Assisted hopping models of the active -absorbing state transition on a line

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Plan of the talk

- Active-absorbing transitions
- Preview of the main results
- Definition of the models
- Properties of the steady state
- Equivalence to gas of defects
- Summary

Introduction

Active-absorbing state form a very important class of non-equilibrium phase transitions [Hinrichsen(2000)]

The best studied model in this class is directed percolation.
The Janssen-Grassberger conjecture: all active -absorbing state transitions with a single order parameter, unique absorbing state, and no additional conservation laws, belong to DP universality class.

Many absorbing states, or additional conservation laws make the critical behavior non-DP.

Models with additional conservation laws:
Simplest is parity-conserving directed percolation (PCDP):
Here the processes are $A \to AAA$ and $A \to \phi$.
A different universality class.

Models with many absorbing states: The number of absorbing states grows exponentially with volume.
Pair -contact process :$AA \to AAA$, $A \to \phi$

Many absorbing states, and conserved number of particles:
Fixed -energy sandpiles
(including Conserved Threshold Transfer Process)
Activated random walkers
Assisted hopping models studied here.

Is $\beta \leq 1$?
In equilibrium and non-equilibrium continuous phase transitions, we usually define the critical exponent β, which describes how the order parameter m tends to zero, in the ordered phase: $m \sim \epsilon^{\beta}$.

In most cases, one finds that $\beta \leq 1$.
Can on prove such an inequality based on some general (e.g. thermodynamics) principles under some restricions?

The general answer to this question has to be No.

Examples with $\beta > 1$:
Conductance of a random mixture of conducting and insulating balls near the percolation point $\sim |\delta\rho|^a$, with $a > 1$.
Also, rigidity percolation, surface tension near gas-liquid critical point...

Clearly, we need to define order parameter carefully.
In the percolation problem, are conductance and prob. of infinite cluster equally good order parameters?

An interesting counter example was provided by De Silva and De Oliveira (2008).
The defined a 1-d model of assisted diffusion of interacting particles, with short ranged interactions, having an active-absorbing transition, with $\beta = 2$.

We construct a generalization of their model. For these transitions, the identification of order parameter seems rather clear.

Preview of the main results

- A class of assisted hopping models in one dimension showing active-absorbing transition
- Finite range interactions
- Exactly determined steady state and critical exponents
- Exponent β can take arbitrarily large positive integer values $\beta = n$ for all $n \geq 1$.
- Equivalence to a gas of defects makes the analysis very simple

Definition of the models

- We consider a lattice gas of N particles, on a ring of L sites.
- Each site has either 0, or at most one particle.
- A configuration may be specified by a binary string 00101110101....
- the model is defined by a range parameter n, which is a positive integer.

Definition of the models (continued)

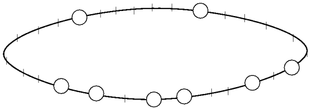

Figure: Particles on a ring.

- Continuous-time Markovian evolution
- Particles diffuse, variable jump size, but cannot cross each other
- If a particle has no occupied neighbor, and sum of lengths of empty intervals on its two sides $> n$, it is immobile
- Rate of jump depends on jump size r, and status of nearly sites.

Definition of the models (continued)

The detailed rates are as follows:

(a)Break-up of 0-clusters: A particle with one occupied neighbor, and next to a 0-interval of length ℓ, can take steps of length r, $r \leq \ell$. The rate is $\Gamma_1(r, \ell)$.

(b)Merging of 0-clusters to form 0-clusters of length $\leq n$: A particle with 0-intervals of lengths ℓ_1 and ℓ_2, with $1 \leq \ell_1 + \ell_2 \leq n$ can jump ℓ_1 steps to left, creating a single 0-interval of length $\ell_1 + \ell_2$, with rate $\Gamma_2(\ell_1, \ell_1 + \ell_2)$

Similarly, rate $\Gamma_2(\ell_2, \ell_1 + \ell_2)$ to jump to end of right 0-interval.

We assume

(i)$\Gamma_1(r,\ell) > 0$, for all $\ell \geq r, r \leq n$

and the detailed balance condition

(ii)$\Gamma_1(r,l) = \Gamma_2(r,l);\ \text{for all} 1 \leq r \leq \ell \leq n$

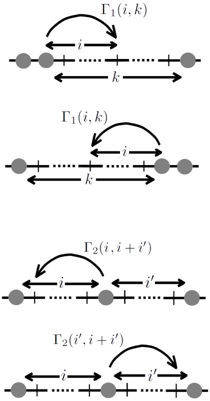

Figure: Figure shows allowed moves.

Characterizing the steady state

0-clusters of length $> \ell$ can break, but cannot form.

$1100000001 \to 1001000001$, but reverse transition is not allowed.

At low densities, the particles spread out, and eventually all of them are immoblie.

Ex: 0010000100001000100001000 ($n = 4$) all particles are immobile.

Clearly, $\rho_c = 1/(n+1)$.

For $\rho = \rho_c$, the system eventually falls in the periodic state $10^n10^n10^n\ldots$

For $\rho > \rho_c$, eventually, there are no 0-clusters of length $> n$, and

Either the smaller 0-clusters keep breaking, and merging,

Or (For $\rho_c < \rho < \rho_{c2}$), system reaches an inactive configuration.

e.g. $n = 4$: 1000100100010010000.... Clearly, $\rho_{c2} = 2/(3+n)$.

Characterizing steady state

Define Height of a configuration = number of 0-clusters of length $> n$.

Then, under evolution, this can only decrease. No detailed balance.

However, in the $h = 0$ "floor level", detailed balance holds, and all accessible configurations in a sector are equally likely.

From $h = 1$ level, one may fall into an immobile state (for intermediate densities), or an active state.

Ex.:$n = 4, L = 10, N = 4$, active configs are 10000110000010000,...

The active sector is unique: the reference state $1^r(10^n)^s$ can be reached from any active state, and moves in this sector are reversible.

All configurations, with at least one mobile particle, and all 0-clusters of length $\leq n$ are in the same sector.

Calculation of number of configurations in the active sector is straightforward.
Use generating function techniques

All words made with substrings $1 + 10 + 100 + 1000 + ...10^n$.

Since all occur with equal probability, mean activity can be also found.

We get mean activity $\sim (\rho - \rho_c)^n$

$\beta = n$

Equivalence to defect gas

For density close to ρ_c, the configurations has large stretches of 10^n repeated.

Take this as a standard configuration.
If 1 is followed by $n-x$ zeroes, we say there are x defects.
Then, an active configuration is given by
...1111x11111x11xx111x111.....
Fixed N, fixed number of defects.
The only constraint is that not more that n defects together.
All configurations are equally likely.
The dilute gas of defects is nearly ideal.
Activity if at least n defects at two adjacent sites.

The density of such sites varies as $(\delta\rho)^n$.

More on defect gas

For transient states, we have 0-clusters of length $> n$.

These correspond to immobile antidefects.

Under dynamics, the defects will diffuse, and annihilate with antidefects, leaving no antidefects in the steady state.

For $\rho < \rho_c$, there are too many antidefects in the beginning. All defects annihilate, leaving only immobile antidefects

The transient dynamics is the dynamics of mobile defects, static antidefects:

defect density$\sim t^{-1/4}$, for $1 \ll t \ll L^4$.

Other results

- Exact expression for mean activity as a function of density
- density-density correlations: $\xi \sim 1/\epsilon$, hence $\nu = 1$
- We can add a field that allows 0-clusters of length $> \ell$ form with rate *h*, keeping detailed balance
  Then calculate exact activity as a function of *h*

Summary

A simple exactly soluble model of active-absorbing state transition

$\beta = n$

Equivalence to a gas of defects

Thank You