Lattice QCD at nonvanishing temperatures and densities

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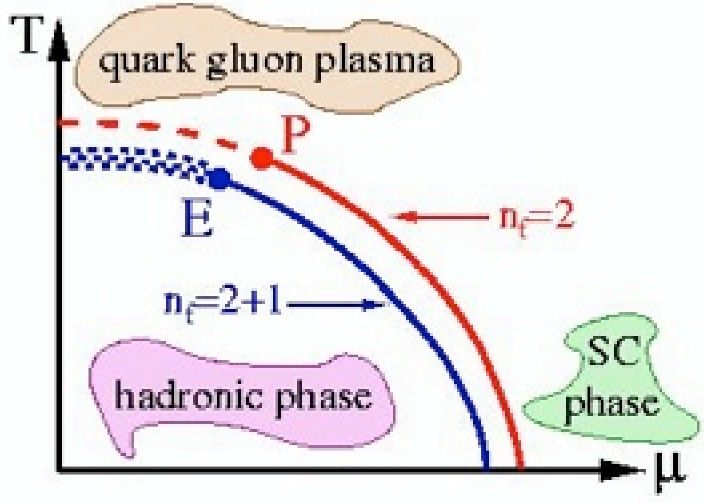
Introduction, experimental motivation



- Chiral phase transition (PT)

$n_f = 2$ with $m_q = 0$ at $\mu = 0 \Rightarrow 2^{nd}$ order PT

$n_f = 2$ with $m_q = 0$ at $T = 0 \Rightarrow 1^{st}$ order PT

$n_f = 2$ with $m_q = 0 \Rightarrow$ tricritical point (P) at μ,T$\neq$0

$n_f = 3$ with $m_q = 0$ at $\mu = 0 \Rightarrow 1^{st}$ order PT

increasing m_q weakens the 1^{st} order PT $\Rightarrow$ cross-over

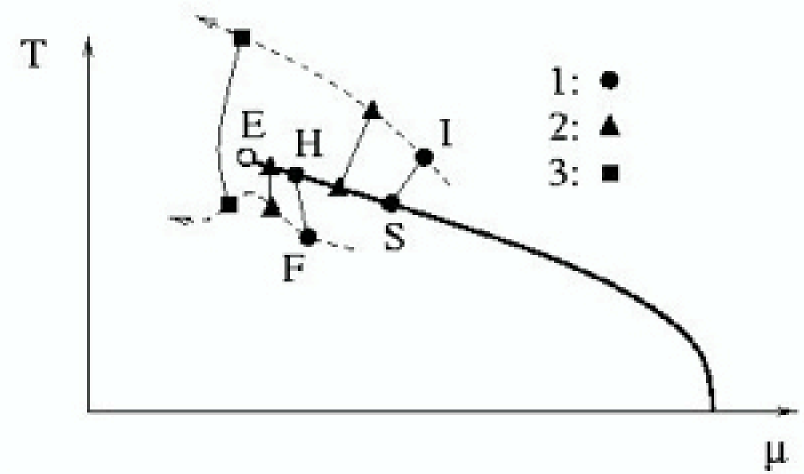
$n_f = 2 + 1$ with physical m_q at $\mu = 0 \Rightarrow$ cross-over

$n_f = 2 + 1$ with physical m_q at $T = 0 \Rightarrow 1^{st}$ order PT

$n_f = 2 + 1$ with physical $m_q \Rightarrow$ critical endpoint (E) at μ,T$\neq$0

Typical trajectory in heavy ion collisions

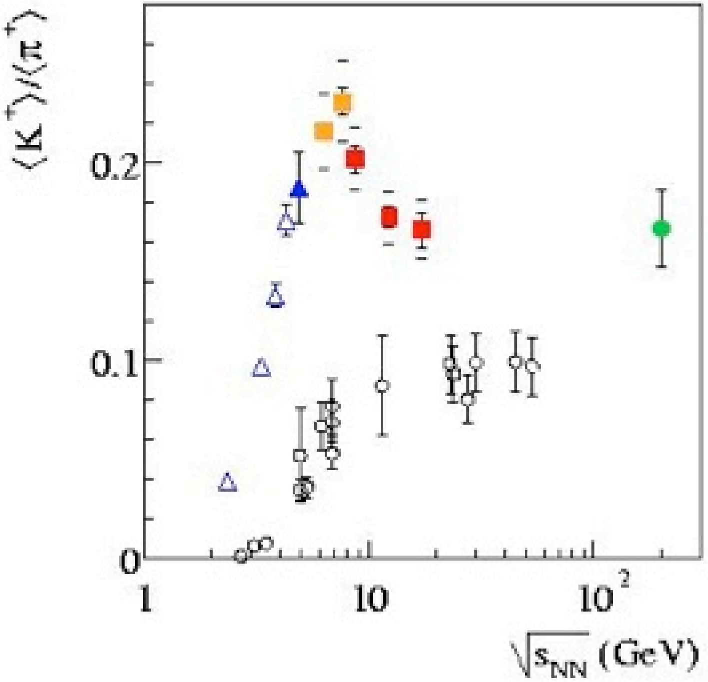
M. Stephanov, K. Rajagopal, E. Shuryak, Phys. Rev. Lett., 81, 4816 (1998)



• control parameters to decrease μ:
increasing the energy of the collision
increasing the centrality of the collision

zigzag trajectories: latent heat reheats the mixed phase
focusing the trajectories towards E (H is closer than S)
no fine-tuning is needed to explore the singular E point

excess in the low p_T pion spectra
non-monotonic behavior: E can be missed on either sides



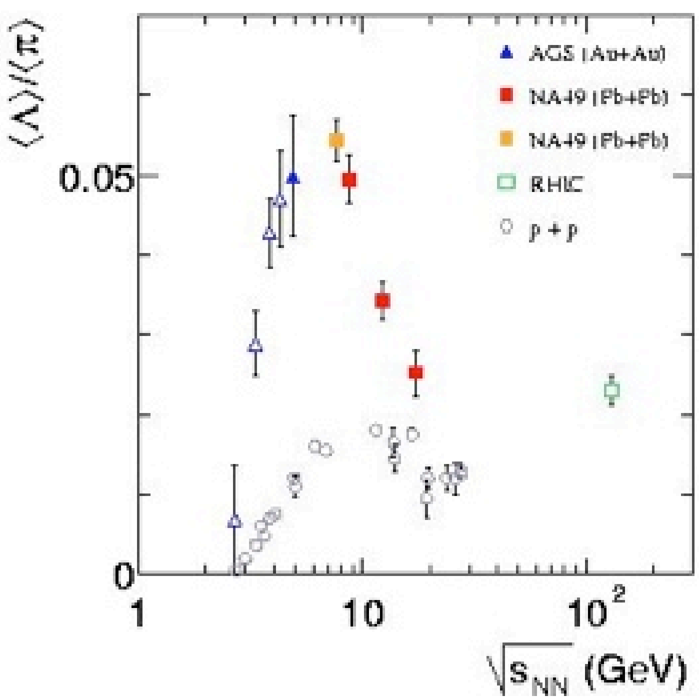
• new excitement

NA49 Collaboration J.Phys.G30:S119-S128,2004

M. Gazdzicki QM04: www-rnc.lbl.gov/qm2004/talks

recent scan: NA49, Pb-Pb $\sqrt{s}$=7-17 GeV
steep maximum observed in the
$\Longleftarrow K^+/\pi^+$ ratio

&



$\Longleftarrow \Lambda/\pi$ ratio

'singlular' behaviour at $\mu_B \approx$400 MeV
might be interpreted as an endpoint signal
or: result of the maximal chemical potential

- location of the endpoint: nonperturbative prediction of QCD

lattice gauge theory: serious problems at $\mu \neq 0$

measure (Dirac determinant) complex $\Rightarrow$ no importance sampling

$\Rightarrow$ sign problem

I.M. Barbour et al., Nucl. Phys. B (Proc. Supl.) 60A, 220 (1998)

Glasgow method: μ reweighting based on an ensemble at $\mu = 0$

after collecting 20 million configurations only unphysical results

$T = \mu = 0$ ensemble does not overlap with the transition states

M.A. Halasz et al., Phys. Rev. D58, 096007 (1998)

random matrix model for the Dirac operator can be solved

$\Rightarrow T_E \approx 120$ MeV and $\mu_E \approx 700$ MeV, can be off by a factor of 2-3

J. Berges, K. Rajagopal, Nucl. Phys. B538, 215 (1999)

Nambu-Jona-Lasinio model, $T - \mu$ phase diagram

lattice QCD in continuum (Gribov copies): zero complexity
though at non-vanishing chemical potential: NP complete problem

quark differencing scheme:

$$\bar{\psi}(x)\gamma^{\mu}\partial_{\mu}\psi(x) \rightarrow \bar{\psi}_n\gamma^{\mu}(\psi_{n+e_{\mu}} - \psi_{n-e_{\mu}})$$
$$\bar{\psi}(x)\gamma^{\mu}D_{\mu}\psi(x) \rightarrow \bar{\psi}_n\gamma^{\mu}U_{\mu}(n)\psi_{n+e_{\mu}} + \dots$$

in continuum the chemical potential acts: $\mu a\bar{\psi}_x\gamma_4\psi_x$
fourth component of an imaginary(!), constant vector potential

fermionic part as a bilinear expression: $S_f = \bar{\psi}_n M_{nm}\psi_m$

Euclidean partition function gives Boltzman weights

$$Z = \int \prod_{n,\mu}[dU_{\mu}(x)][d\bar{\psi}_n][d\psi_n]e^{-S_g-S_f} = \int \prod_{n,\mu}[dU_{\mu}(n)]e^{-S_g}\det(M[U])$$

Metropolis step for importance sampling:

$$P(U \rightarrow U') = \min\left[1, \exp(-\Delta S_g)\det(M[U'])/\det(M[U])\right]$$

for μ=0 the determinant is positive, for $\mu \neq 0$ it is complex
$\Rightarrow$ no probability interpretation, no Monte-Carlo method

Overlap improving multi-parameter reweighting

Z. Fodor and S.D. Katz, Phys. Lett. B534 (2002) 87

$$Z(m,\mu,\beta) = \int \mathscr{D}U \exp[-S_g(\beta,U)] \det M(m,\mu,U) =$$

$$\int \mathscr{D}U \exp[-S_g(\beta_0,U)] \det M(m_0,\mu=0,U)$$

$$\left\{ \exp[-S_g(\beta,U)+S_g(\beta_0,U)] \frac{\det M(m,\mu,U)}{\det M(m_0,\mu=0,U)} \right\}$$

first line = measure, field configurations of the Monte-Carlo
curly bracket = can be measured on each configuration, weight

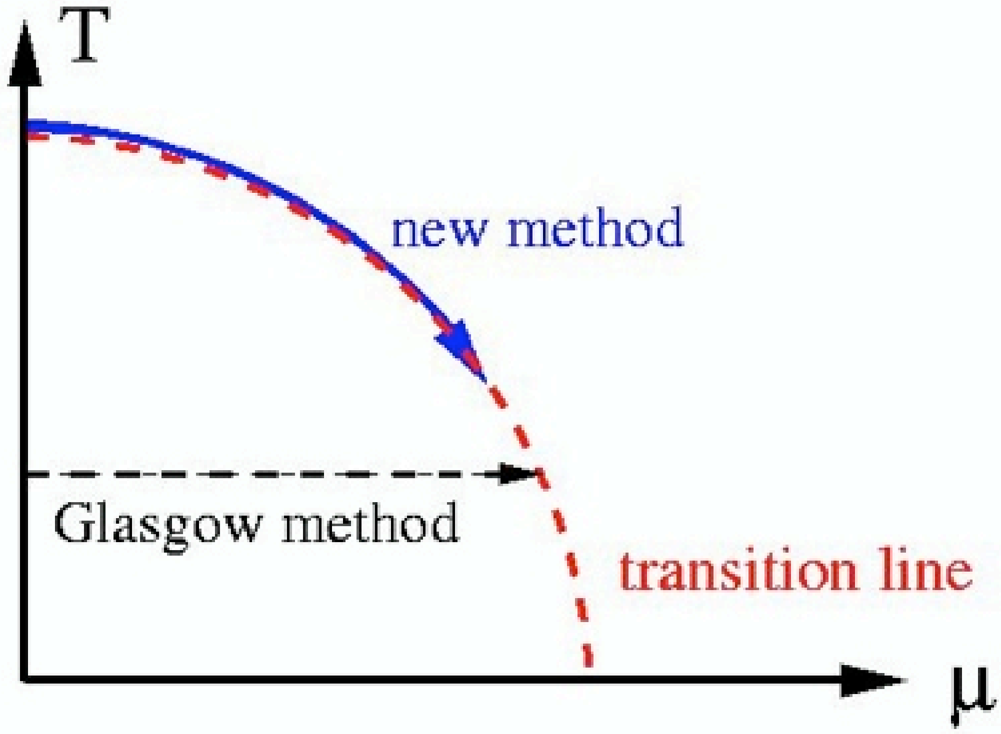
expectation value of an observable O:

$$\langle 0 \rangle_{\beta,\mu,m} = \frac{\sum w(\beta,\mu,m) O(\mu,m)}{\sum w(\beta,\mu,m)}$$

observables to get the transition points at $\mu \neq 0$ (susceptibilities)

simultaneously changing several parameters: better overlap
e.g. transition configurations are mapped to transition ones

Comparison with the Glasgow method



one parameter reweighting	New method
single parameter (μ)	two parameters (μ and β)
purely hadronic configurations	transition configurations


ALICEnext

Endpoint with physical quark masses on $L_t = 4$ lattices

Z.Fodor, S.D.Katz, hep-lat/0402006, JHEP 04 (2004) 050

• three basic steps of the analysis

m_s=0.25, m_{ud}=0.0092: physical ones, T=0 measurements show

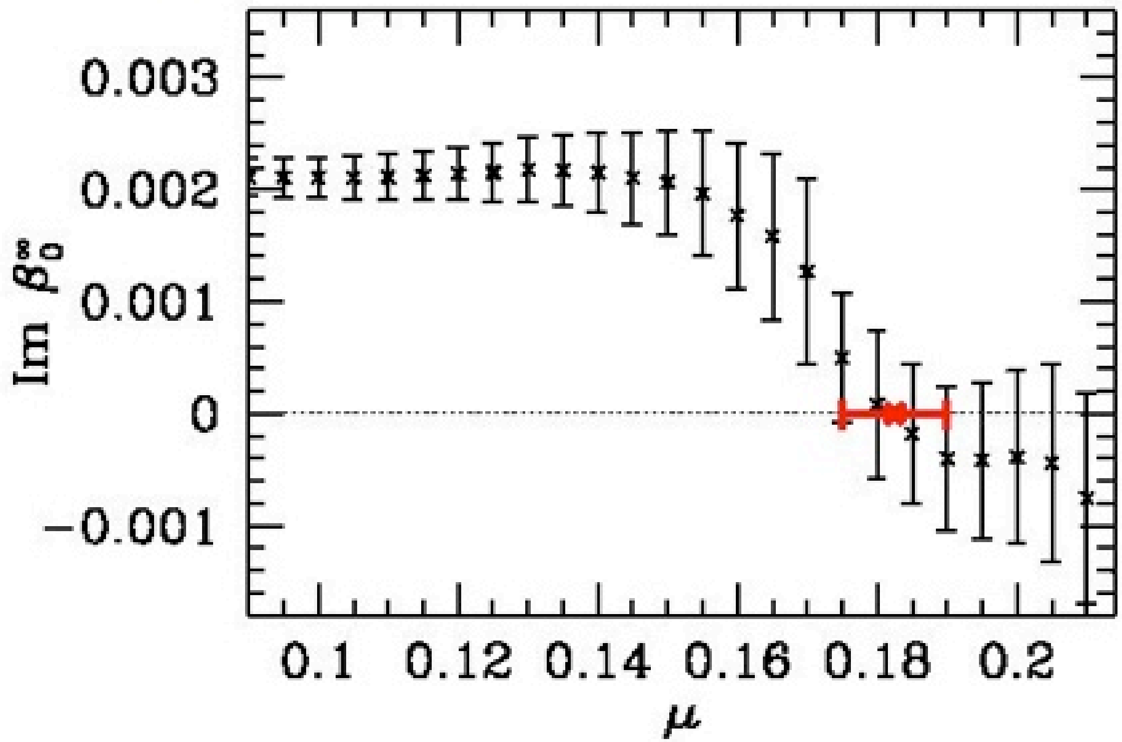
a. determine the transition points, Re(β_0), on L_s=6,8,10,12
β_c as a function of μ by the Lee-Yang zeros
for $\mu \neq 0$ overlap improving multi-parameter reweighting
100k,100k,100k,150k configurations, respectively
every 50th configuration treated as independent (few thousend)

b. by inspecting the $V \to \infty$ limit of Im(β_0)
separate the crossover and the 1^{st} order PT regions in μ

c. connect μ=T=0 lattice parameters with observables:
physical scale by R_0 (1/403 MeV) and m_ρ (770 MeV)
(3×3000 configurations on $12^3 \cdot 24$ lattices)

- separate the crossover and the 1^{st} order PT

$V \to \infty$ limit of Im(β_0) as a function of μ



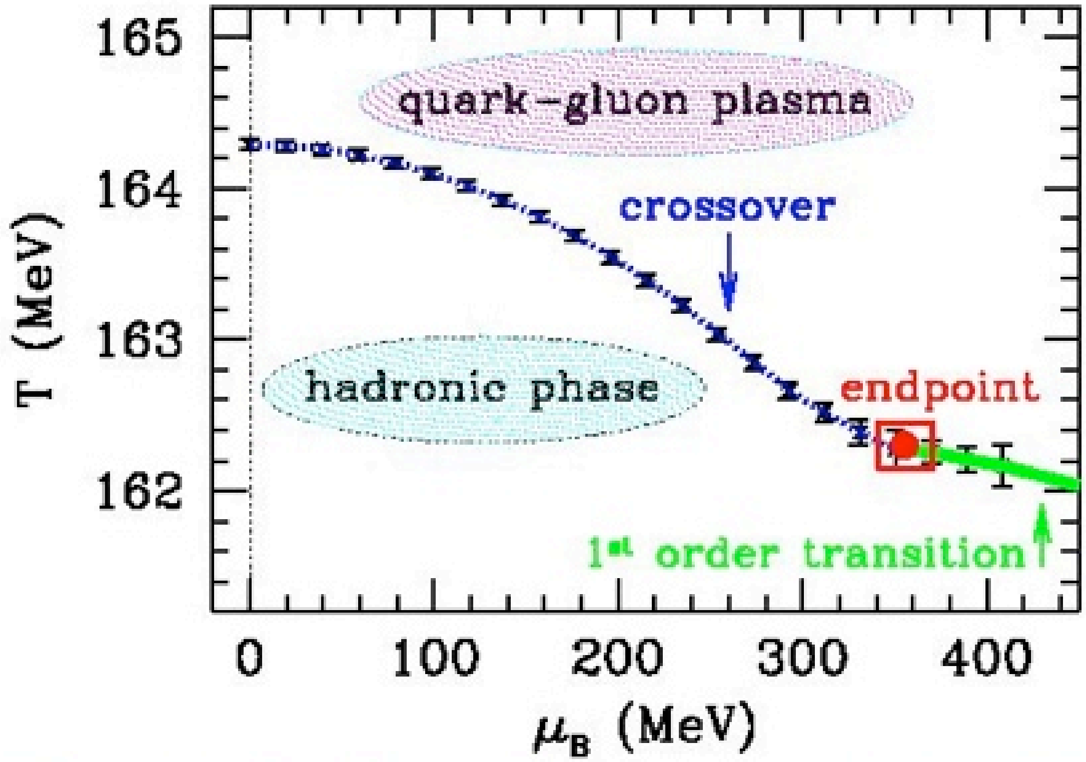
small μ: Im(β_0^∞) inconsistent with 0 $\Rightarrow$ crossover

increasing μ: Im(β_0^∞) decreases $\Rightarrow$

transition becomes consistent with a 1^{st} order PT

endpoint chemical potential: $\mu_{end} = 0.183(8)$

- T as a function of the baryonic chemical potential μ_B



- lattice result for physical quark masses at $L_t = 4$

endpoint: $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV

at μ_B=0 transition temperature: $T_c = 164 \pm 2$ MeV.

$T/T_c = 1 - C\mu_B^2/T_c^2$ wit C=0.0032(1)

Equation of state along the line of constant physics (LCP)

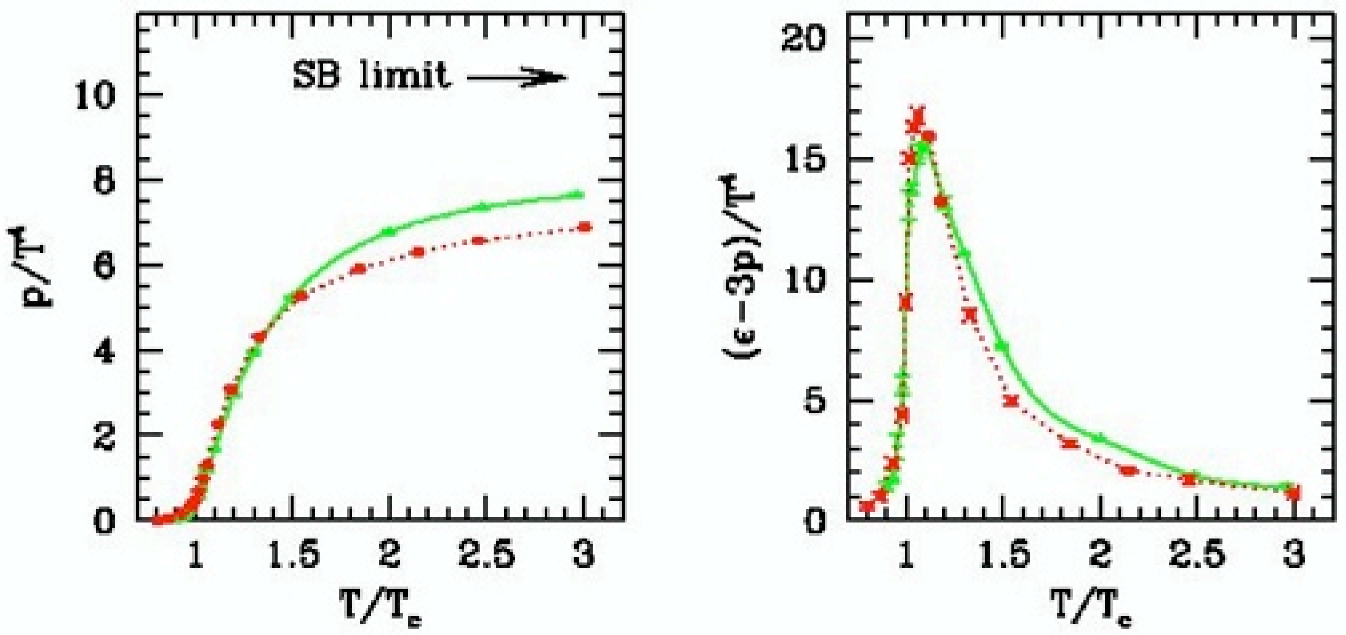
Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B568 (2003) 73

F. Csikor et al. JHEP 05 (2004) 046

the pressure ($p \propto \log[Z]$) along the LCP by the integral method:

$$\frac{p}{T^4} = -L_t^4 \int d(\beta, ma) \left(\frac{\partial(\log Z)}{\partial \beta}, \frac{\partial(\log Z)}{\partial(ma)} \right) =$$

$$-L_t^4 \int d\beta \left[< \mathrm{P} > + m_u \frac{\partial a}{\partial \beta} < \bar{u}u > + m_s \frac{\partial a}{\partial \beta} < \bar{s}s > \right]$$

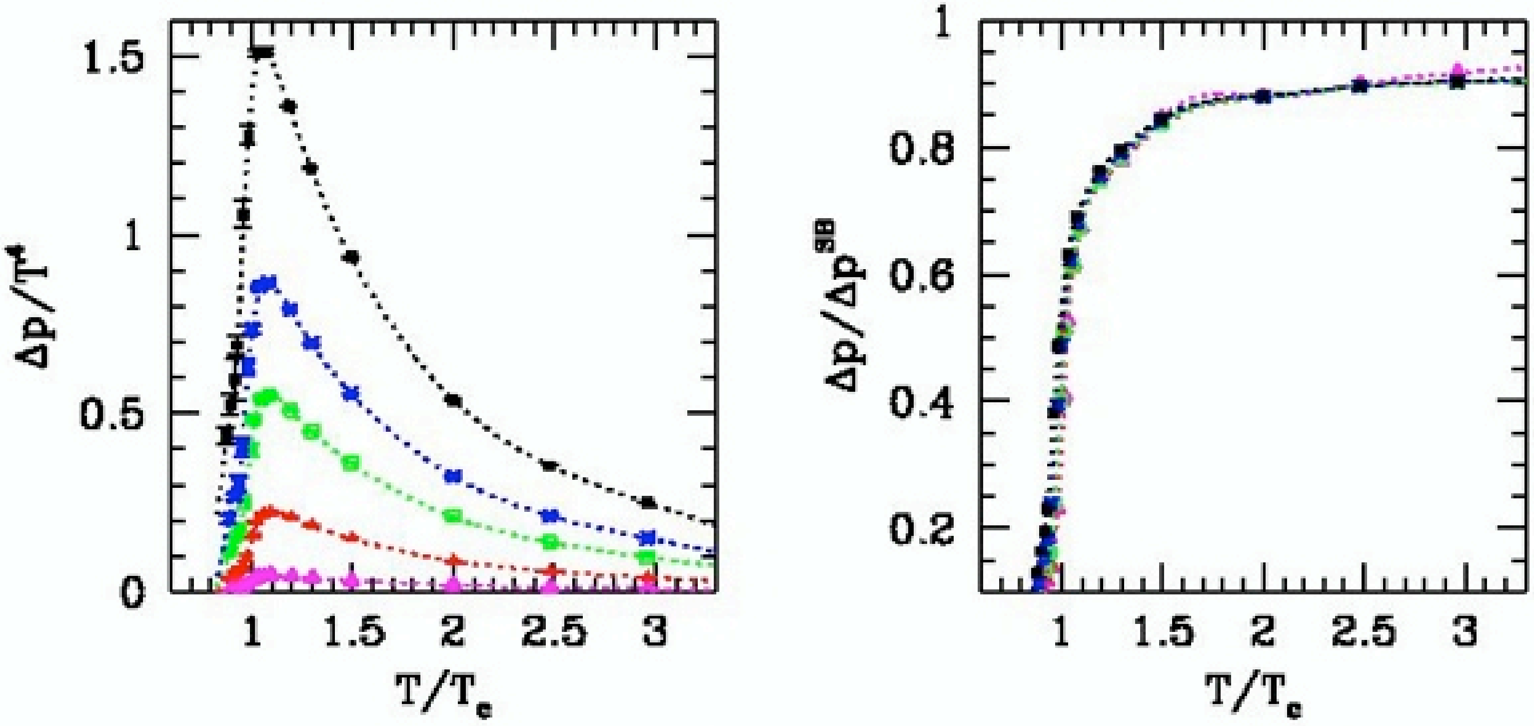


- analogous equations, pressure is given by the integral method

$$-\frac{p}{T^4} = L_t^4 \int d(\beta, ma, \mu a) \left(\frac{\partial(\log Z)}{\partial \beta}, \frac{\partial(\log Z)}{\partial(ma)}, \frac{\partial(\log Z)}{\partial(\mu a)} \right)$$

- equation of state at finite chemical potential upto $\mu_q \approx T_c$

  Δp=p($\mu \neq 0$)−p(μ=0) for μ_B=50,...,500 MeV



almost universal T dependence for the normalised $\Delta p / \Delta p^{SB}$

$\mu \neq 0$ multi-parameter reweighting with Taylor expansion

C.R. Allton et al., Phys. Rev. D66 074507,'02, D68 014507,'03

$$Z(m,\mu,\beta) = \int \mathscr{D}U \exp[-S_g(\beta,U)] \det M(m,\mu,U) =$$

$$\int \mathscr{D}U \exp[-S_g(\beta_0,U)] \det M(m_0,\mu=0,U)$$

$$\left\{ \exp[-S_g(\beta,U)+S_g(\beta_0,U)] \frac{\det M(m,\mu,U)}{\det M(m_0,\mu=0,U)} \right\}$$

instead of evaulating determinants expand them in μ or $\exp(\mu)$:

$$\ln\left(\frac{\det M(\mu)}{\det M(0)}\right) = \sum_{n=1}^{\infty} \frac{\mu^n}{n!} \frac{\partial^n \ln \det M(0)}{\partial \mu^n} \equiv \sum_{n=1}^{\infty} R_n \mu^n$$

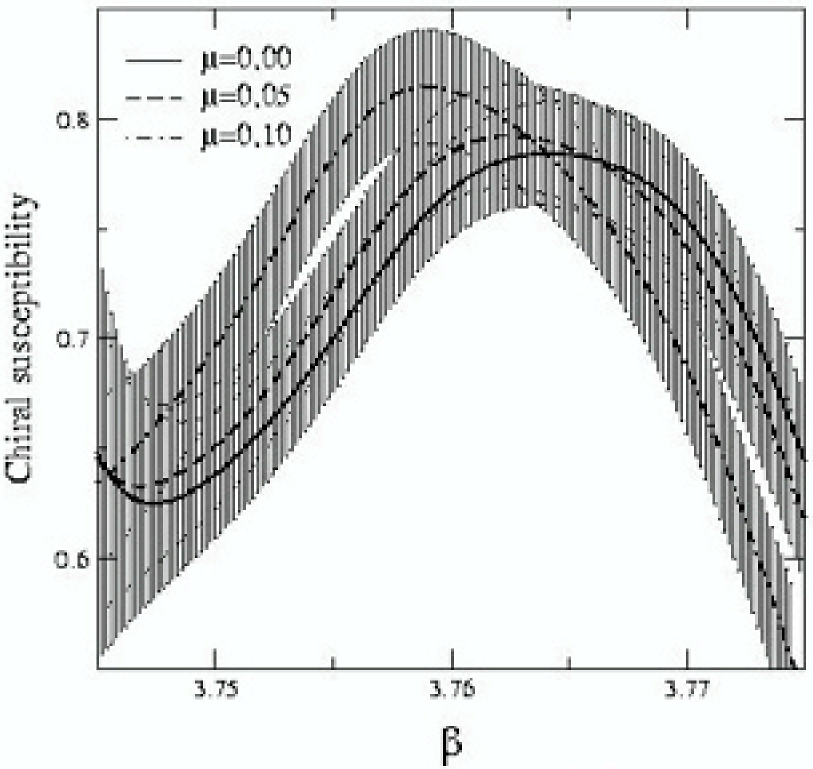
faster than the complete evaluation of the determinants

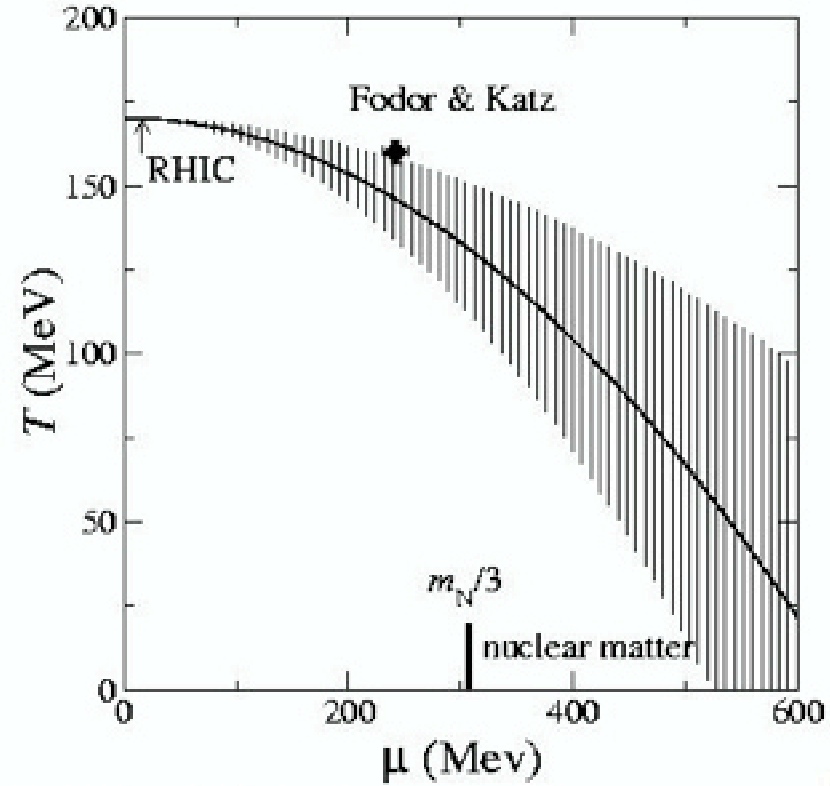
only valid for somewhat smaller μ values than the full technique

• trace out the transition points $\beta_c(\mu)$ in 2 flavour QCD

by looking for the susceptibility peaks of Polyakov or $\langle\bar{\psi}\psi\rangle$

convert it into physical units (T and μ_B in MeV)





⇒ curvature is consistent with other results

presence of higher order terems in the Taylor expansion

⇒ uncertainties at small T and large μ

QCD phase diagram from imaginary chemical potential

P.deForcrand, O.Philipsen, Nucl. Phys. B642 290,'02; B673 170, '03

fermion determinant: real for imaginary chemical potential (μ_I)
$\Rightarrow$ no sign problem, no need for reweighting

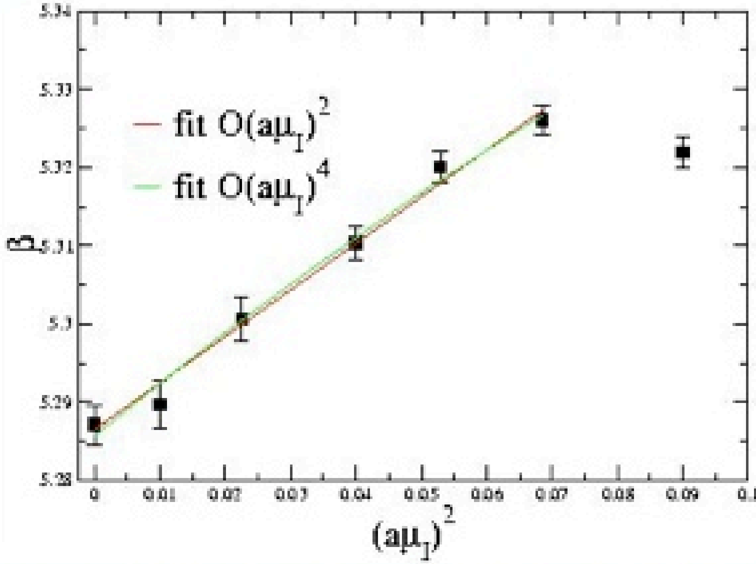
directly obtain the (β_c,μ_I) transition line
analytically continue it to get the physical (β_c,μ) line

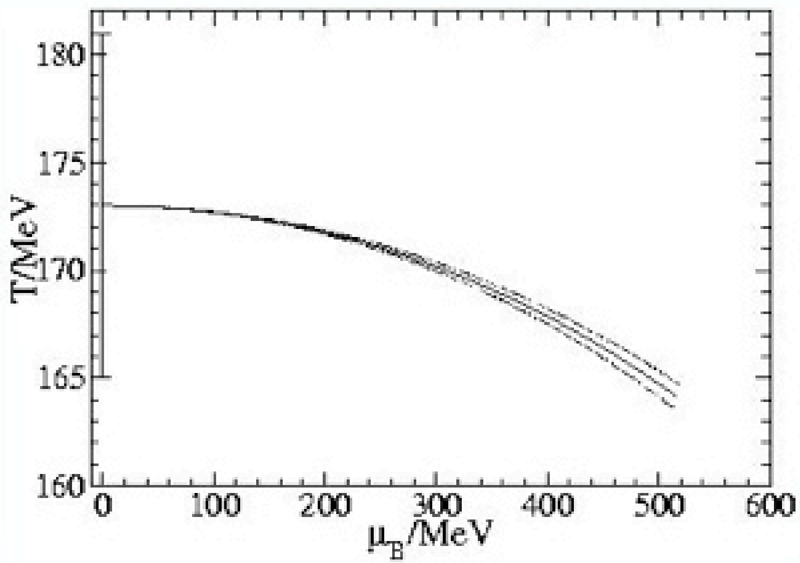
transition line (β_c, μ_I) is given by the susceptibility-peak

$$\chi = VN_t \langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \rangle, \qquad \partial\chi/\partial\beta = 0 \qquad \partial^2\chi/\partial\beta^2 < 0$$

on finite V the analytic $\chi(\mu_I, \beta)$ can be measured
using the implicitely given $\beta_c(\mu_I)$ one gets

$$\partial\beta_c/\partial\mu = -i\partial\beta_c/\partial\mu_I$$





- curvature is consistent with other results

$$T_c(\mu)/T_c(0) = 1 - 0.500(67)(\mu/\pi T_c)^2$$

- mass dependence in n_f=3 QCD for the critical endpoint:

$$m_c(\mu)/m_c(0) = 1 + 0.84(36)(\mu/\pi T_c)^2$$

- the equation of state can be determined, too

Summary, outlook

- critical endpoint in the μ-T plane: unambiguous, non-perturbative prediction of the QCD Lagrangian $\Rightarrow$ important experimental consequences for heavy ion collisions

- lattice QCD at finite μ is an old, unsolved problem
recent method: overlap improving multi-parameter reweighting
presumably good enough to locate the above endpoint

- overlap improving multi-parameter reweighting:
standard importance sampling with reweighting in β, m and μ
maps transition ensemble to a transition ensemble
(or hadronic/QGP ones to hadronic/QGP ones)

- can be applied to any number of Wilson or staggered quarks

• T=0 and T≠0 simulations in QCD with n_f=2+1 quarks
infinite volume behavior of the Lee-Yang zeros
tells the difference between a 1^{st} order PT and a crossover

physical quark masses on L_t=4 lattices:

endpoint: $T_E = 162 \pm 2$ MeV, $\mu_E = 360 \pm 40$ MeV

at μ_B=0 transition temperature: $T_c = 164 \pm 2$ MeV.

• equation of state is obtained at finite temperature
(T=0.8 ... 3·T_c) and chemical potential (μ_B=0...500 MeV)

• several other new ideas and techniques:
Taylor expansion in the chemical potential
analytic continuation from imaginary chemical potential