# Effects of breaking vibrational energy equipartition on measurements of temperature in macroscopic oscillators

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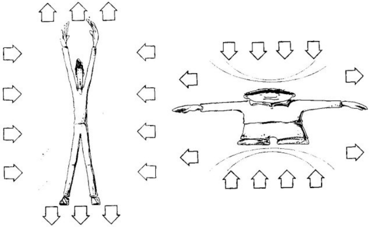
PRL 2009; J. Stat. Mech. 2009 and 2013;
Class. Quant. Grav. 2010; PRB 2011; PRE 2011 and 2012

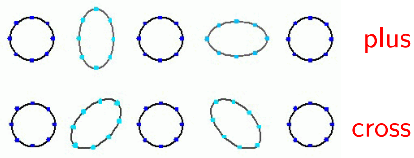
http://www.rarenoise.lnl.infn.it/

# Outline

# Thermal fluctuations unobservable in macroscopic objects?

General Relativity predicts gravitational waves (GW): e.g. accelerating binary systems of neutron stars or black holes; vibrations of black holes or neutron stars.
Hulse-Taylor measurement of orbits of two neutron stars, spiralling as if losing energy by GW emission; in excellent agreement with predictions, were awarded Nobel prize in 1993.
GW: kind of space-time ripples, in two fundamental states of polarization, *cross* and *plus*. Effect of GW on matter: squeezing and stretching, depending on phase.





# The idea which was behind the RareNoise project

## Ground-based Detectors

- Can detect thermal fluctuations intrinsic to the test mass.
- Expected to approach the quantum limit in the future.

## Nonequilibrium stationary states and noise

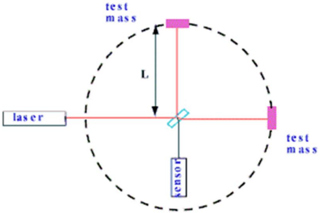
- Past studies had assumed the noise be Gaussian. However the experimentalists' interest is in the tails of the distributions. There, they may be not.

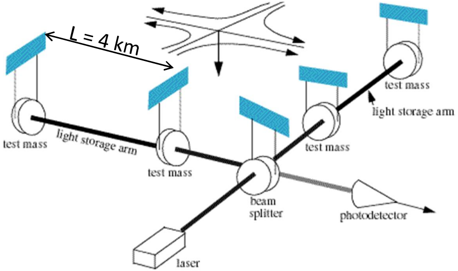
## Then the question

- We detect a rare burst. Is it of an external source? Or false positive due to rare nonequilibrium (and non-Gaussian) fluctuations? Knowing correct statistics is mandatory.

# Gravitational Wave detector

Motivation: GWs will provide new and unique information about astrophysical processes




GW amplitude: $h \sim \frac{1}{2}\frac{\Delta L}{L}$

very tiny effect!!

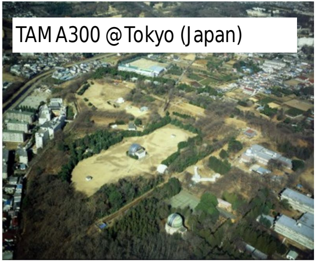
A detection rate of few events/year requires

sensitivity of $h \sim 10^{-22}$ over timescales as short as 1msec

small signal noise ⇒

noise sources must be reduced to very low levels

# GW detectors (interferometers)

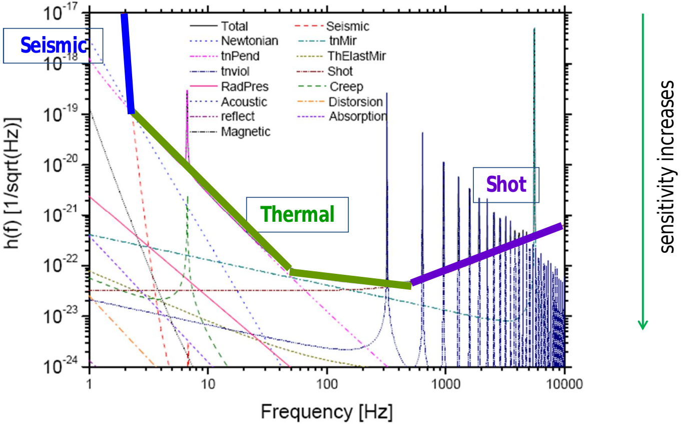





# GW detector noise budget

**Dominant Sources of Noise:**

- seismic noise
- thermal noise
- photon shot noise

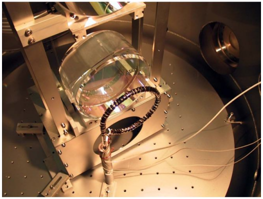


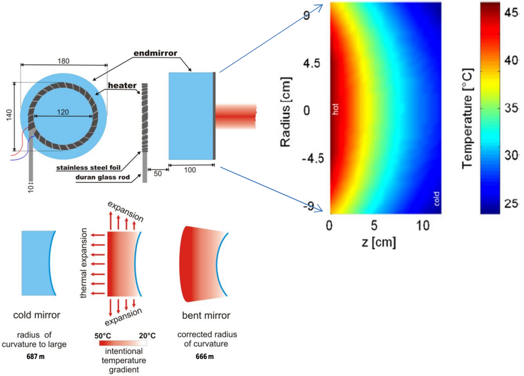
# Thermal compensation

to correct mismatch of the mirror Radius Of Curvature (ROC) due to:
- fabrication
- thermal lensing
- thermo-elastic deformation

} due to absorbed power (up to ~0.5W)

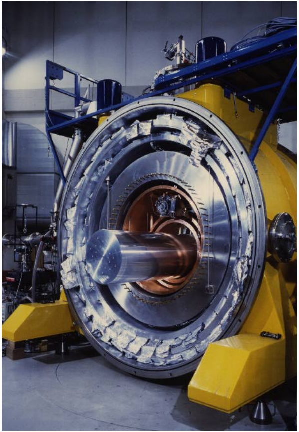
Applied thermal gradient deforms the mirror and corrects the ROC

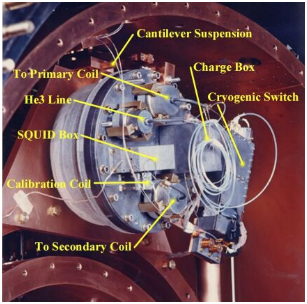





**What is the 'thermal noise' of such a non-equilibrium body?**

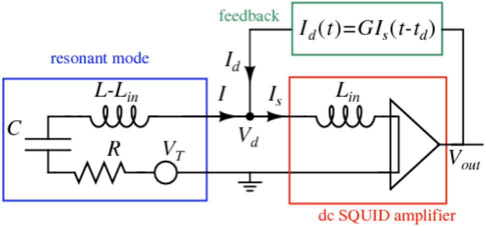
Resonant-bar GW detectors: feedback cooling down to mK: viscous force reduces thermal noise on length of resonant-bar detector AURIGA (PRL top ten stories, 2008).






Steady state modelled by 3 electro-mechanical oscillators with stochastic driving.

$$L\frac{dI_s(t)}{dt} + I_s(t)\left[R + R_d\right] + \frac{q_s(t)}{C} = \sqrt{2k_B T_0 R}\,\Gamma(t)$$




$$I_d(t) = GI_s(t - t_d)$$

$$t_d = \frac{\pi}{2\omega_r}$$

$$G \ll 1$$

$R_d = G\omega_r L_{in}$ expresses viscous damping due to feedback;

No time reversal invariance ($q_s' = q_s$, $I_s' = -I_s$, $t' = -t$), violates Einstein relation, but *formally* identical to equilibrium oscillator at fictitious temperature $T_{\text{eff}} = T_0/(1+g)$

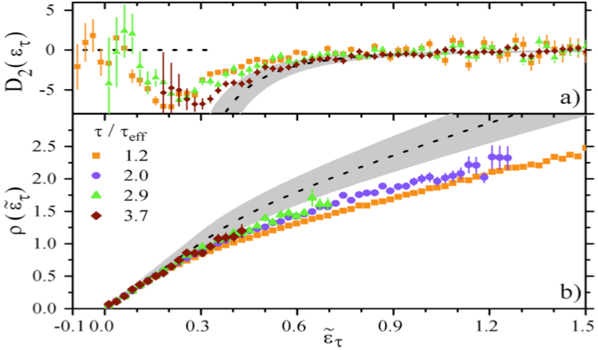
with feedback efficiency $g = R_d/R$, so that: $\langle I_s^2\rangle = 2k_B T_{\text{eff}}/L$

Hence, usually treated as **equilibrium system!**

PDF and fluctuation relation of injected power $P_\tau$: Farago, '02

$$\rho(\tilde{\epsilon}_\tau) = \lim_{\tau\to\infty} \frac{1}{\tau} \ln \frac{\mathrm{PDF}(\tilde{\epsilon}_\tau)}{\mathrm{PDF}(-\tilde{\epsilon}_\tau)} = \begin{cases} 4\gamma\tilde{\epsilon}_\tau, & \tilde{\epsilon}_\tau < \frac{1}{3}; \\ \gamma\tilde{\epsilon}_\tau \left(\frac{7}{4} + \frac{3}{2\tilde{\epsilon}_\tau} - \frac{1}{4\tilde{\epsilon}_\tau^2}\right), & \tilde{\epsilon}_\tau \geq \frac{1}{3}. \end{cases}$$

$\tilde{\epsilon}_\tau = P_\tau L/(k_B T_0 R) =$; $\gamma = (R + R_d)/L$, $T_{\mathrm{eff}} = (22 \pm 1)$ mK



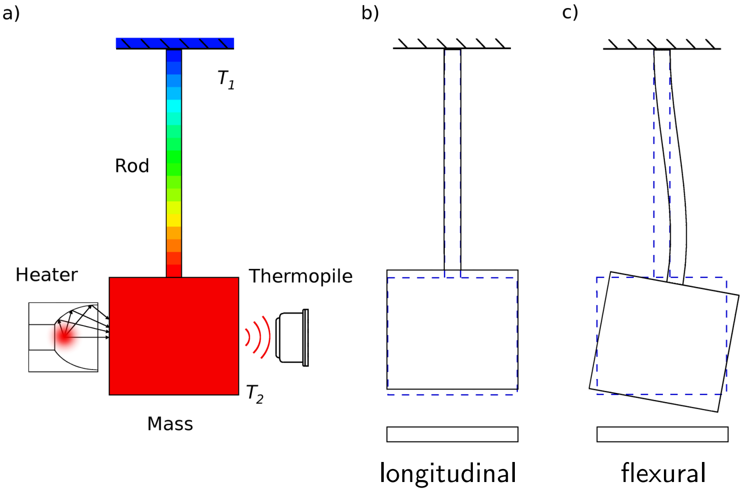

second derivative of PDF;

b) $\rho(\tilde{\epsilon}_\tau)$.

Data from May 2005 to May 2008.

Shades = experimental uncertainty on $\tau_{\mathrm{eff}}$, $T_{\mathrm{eff}}$, $T_0$.

# RN aluminum exp. - longitudinal and flexural oscillations



For macroscopic systems in local thermodynamic equilibrium (LTE)

*"the properties of a 'long' metal bar should not depend on whether its ends are in contact with water or with wine 'heat reservoirs' at temperature $T_1$ and $T_2$"* (Rieder, Lebowitz, Lieb, JMP 1967)

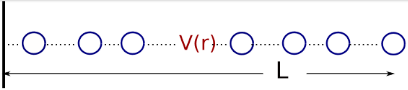
But modelling by 1-dimensional systems incurs in violations of conditions of LTE, hence strong dependence on details of microscopic dynamics: care must be taken in tuning parameters to obtain

"proper thermo-mechanical" behaviour.

Wanted "realistic" equilibrium properties:

**thermal expansion, and temperature dependent elasticity, resonance frequencies and quality factor.**

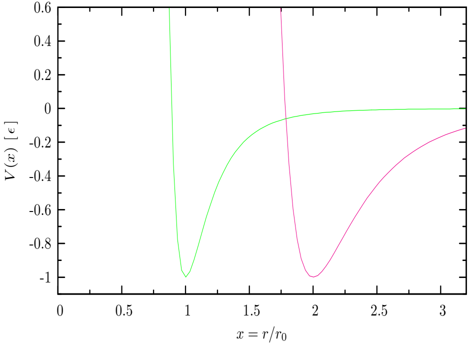
and non-equilibrium: **linear "temperature" profile.**




Deterministic reversible semi-open Nosé-Hoover at $T_1$ and $T_1 + \Delta T$

Nearest- and next-nearest-neighbors L-J. $N = 128, 256, 512$

$$V(r_i, r_{i\pm\ell}) = \epsilon \left[ \left( \frac{\ell r_0}{|r_i - r_{i\pm\ell}|} \right)^{12} - 2 \left( \frac{\ell r_0}{|r_i - r_{i\pm\ell}|} \right)^{6} \right] \quad ; \quad \ell = 1, 2$$




$$m\ddot{r}_i = F_i^{\rm int}(r_i, r_{i\pm1}, r_{i\pm2}) - \chi_i \dot{r}_i$$

$$\dot{\chi}_i = \frac{m}{\tau^2}\left(\frac{K}{k_B T_i} - 1\right) \; ; \; K_i = m\dot{r}_i^2$$

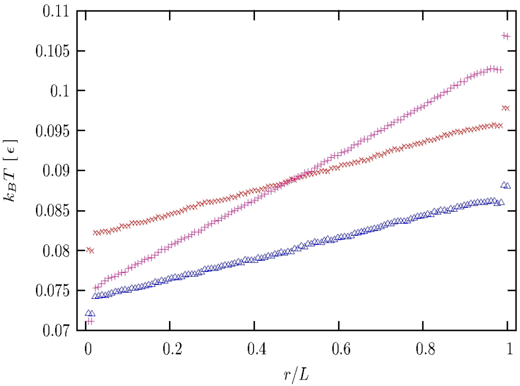
for $i = 1, 2$ and $N-1, N$

$$\chi_i = 0 \quad \text{for} \quad i \neq 1, 2, N-1, N$$

**Looks more like 3D**

Canonical and local canonical appear consistent with observed results from simulations (elasticity etc.)

Kinetic temperature profile straight apart from thermostatted borders, $i = 1, 2, N-1, N$




Maybe better mixing?

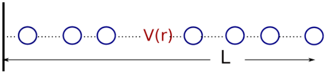
# Spectral density - Experiment and Simulations

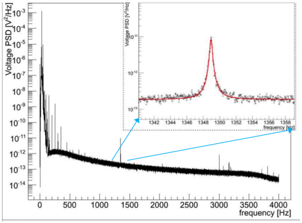

For given $z = z(t)$ real,

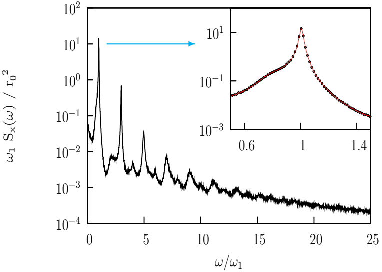
$$S_z(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t} \langle z(t) z(0) \rangle \mathrm{d}t$$

e.g. $z \to x(t) = L(t) - \langle L \rangle$,

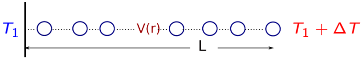
or $z \to v(t) = \dot{x}(t)$; $z \to V(t)$



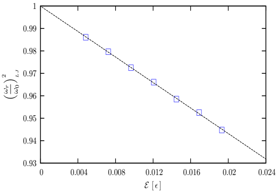



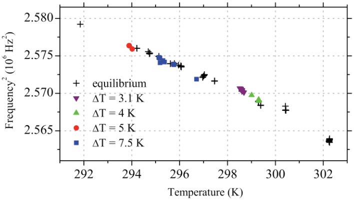



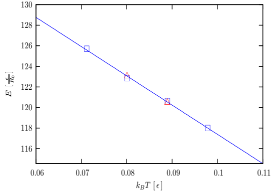
# Equilibrium & Nonequilibrium thermo-elasticity - Exp+Sim

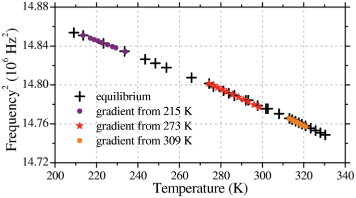






# Equilibrium & Nonequilibrium thermo-elasticity

1) 1D model reproduces thermo-elastic properties at equilibrium, e.g. linearity of elastic modulus E or of $\omega_{res}$ with $T$;

2) It works out of equilibrium as well:
e.g. $\omega_r = \omega_r(\overline{T})$, with average temperature $\overline{T} = (T_1 + T_2)/2$, and $\omega_r(T)$ the equilibrium resonance frequency.

3) Non trivial: for larger $\Delta T$, theory does not apply. Explanation in terms of local canonical,

$$\psi_i = \exp\left(-E_i/k_B T_i\right)$$

i.e. under local equilibrium.

Experiment: low-loss (high quality factor) bar.

⇒ dynamics: independent damped oscillators forced by thermal noise (PSD sum of Lorentzian curves). Equilibrium is canonical and independent of damping.

⇒ normal modes of reduced mass $\mu_i$, resonating at $\omega_i$:

$$H(\mathbf{x},\mathbf{v}) = \frac{1}{2}\sum_i \mu_i(\omega_i^2 x_i^2 + v_i^2) \; ; \quad P(\mathbf{x},\mathbf{v}) = e^{-H(\mathbf{x},\mathbf{v})/k_B T}/Z$$

Experiment: one end fixed and nearly all mass at other end. Hence numerical simulations with $\mu_1 \approx M$. At equilibrium, averaging over $P$:

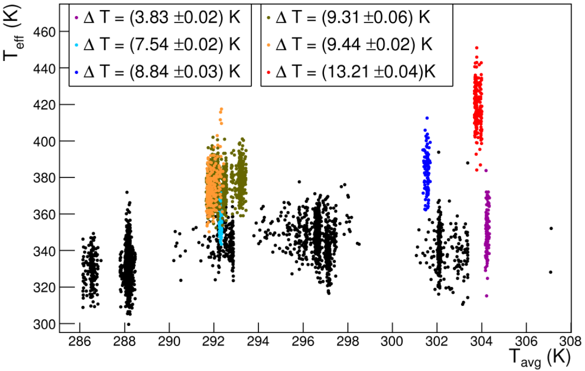
$$\langle x_1^2 \rangle = \frac{k_B T}{M\omega_1^2}$$

i.e. $x_1$ yields a measurement of temperature.

On previous grounds, could one just use $\overline{T}$ in place of $T$, in general, if moderately out of equilibrium?

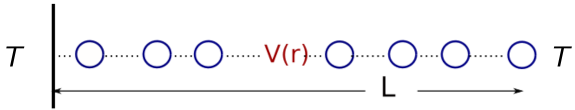
# Experiment says NO

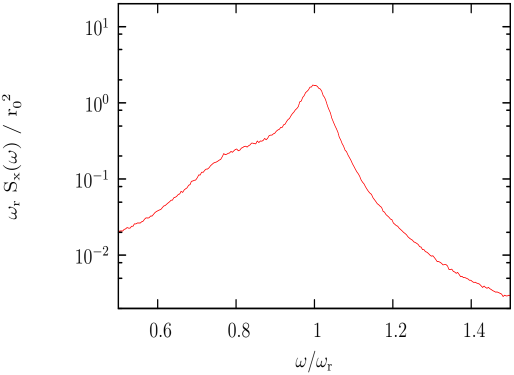

$$\langle x^2 \rangle = \frac{k_B T_{\rm eff}}{m\omega^2}$$



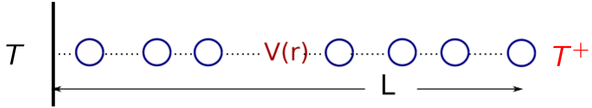

For growing gradients $\overline{T}$ separates from $T_{eff}$ given by spectrum!

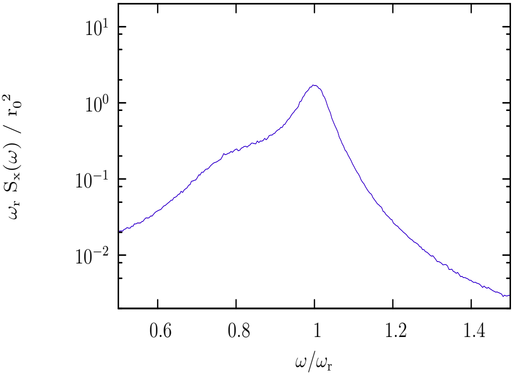
# Simulations



# Effects of growing gradients: $\nabla T \uparrow$ at same $\overline{T}$



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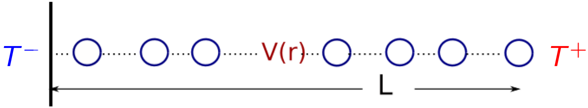


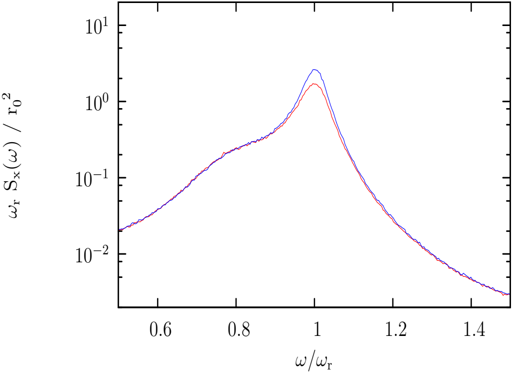

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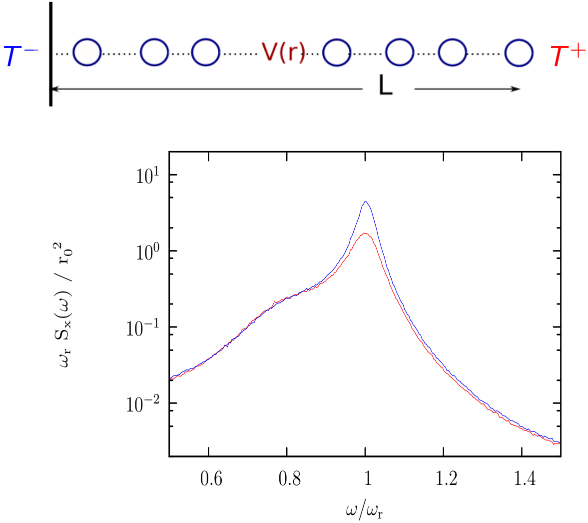

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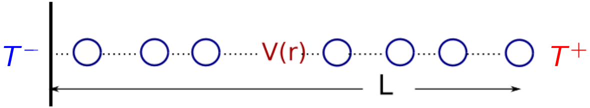


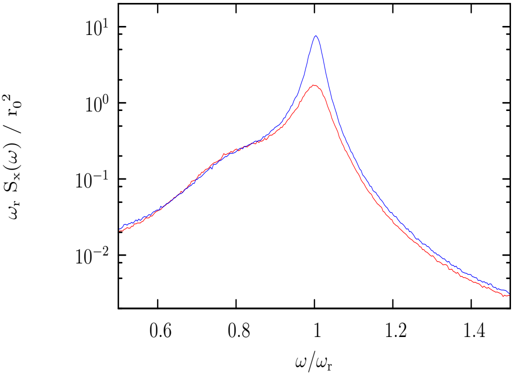

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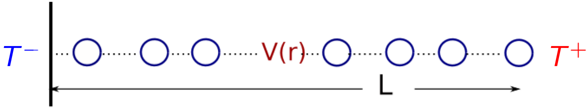
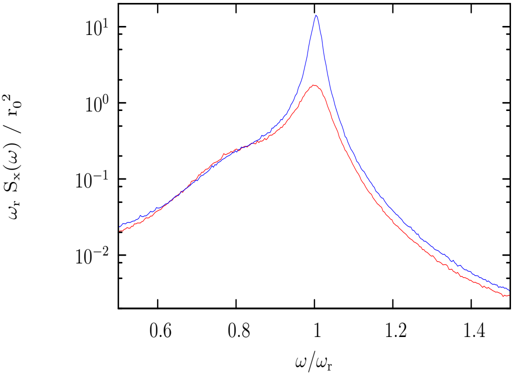



To be published

# Mode-mode correlations

In 1D models, a current $J \neq 0$ means $\langle x_i v_j \rangle \neq 0$ for some $i, j$

Hyp.: For steady state, in canonical ensemble (under harmonic approximation), Jou et al. $\boxed{\beta H \Rightarrow \beta H + \gamma J}$ with

$$e^{-\beta H(\mathbf{x},\mathbf{v})} \Rightarrow e^{-\beta H(\mathbf{x},\mathbf{v}) - \gamma J(\mathbf{x},\mathbf{v})}; \quad \text{with} \quad J = -\frac{1}{N} \sum_{i \neq k}^{1,N} j_{ik} \, x_i v_k$$

$\gamma$ = Lagrange multiplier of heat flux. $J \propto \nabla T$ for small $\nabla T$.

Guess $\beta$ and make even simpler, more general, assumption on $x_i v_k$:
if $w$ is one velocity correlated with $x_1$, consider:

$$P_{NEQ}(x_1, w) = \exp\left(-M\omega_1^2 \, x_1^2 / 2k_B \overline{T} - \mu \, w^2 / 2k_B \overline{T} + \lambda M \omega_1^2 \, x_1 w\right) / \kappa$$

where

$$\kappa = \frac{2\pi}{\sqrt{M\omega_1^2 \left[\mu/(k_B \overline{T})^2 - \lambda^2 M \omega_1^2\right]}}; \quad \text{and} \quad \overline{T} = (T_1 + T_2)/2$$

$$\langle x_1 w \rangle = \frac{\lambda}{\mu/(k_B \overline{T})^2 - \lambda^2 M \omega_1^2} \; ; \qquad \langle x_1^2 \rangle = \frac{\mu \langle x_1 w \rangle}{\lambda M \omega_1^2 k_B \overline{T}}$$

Introduce

$$\phi = -M\omega_1^2 \langle x_1 w \rangle \; ; \quad \eta = \frac{\mu}{M\omega_1^2 (k_B \overline{T})^2} \; ; \quad \lambda(\phi) = \frac{1 - \sqrt{1 + 4\eta\phi^2}}{2\phi}$$

then

$$\left\langle x_1^2 \right\rangle = \frac{\eta}{\eta - \lambda(\phi)^2} \left\langle x_1^2 \right\rangle^{(eq)} (\overline{T})$$

with limit cases

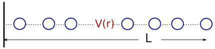
$$\left\langle x_1^2 \right\rangle \simeq \left(1 + \eta\phi^2\right) \left\langle x_1^2 \right\rangle^{(eq)} (\overline{T}) \; , \quad |\phi| \ll 1/\sqrt{\eta}$$

$$\left\langle x_1^2 \right\rangle \simeq \sqrt{\eta} |\phi| \left\langle x_1^2 \right\rangle^{(eq)} (\overline{T}) \; , \quad |\phi| \gg 1/\sqrt{\eta}$$

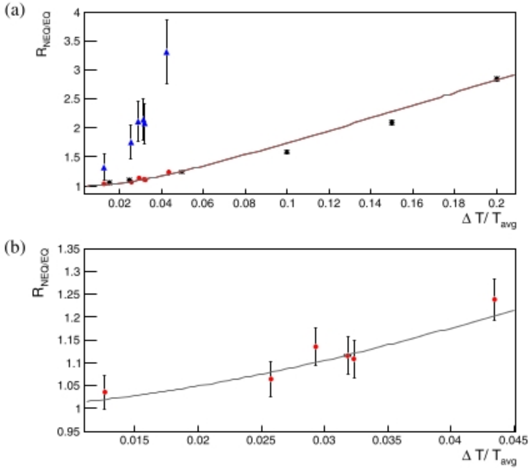
i.e.

$$\frac{\left\langle x_1^2 \right\rangle}{\left\langle x_1^2 \right\rangle_{eq}} - 1 \propto (\Delta T)^2 \; , \quad \Delta T \ll \overline{T}$$

# Simulations and experiment







# Discussion and open questions

- In experiment, normal-mode analysis justified by high $Q$; Fourier law by small gradients;
- experimental data agree with numerical results for such simple model, for thermo-mechanical properties and as well as for vibrational energy of solids, as functions of $\overline{T}$, at small $\nabla T$;
- temperature immediately ceases to be the sole parameter characterizing fluctuations of long-wavelength modes: indeed strong dependence of "$T_{eff}$", i.e. of $\langle x^2 \rangle$, on $\nabla T$;
- **Experiment constitutes protocol to measure value of Lagrange multiplier $\lambda$, the "heatability" of the mode;**
- **dependence on initial conditions?**
- **theory and range of applicability?**