Current fluctuations in non equilibrium systems

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Collaborators

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A. Dhar and K. Saito

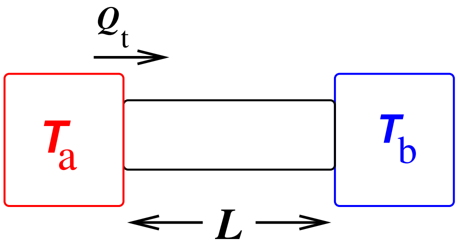
OUTLINE

Diffusive systems

Mechanical versus Diffusive systems

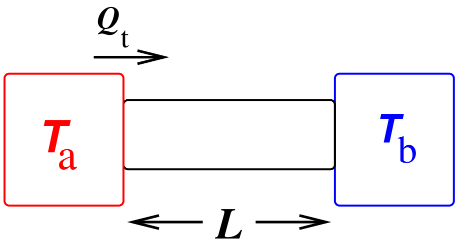
Levy walk model

NON EQUILIBRIUM STEADY STATE



2 heat baths

NON EQUILIBRIUM STEADY STATE



2 heat baths

$\langle Q_t \rangle$?

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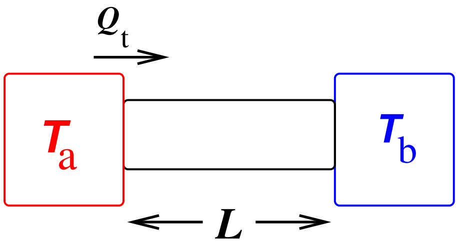

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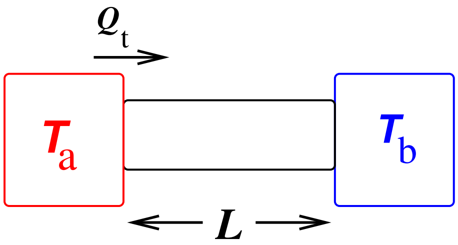
$\langle Q_t \rangle$?

$\langle Q_t^2 \rangle - \langle Q_t \rangle^2$?



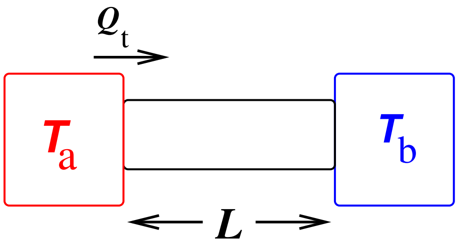
2 heat baths

$\langle Q_t \rangle$?

$\langle Q_t^2 \rangle - \langle Q_t \rangle^2$?

Distribution $P(Q_t)$ of Q_t

NON EQUILIBRIUM STEADY STATE



2 heat baths

$\langle Q_t \rangle$?

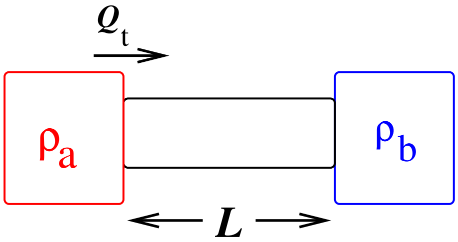
$\langle Q_t^2 \rangle - \langle Q_t \rangle^2$?

Distribution $P(Q_t)$ of Q_t

Does it satisfy Fourier's law?

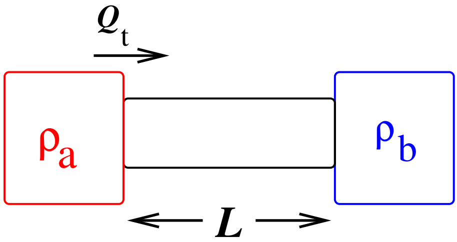
the Fluctuation theorem?

NON EQUILIBRIUM STEADY STATE



2 reservoirs of particles

NON EQUILIBRIUM STEADY STATE



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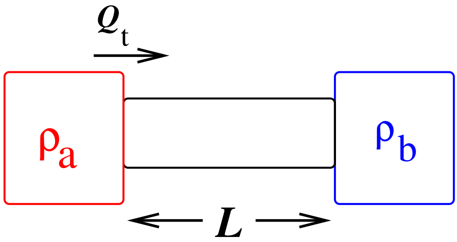

2 reservoirs of particles

$\langle Q_t \rangle$?

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Distribution $P(Q_t)$ of Q_t

Generating function (for large t)

$$\langle \exp [\lambda Q_t] \rangle \sim \exp [t \ \mu(\lambda)]$$

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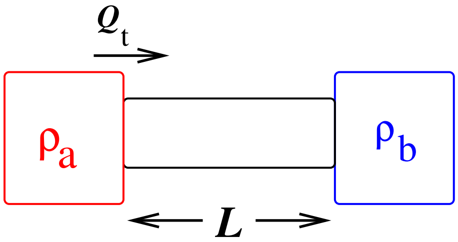

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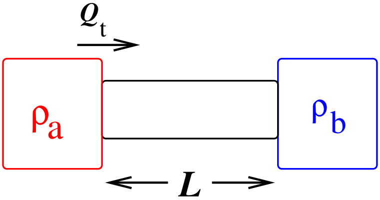
$$\langle \exp[\lambda Q_t] \rangle \sim \exp[t\ \mu(\lambda)]$$

($Q_t \simeq$ Sum of t/τ independent random variables)

INDEPENDENT PARTICLES

(Ballistic, random walkers, Levy flights,..)



with $\rho_a > 0$ and $\rho_b = 0$

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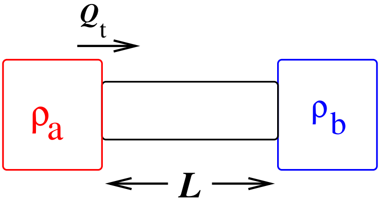

with $\rho_a > 0$ and $\rho_b = 0$

Poisson process:

$\langle \exp[\lambda Q_t] \rangle \sim \exp[t\ \mu(\lambda)]$ with $\mu(\lambda) = \kappa \times (e^{\lambda} - 1)$

INDEPENDENT PARTICLES

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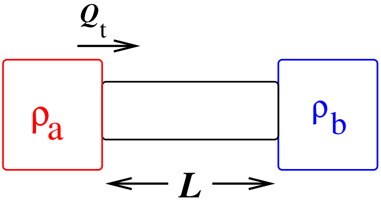

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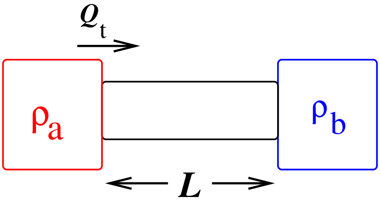
$$\langle \exp[\lambda Q_t] \rangle \sim \exp[t \; \mu(\lambda)]$$

with $\mu(\lambda) = \kappa \times (e^{\lambda} - 1)$

Fano factor $F = \frac{\langle Q_t^2 \rangle - \langle Q_t \rangle^2}{\langle Q_t \rangle} = 1$

INDEPENDENT PARTICLES

(Ballistic, random walkers, Levy flights,..)



with $\rho_{\mathrm{a}} > 0$ and $\rho_{\mathrm{b}} = 0$

Poisson process:

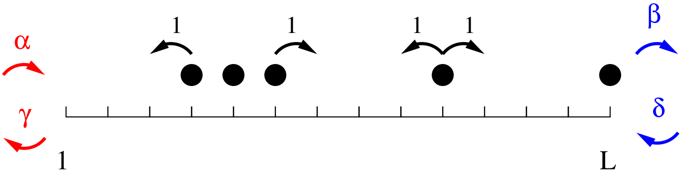
$\langle \exp[\lambda Q_t] \rangle \sim \exp[t \ \mu(\lambda)]$ with $\mu(\lambda) = \kappa \times (e^{\lambda} - 1)$

If during dt, there is a probability pdt that a particle is emitted by the left reservoir which will hit the right resevoir before the left reservoir

$$\left\langle e^{\lambda Q_t} \right\rangle = \prod_{dt} \left(1 - pdt + pdte^{\lambda} \right) = \exp\left[t \times p(e^{\lambda} - 1) \right]$$

EXCLUSION PROCESSES

SSEP (Symmetric simple exclusion process)

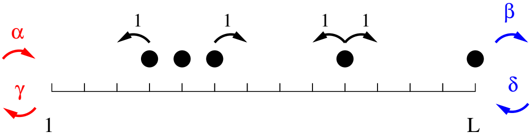


$$\rho_a = \frac{\alpha}{\alpha + \gamma} \,, \qquad \qquad \rho_b = \frac{\delta}{\beta + \delta}$$

$$\boxed{\langle \exp\left[\lambda Q_t\right] \rangle \sim \exp\left[t \ \mu(\lambda)\right]} \quad ?$$

TWO APPROACHES

SSEP (Symmetric simple exclusion process)



Microscopic

Bethe ansatz, Perturbation theory, Computer,...

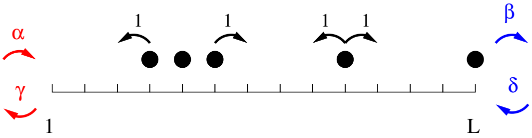
Macroscopic

$i = Lx \ , \ t = L^2\tau$

$$\text{Pro}(\{\rho(x,\tau), j(x,\tau)\}) \sim \exp\left[-L\int_0^{T/L^2} dt \int_0^1 dx \, \frac{[j+\rho']^2}{4\rho(1-\rho)}\right]$$

SSEP (Symmetric simple exclusion process)

D. Douçot Roche 2004



$$\lim_{t\to\infty} \frac{\langle Q(t)\rangle}{t} \simeq \frac{1}{L}[\rho_a - \rho_b] \qquad \text{Fick's law}$$

$$\lim_{t\to\infty} \frac{\langle Q^2(t)\rangle_c}{t} \simeq \frac{1}{L}\left[\rho_a + \rho_b - \frac{2(\rho_a^2 + \rho_a\rho_b + \rho_b^2)}{3}\right]$$

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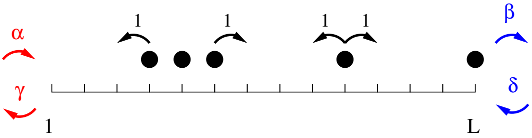

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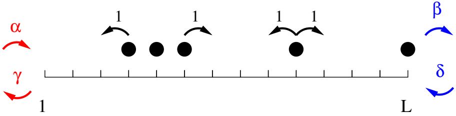
SSEP with $\rho_a = 1$ and $\rho_b = 0$ $\Rightarrow$

$$F = \frac{\langle Q_t^2\rangle_c}{\langle Q_t\rangle} = \frac{1}{3} \quad ; \quad \frac{\langle Q_t^3\rangle_c}{\langle Q_t\rangle} = \frac{1}{15} \quad \cdots$$

CURRENT FLUCTUATIONS IN THE SSEP



$$\left\langle e^{\lambda Q_t} \right\rangle \sim e^{t\ \mu(\lambda)}$$

$\mu(\lambda)$ gives all the cumulants of Q_t

For large L

$$\mu(\lambda, \rho_a, \rho_b, \text{contacts}) = \frac{1}{L} R(\omega)$$

where

$$\omega = \rho_a\,(e^{\lambda} - 1) + \rho_b\,(e^{-\lambda} - 1) + \rho_a\,\rho_b\,(e^{\lambda} - 1)(e^{-\lambda} - 1)$$

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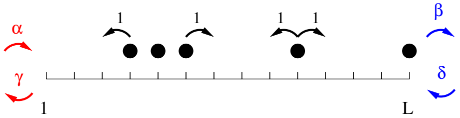

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Result: $$R(\omega) = [\log(\sqrt{1+\omega} + \sqrt{\omega})\ ]^2$$

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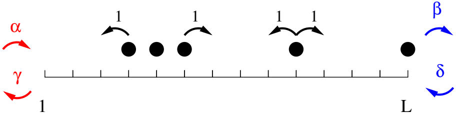

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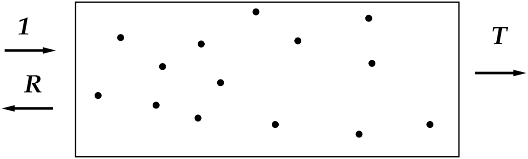
Result:

$$\boxed{R(\omega) = [\log(\sqrt{1+\omega} + \sqrt{\omega})\ ]^2}$$

Same as the universal statistics of transport of fermions in disordered conductors (suppression of shot noise)

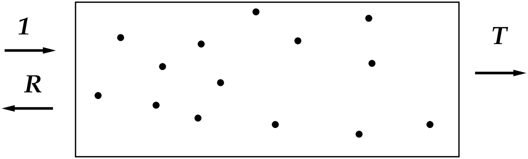
Beenakker, Buttiker 1992, Lee, Levitov, Yakovets 1995,.....



Many channels

$$\frac{\langle Q_t \rangle}{t} = \sum_n T_n \quad ; \quad \frac{\langle Q_t^2 \rangle - \langle Q_t \rangle^2}{t} = \sum_n T_n (1 - T_n)$$

QUANTUM TRANSPORT AND FULL COUNTING STATISTICS



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t electrons scatterred in each channel

$$\left\langle e^{\lambda Q_t} \right\rangle = \prod_n \left(1 - T_n + T_n e^{\lambda}\right)^t$$

STATISTICS OF THE T_n's

$$\left\langle e^{\lambda Q_t} \right\rangle = \prod_n \left(1 - T_n + T_n e^{\lambda}\right)^t$$

Many channels

$$\rho(T) = \sum_n \delta(T - T_n) = \frac{A}{T\sqrt{1-T}}$$

leads to the same result as for the SSEP

Beenakker, Buttiker 1992, Lee, Levitov, Yakovets 1995

STATISTICS OF THE T_n's

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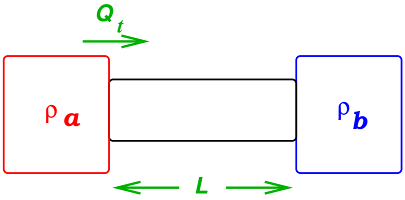
Fokker Planck equation for $P_L(T_1, \cdots T_n)$ when $L \to L + DL$ is similar to the Fokker Planck equation of Dyson's Brownian motion for random matrices.

Dorokhov 82
Mello Pereyra and Kumar 88
Mello Pichard 89, 91
Beenakker 92
Nazarov 94

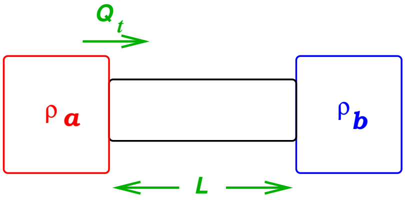
DIFFUSIVE SYSTEMS



Bodineau D. 2004

- For $\rho_a - \rho_b$ small: $$\frac{\langle Q_t \rangle}{t} = \frac{D(\rho)(\rho_a - \rho_b)}{L}$$
- $\rho_a = \rho_b = \rho$: $$\frac{\langle Q_t^2 \rangle}{t} = \frac{\sigma(\rho)}{L}$$

DIFFUSIVE SYSTEMS



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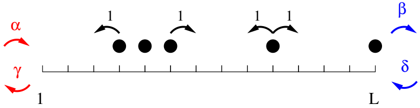
Then one can calculate:

1. $\mu(\lambda)$
2. All cumulants of Q_t for arbitrary ρ_a and ρ_b

DIFFUSIVE SYSTEMS

Symmetric exclusion



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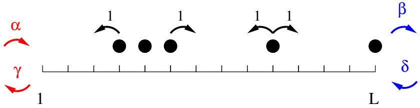


General lattice gas

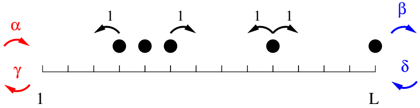
Random walkers

KMP model Kipnis Marchioro Presutti

Weakly driven lattice gases (field $\sim L^{-1}$)

DIFFUSIVE SYSTEMS

Symmetric exclusion



Fourier's law

$$\frac{\langle Q_t \rangle}{t} \simeq \frac{G(T_a, T_b)}{L}$$

General lattice gas

Fick's law

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KMP model Kipnis Marchioro Presutti

Weakly driven lattice gases (field $\sim L^{-1}$)

MACROSCOPIC FLUCTUATION THEORY

Onsager Machlup theory for non equilibrium

Bertini De Sole Gabrielli Jona-Lasinio Landim 2001 $\rightarrow$

$\frac{d\rho}{dt} = -\frac{dj}{dx}$ (conservation law)

Evolution of a profile $\rho(x,t)$ for $0 \leq t \leq T$

$$\mathrm{Pro}(\{\rho(x,t), j(x,t)\})$$
$$\exp\left[-L\int_0^{T/L^2} dt \int_0^1 dx \, \frac{[j(x,t)+\rho'(x,t)D(\rho(x,t))]^2}{2\sigma(\rho(x,t))}\right]$$

$$\Updownarrow$$

$$j(x,t) = -\rho'(x,t)D(\rho(x,t)) + \frac{1}{\sqrt{L}}\eta(x,t)$$

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$$\Updownarrow$$

$$j(x,t) = -\rho'(x,t) D(\rho(x,t)) + \frac{1}{\sqrt{L}} \eta(x,t)$$

with the white noise $\langle \eta(x,t)\eta(x',t')\rangle = 2\sigma(\rho(x,t))\delta(x-x')\delta(t-t')$

TRUE VARIATIONAL PRINCIPLE

Bertini De Sole Gabrielli
Jona-Lasinio Landim 2005

$$\mu(\lambda) = \frac{1}{L} \lim_{T\to\infty} \max_{\rho,j} \frac{1}{T} \int_0^T dt \int_0^1 dx \; \lambda j(x,t) - \frac{[j(x,t) + \rho'(x,t) D(\rho(x,t))]^2}{2\sigma(\rho(x,t))}$$

with $\frac{d\rho}{dt} = -\frac{dj}{dx}$ (conservation), $\rho_t(0) = \rho_a$, $\rho_t(1) = \rho_b$

- Sufficient condition for the optimal profile to be time independent
- Dynamical phase transition

the optimal $\rho_t(x)$ starts to become time dependent

Bodineau D. 2005, 2007
Hurtado, Garrido 2009

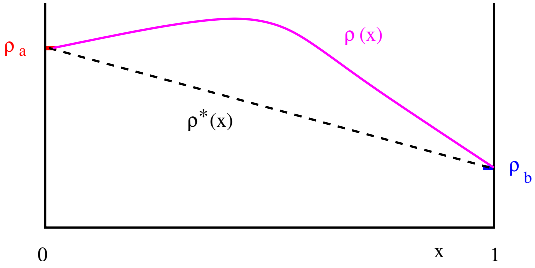
VARIATIONAL PRINCIPLE IN ONE DIMENSION

Bodineau D. 2004

Assuming that $j(x,t) = j$

Generating function $\langle e^{\lambda Q_t} \rangle \sim \exp[t\mu_L(\lambda)]$

$$\mu_L(\lambda, \rho_a, \rho_b) = \frac{1}{L} \max_{j,\rho(x)} \left[\lambda j - \int_0^1 dx \, \frac{[j\ L\ + \rho'(x)\ D(\rho(x))]^2}{2\sigma(\rho(x))} \right]$$



DIFFUSIVE SYSTEMS: all the cumulants

Bodineau D. 2004

$$\frac{\langle Q_t \rangle_c}{t} = \frac{1}{L}\mathcal{I}_1$$

$$\frac{\langle Q_t^2 \rangle_c}{t} = \frac{1}{L}\ \frac{\mathcal{I}_2}{\mathcal{I}_1}$$

$$\frac{\langle Q_t^3 \rangle_c}{t} = \frac{1}{L}\ \frac{3\ (\ \mathcal{I}_3\ \mathcal{I}_1 - \mathcal{I}_2^2\ )}{\mathcal{I}_1^3}$$

$$\frac{\langle Q_t^4 \rangle_c}{t} = \frac{1}{L}\ \frac{3\ (\ 5\ \mathcal{I}_4\ \mathcal{I}_1^2 - 14\ \mathcal{I}_1\mathcal{I}_2\ \mathcal{I}_3 +\ 9\ \mathcal{I}_2^3\ )}{\mathcal{I}_1^5}$$

where

$$\mathcal{I}_n = \int_{\rho_b}^{\rho_a} D(\rho)\sigma(\rho)^{n-1} d\rho$$

For the SSEP $\quad D(\rho) = 1$ and $\sigma(\rho) = 2\rho(1-\rho)$

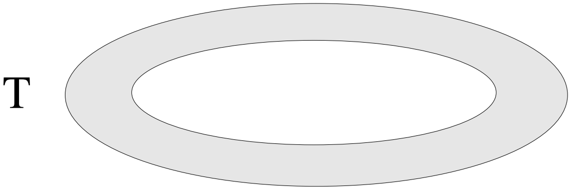
For the KMP $\quad D(\rho) = 1$ and $\sigma(\rho) = 4\rho^2$

For random walkers $\quad D(\rho) = 1$ and $\sigma(\rho) = 2\rho$

Current fluctuations in non-equilibrium steady states

Current fluctuations in non-equilibrium steady states

Current fluctuations on a ring



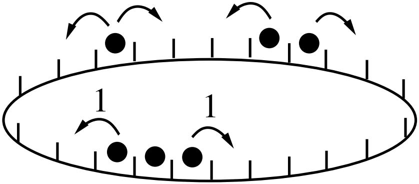
SSEP ON A RING

Appert D Lecomte Van Wijland 2008

N particles

L sites

$$\rho = \frac{N}{L}$$



Q_t flux through a bond during time t

UNIVERSAL CUMULANTS OF THE CURRENT

$$\frac{\langle Q^2\rangle_c}{t} = \frac{\sigma}{L}$$

Gaussian

$$\frac{\langle Q^4\rangle_c}{t} \simeq \frac{\sigma^2}{2L^2}, \quad \frac{\langle Q^6\rangle_c}{t} \simeq -\frac{\sigma^3}{4L^2}, \quad \frac{\langle Q^8\rangle_c}{t} \simeq \frac{5\sigma^4}{12L^2}$$

Universal

UNIVERSAL CUMULANTS OF THE CURRENT

$$\frac{\langle Q^2 \rangle_c}{t} = \frac{\sigma}{L}$$

Gaussian

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Universal

$$\left\langle e^{\lambda Q_t} \right\rangle \sim e^{t\,\mu(\lambda)}$$

with

$$\mu(\lambda) - \frac{\lambda^2}{2} \frac{\langle Q^2 \rangle_c}{t} = \frac{1}{L^2}\, \mathcal{F}\left(-\frac{\sigma\lambda^2}{4}\right)$$

$$\mathcal{F}(u) = -4 \sum_{n \geq 1} \left[n\pi\sqrt{n^2\pi^2 - 2u} - n^2\pi^2 + u \right] = \frac{1}{3}u^2 + \frac{1}{45}u^3 + \frac{1}{378}u^4 + \ldots$$

$\mathcal{F}$ universal

Singularity as $u \to \frac{\pi^2}{2}$

RESULTS FOR A GENERAL DIFFUSIVE SYSTEM

$$\left\langle e^{\lambda Q_t} \right\rangle \sim e^{t\,\mu(\lambda)}$$

$$\mu(\lambda) - \frac{\lambda^2 \langle Q^2 \rangle}{2t} = \frac{1}{L^2} D \mathcal{F}\left(\frac{\sigma \sigma''}{16 D^2} \lambda^2 \right)$$

$$\mathcal{F}(u) = -4 \sum_{n \geq 1} \left[n\pi \sqrt{n^2\pi^2 - 2u} - n^2\pi^2 + u \right] = \frac{1}{3} u^2 + \frac{1}{45} u^3 + \frac{1}{378} u^4 + \ldots$$

- Phase transition as $u \to \pi^2/2$
- For $n \geq 2$

$$\frac{\langle Q^{2n} \rangle_c}{t} \sim \frac{1}{L^2} \; \frac{(2n)! B_{2n-2}}{n! \; (n-1)!} \; D \left(\frac{\sigma \sigma''}{8 D^2} \right)^n$$

B_n Bernoulli numbers

FLUCTUATING HYDRODYNAMICS

Gaussian expansion of the **macroscopic fluctuation theory** around a constant current and a flat profile.

$$\rho(x,t) = \rho + \sum_{k,\omega} k[a_{k,\omega} e^{i\omega\tau + ikx} + a^*_{k,\omega} e^{-i\omega t - ikx}]$$

$$j = j_0 - \omega[a_{k,\omega} e^{i\omega\tau + ikx} + a^*_{k,\omega} e^{-i\omega t - ikx}] \; .$$

Gaussian fluctuations

$$\mathrm{Pro}(Q_t = j_0 t, \{a_{k,\omega}\}) \sim \exp\left[-\frac{j_0^2}{2\sigma}\frac{t}{L} - \frac{t}{L}\sum_{\omega,k} |a_{k,\omega}|^2 \left(\frac{(\sigma\omega + j_0\sigma' k)^2}{\sigma^3} + \frac{D^2 k^4}{\sigma} - \frac{j_0^2 \sigma'' k^2}{2\sigma^2} \right) \right]$$

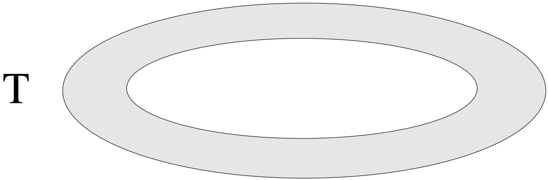
- Integrate over the fluctuations
- Sum over the discrete modes k

MACROSC. FLUCT. TH. and FINITE SIZE EFFECTS

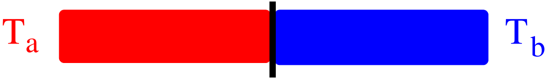
- General theory
- Large deviations of the current in the steady state
  - All diffusive systems if the optimal profile is time independent (see the talk by Thierry)
  - Finite size corrections
- Large deviations of the density in the steady state
  - Few cases have been solved
  - Finite size corrections? D. Retaux 2013
- Non steady state situations

Current fluctuations in non-equilibrium steady states

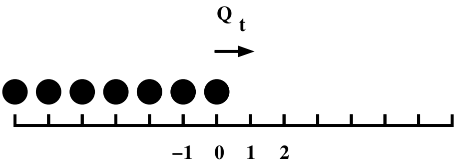
Universal fluctuations on a ring



Non-equilibrium initial condition



STEP INITIAL CONDITION



ASEP

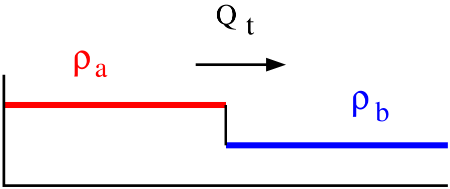
G. Schütz 98

⋮

Prähofer Spohn 2000-2002

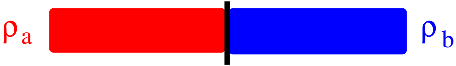
⋮

Tracy Widom 2008



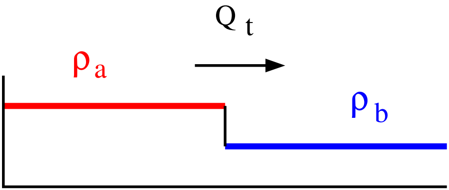
SSEP

D Gerschenfeld 2009



$$\boxed{\left\langle e^{\lambda Q_t} \right\rangle = ?}$$

STEP INITIAL CONDITION



SSEP

D Gerschenfeld 2009

$$\boxed{\left\langle e^{\lambda Q_t} \right\rangle \simeq \exp[\sqrt{t}\ F(\omega)]}$$

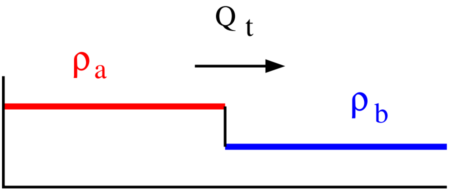
where

$$\boxed{\omega = \rho_a(e^{\lambda} - 1) + \rho_b(e^{-\lambda} - 1) + \rho_a\rho_b(e^{\lambda} - 1)(e^{-\lambda} - 1)}$$

and

$$F(\omega) = \frac{1}{\sqrt{\pi}} \sum_{n\geq 1} \frac{(-)^{n+1}}{n^{3/2}}\ \omega^n \equiv \frac{1}{\pi} \int_{\infty}^{\infty} dk \log\left[1 + \omega e^{-k^2}\right]$$

STEP INITIAL CONDITION



SSSEP

D Gerschenfeld 2009

$$\langle e^{\lambda Q_t} \rangle \simeq \exp[\sqrt{t}\; F(\omega)]$$

with

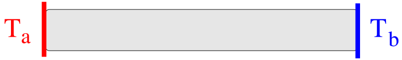
$$F(\omega) = \frac{1}{\pi} \int_{\infty}^{\infty} dk \log\left[1 + \omega e^{-k^2}\right]$$

For large Q_t

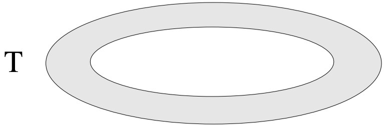
$$\mathrm{Pro}(Q_t) \sim \exp[-\frac{\pi^2}{12}\; Q_t^3/t]$$

Meerson, Sasorov 2013-14

DIFFUSIVE SYSTEMS: summary

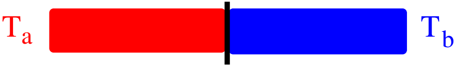


$$\langle Q^n \rangle_c / t \sim L^{-1}$$



$$\langle Q^2 \rangle_c / t \sim L^{-1}$$

$$\langle Q^{2n} \rangle_c / t \sim L^{-2} \quad \text{for} \quad n \geq 2$$



$$\langle Q^n \rangle_c \sim t^{1/2}$$

ONE DIMENSIONAL MECHANICAL SYSTEMS

Ideal gas

No Fourier's law

Harmonic chain

$$E = \sum \frac{p_i^2}{2m} + \sum_i g(x_{i+1} - x_i)^2$$

$$\frac{\langle Q_t \rangle}{t} \simeq G(T_a, T_b)$$

ONE DIMENSIONAL MECHANICAL SYSTEMS

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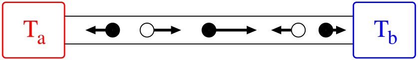
Hard particle gas



Anomalous Fourier's law

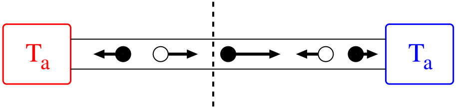
$$\frac{\langle Q_t \rangle}{t} \simeq \frac{G(T_a, T_b)}{L^{1-\alpha}}$$

$.3 \leq \alpha \leq .5$ Delfini Lepri Livi Politi Livi, Spohn, Grassberger, Dhar ...

Anharmonic chain

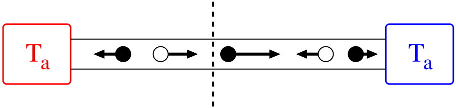
HARD PARTICLE GAS: the second cumulant

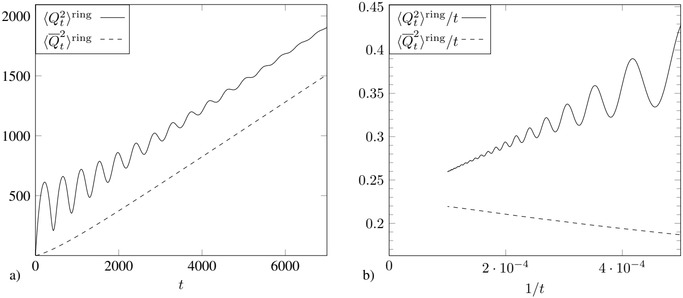
Brunet D. Gerschenfeld 2010



HARD PARTICLE GAS: the second cumulant

Brunet D. Gerschenfeld 2010

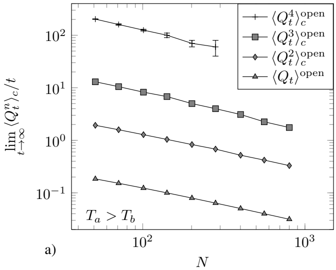


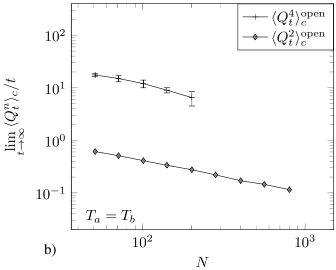


$$\frac{\langle Q^2 \rangle}{t} \text{ has a limit}$$

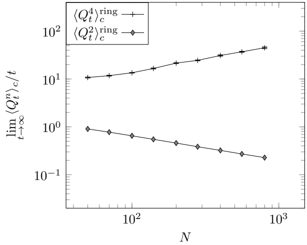
HARD PARTICLE GAS: Size dependence of the cumulants

OPEN SYSTEM



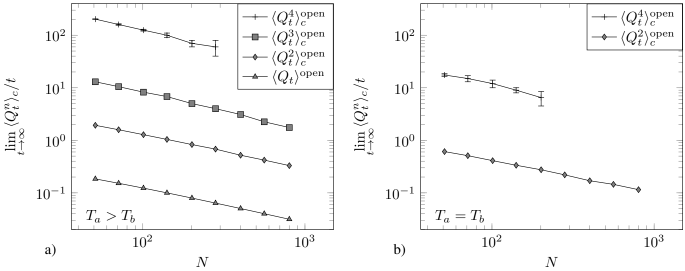


RING

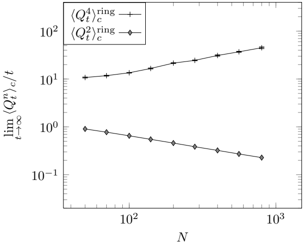


HARD PARTICLE GAS: Size dependence of the cumulants

OPEN SYSTEM

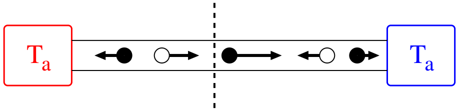


RING

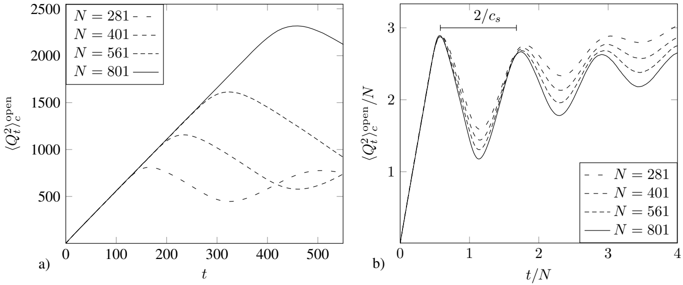


ANOMALOUS FOURIER'S LAW

HARD PARTICLE GAS

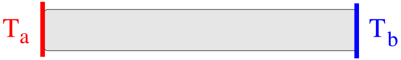


INFINITE SYSTEM

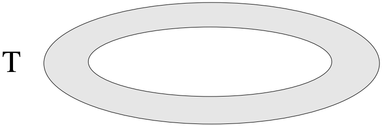


$$\langle Q^{2n} \rangle_c \sim t$$

MECHANICAL SYSTEMS: summary

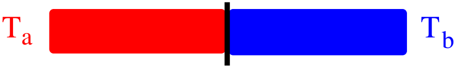


$$\langle Q^n \rangle_c / t \sim L^{-2/3}$$



$$\langle Q^2 \rangle_c / t \sim L^{-1/2}$$

$$\langle Q^4 \rangle_c / t \sim L^{1/2}$$



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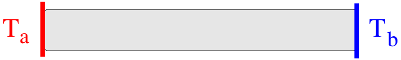
DIFFUSIVE SYSTEMS: summary



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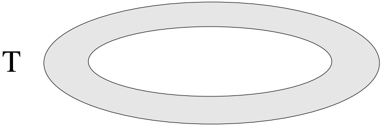

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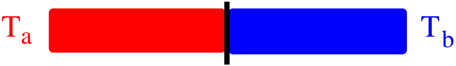
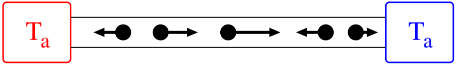


$$\langle Q^n \rangle_c \sim t^{1/2}$$

LEVY WALKERS

Dhar, Saito, D. 2012



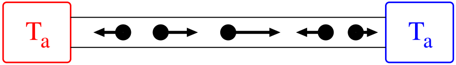
All walkers have a unit velocity with random flying times τ

$$\rho(\tau) \sim A\,\tau^{-\beta-1} \qquad \text{with} \qquad 1 < \beta < 2$$

Cipriani, Denisov, Politi 2005

LEVY WALKERS

Dhar, Saito, D. 2012



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Open geometry

$$\frac{\langle Q_t^n \rangle_c}{t} \propto L^{1-\beta}$$

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Infinite line $\quad \langle x^2 \rangle \sim t^{3-\beta}$

Ring

$$\frac{\langle Q_t^2 \rangle_c}{t} \propto t^{3-\beta} L^{-1}$$

LEVY WALKERS

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Infinite line $\quad \langle x^2 \rangle \sim t^{3-\beta}$

Ring

$$\frac{\langle Q_t^2 \rangle_c}{t} \propto t^{3-\beta} L^{-1}$$

Ring + cut-off time $\tau_{\max} \sim L^\delta$ with $\beta = 5/3$ and $\delta = 3/2$

$$\frac{\langle Q_t^2 \rangle_c}{t} \propto L^{-1/2} \quad ; \quad \frac{\langle Q_t^4 \rangle_c}{t} \propto L^{1/2}$$

OPEN QUESTIONS

Macroscopic fluctuation theory and finite size corrections

Diffusive systems in non stationnary situations

When the optimal profile is time dependent

Theory for mechanical systems

Link with Bethe ansatz calculation (Kirone's talk?)