AGING OF CLASSICAL OSCILLATORS DURING A NOISE-DRIVEN MIGRATION OF OSCILLATOR PHASES

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Work in collaboration with F. Ionita (JU), D. Labavic (JU) and M. Zaks (HU, Berlin)

Order-by-disorder in oscillatory systems

> Noise-driven migration of oscillator phases

Aging in classical nonlinear oscillators

P. Kaluza and HMO, Chaos 20, 043111 (2010).

F. Ionita, D. Labavic, M. Zaks and HMO, Eur. Phys. J. B 86(12), 511(2013).

F. Ionita, and HMO, Phys.Rev.Lett.112, 094101 (2014)



Outline: In analogy to spin systems we shall see how



here in excitable and oscillatory systems

CONCEPT AND IMPACT OF FRUSTRATION IN DYNAMICAL SYSTEMS

Physics {	Gauge theories	Field strength	
	General relativity	Curvature	
	Spin glasses Oscillatory systems: phase oscillators, excitable systems	Frustration	
Social systems	Approach to balance	Imbalance	
Communication	Assigning a meaning	Misunderstanding	
Economics	Financial markets	Arbitrage	



G. Mack, Commun.Math.Phys.219, 141 (2001).
& Fortschritte der Physik, 81: 135-185 (1981): Physical Principles,
Geometrical Aspects and Locality of Gauge Field Theories.



THE NOTION OF FRUSTRATION FOR OSCILLATORY AND EXCITABLE SYSTEMS

Criterion for undirected couplings

Consider a loop with undirected interaction bonds and couplings that can be either

attractive or repulsive ferromagnetic or antiferromagnetic excitatory or inhibitory repressive or supportive

Consider a path from A to B along the shortest connection and along the complementary path in the loop from B to A.

The bond from A to B is not frustrated if A acts upon B in the same way as B upon A (e.g. attractive), otherwise it is.



A in phase with B, B with C \rightarrow C with A, but if C wants to be antiphase with A, the link CA or CB is frustrated

Result of Daido: three Kuramoto oscillators coupled in a "frustrating way" lead to multistable behavior (Progr. Theor. Phys. 1987)

CRITERION FOR FRUSTRATION IN CASE OF DIRECTED COUPLINGS IN VIEW OF EXCITABLE SYSTEMS (Kaluza & HMO, Chaos 20, 043111 (2010))

- Consider a loop with directed interaction bonds and couplings that can be either
- repressing or activating
- excitatory or inhibitory

Consider a path from A to B along the shortest connection and along the complementary path in the loop from B to A.

The bond from A to B is not frustrated if A acts upon B in the opposite way as B upon A (e.g. A to B activating, B to A via C and D repressing), otherwise it is.

Different realizations of the frustration

• Via the number of couplings

• Via the type of couplings along with the number



Conjecture on multistability confirmed in coupled genetic circuits

$$\frac{dA_i}{dt} = \frac{\alpha}{1+(B_i/K)} \cdot \left(\frac{b+A_i^2}{1+A_i^2}\right) - A_i$$
$$+\beta_R \sum_{j=1}^N B_{ij} \frac{1}{1+(A_j/K)^2} + \beta_A \sum_{j=1}^N Q_{ij} \frac{(A_j/K)^2}{1+(A_j/K)^2}$$
$$\frac{dB_i}{dt} = \gamma A_i - \gamma B_i , \qquad i = 1, \dots, N.$$

Adjacency matrix of repressing couplings R_{ij}

Adjacency matrix of activating couplings Q_{ii}

Consider most simple motifs with and without frustration for which the frustration is implemented either:

- via the topology (even number of repressing couplings) or
- via the type of coupling (replace repressing by activating ones)

MOST SIMPLE MOTIFS



Frustration on two levels



For the frustrated plaquette we obtain:

$$\begin{aligned} \frac{dA_i}{dt} &= \frac{\alpha}{1 + (B_i/K)} \cdot \left(\frac{b + A_i^2}{1 + A_i^2}\right) - A_i \\ &+ \beta_R \frac{R_{ii+1}}{1 + (A_{i+1}/K)^2} \\ \frac{dB_i}{dt} &= \gamma(A_i - B_i) , \qquad i = 1, 2, 3, 4 \mod 4 \end{aligned}$$

Individual nodes in the oscillatory regime: α =80, β_R =0.01



3 patterns of phase-locked motion:

- 3 different phases out of four or
- 4 different phases or
- 2 different out of four coincide

multistable behavior for β_R =0.01, 0.1

Multistability in synthetic genetic circuits could be explained this way

HERE INSTEAD: CLASSICAL ROTATORS WITH FRUSTRATION

The model: N active rotators

$$\frac{d\varphi_i}{dt} = \omega_i - b\sin\varphi_i + \sigma_A\xi_i(t) + \frac{(\kappa + \sigma_M\eta_i(t))}{\mathcal{N}_i}\sum_j A_{ij}\sin(\varphi_j - \varphi_i).$$



Fig. 1: Hexagonal lattice with all triangles frustrated for all couplings being negative (a) and not frustrated (b) for positive couplings along the horizontal links and negative ones otherwise.

1. N identical oscillators without noise

The phase diagram as a function of ω , b, and κ

The versatility of attractors in comparison to spin systems

A particularly rich attractor space for the 4x4 system

2. N identical oscillators with additive or multiplicative noise

Order-by-disorder repeatedly induced for increasing noise strength

Noise-induced migration of oscillator phases

Indications for a rough landscape with hierarchies in the potential barriers

A multitude of escape times from metastable states

1. N IDENTICAL OSCILLATORS WITHOUT NOISE

The phase diagram as a function of ω and κ for b=1: so far along a few sections, but ongoing work by M. Zaks et al.



Kuramoto case with b=0 separately presented

The versatility of attractors in comparison to spin systems

In spin glasses: fixed points In these systems: various "collective" fixed points a variety of "collective" limit cycles differing by their correlation between individual phases frequency pattern of phase-locked motion basin of attraction stability symmetry quasiperiodic solutions chaotic solutions

as a combined effect of frustration, lattice size and lattice symmetry.

WHAT CAN WE PROVE ABOUT MULTISTABILITY?

Special case: N Kuramoto oscillators. The system can be reduced to

$$\dot{\varphi}_i = -\sum_j A_{ij} \sin(\varphi_j - \varphi_i).$$

Consider as a special set of solutions plane waves with fronts of constant phases along parallel lines on the hexagonal lattice. Their spatial distribution is characterized by

$$\varphi_{m,n} = \frac{2\pi}{M}k_1m + \frac{2\pi}{L}k_2n$$

m=1,...,M, n=1..., L coordinates and k1, k2 integers because of p.b.c. Then it can be shown that for a sufficient large extension M and L and M=L=even, there are always two sets of wave vectors k1=k2=k and k1=k2=k+1, such that the plane waves correspond to different solutions, differing by the number f clusters of coinciding phases.

SIMILARLY FOR ACTIVE ROTATORS, IN PARTICULAR PLANE WAVES AND SPHERICAL WAVES:



The number and variety of attractors is extremely rich already for a 4x4-lattice.

The classification according to p_n-patterns with n denoting the number of clusters of coalescing phases is **not unique**, but suited for our notion of order.

- p4-solutions: 4 clusters, originally only 75 such solutions could be identified, differing by their frequency, in particular plane waves (without counting the degeneracy due to the lattice symmetries), meanwhile a continuum of these solutions
- **p6-solutions**: spherical waves (degeneracy 96 due to 16 sites for the center and 6 rotations about 60 degrees due to the lattice symmetry)
- p16-solutions of individual limit cycles
- quasiperiodic solutions

using the numerically obtained Poincaré-mapping on the hypersurface Φ_1 = const.

Of particular interest: p4-patterns of 4 clusters with 4 identical phases each (ongoing work by M. Zaks et al.)

Same color – same phase

Note: each oscillator is coupled by two links to representatives of all other three clusters. If we do not distinguish the individual members of a cluster, we see a global coupling between the clusters of identical members with double the strength than on the original fine-grained lattice



The set of 16 equations reduces into 4 sets (one for each color) of 4 identical equations . The representative set of four equations describes a set of globally coupled set of identically equipped oscillators and with sinusoidal coupling. According to **Watanabe and Strogatz** we should expect an infinite number of conserved quantities and a continuum of frequencies for our limit-cycle solutions \implies dynamically generated reduction of d.o.f.

2. N IDENTICAL OSCILLATORS WITH ADDITIVE OR MULTIPLICATIVE NOISE

$$\frac{d\varphi_i}{dt} = \omega_i - b\sin\varphi_i + \sigma_A\xi_i(t) + \frac{(\kappa + \sigma_M\eta_i(t))}{\mathcal{N}_i}\sum_j A_{ij}\sin(\varphi_j - \varphi_i).$$

ORDER-BY-DISORDER, IN WHAT SENSE?

Usually: Order-by-disorder is considered in spin systems.

Generic: The ground state is degenerate due to competitions among the interactions. The degeneracy is lifted due to disorder.

The lifting can be temperature driven (Villain et al. J. Phys. 1980, Bergman et al., Nature Physics 2007) or quantum driven (Chubukov, PRL (1992), Reimers et al. PRB (1993)) or due to dilution (Henley PRL 1989)

The effect is observed in classical spin models, quantum magnetism, and in ultracold atoms (Turner et al., PRL98, 2007). It depends on the degree of degeneracy whether the effect is observed.

Order-by-disorder in classical oscillatory systems:

Disorder: additive noise or multiplicative noise

Order: the "degree" of synchronization: either a disordered stationary solution with all units oscillating with their own phase changes towards a solution with partially coinciding phases, or, the number of phase-synchronized clusters decreases, so that more phases coincide for an intermediate noise strength

IN NEED FOR A SUITABLE ORDER PARAMETER that can distinguish between "order" in the sense of how many phases coincide. Generalized Kuramoto order parameters are not suited

$$\rho_n = 1/N \sum_{j=1}^N \exp i n\varphi_j$$

Use the peak structure of histograms instead for larger sizes.



σ_A	panel	ρ_1	ρ_2	$ ho_4$	$ ho_6$
0.00	a	0.001	0.102	0.060	0.149
0.01	b	0.002	0.279	0.300	0.339
0.02	с	0.003	0.868	0.514	0.104
0.03	d	0.005	0.489	0.803	0.414
0.04	е	0.006	0.420	0.503	0.366
0.05	f	0.007	0.372	0.334	0.393
0.06	g	0.009	0.838	0.461	0.235
0.07	h	0.009	0.280	0.302	0.258
0.08	i	0.011	0.767	0.303	0.227
0.09	j	0.012	0.644	0.308	0.417
0.10	k	0.013	0.787	0.346	0.224
1.00	1	0.122	0.256	0.223	0.222

The table illustrates that ρ_n does not work in all cases as compared to the number of coalescing phases on the phase plots.

WE SEE REPEATEDLY ORDER-BY-DISORDER IN THE FOLLOWING SENSE:



Fig. 3: Order by disorder on a 4×4 lattice, for $\omega = 0.7$ b = 1, $\kappa = -2$, $\sigma_M = 0$ and monotonically increasing noise intensity σ_A between panels (a) to (l). For further explanations see the text.

Here: Fixed identical initial conditions, but increasing the noise intensity. The snapshots are representative for a certain time interval of some hundred or thousand time units, afterwards the patterns of synchronization may have changed.

Note: Different from the action of noise in coherence resonance:

Coherence resonance:

For an intermediate strength of noise, oscillatory response in an excitable system is most coherent (here no external field)

System size resonance

For an optimal size of the system, the system's response is most regular, illustrated in a system of coupled nonlinear noisy oscillators (ensemble averages fluctuate with $(D/N)^{1/2}$)



D the (effective) noise intensity





Low noise

lattice of 10x10 active rotators clusters 100—10--100

Varying the noise intensity between 0.0001-0.1

Intermediate noise



Strong noise

Zoom into the noise intensity:

Increasing monotonically the noise intensity from left to right and top to bottom one observes disorder (d) and order (o) as the sequence:

d o d o d o d

The system is very sensitive to the initial condition.

Varying the noise intensity between 0.001-0.1 in steps of 0.01



lattice of 10x10 active rotators clusters 100-10-100-10-100-10---100

Zoom further into the noise intensity:

Increasing monotonically the noise intensity from left to right and top to bottom, one observes disorder (d) and order (o) as the sequence:

d o d o d



lattice of 10x10 active rotators clusters 100-10-100-10-100

Varying the noise intensity between 0.06-0.07 in steps of 0.001





Low noise

Lattice of 4x4

Kuramoto oscillators clusters 16–2–16

Intermediate noise



How representative are these plots? Similar plots are obtained

- if the noise realization is varied
- > the additive noise is replaced by multiplicative one
- > the oscillators are reduced to Kuramoto oscillators
- the lattice size is increased
- the time windows for snapshots over 200 time units are varied.

What is the explanation for the repeatedly increasing order when the noise strength is monotonically varied?

The system obeys a gradient dynamics with potential V. The resolution of its shape depends on the noise intensity.

$$V = -\omega \sum_{i} \varphi_{i} - b \sum_{i} \cos \varphi_{i} - \frac{\kappa}{2N} \sum_{i,j} A_{ij} \cos(\varphi_{j} - \varphi_{i}).$$

Consider the oscillatory part:



barriers of different height in the energy landscape

Due to noise, the "fine-structure" of the "potential" can no longer be resolved, but still the overall structure, while for even stronger noise the shape of the deterministic potential gets buried under the noise. From zooming into the noise intervals: The energy landscape seems to have a similar structure on different scales of resolution.

EXPLANATION OF SLOW DYNAMICS AND AGING IN SPIN GLASSES



Hierarchical structure of the metastable states as a function of temperature From E. Vincent et al. arXiv: cond-mat/9607224



Subthreshold periodic perturbation, does not allow the particle to leave any of the four local minima without noise but with noise it triggers the switching between the different minima.



Fig. 9: Stochastic resonance for a potential (a) with two barrier heights: Panels (b) - (f) show the value of the force (in blue) and the response of the system (in red) as functions of time, for different noise intensities $\sigma = 0.22, 0.53, 0.84, 1.22$, and 2.0. For further explanations see the text.

NOISE-DRIVEN MIGRATION OF OSCILLATOR PHASES

On a longer time scale we see the following "stationary" state keeping the noise intensity fixed:



0-200 time units







600-800







1400-1600





4200-4400







Escape via an unstable limit cycle (unstable also without noise)

So the "stationary state" in the presence of noise is characterized by ongoing transitions between the different pattern of phase locked motion, characterized as p4, p6,p16, disordered, or transient p3 states.



9600-9800 and so on for ever

What is a time-independent feature of the stationary state?

The distribution of escape times from a 16- to a 4-cluster solution

- Consider an initial condition that leads to a 16-cluster solution .
- For a fixed noise intensity, here σ=0.05, solve the differential equation for 1000 noise realizations.
- For each solution, the first escape to another attractor occurs when V_{osc}>-6.5 (numerically verified).
- Register this event of crossing the potential threshold if there are at least 50 time units between two crossings.



The histogram depends on the selection of metastable states, so far also on the overall time span: first escape during the first 10000 or the second 10000 time units

What might be called "**stationary**" in the sense of being time independent is only the feature of the multi-peak structure of the histogram of escape times , for a given noise strength.

INTERPRETATION OF THE MULTI-PEAK STRUCTURE AND THE ITERATED ORDER AND DISORDER

The multi-peak-structure seems to be typical for situations, where unstable limit cycles separate stable attractors. A multi-peak structure was predicted for a two-variable system with stochastic transitions through an unstable limit cycle[1].

Moreover, if two stable attractors are separated by an unstable limit cycle, for low noise the escape rate should be modulated by an oscillatory factor [2]. This may explain why we see as a function of a monotonically increasing noise strength more or less escapes to other metastable states. Our system is high-dimensional.

[1] A.L.Kawczynski et al., Phys.Chem.Chem.Phys. 10,289 (2008)

[2] R.S.Maier etal., Phys.Rev.Lett.77 4860 (1996)

SUMMARY SO FAR

Active rotators and Kuramoto oscillators on a hexagonal lattice with frustrated bonds

- show a large number and a variety of coexisting attractors.
- Under noise we see the ongoing migration of phases through the potential landscape.
- The escape times between the metastable states define a multitude of time scales.



We expect to see aging of these oscillators.

DEFINING CRITERIA FOR PHYSICAL AGING (IN CONTRAST TO BIOLOGICAL AGING):

Do relaxation processes towards the stationary state show

- **slow dynamics** (slow in the sense of non-exponential relaxation). The slow dynamics would be visible after a quench into the regime of multistable states in appropriate correlation functions.
- breaking of time-translation invariance. If we distinguish between the waiting time t_w after a quench when a measurement of an observable starts, and the observation time t, when the observable is measured again, at t > t_w, an observable such as the (auto)correlation function depends on both times.
- dynamical scaling. Dynamical scaling is observed if the individual curves for the correlation functions can be superimposed onto a single master curve by an appropriate rescaling of the argument, depending on t and t_w. If we observe dynamical scaling for a given excitable or oscillatory system, the question arises of how universal this scaling behavior is between different realizations of such systems, differing by their individual dynamics.

MEASURE AGING OF ACTIVE ROTATORS AND KURAMOTO OSCILLATORS

via the autocorrelation functions:

- **Prepare the system in the vicinity of the unique fixed point at \kappa > 0.**
- Quench the system towards κ < 0 in the regime of coexisting synchronized oscillations to push it out-of-equilibrium.</p>
- Wait and let it evolve under the action of additive noise.
- Perform a first measurement of the autocorrelation function at time t_w.
- Perform a second measurement at time t > t_w

for two lattice sizes (32x32 and 4x4) and three noise intensities $\sigma = 0.01$, 0.1, 0.5 with and without frustration.

Note that temperature in spin systems plays a twofold role: driving the transition to a phase with multistable behavior and providing fluctuations. In our case the coupling provides the bifurcation parameter from one phase into the other and the noise creates the fluctuations.

The state of the system at time t is specified by the vector of all phases

$$\vec{\phi}$$
 = ($\Phi_1(t), \Phi_2(t),, \Phi_N(t)$).

We compute the two-time autocorrelation function defined as

$$C(t,t_w) := \frac{\langle \vec{\phi}(t) \vec{\phi}(t_w) \rangle - \langle \vec{\phi}(t) \rangle \langle \vec{\phi}(t_w) \rangle}{\sigma_t \sigma_{t_w}}$$

with standard deviations
$$\sigma_t^2 = \langle \vec{\phi}(t)\vec{\phi}(t) \rangle - \langle \vec{\phi}(t) \rangle \langle \vec{\phi}(t) \rangle$$
,

where the averages are calculated over a sufficient number of noise realizations.

AGING OF ACTIVE ROTATORS: 4x4 AND 32x32 WITH NOISE



DYNAMICAL SCALING OF ACTIVE ROTATORS FOR 4x4 AND 32x32



 $t_w^{|b|}C(t,t_w) = f(\frac{t}{t_w})$

 $b \in [0, 0.02] (4 \times 4) \text{ and } b \in [0, 0.03] (32 \times 32)$

AGING FOR KURAMOTO OSCILLATORS ON 4X4 AND 32X32 LATTICES



ACTIVE ROTATORS ON A 32x32 LATTICE WITHOUT FRUSTRATION BUT WITH NOISE



So no dependence on the waiting time t_w , neither for the 4x4 case.

Summary of the results on aging:

We do see physical aging

even for a very small system size of 4x4 oscillators with a short transient time, but a very rough and structured attractor landscape

> also for larger systems

- for classical rotators and Kuramoto oscillators
- for a different choice of autocorrelation functions
- No signatures of aging for a system with disorder in the coupling signs, but no frustration.
- The mechanism seems to be the same as for spin glasses, but the attractor landscape is much more versatile in which the phases continue to move from one metastable state to another.

OUTLOOK TO NEXT STEPS

- Other manifestations of aging in the response to external forces
- Aging in other oscillatory systems like genetic circuits
- Predictions of aging and scaling behavior in simpler models
- Questions about universality w.r.t. the exponents
- Memory and rejuvenation effects
- Determination of "critical ages"

in view of the following more interesting questions:

What are different aging mechanisms?

Here we have found no new mechanism, but a very different realization of the same mechanism that is acting in spin glasses.

What is the role of noise in aging? (Montemurro et al. PRE67,031106 (2003)) There Hamiltonian of oscillators like ours, no noise, no frustration, no disorder, but for a particular family of initial conditions in the limit of infinite range couplings and $N \rightarrow \infty$ before $t \rightarrow \infty$.

What is the relation between physical aging and biological aging? physical aging in the sense of age dependent response to perturbations biological aging e.g. in the sense of deterioration of pacemaker cells.

Can physical aging of "soft matter" contribute to biological aging of cells and whole organisms?

What are independent sources and independent aging mechanisms on different biological scales, are the mechanisms the same?

Our excitable and oscillatory systems have applications to biological systems.

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Invited speakers

Topics include

Michael Assaf * Racah Institute of Physics, Hebrew University of Jerusalem Jan Benda * Eberhard Karls University, Tübingen Michael Breakspear * QIMR and UNSW Berghofer, Sidney, Australia Thierry Emonet * Yale University, New Haven, U.S.A. **Tobias Galla *** Manchester University, Manchester Jordi Garcia-Ojalvo 🏞 Universitat Pompeu Fabra, Barcelona **Benjamin Lindner*** Humboldt University, Berlin Wolfgang Maass * Graz University of Technology, Graz **Ralf Metzler*** Potsdam University, Potsdam Sidney R. Nagel The University of Chicago, U.S.A Simone Pigolotti* Universitat Politecnica de Catalunya, Barcelona Joachim Rädler * Ludwig Maximilians University, Munich Jaime de la Rocha * Institut D' Investigacions Biomédiques August Pi i Sunyer, Barcelona Lutz Schimansky-Geier* Humboldt University, Berlin Susanne Schreiber * Humboldt University, Berlin Pieter Rein Ten Wolde * FOM Institute AMOLF, Amsterdam Raúl Toral * IFISC, UIB-CSIC, Palma de Mallorca

The effects of extrinsic noise on cellular decision making Correlated fluctuations in genetic networks Propagation of noise of sequential gene regulation Error rates in biological copying Single cell response and decision making under noise The generation of transcriptional noise in bacteria Costs and benefits of biochemical noise Noise in large-scale cortical rhythms Noise-induced order in collective neural populations Computations by noisy networks of spiking neurons Noise and irregular firing in cortical circuits Cellular mechanisms of temperature-compensation in receptor neurons Coherent noise, scale invariance and intermittency in

large systems

Shaping noise for population success

Quasi-cycles induced by noise

Interplay between noise and delay

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We invite applications from graduate students, PhD students and postdocs with a background in theoretical aspects of physics, neuroscience, or biology. Applicants from the experimental side should be interested in the mathematical modeling and analysis of experimental data. For further details please visit: https://www.iacobs-university.de/noise



*=confirmed

Application deadline: April 30, 2014 Contact: <u>s.meier@jacobs-university.de</u>



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