Firenze 26-30 Mai 2014
Advances in Nonequilibrium Statistical Mechanics

Jarzynski Equality and the Landauer's bound: an experimental approach

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Nature 483, 187-189 (2012)

2013 *EPL* **103** 60002 ; arXiv:1302.4417 ;
Detailed Jarzynski Equality applied on a Logically Irreversible Procedure

About Stefano

Workshop on Chaos and Complexity

1988, Villa Gualino

[R. Livi]; [S. Ciliberto]; [S. Ruffo]

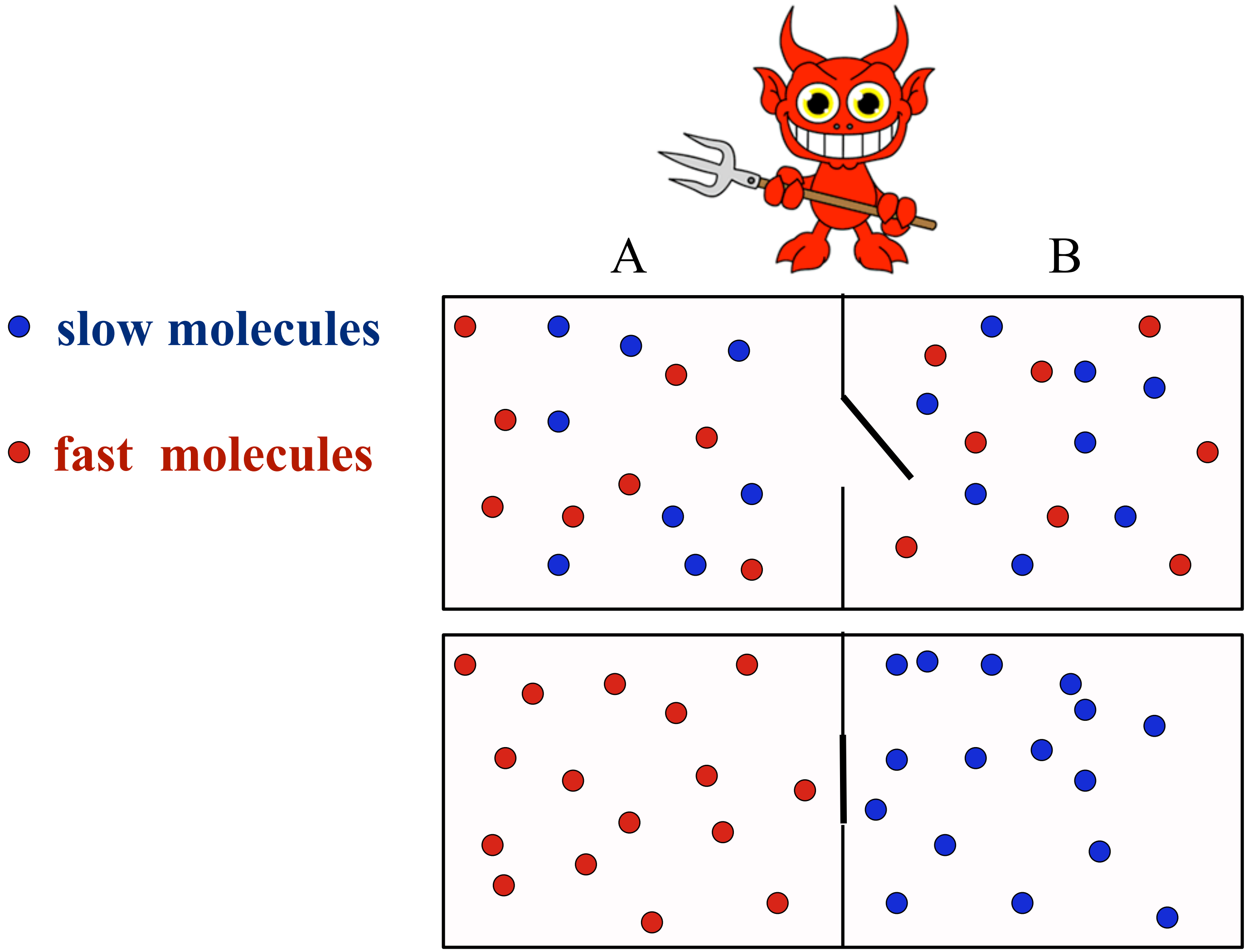
A cellular automaton model of a fluid experiment,
F. Bagnoli, Francescato, S. Ciliberto, R.Livi, S.Ruffo

Phase transitions in convection experiments,
F. Bagnoli, S. Ciliberto, R. Livi and S. Ruffo ,
Les Houches (Marzo 1989). "Relaxation in complex systems" (Plenum 1990).

Outline

- Landauer's principle
- How to realise it ?
- Experimental set-up
- Data analysis
- Comparison with numerical results
- Landauer's limit and the Jarzynski equality
- Conclusions

Landauer's Principle and The Maxwell's Demon



The Landauer's principle (I)

Any logically irreversible transformation of classical information is necessarily accompanied by the dissipation of at least $k_B T \cdot \ln 2$ of heat per lost bit (about $3 \cdot 10^{-21}$ Joules at room temperature)

Typical examples of logically irreversible transformations are Boolean functions such as AND, NAND, OR and NOR

They map several input states onto the same output state

The erasure of information, the RESET TO ONE operation, is logically irreversible and leads to an entropy production of $k_B \cdot \ln 2$ per erased bit

Landauer's principle II

Landauer's principle is a central result which exorcises the Maxwell's demon

It has been criticised and never tested in a real experiment

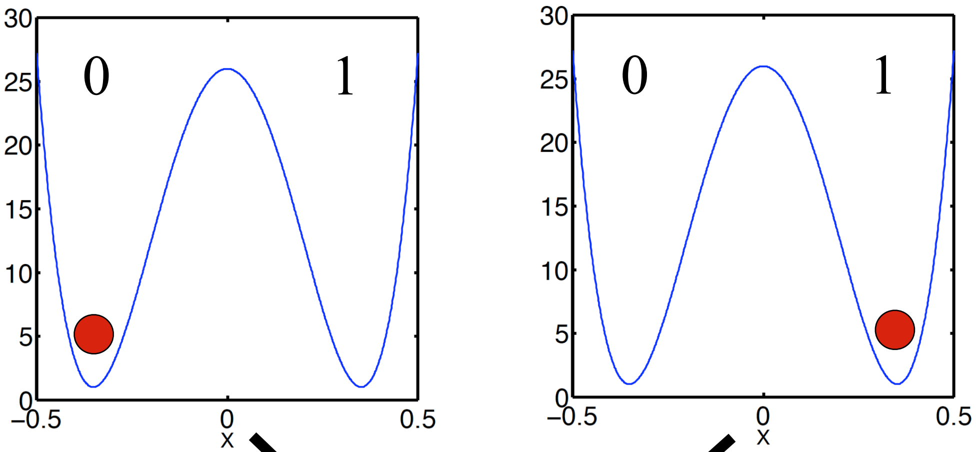
Questions

- Can the Landauer's limit be reached in any experiment ?
- Does any experimentally feasable procedure allow us to reach the limit ?

Following Bennett we use in our experiment the RESET to ONE operation

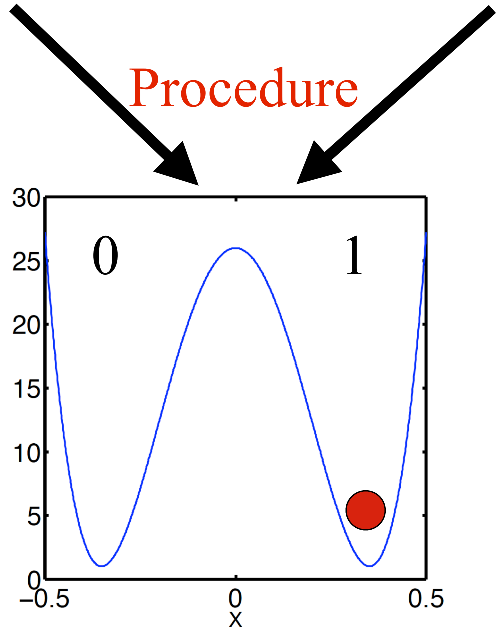
Bennett, C. H. The thermodynamics of computation, a review. Int. J. Theor. Phys. 21, 905-940 (1982).

The Bennett's erasure procedure



Initial state is 0 or 1 with equal probability 1/2

$S_i = k_B \cdot \ln 2$



Final state is 1 with probability 1

$S_f = 0$

Thus $\Delta S_{min} = -k_B \cdot \ln 2$

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Procedure

Quasi Static : $-T\Delta S=Q$

Energy variation : $\Delta U=0$

First principle : $\Delta U=-Q+W$

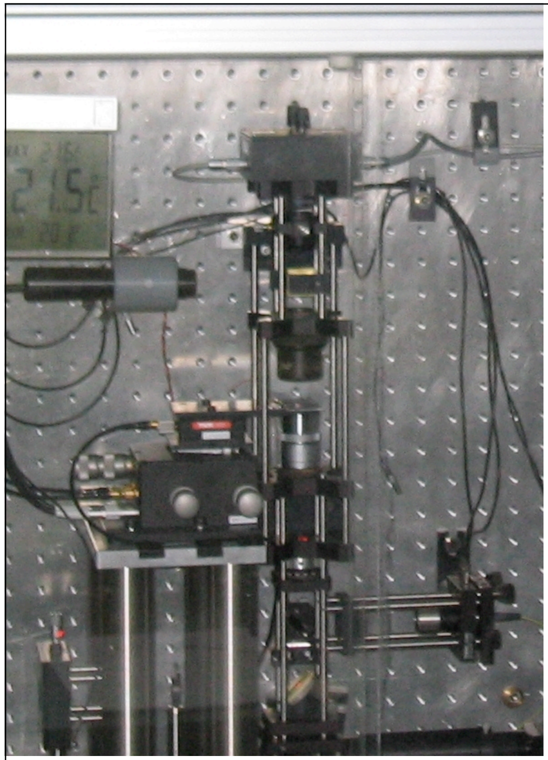
$$\Downarrow$$

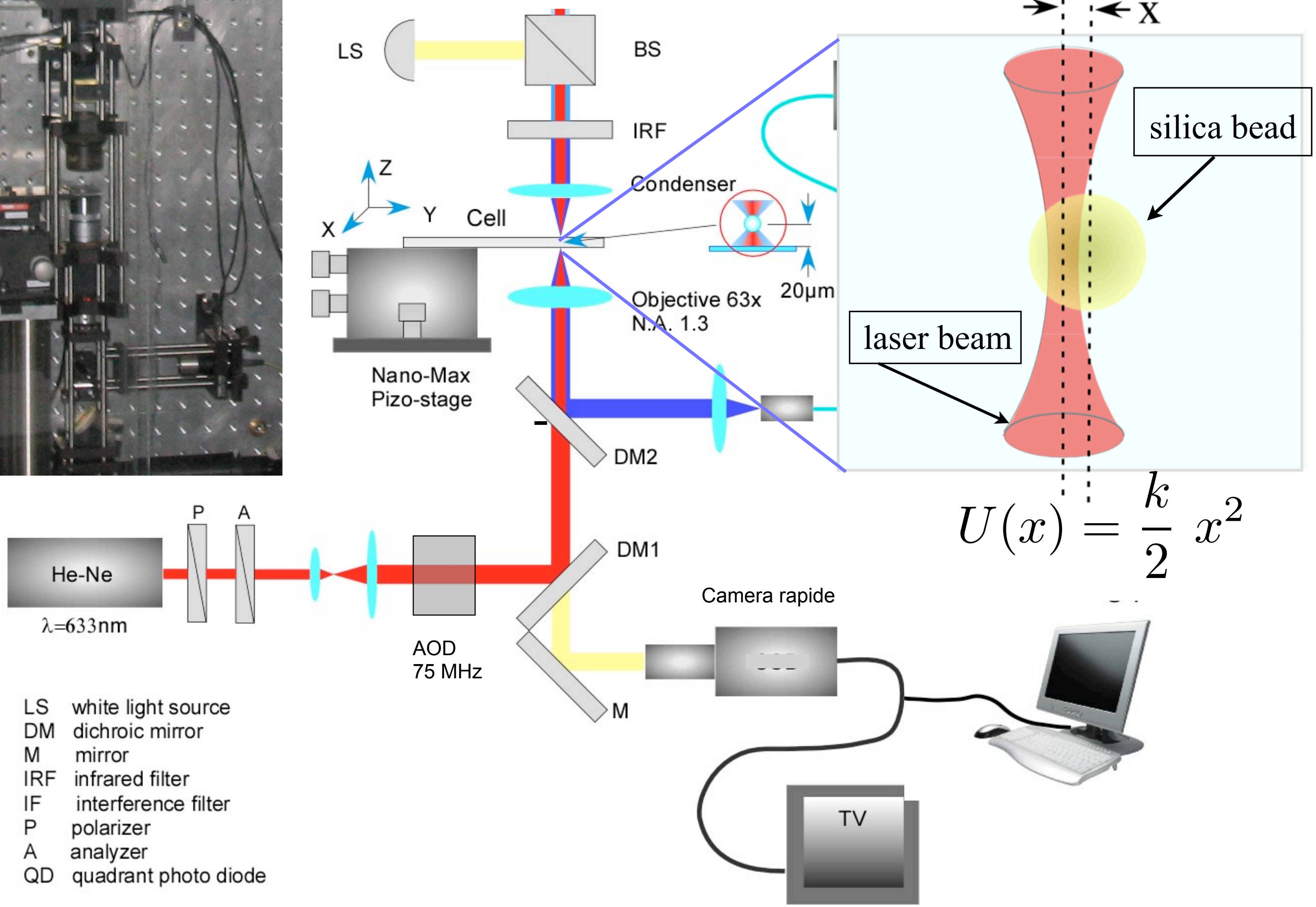
In average : $\langle W\rangle = \langle Q\rangle = -T\,\Delta S \geq k_B T\ln(2)$

Numerical result :

Memory Erasure in Small Systems,
R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

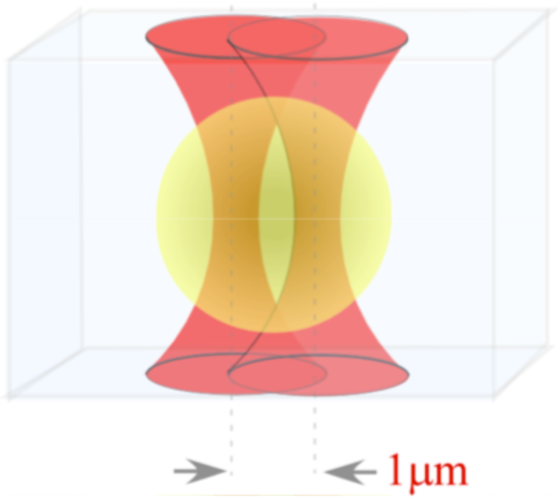
Experimental set-up Optical trap

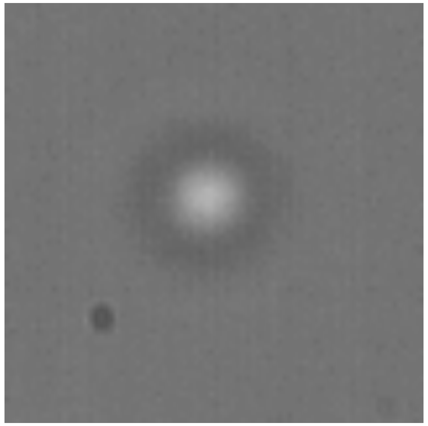


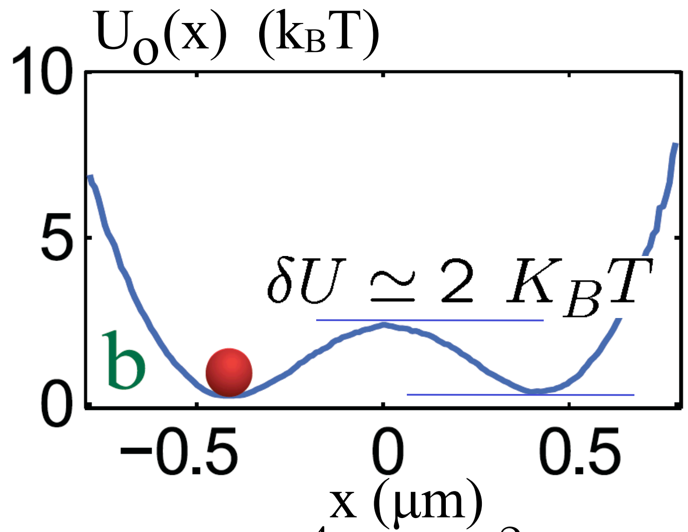


$$U(x) = \frac{k}{2}\ x^2$$

Brownian particle trapped by two laser beams







$$U_o(x) = a\ x^4 - b\ x^2\ +\ d\ x$$

The Kramers time

$$\tau_K = \tau_o\ \exp[\frac{\delta U}{k_B T}]$$

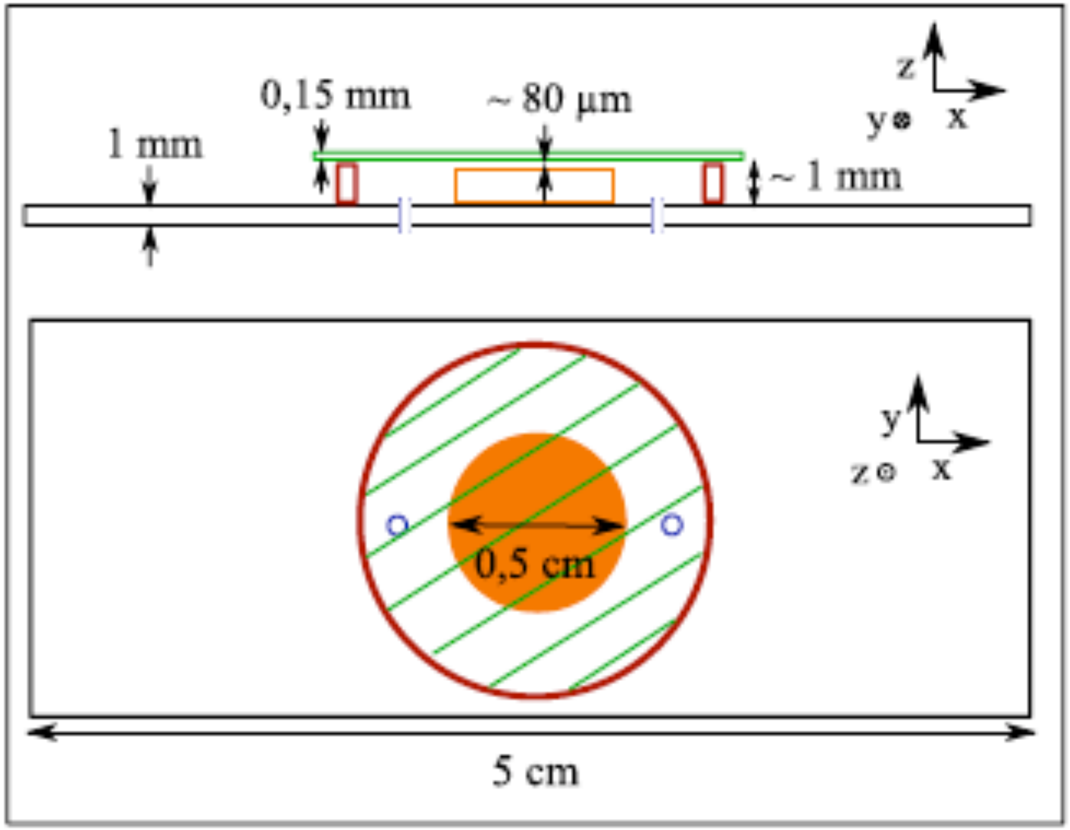
with $\tau_o = 1\ s$

Potential measured using detailed balance

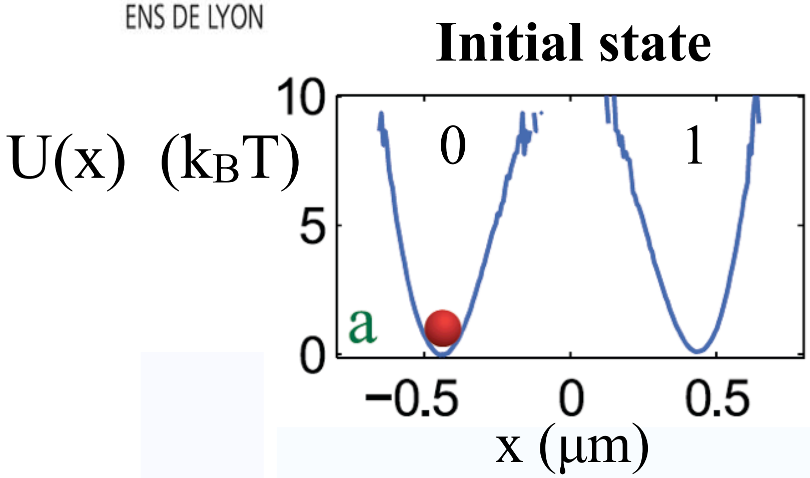
$$\frac{\omega_{i \to j}}{\omega_{j \to i}} = e^{-\frac{\Delta U_{ij}}{k_B T}}$$

with $\Delta U_{i,j} = U(x_i) - U(x_j)$

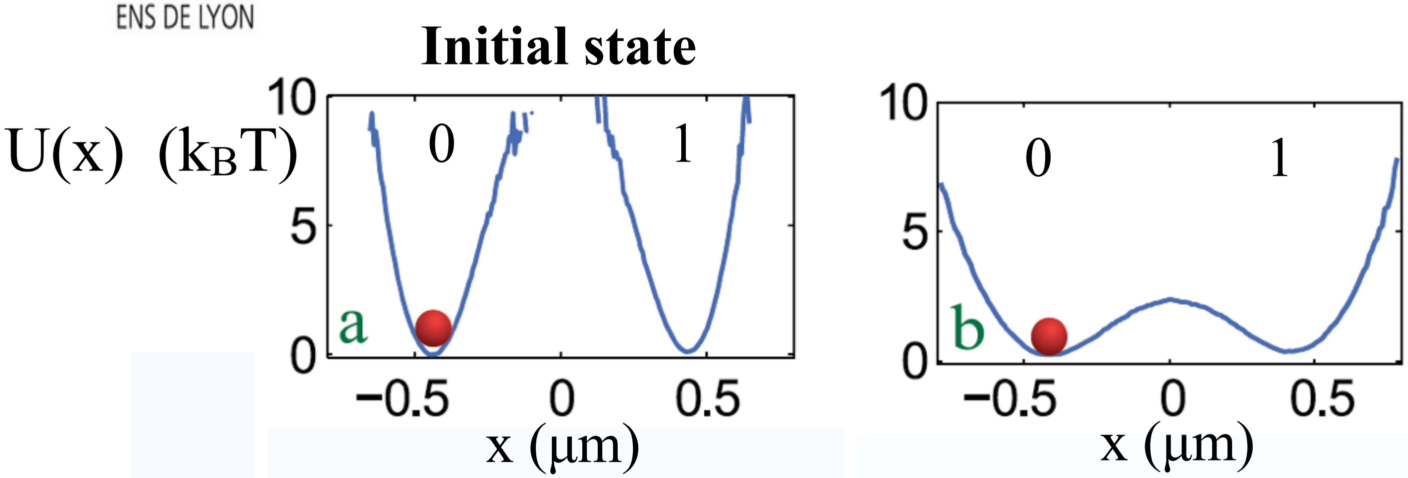
The cell for the bead



The Erasure Procedure

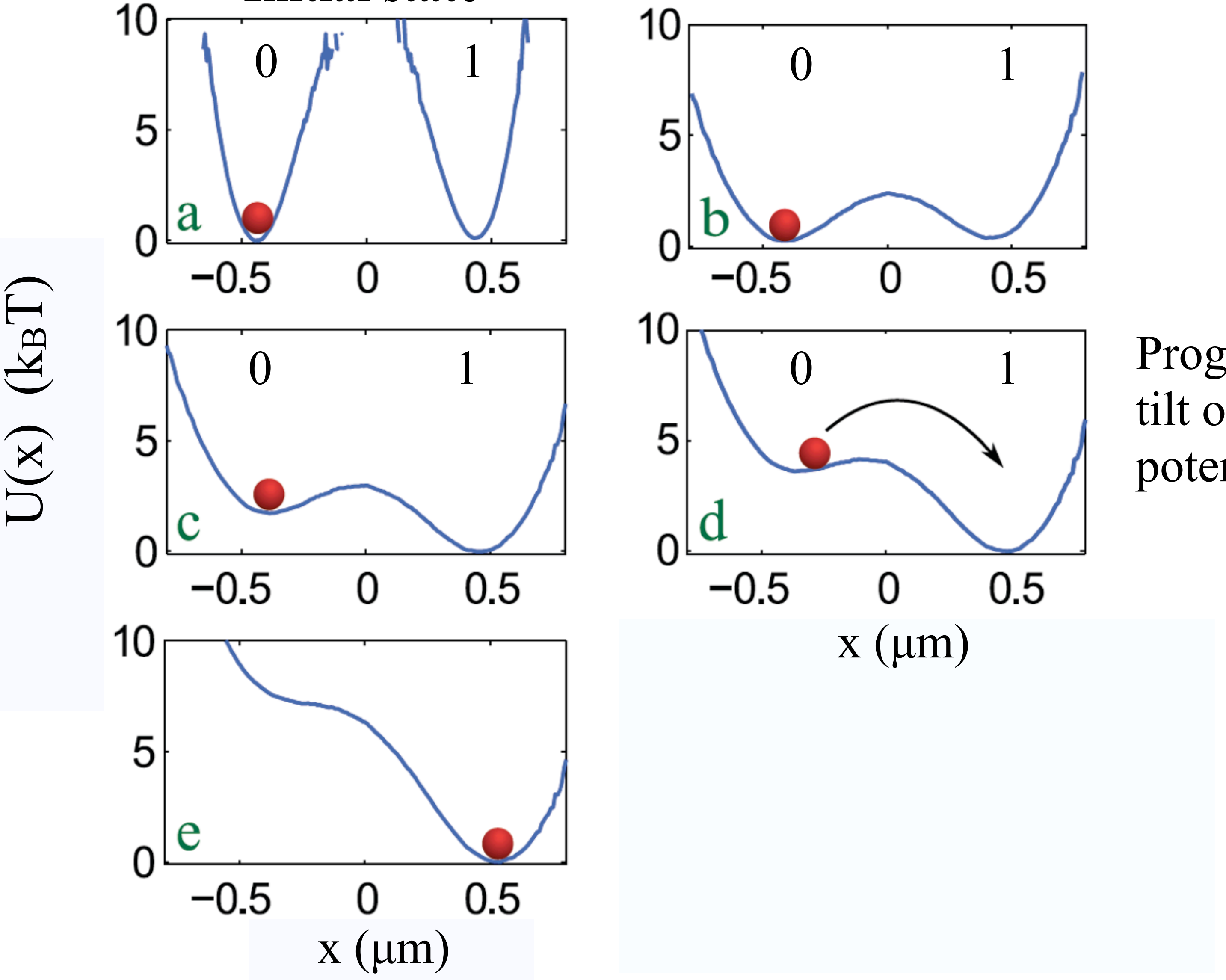


The Erasure Procedure



reduction of the barrier

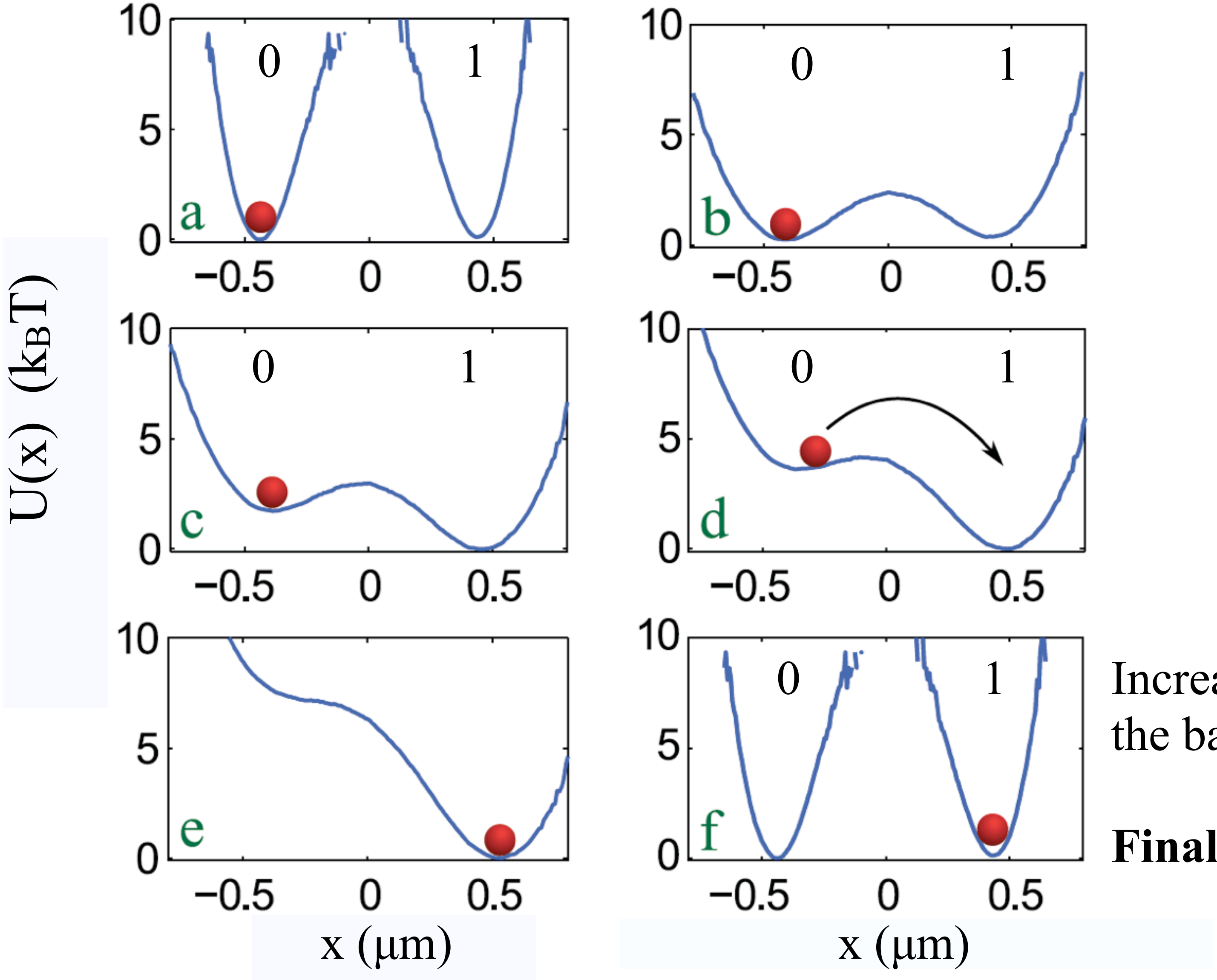
The Erasure Procedure



Progressive tilt of the potential

The Erasure Procedure

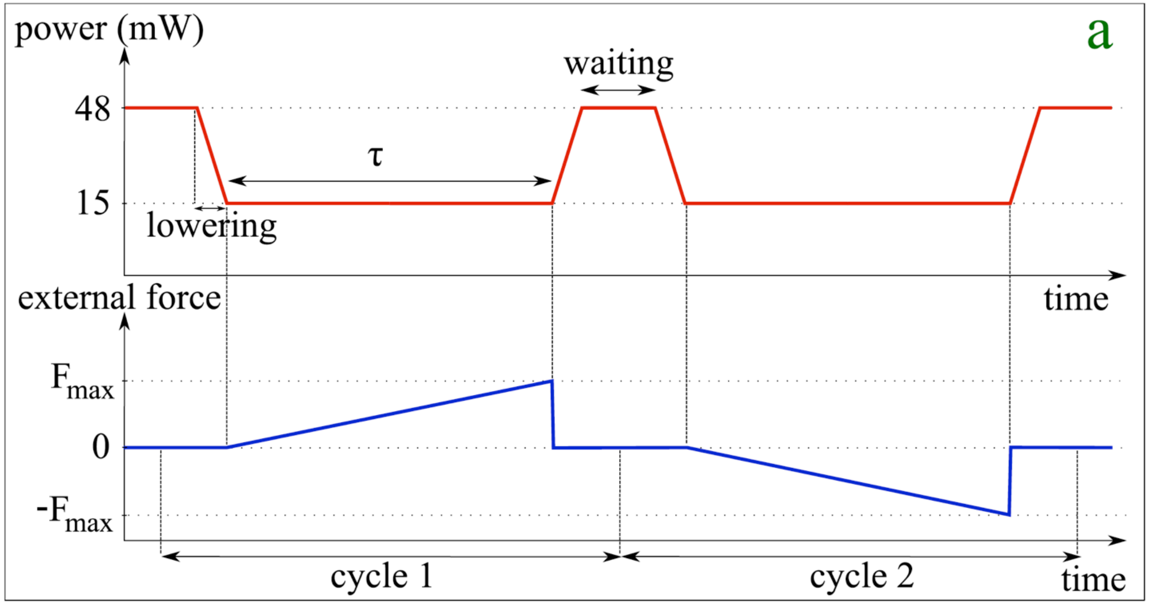
Initial state



Increasing of the barrier

Final state

Potential external control as a function of time



The laser intensity controls the barrier height

The potential tilt is produceed by a linearly increasing external force f, applied on the time τ.

$$\tau_{cycle} = \tau + 2\ \mathrm{s}$$

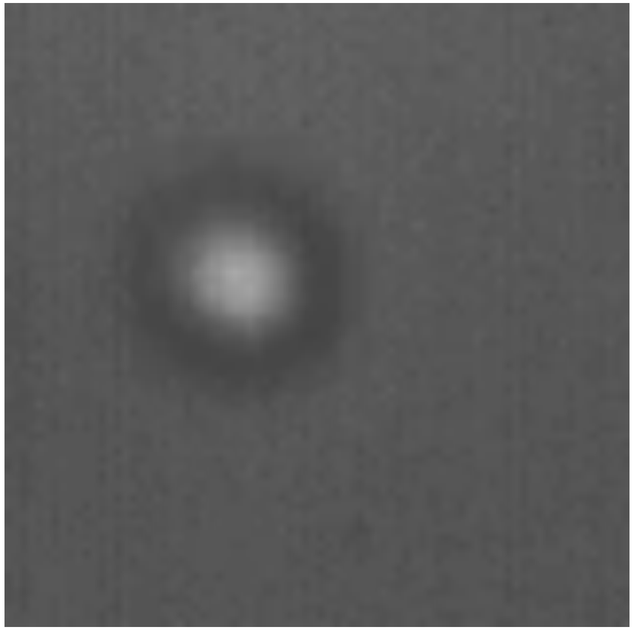
The force f is created by displacing the cell with respect to the laser, thus

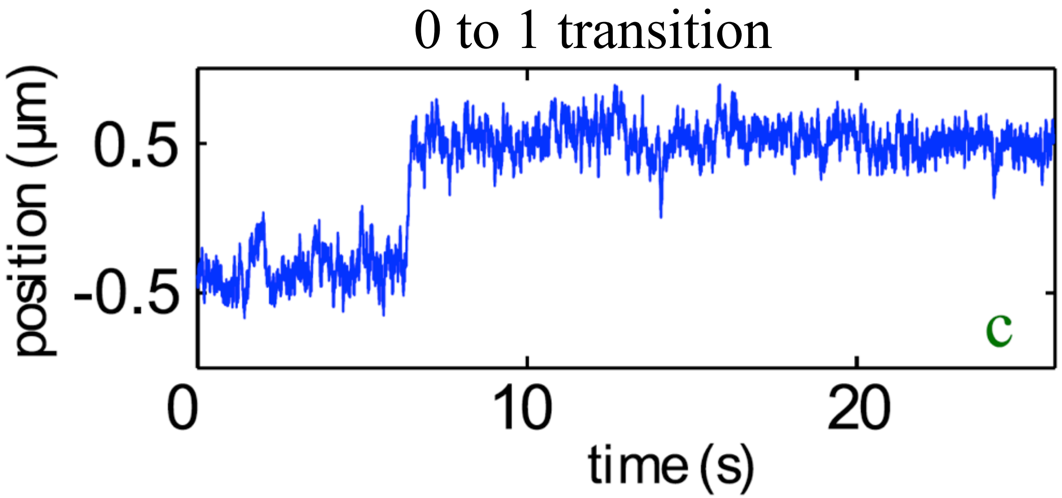
$$f = -\nu\ v \qquad \text{with} \qquad \nu = 6\pi R\mu$$

Two control parameters: τ the time of application of f

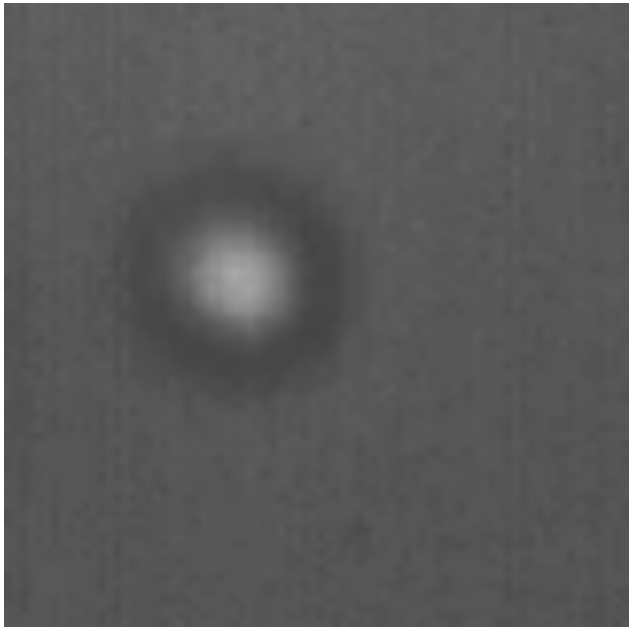
F_{max} the maximum applied force

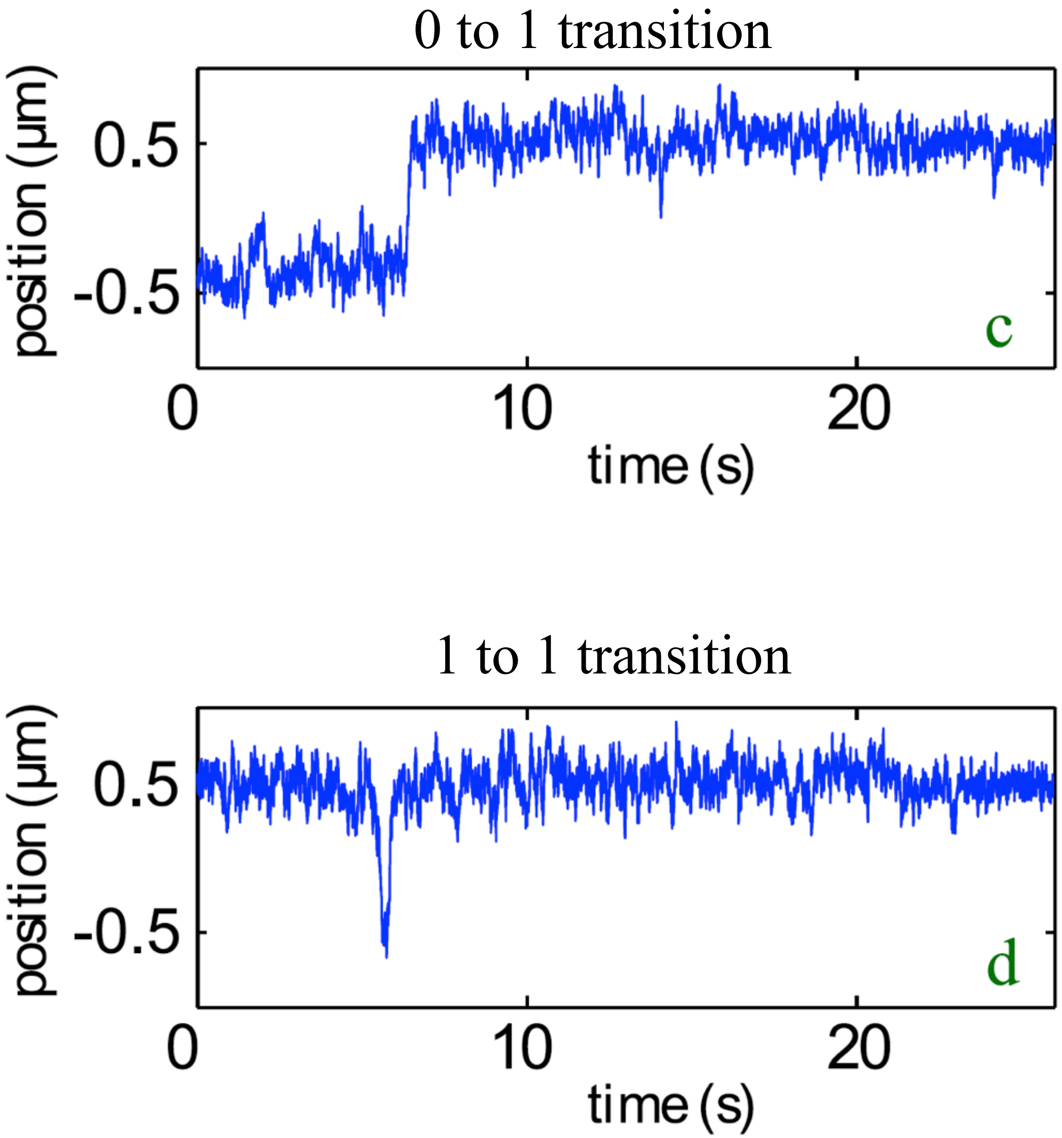
Bead trajectories





Bead trajectories





The work on the erasure cycle

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + f(t) + \eta$$

multiplying by ẋ and integrating for a time τ we get :

$$\Delta U_\tau = W_\tau - Q_\tau$$

Stochastic thermodynamics

$$\Delta U_\tau = -\int_0^\tau \frac{\partial U_o}{\partial x}\ \dot{x}\ dt \qquad W_\tau = \int_0^\tau f\ \dot{x}\ dt$$

$$Q_\tau = \int_0^\tau \nu \dot{x}^2\ dt - \int_0^\tau \eta \dot{x}\ dt$$

Sekimoto K, Progress of Theoretical Phys. supplement (130), 17 (1998).

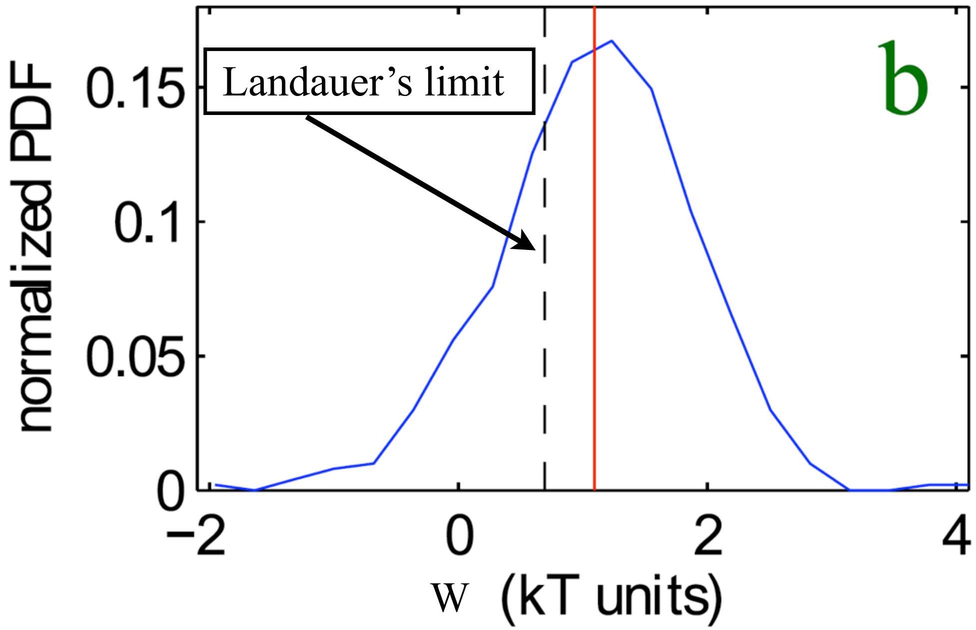
The work on the erasure cycle

The two erasure cycles have been considered

$$\begin{matrix} 0 \searrow & \\ & 1 \\ 1 \nearrow & \end{matrix} \qquad \text{and} \qquad \begin{matrix} 0 \searrow & \\ & 0 \\ 1 \nearrow & \end{matrix}$$

$$W_F = -\int_0^{\tau_{cycle}} \nu \; v(t) \dot{x} \; dt = \int_0^{\tau_{cycle}} F_{max} \frac{t}{\tau} \dot{x} \; dt$$

$$\xcancel{\Delta U_\tau = -\int_0^{\tau_{cycle}} \frac{\partial U_o}{\partial x} \; \dot{x} \; dt}$$

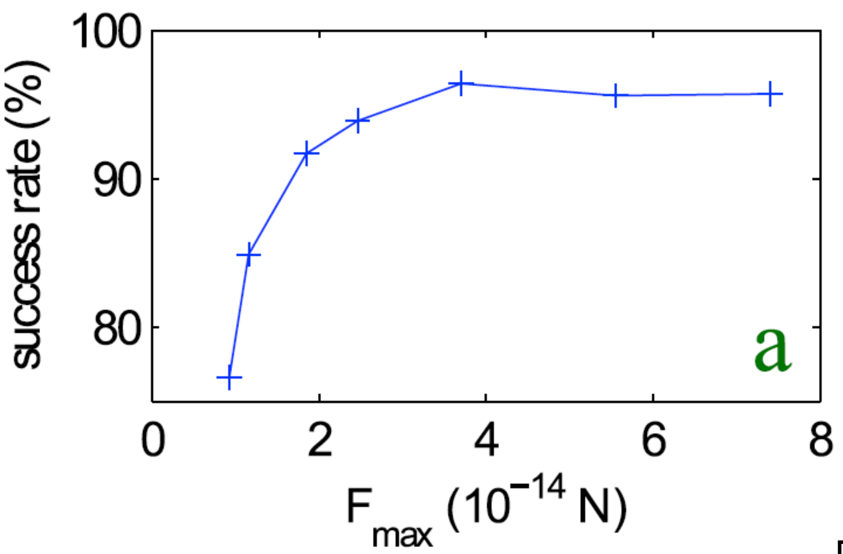


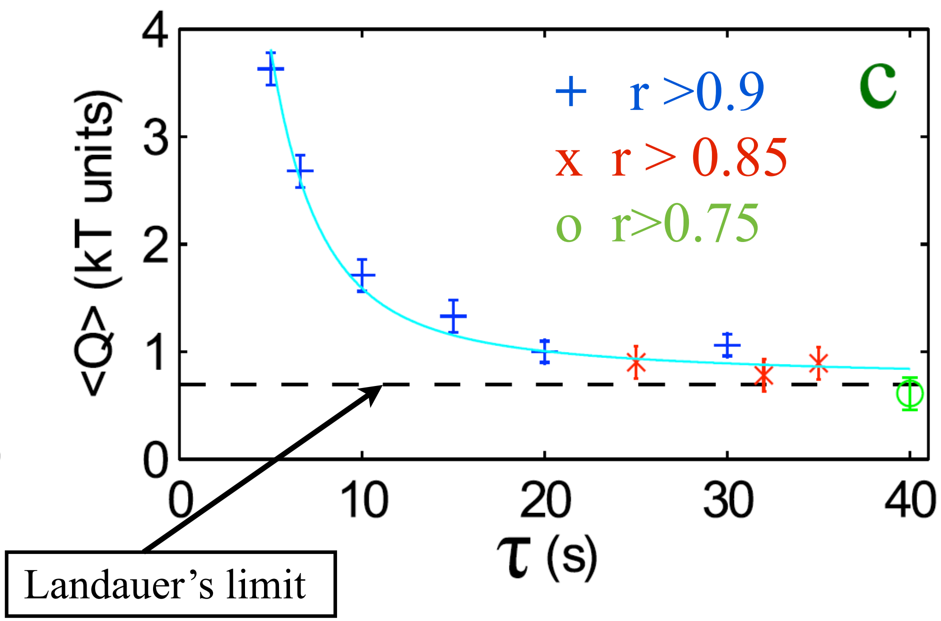
Results of the erasure procedure

$$\text{Success rate } r = \frac{\text{number of successful cycles}}{\text{Total number of cycle}}$$

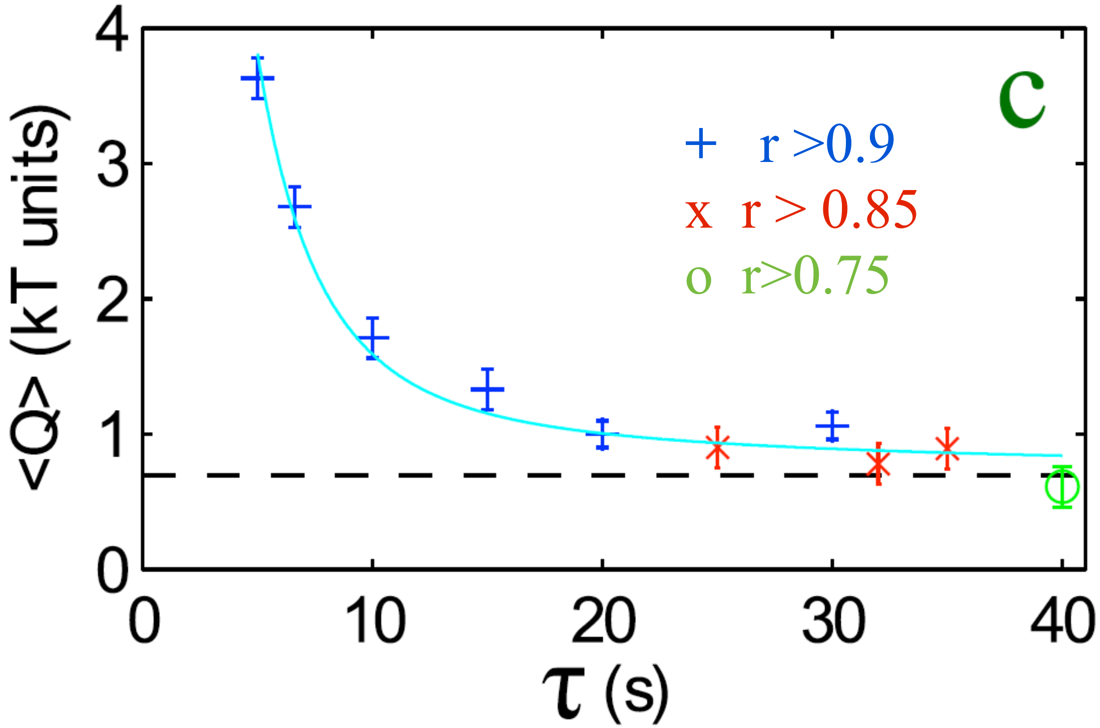
Qualitative observations :

- At constant τ : W and r increase with F_{max}
- At constant F_{max} : W decreases and r increases for increasing τ





Landauer's limit



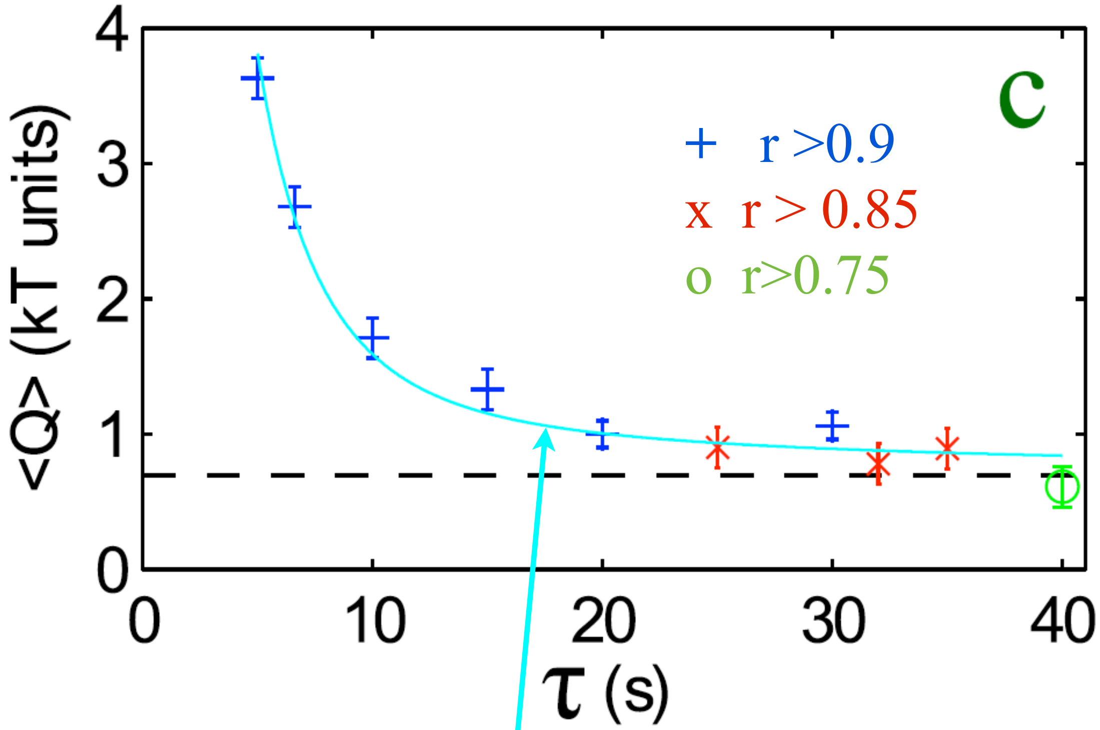
Landauer's limit as a function of r

$$< Q >^r_{\text{Landauer}} = kT[\ln 2 + r \ln r + (1-r)\ln(1-r)]$$

At r=0.5 $$< Q >^r_{\text{Landauer}} = 0$$

Indeed the Erasure Procedure left the initial state unchanged

Landauer's limit



Asymptotic behaviour

$$\tau \to \infty$$

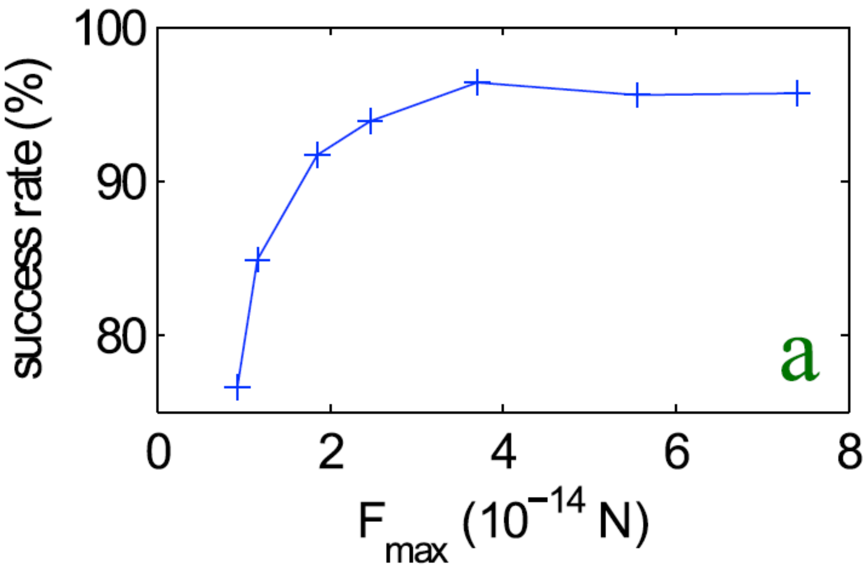
Sekimoto -Sasa J. Phys. Soc. Jpn. 66, 3326 (1997).

$$< W > \simeq \Delta F + B/\tau$$

As $< \Delta U >= 0$ then $\Delta F = -T \Delta S$ and $< Q >=< W > \simeq kT \ln 2 + B/\tau$

$$\boxed{< Q >=< Q >_{\rm Landauer} + B/\tau}$$

The success rate r

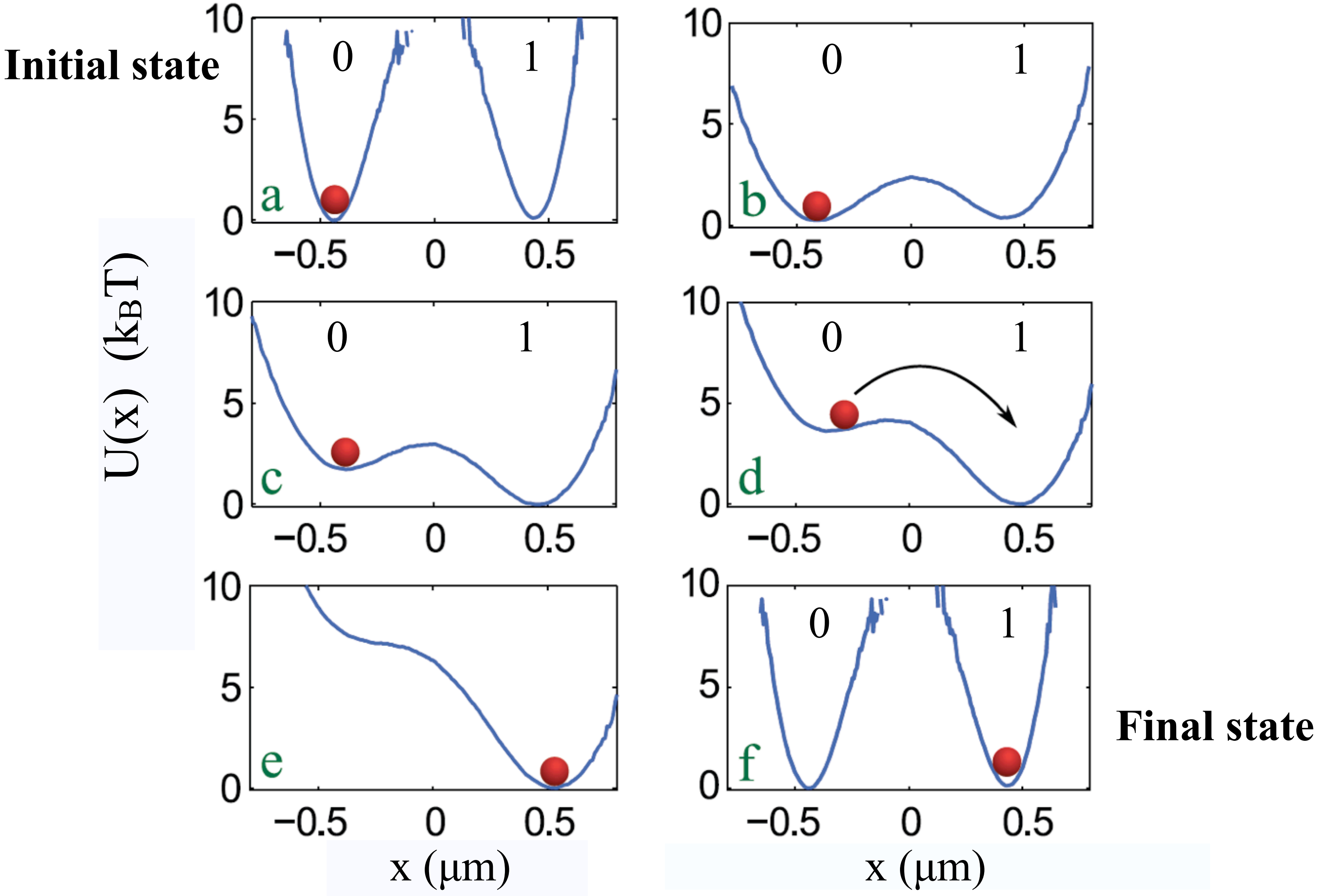


Why in the experiment r< 1 ?

Is this result produced by 3D effects of the trap ?

Is the finite height of the initial barrier responsible of r<1 ?

The Erasure Procedure



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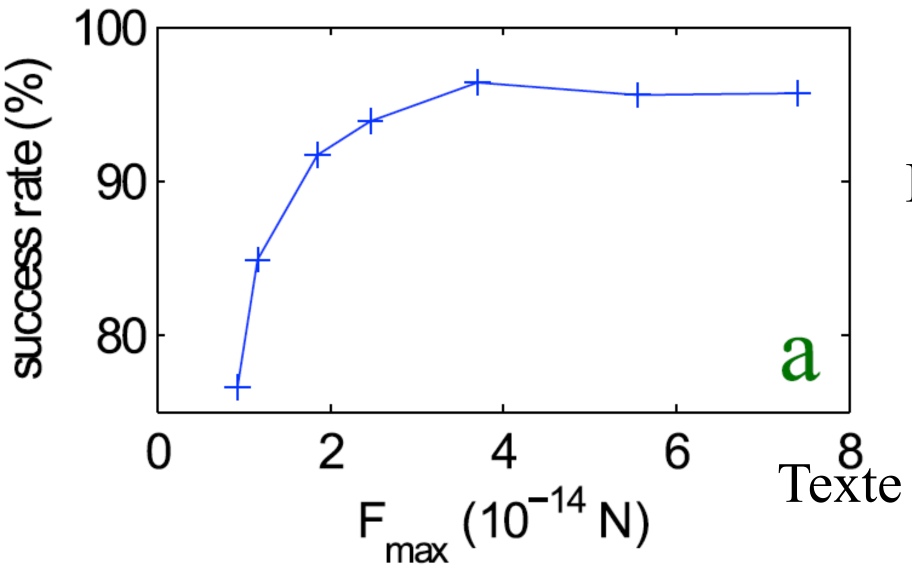
The success rate r



Why in the experiment r< 1 ?

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Is the finite height of the initial barrier responsible of r<1 ?

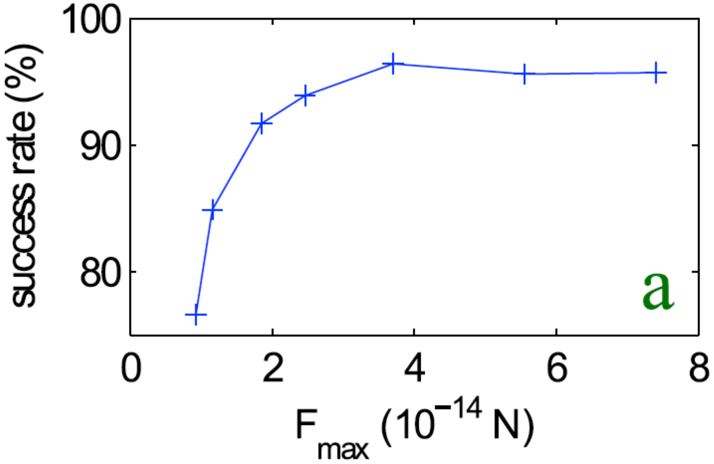
Numerical test

$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + \eta$$

We use all the experimental parameters and procedure

with two different initial barriers $8k_BT$ and $15k_BT$

The success rate r



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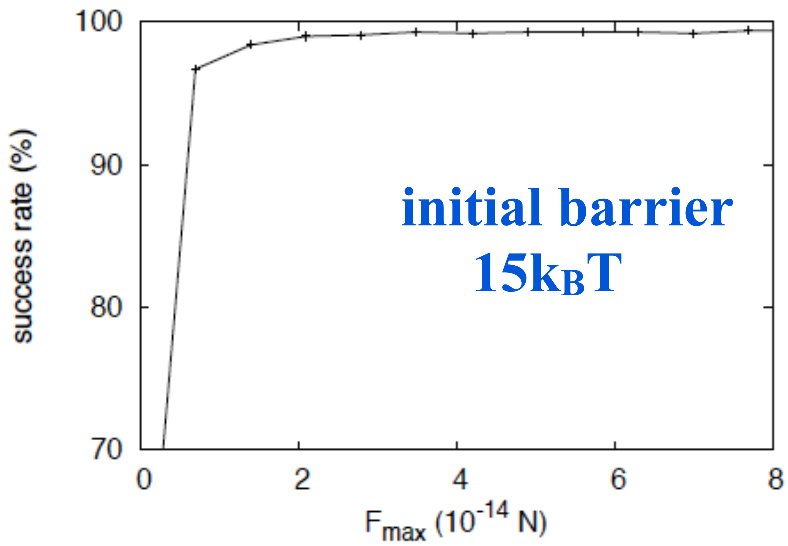
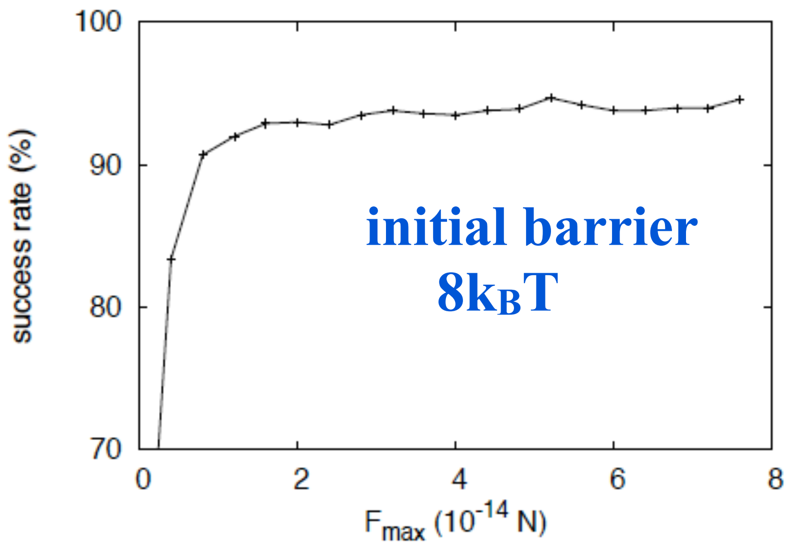
$$\nu \dot{x} = -\frac{\partial U_o(x,t)}{\partial x} + \eta$$

Numerical test





Conclusions (partials)

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

Question: Does Jarzynski equality compute the right ΔF?

Landauer's limit and the Jarzynski equality

$$< \exp(-W_s) >= \exp(-\Delta F)$$

with $$W_s = -\int_o^{\tau_{cycle}} \dot{\lambda} \frac{\partial H(x, \lambda)}{\partial \lambda} dt$$

In our case this equality transforms

$$W_s = \int_o^{\tau} \dot{f} \; x \; dt = [f \; x]_o^{\tau} - \int_o^{\tau} f \; \dot{x} \; dt = -W_f$$

Since the height of the barrier is always finite there is no change in the ***equilibrium*** **F** of the system between **the beginning and the end of the procedure**.

$$< \exp(-W_s) >= \frac{\rho_{eq}(\tau)}{\rho(\tau)} \exp(-\Delta F)$$

S. Vaikuntanathan and C. Jarzynski, Euro. Phys. Lett. 87, 60005 (2009).

Generalized Jarzynski

Landauer's limit and the Jarzynski equality

We consider the erasure procedure
$$\begin{matrix} 0 \searrow & \\ & 0 \\ 1 \nearrow & \end{matrix}$$

If the final state is 0 then $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

and the Generalized Jarzynski is : $< \exp(-W_s) >_{\to 0} = \dfrac{1/2}{r}$

from Jensen inequality $< W_s >_{\to 0} \geq (\ln 2 \; + \; \ln r)$

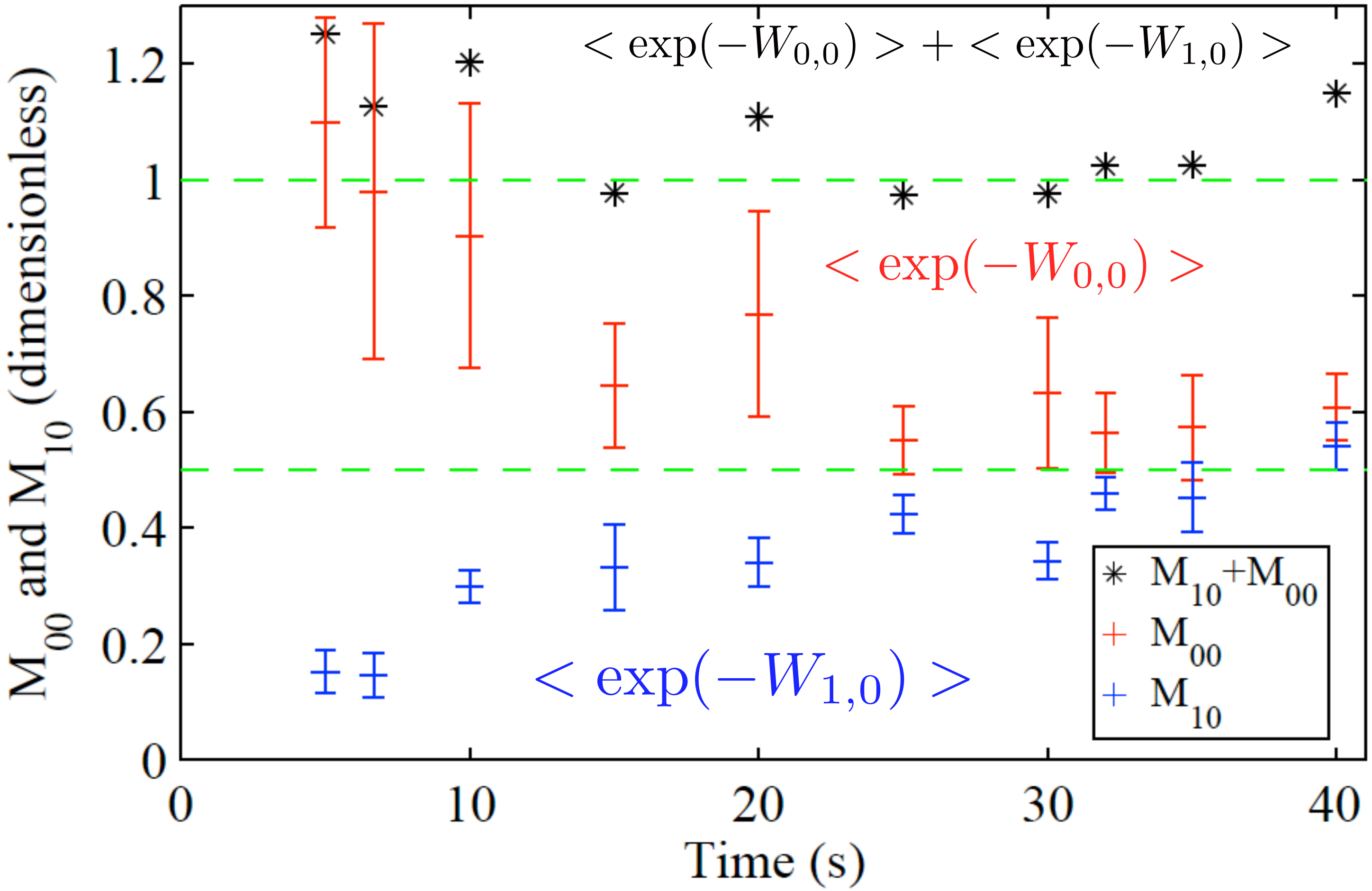
$$\frac{1}{2} < \exp(-W_{1,0}) > + \frac{1}{2} < \exp(-W_{0,0}) > = \frac{1}{2}$$

Work done if the particle makes the jump from 1 to 0

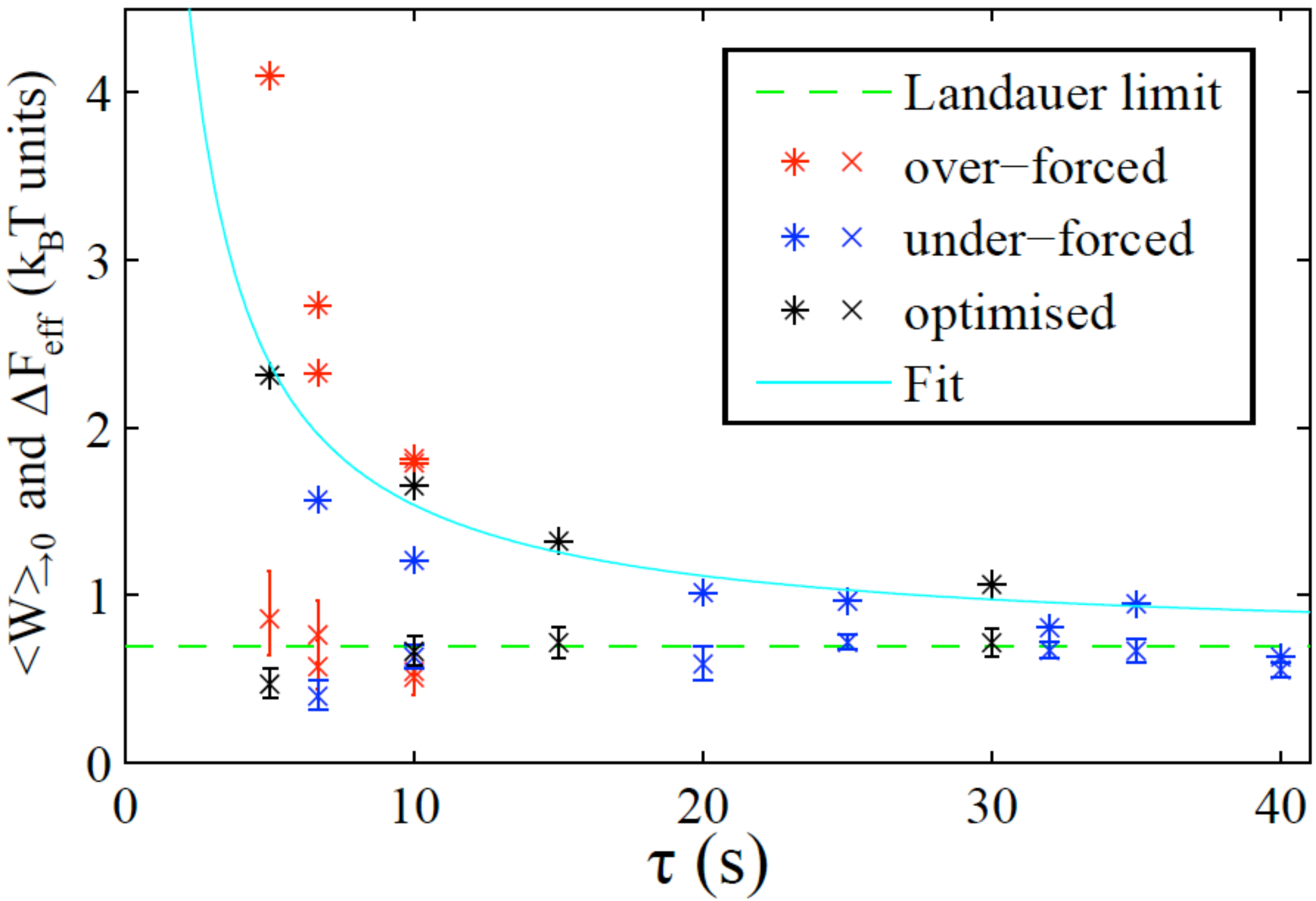
Work done when the particle starts in the final state

Landauer's limit and the Jarzynski equality

$$-\ln\left(\frac{<\exp(-W_{0,0})> + <\exp(-W_{1,0})>}{2}\right) = \Delta F_{eff}$$



Landauer's limit and the Jarzynski equality



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Landauer's limit and the Jarzynski equality

We consider the erasure procedure $\begin{matrix} 0 \searrow \\ \\ 1 \nearrow \end{matrix} 0$

If the final state is 0 **then** $\rho = r \simeq 1, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

$$< \exp(-W_s) >_{\to 0} = \frac{1/2}{r} \qquad \text{and} \qquad < W_s >_{\to 0} \geq (\ln 2 \; + \; \ln r)$$

If the final state is 1 then $\rho = (1 - r) \simeq 0, \quad \rho_{eq} = 1/2, \quad \Delta F = 0$

$$< \exp(-W_s) >_{\to 1} = \frac{1/2}{1 - r} \qquad \text{and} \quad < W_s >_{\to 1} \geq \ln 2 \; + \; \ln(1 - r)$$

Total work $\quad < W_s >= r \;\; < W_s >_{\to 0} + (1 - r) \;\; < W_s >_{\to 1}$

using the inequalities $\quad \boxed{< W_s > \geq \ln 2 \; + r \;\; \ln r + (1 - r) \;\; \ln(1 - r)}$

The generalized Landauer's bound

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Conclusions

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- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

- Jarzinsky equality computes the Landauer limit independently of the rapidity of the procedure

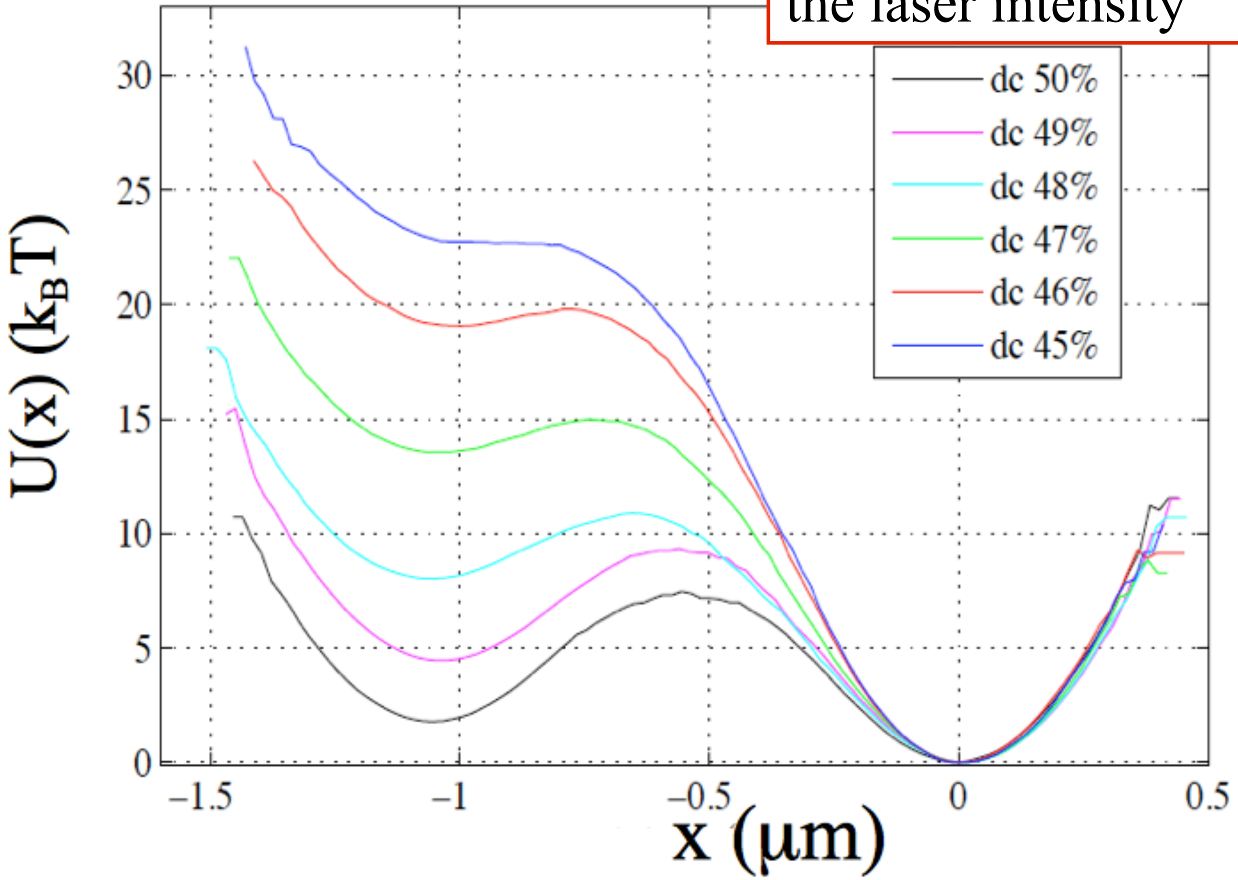
Question : Does any procedure allows us to reach the Landauer's limit ?

Answer : NO. The barrier reduction and tilt must be two separate process

See recent paper on optimisation :
E. Aurell, et al. J. Stat. Phys.147, 487-505 (2012).

Other procedure (I)

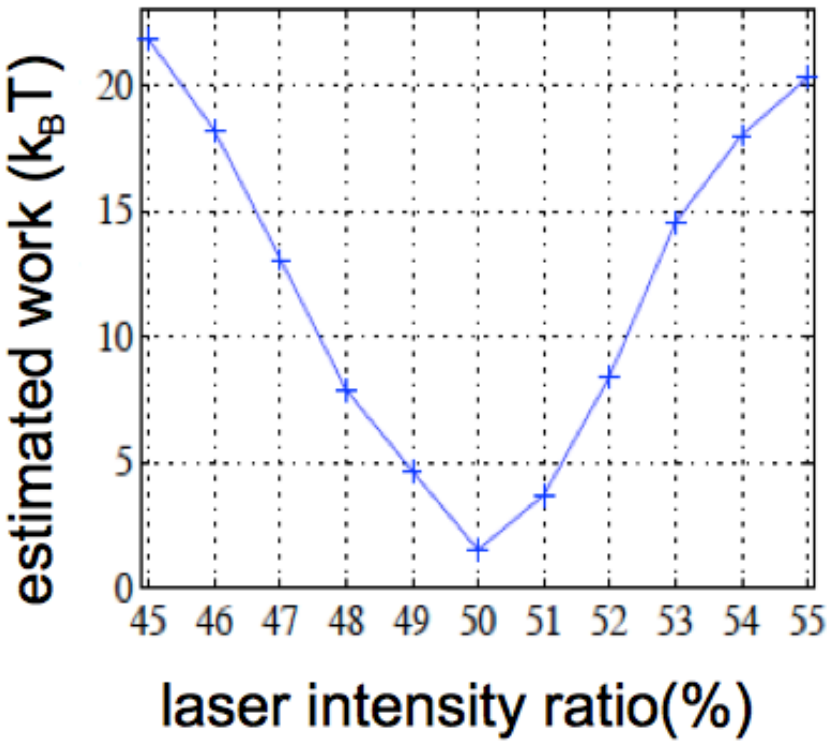
Relative change of the laser intensity



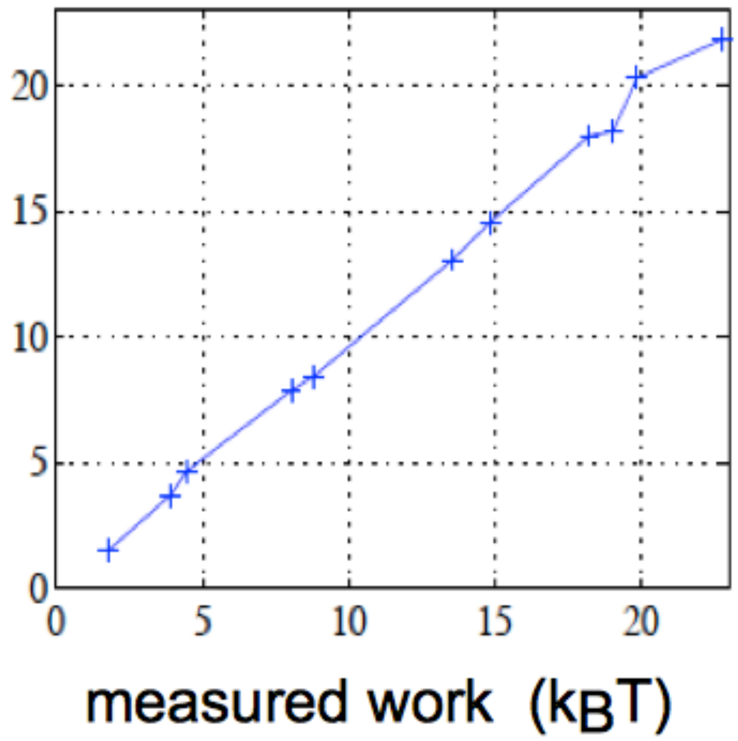
The ramping time of the laser intensity has been changed from 1s to 50s

Other procedure (II)

Fixed intensity ratio



Ramp of intensity ratio



The work is mainly due to the jump of the particle

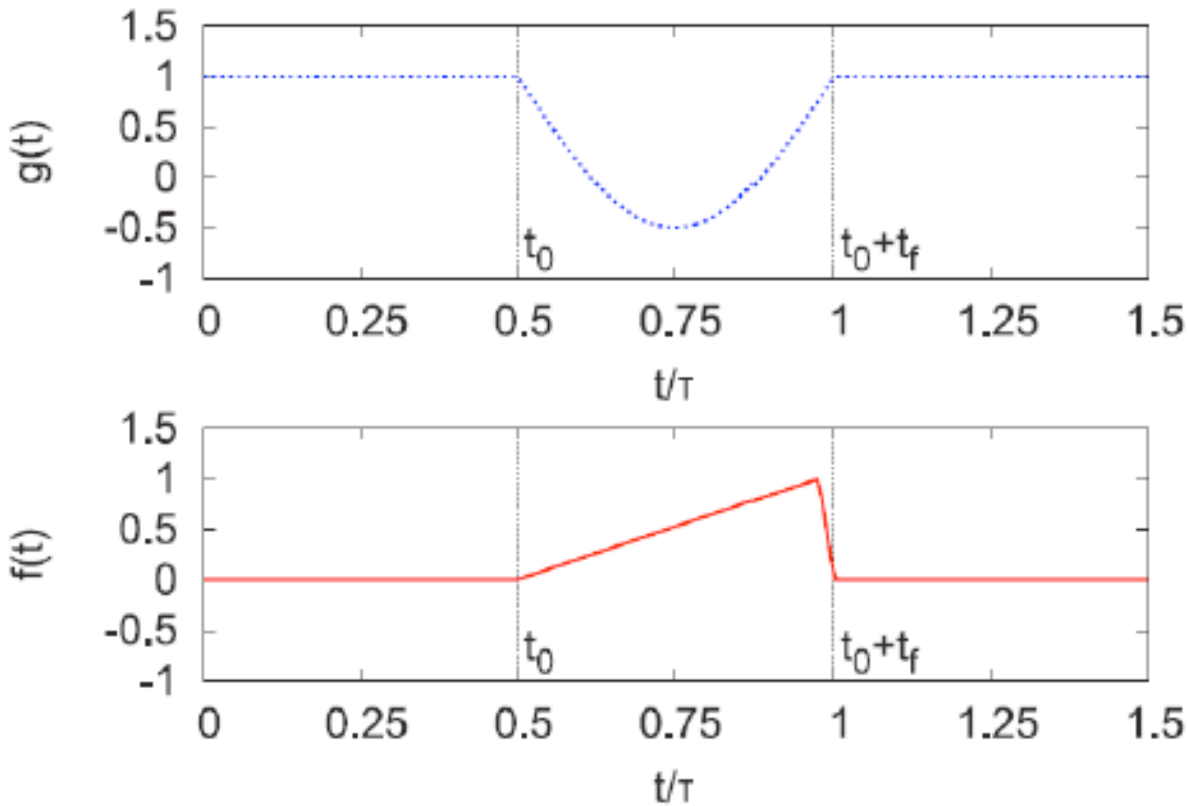
The Landauer limit can never be reached

Numerical results

Memory Erasure in Small Systems,
R. Dillenschneiderand E. Lutz, Phys. Rev. Lett. 102, 210601 (2009)

Potential : $V(x,t) = -\frac{1}{2}g(t)x^2 + \frac{1}{4}x^4$

External force : $Af(t)$



Non-dimensional numbers and the success rate

$$\bar{\tau} = \frac{\tau}{\tau_k}$$

Possibility of jumping the barrier without force

$$\bar{F} = \frac{\delta x \ F_{max}}{\Delta U}$$

The maximum external work overcomes the barrier

$\tau_K = \tau_o \exp[\frac{\Delta U}{k_B T}]$ is the Kramers time with $\tau_o \simeq 1 s$

δx is the distance of the potential minima

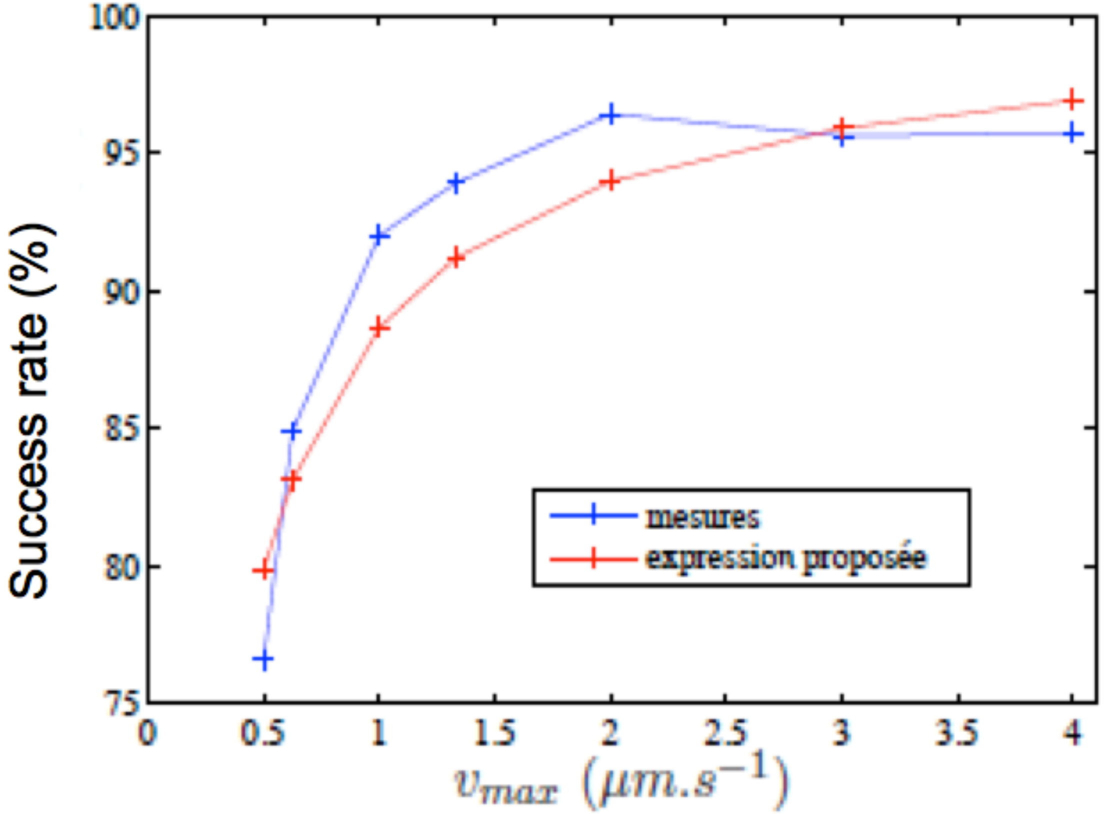
One can think that the success rate is :

$$r = \frac{1}{2}[1 + \exp(-\frac{1}{\bar{\tau}\bar{F}})]$$

Non-dimensional numbers and the success rate

Experimentally

$$r = \frac{1}{2}[1 + \exp(-\frac{a}{\bar{\tau}\bar{F}^2})]$$



Conclusions

- Our experimental results indicate that the thermodynamic limit to information erasure, the Landauer bound, can be approached in the quasistatic regime, but not exceeded.

- The asymptotic limit is reached in $1/\tau$ for $\tau > 3\,\tau_k$

- The fact that $r<1$ is due to the finite height of the initial barrier

- Thermal fluctuations play an important role to reach the limit

Question : Does any procedure allows us to reach the Landauer's limit ?

Answer : NO. The barrier reduction and tilt must be two separate process