Accelerating Branes
and
The String/Black hole transition

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String theory contains two natural scale:

$\ell_s = \sqrt{\alpha'}$; Tension of string $= \frac{1}{2\pi\alpha'}$

ℓ_p ; $G_N = \ell_p^{d-2}$

If $\ell_s \simeq \ell_p$, string theory is typically hard

It is most helpfull for $\ell_s \gg \ell_p$, i.e. small string coupling.

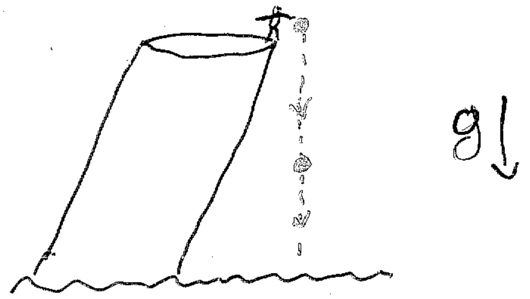
We have, in principle good control over stringy phenomena, which occur at distances $L \simeq \ell_s$, and can be analyzed using worldsheet methods. Quantum phenomena, which occur at $L \simeq \ell_p$ are hard.

In practice, our understanding of stringy phenomena is best in Euclidean space. One of the reasons is that there are many solvable examples of spaces whose size and/or scale of variation is the string scale.

It would be nice to develop a comparable level of understanding of Minkowski backgrounds.

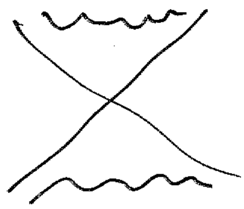
Examples we will consider today:

(1)



What happens when $g \approx \frac{1}{\ell_s}$?

(2) Black holes:



What happens when Hawking temperature T_{BH} is of order the Hagedorn temperature $\approx \frac{1}{\ell_s}$?

Our strategy will be to try to "eat the cake and have it too".

The solutions in question have Euclidean continuations. We will study string effects in Euclidean space, and then continue them to the Minkowski problem.

Accelerating D-branes

To study acceleration effects, need a well localized object. Strings typically have size l_s or more, but D-branes are much smaller.

A stationary D-brane localized in a direction ϕ is described by a wavefunction $\psi(\phi) \propto \delta(\phi)$.

We would like to study accelerating D-branes, and in particular examine their wave functions.

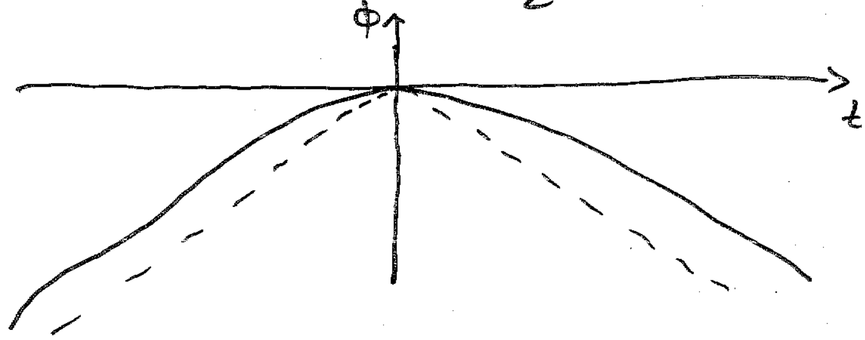
Consider a spacetime of the form:

$$\mathbb{R}_\phi \times \underset{\substack{\uparrow \\ \text{time}}}{\mathbb{R}_t} \times (\ldots)$$

Dilaton: $\Phi = -\frac{Q}{2}\phi \implies g_s \sim e^{-\frac{Q}{2}\phi}$

A D-brane localized in ϕ has energy $E \sim \frac{1}{g_s(\phi)} \sim e^{\frac{Q}{2}\phi}$, and so experiences a force in negative ϕ direction. Its classical trajectory (from DBI analysis)

$$e^{-\frac{Q}{2}\phi} = \cosh \frac{Q}{2} t$$

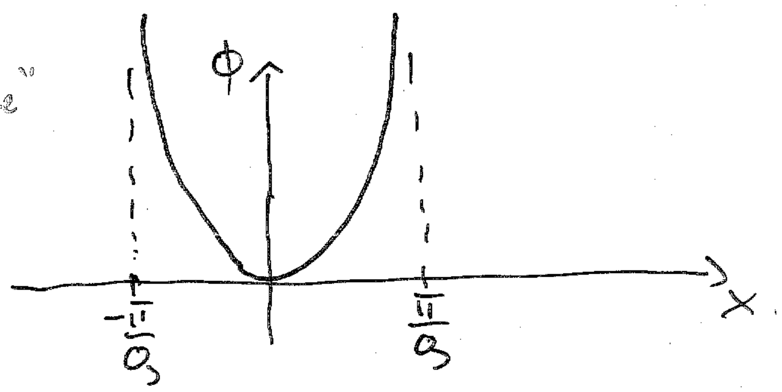


For small Q, expect this picture to be accurate; $Q \gtrsim 1$ can have large string effects. What are they?

Rotate to Euclidean space

$$t \to ix \Rightarrow \qquad e^{-\frac{Q}{2}\phi} = \cos\frac{Q}{2}x$$

"Hairpin brane"
akyanov
itchev
amolodchikov



Exact boundary state known.

Can caculate T_{xx} ($\leftrightarrow T_{00}$)

akayama
amigachi
ey
akayanagi

$$T_{xx}(\phi, x) \sim -y^{\frac{2}{g^2}-1} e^{-y^{\frac{2}{g^2}}}$$

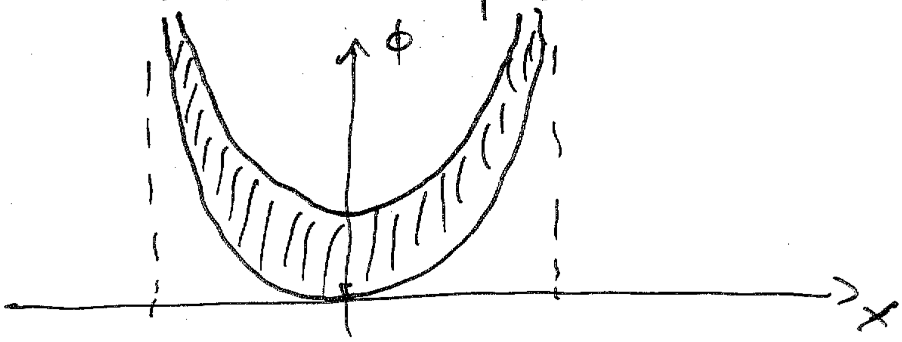
$$y \equiv \frac{e^{-\frac{Q}{2}\phi}}{\cos\frac{Q}{2}x}$$

Without string corrections, $y=1$.

Indeed, as $Q \to 0$,

$$T_{xx} \sim -\delta\left(\phi + \frac{2}{Q} \ln \cos \frac{Q}{2} x\right)$$

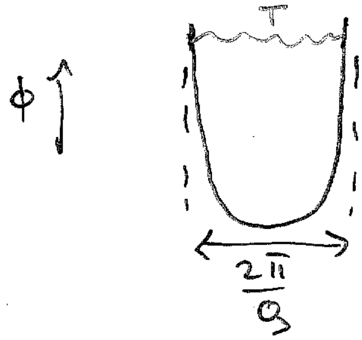
For finite Q, string effects smear the hairpin:



- Smearing mostly near the tip.
- For small Q, $\delta\phi \sim Q$.
- As Q increases, $\delta\phi$ grows. It diverges when $Q^2 \to 2$, where hairpin becomes non-normalizable.

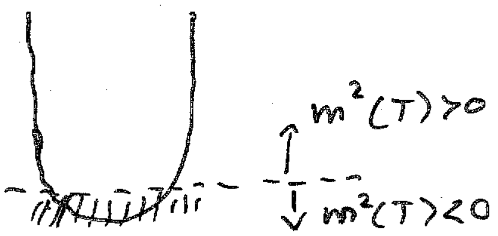
This smearing has a simple interpretation:

In addition to its curved shape, the hairpin has a condensate of the stretched tachyon

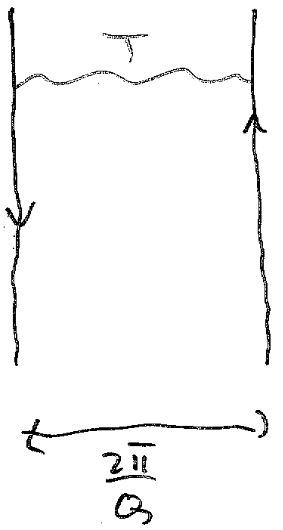


$T \sim \mu e^{-\frac{1}{2Q}\phi}$

* For small Q, tachyon is heavy at $\phi \to \infty$, so condensate goes rapidly to zero there. Near tip, tachyon is light and smears geometry



As Q increases, region where T is light becomes larger. When $Q^2 \to 2$, mass of T at $\phi \to \infty$ goes to zero and hairpin geometry is completely smeared. For $Q^2 > 2$ a better description is



$T \sim \mu e^{-\frac{1}{2Q}\phi}$

Back to Minkowski problem

$x \to it$

$$y = \frac{e^{-\frac{g}{2}\phi}}{\cosh \frac{g}{2} t}$$

Energy density:

$$T_{oo}(\phi,t) = \frac{E}{g\Gamma(1-\frac{g^2}{2})}\, y^{\frac{2}{g^2}-1} e^{-y^{\frac{2}{g^2}}}$$

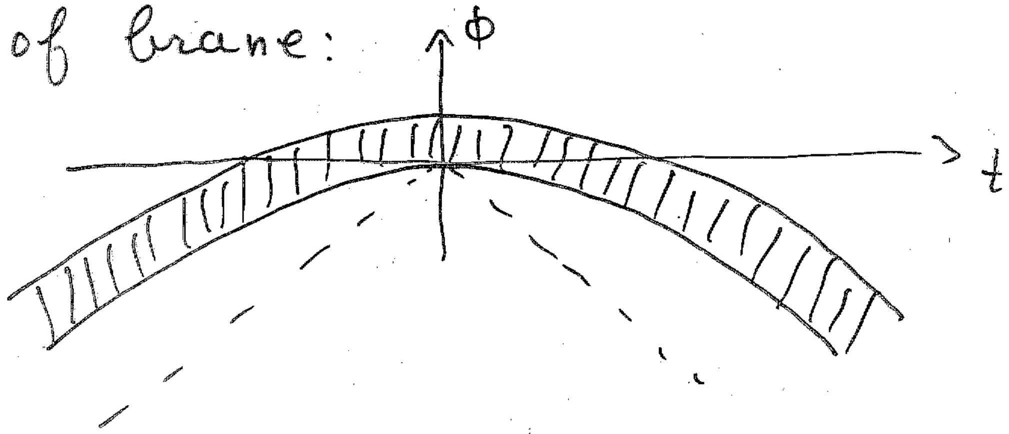
As $g \to 0$:

$$T_{oo}(\phi,t) \cong E\,\delta\left(\phi + \frac{2}{g}\ln\cosh\frac{g}{2}t\right)$$

Energy density is concentrated on the classical trajectory.

Finite g:

Stringy effects smear trajectory of brane:



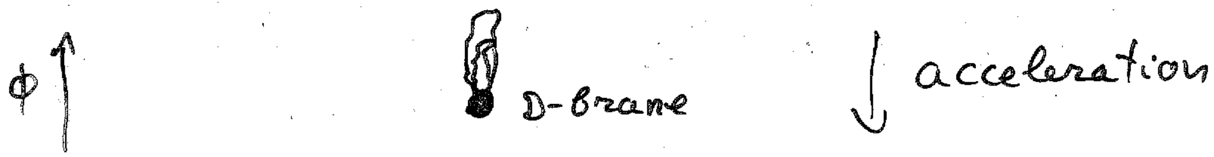
For small g, amount of smearing is $\delta\phi \sim g$.

As $g^2 \to 2$, smearing grows without bound, and brane becomes non-normalizable.

$\downarrow$

Maximal acceleration

Interpretation of smearing in Minkowski spac



* Small excitations of D-brane are open strings.

* Continuation from Euclidean gives state with open strings oscillating in ϕ.

* Acceleration in $-\phi$ direction gives force in $+\phi$ direction on strings $\Rightarrow$ D-brane has stringy halo at a larger ϕ.

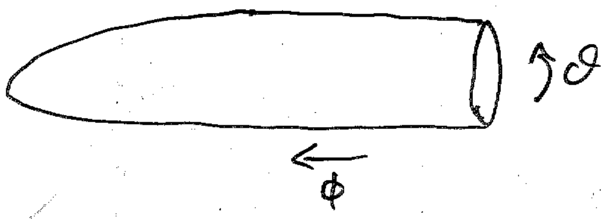
* At critical value of acceleration, open strings extend to infinity.

Two dimensional black hole

In 1+1 dimensions with linear dilaton, there is a black hole

$$ds^2 = d\phi^2 - \left(\tanh^2 \tfrac{Q}{2}\phi\right) dt^2$$

Euclidean continuation $t \to i\vartheta$ gives cigar

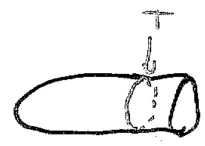


Hawking temperature:

$$T_{BH} = \frac{Q}{4\pi}$$

String effects are understood in this case. Like for hairpin, tachyon winding around θ has a condensate

$$T \sim \mu e^{-\frac{1}{Q}\phi}$$



Region where $m^2(T) < 0$ is smeared by stringy effects.

For small Q, size of region is: $\delta\phi \sim l_s$.
For $Q^2 \to 2$ its size diverges, since tachyon mass at infinity goes to zero

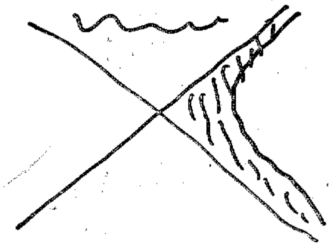
Minkowski black hole

Small Q: $T_{BH} = \frac{Q}{4\pi}$ is low.

An observer at fixed ϕ sees higher Hawking temperature:

$$T_{BH}(\phi) = \frac{Q}{4\pi} \frac{1}{\tanh \frac{Q}{2}\phi}$$

Region in which $T_{BH}(\phi) > T_{\text{Hagedorn}}$ is smeared by classical condensate of closed strings.



Susskind

This stretched horizon is same as region in which winding tachyon has $m^2 < 0$ in Euclidean black hole.

As Q increases, size of stretched horizon grows. It diverges when $Q^2 \to 2$ since Hawking temperature at ∞ approaches Hagedorn temperature.

In this limit have small black hole surrounded by a very large smeared layer at Hagedorn temperature.

$\Rightarrow$ Can't tell black hole from gas of fundamental strings with same E

Therefore, as $T_{BH} \to T_{Hagedorn}$, expect

$$S_{BH} \to S_{\substack{\text{fundamental} \\ \text{strings}}}$$

(Bekenstein-Hawking entropy = Hagedorn)

This is indeed the case

Giveon
Kutasov
Rabinovici
Sever

Higher dimensional black holes

Can repeat the discussion. Get following predictions:

i) Euclidean Schwarzschild solution in d-dimensional string theory becomes non-normalizable when $T_{BH} > T_{\text{Hagedorn}}$, due to existence of a winding tachyon condensate.

2) When $T_{BH} = T_{Hagedorn}$, black hole entropy must coincide with fundamental string entropy. This is a more precise version of string/black hole correspondence of Susskind; Horowitz, Polchinski.

For d dimensional Schwarzschild without α' corrections find

$$\left.\frac{S_{string}}{S_{BH}}\right|_{T_{BH}=T_{Hagedorn}} \simeq \frac{d-2}{d-3}$$

Presumably, α' corrections remove discrepancy.