# A boundary-induced transition in chains of coupled oscillators

**Antonio Politi**

Department of Physics, University of Aberdeen, UK

Florence, 27 May 2014

# Collaborators

Stefano Iubini (ISC-CNR Florence)

Stefano Lepri (ISC-CNR Florence)

Roberto Livi (Dept. of Physics, Florence)

# Outline

- The Discrete NonLinear Schrödinger (DNLS) equation: non-equilibrium stationary states.

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- Coupled-transport processes at zero temperature.
- The XY model: extensive analysis
- Back to the DNLS

# The DNLS model

$$\mathcal{H} = \sum_n \left[ |z_n|^4 + z_n^* z_{n+1} + z_n z_{n+1}^* \right]$$

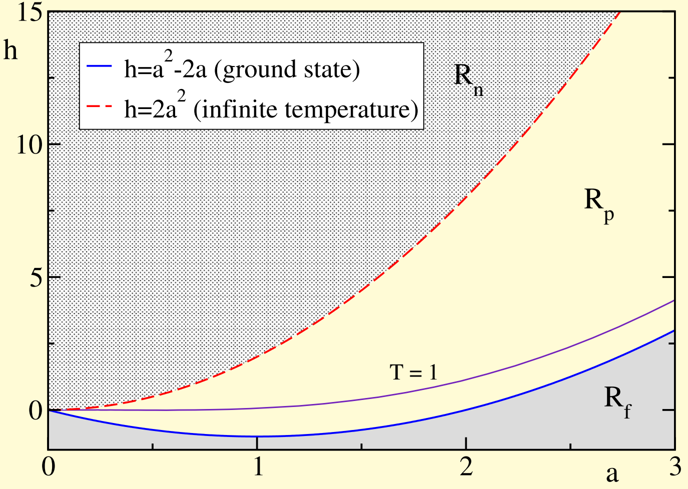
$$i \frac{dz_n}{dt} = 2|z|^2 z_n + (z_{n-1} + z_{n+1})$$

TWO CONSERVATION LAWS

Energy: $\mathcal{H}$ Mass: $A = \sum_n |z_n|^2$

$h$: energy density $a$: mass density

# DNLS equilibrium phase diagram (Rasmussen, 2001)



# Operative definition of the relavant observables

$$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial H} \qquad \frac{\mu}{T} = -\frac{\partial \mathcal{S}}{\partial A}$$

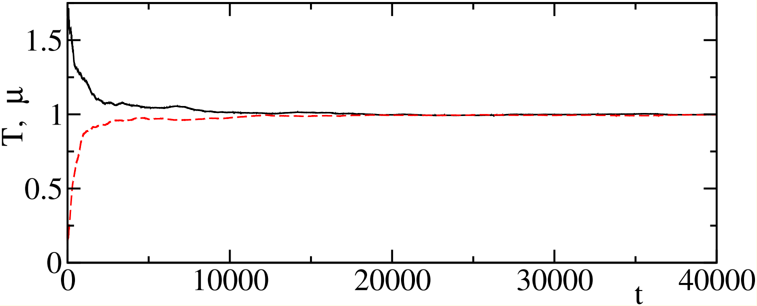
$$\frac{\partial \mathcal{S}}{\partial C_1} = \left\langle \frac{W\|\vec{\xi}\|}{\vec{\nabla} C_1 \cdot \vec{\xi}} \vec{\nabla} \cdot \left( \frac{\vec{\xi}}{\|\vec{\xi}\| W} \right) \right\rangle$$

$$\vec{\xi} = \frac{\vec{\nabla} C_1}{\|\vec{\nabla} C_1\|} - \frac{(\vec{\nabla} C_1 \cdot \vec{\nabla} C_2)\vec{\nabla} C_2}{\|\vec{\nabla} C_1\| \|\vec{\nabla} C_2\|^2} \; ; \; W^2 = \sum_{j<k}^{2N} \left[ \frac{\partial C_1}{\partial x_j} \frac{\partial C_2}{\partial x_k} - \frac{\partial C_1}{\partial x_k} \frac{\partial C_2}{\partial x_j} \right]^2$$

$$C_{1,2} = H, A$$

$z_n = x_{2n} + i x_{2n+1}$ (Franzosi 2011, Iubini et al. 2012)

# Positive temperatures: numerical check

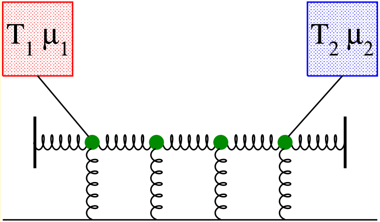



Length 50

System in contact with two Monte Carlo thermostats

$$\exp[-(\Delta H - \mu \Delta A)/T]$$

# The non-equilibrium setup: steady states




Fluxes

$$J_a(n) = 2\,(p_{n+1}q_n - p_n q_{n+1}) \qquad J_h(n) = -\,(\dot{p}_n p_{n-1} + \dot{q}_n q_{n-1})$$

$$z_n = p_n + iq_n$$

# Langevin dynamics

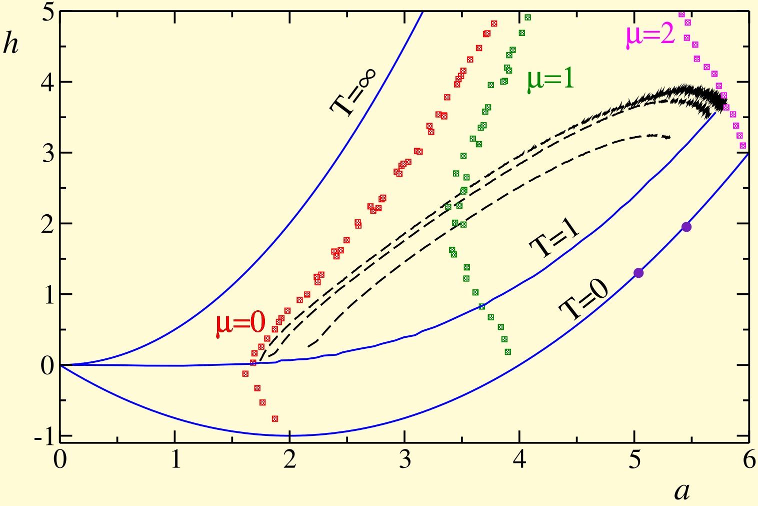
$$i\dot{z}_n = (1+i\gamma)\left[-2|z_n|^2 z_n - z_{n+1} - z_{n-1}\right] + i\gamma\mu z_n + \sqrt{\gamma T}\,\xi_n(t)$$

$\xi_n(t) = \xi'_n + i\xi''_n$

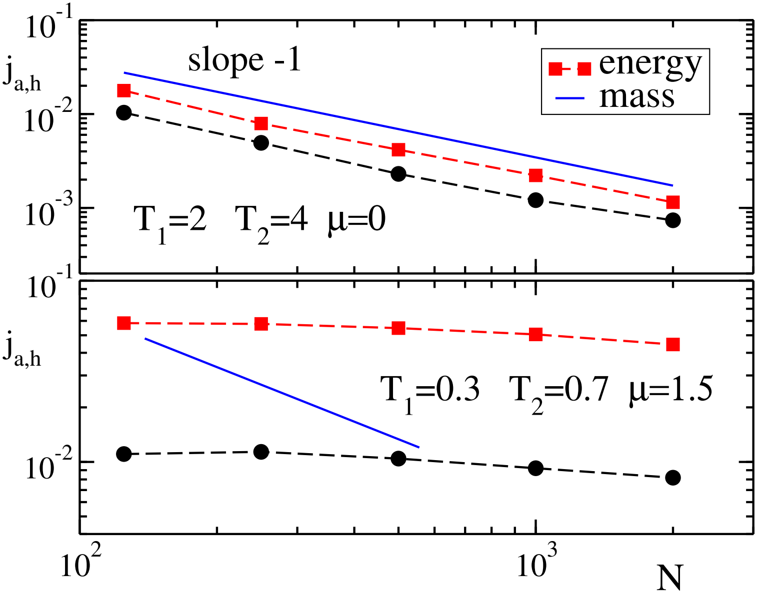
$$\begin{aligned}
\dot{p}_n &= -\frac{\partial H}{\partial q_n} - \gamma\frac{\partial H_\mu}{\partial p_n} + \sqrt{2\gamma T}\xi'_n(t) \\
\dot{q}_n &= \frac{\partial H}{\partial p_n} - \gamma\frac{\partial H_\mu}{\partial q_n} + \sqrt{2\gamma T}\xi''_n(t)
\end{aligned}$$

$H_\mu$ is the effective Hamiltonian $H_\mu = H - \mu A$

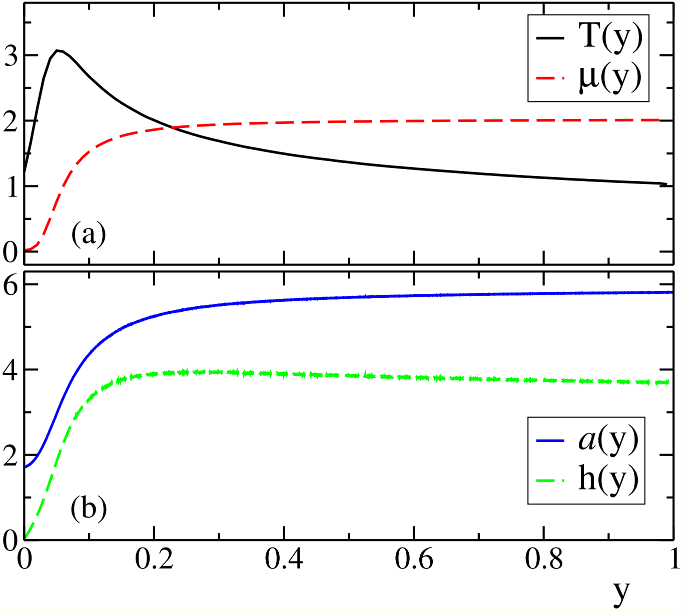
# Non-equilibrium steady states



# Normal transport



# Non-monotonous temperature profile



# Large mass-density limit

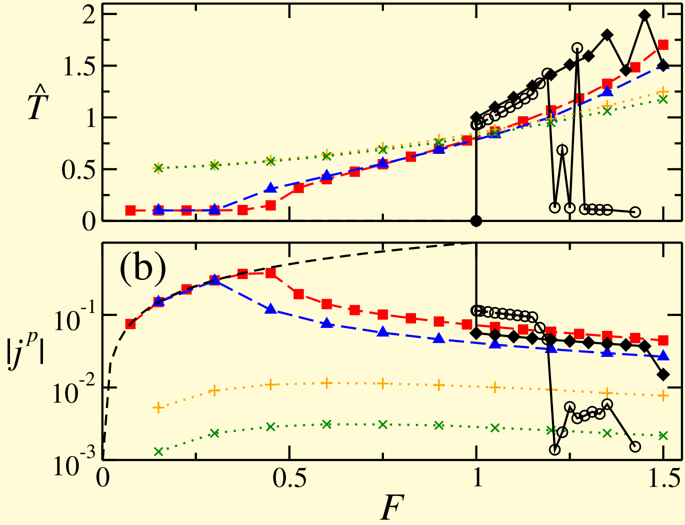
Decompose the variable $z_n$ into $\lambda_n$ and $\phi_n$

$$z_n = \sqrt{a}(1 + \lambda_n/4a)\exp[i(2(a-1)t + \phi_n + n\pi)]$$

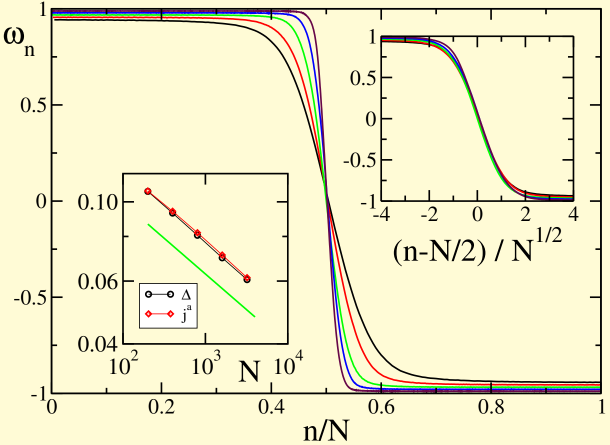
$$\begin{aligned} \dot{\phi}_n &= \lambda_n \\ \dot{\lambda}_n &= 4a\left[\sin(\phi_{n+1} - \phi_n) - \sin(\phi_n - \phi_{n-1})\right] - \gamma'(\lambda_n - \delta\mu) + \sqrt{4\gamma T}\xi_n \end{aligned}$$

$$\mathcal{H}_{XY} = \sum_n \frac{\lambda_n^2}{2} - \sum_n 4a\cos(\phi_{n+1} - \phi_n)$$

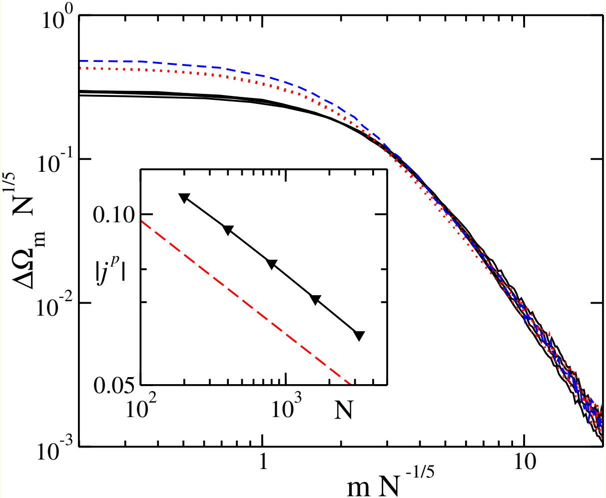
# The XY model



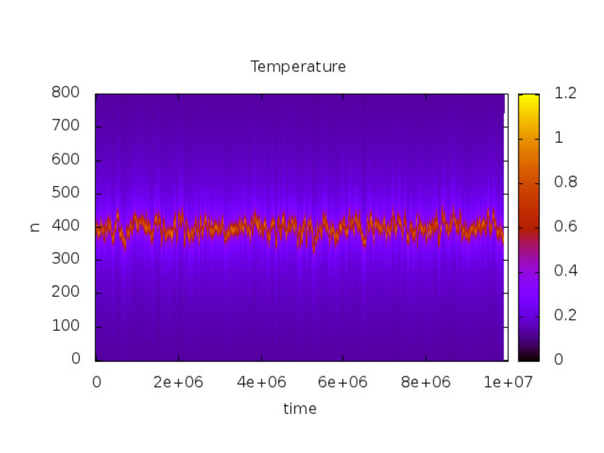
# Frequency profile



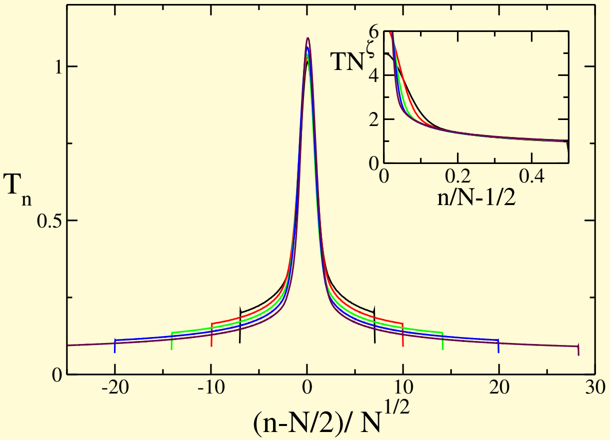
# Frequency gradient



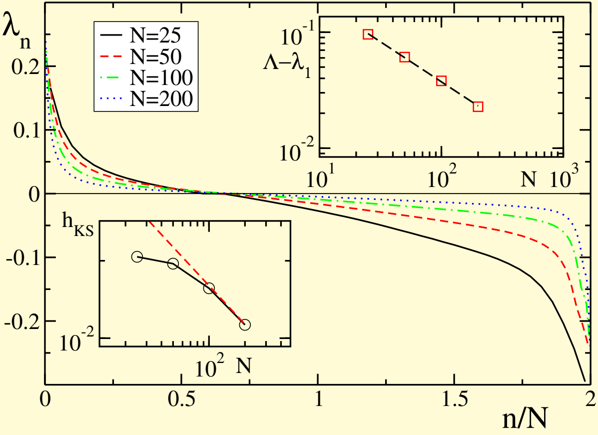
# Space-time pattern



# Temperature profile



# Lyapunov exponents



# Back to DNLS dynamics