balancing accretion, the signal could be “detectable” with a few months integration (although there are serious caveats).

r-mode puzzle

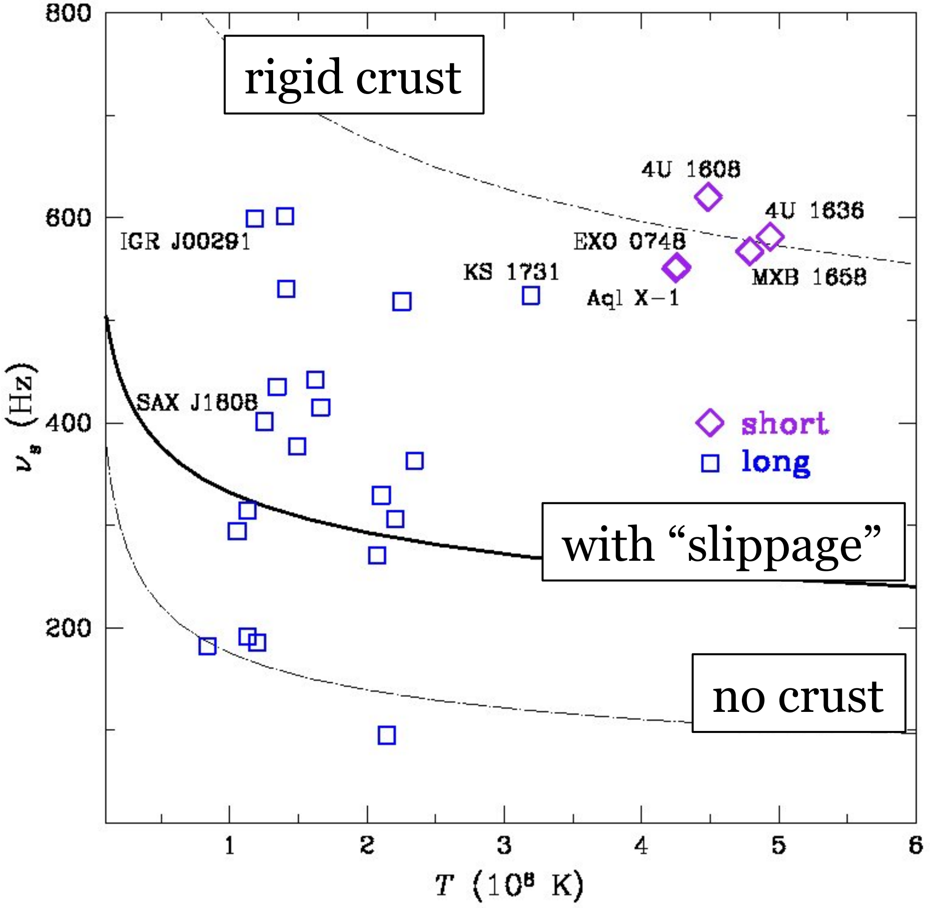
Demonstrates that our understanding of the r-modes is incomplete. Given the “best estimate” for the main damping mechanisms, many observed LMXBs should be unstable.

Rigid crust with viscous (Ekman) boundary layer would lead to sufficient damping…

…but the crust is more like jelly, so the effect is reduced (“slippage”).

Magnetic field is too weak to alter the nature of the boundary layer.

Superfluid “mutual friction” (due to electrons scattered off vortices) has no effect.



Saturation amplitude due to mode-coupling is too large to allow evolution far into instability region.

Equation of state	What is the role of hyperons/deconfined quarks?
	How do we model finite temperature systems?
Superfluid dynamics	When do we need to “worry” about multifluid dynamics?
	What is the true nature of mutual friction, and how does vortex pinning work?
	How do we model the different interfaces?
The crust	Can we model viscous boundary layers in “realistic” neutron stars?
	How “important” is the crust superfluid?
Magnetic field	What is the nature of the magnetic r-modes, and how do they interact with the magnetic field during the unstable phase?
	How do we describe superconducting systems?
Saturation/evolution	How can we “confirm” the mode-coupling results, and “improve” them to account for more realistic physics?