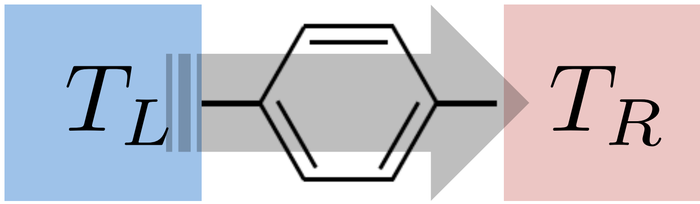
Kondo-Signature in heat exchange process via local two-state system



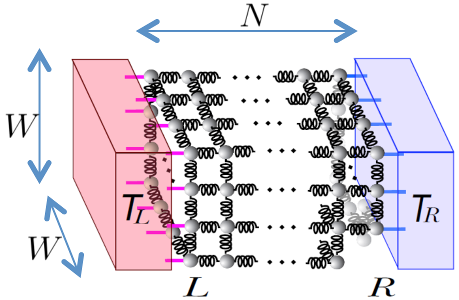
Keiji Saito (Keio University)

Takeo Kato (ISSP)

Phys. Rev. Lett. vol.111, 214301 (2013)

Heat Conduction

◇3D systems



Average current: Fourier law

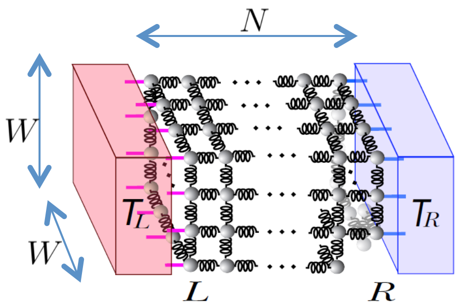
e.g., KS, Dhar, PRL(2010),
Wang, Hu, PRL (2011)

Current fluctuation: MFT

e.g., KS, Dhar, PRL(2011)

Heat Conduction

◇3D systems



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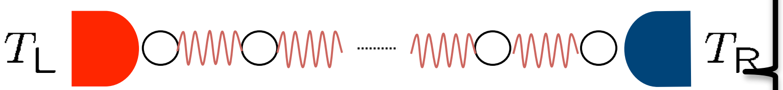
◇1D & 2D systems



Average current: nonFourier law

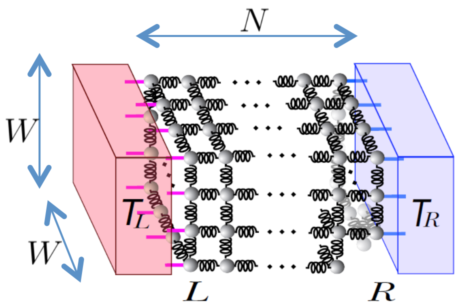
Lepri,Livi,Politi, PRL (1997)

Current fluctuation:

e.g., Dhar, KS, Derrida PRE (2013)

Heat Conduction

◇3D systems



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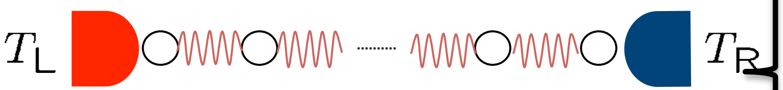


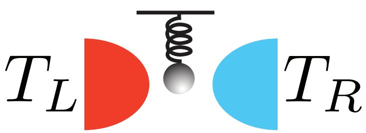
Average current: nonFourier law

Lepri,Livi,Politi, PRL (1997)

Current fluctuation:

e.g., Dhar, KS, Derrida PRE (2013)

◇0D systems



Average current: nontrivial or trivial ??

Current fluctuation: Many studies

Further motivation : Electron–Heat correspondence

Electric conduction vs. Heat conduction

....

▷ Diffusive transport

Ohm's law $I = -\sigma \frac{dV}{dx}$ ⟷ Fourier's law $I = -\kappa \frac{dT}{dx}$

Further motivation : Electron–Heat correspondence

Electric conduction vs. Heat conduction

. . . .

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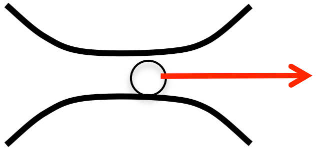
▷ Ballistic transport

Conductance quantum of electric transport ⟷ Conductance quantum of thermal transport

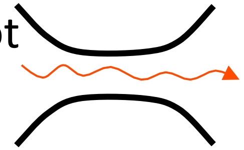
▷ Ballistic transport

Conductance quantum of electric transport



$$I_{el} = \frac{e}{\pi\hbar}\int_{-\infty}^{\infty} d\omega \mathcal{T}(\omega)(F_L(\omega) - F_R(\omega))$$

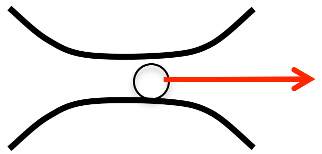
Conductance quantum of thermal transport



$$I_{th} = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T}(\omega)(n_L(\omega) - n_R(\omega))$$

▷ Ballistic transport

Conductance quantum of electric transport



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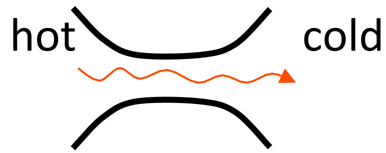


$$I_{th} = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T}(\omega)(n_L(\omega) - n_R(\omega))$$

1. Perfect transmission

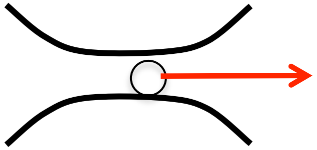
$$\mathcal{T} \longrightarrow 1$$

2. Low temperature

$$T \longrightarrow 0$$

▷ Ballistic transport

Conductance quantum of electric transport



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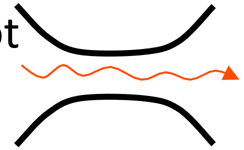

$$I_{th} = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T}(\omega)(n_L(\omega) - n_R(\omega))$$

1. Perfect transmission

$$\mathcal{T} \to 1$$

2. Low temperature

$$T \to 0$$

$$\boxed{\frac{dI_{el}}{dV} \to G_{el} = \frac{e^2}{\pi\hbar}}$$

Experiment: 1988

$$\boxed{\frac{dI_{th}}{dT} \to G_{th} = \frac{\pi^2 k_B^2 T}{3h}}$$

Experiment: 2000

Electric conduction vs. Heat conduction

- Diffusive transport

Ohm's law $I = -\sigma \frac{dV}{dx}$ ⟷ Fourier's law $I = -\kappa \frac{dT}{dx}$

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▷ Controlling

Diode ⟷ Thermal diode (Experiment: 2006)

Electric conduction vs. Heat conduction

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Diode ⟷ Thermal diode (Experiment: 2006)

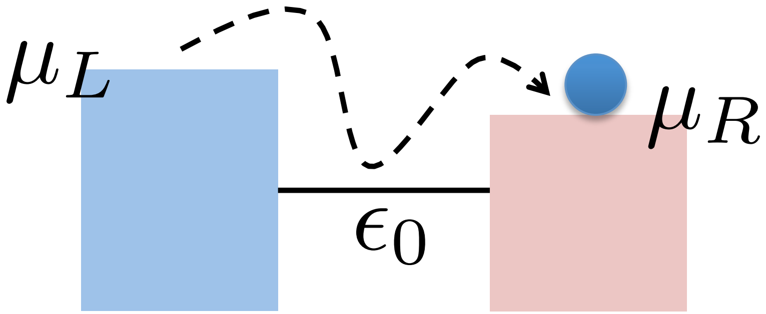
▷ Zero-dimensional transport

Kondo Effect ⟷ ????

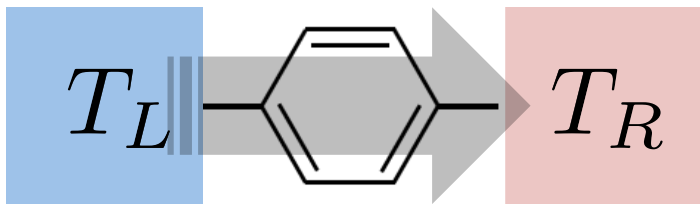
2. Transport via zero-dimensional object

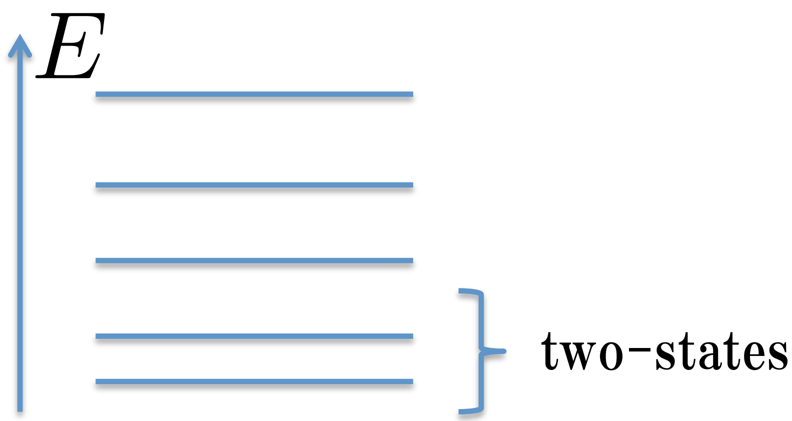
Quantum-dot vs. Spin-Boson

◇Electric transport via Quantum-dot



◇ Thermal transport via local two-level system





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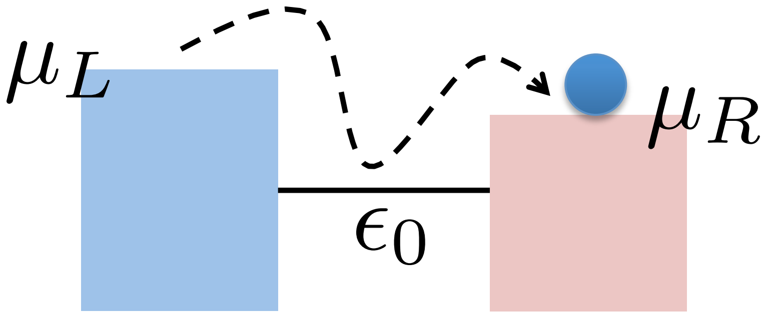

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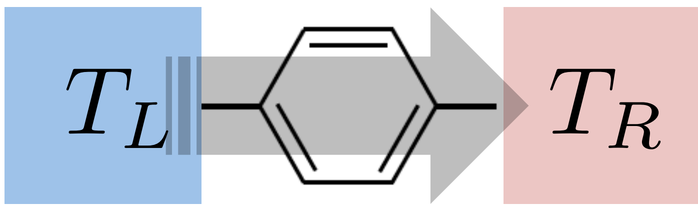
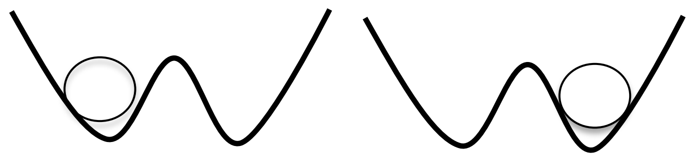


Two-state Hamiltonian

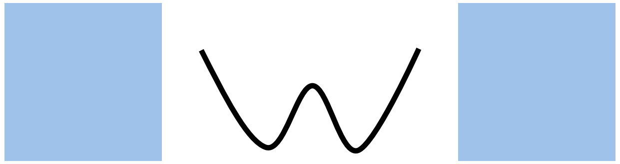


$$H_S = \Delta\sigma_x$$

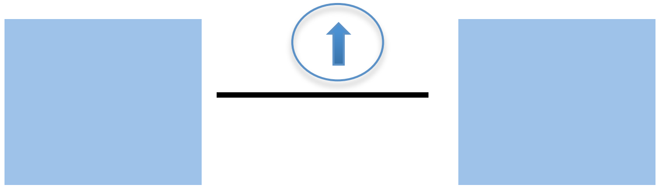
Equilibrium physics of Kondo model and spin–boson model

◇ Kondo model of quantum-dot

◇Spin-Boson model



$$H = \frac{\hbar\Delta}{2}\sigma_x + \frac{\sigma_z}{2}\sum_{\nu=L,R}\sum_k \hbar\lambda_k(b_{\nu k} + b^\dagger_{\nu k}) + \sum_{\nu=L,R}\sum_k \hbar\omega_{\nu k} b^\dagger_{\nu k} b_{\nu k}$$



◇ Ohmic Spectral densiy

$$I(\omega) = \sum_k \lambda_k^2\,\delta(\omega - \omega_k)$$
$$= 2\alpha\omega\,\theta(\omega_c - \omega)\,\theta(\omega)$$

$$H_{AK} = \sum_{\nu=L,R}\sum_{\nu k\,\sigma} c^\dagger_{\nu k\,\sigma} c_{\nu k\,\sigma} + J_\perp \sum_{k,k'}(c^\dagger_{\nu k\uparrow} c_{\nu k'\downarrow} S^- + c^\dagger_{\nu k\downarrow} c_{\nu k'\uparrow} S^+)$$

$$+\frac{J_\parallel}{2}\sum_{k,k'}(c^\dagger_{\nu k\uparrow} c_{\nu k'\uparrow} - c^\dagger_{\nu k\downarrow} c_{\nu k'\downarrow}) S^z$$

Equilibrium physics of Kondo model and spin-boson model

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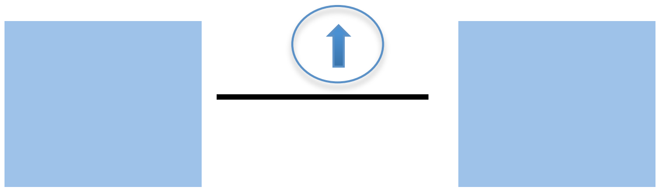
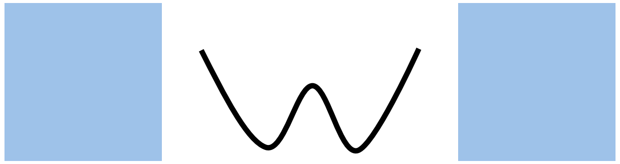


◇Spin-Boson model



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One to one mapping!

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◇Spin-Boson model

$$H = \frac{\hbar\Delta}{2}\sigma_x + \frac{\sigma_z}{2}\sum_k \hbar\lambda_k (b_k + b^\dagger_k) + \sum_k \hbar\omega_k b^\dagger_k b_k$$

$$I(\omega) = \sum_k \lambda_k^2\, \delta(\omega - \omega_k)$$
$$= 2\alpha\omega\, \theta(\omega_c - \omega)\, \theta(\omega)$$

$$\Delta = \rho_0 \omega_c J_\perp$$

$$\alpha = \left[1 - \frac{2}{\pi}\arctan(\pi\rho_0 J_\parallel/4)\right]^2$$

Cf. Legget, Chakravarty, Dorsey, Fisher, Garg, and Zwerger, RMP (1987)

The same Kondo physics in Equilibrium situation

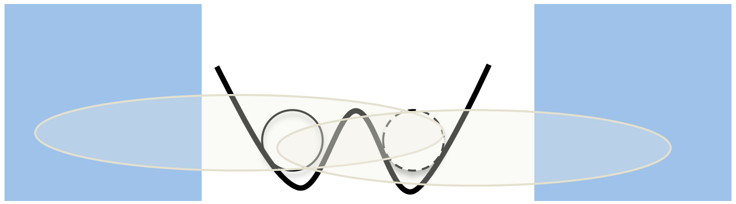
1) Existence of Kondo temperature T_K

2) Formation of highly entangled state

$T \ll T_K$

Highly entangled state
between localized spin in quantum-dot
and electron leads (Kondo-singlet)

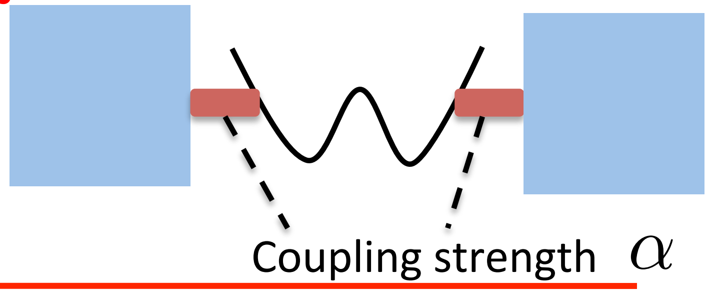
$T \ll T_K$

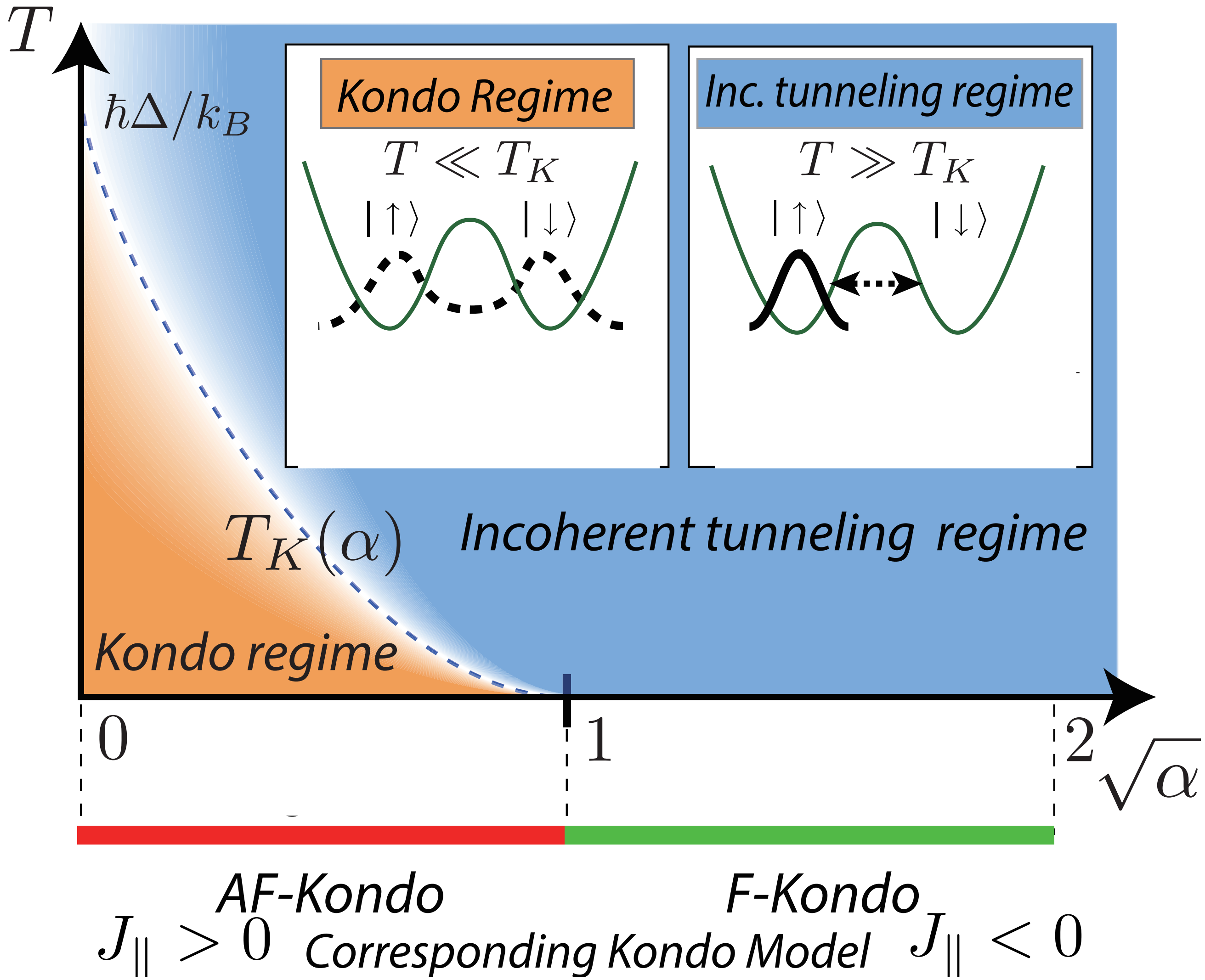


Highly entangled state
between two level system and
bosonic reservoirs

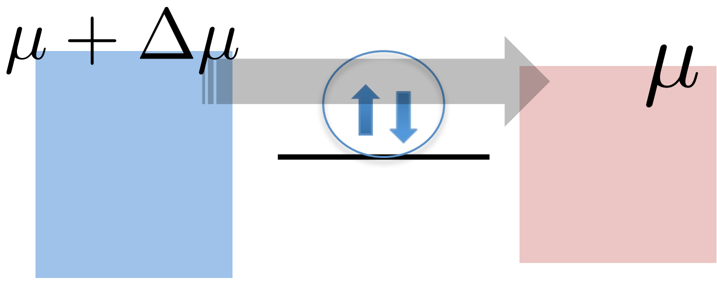
Equilibrium properties in Spin–Boson system

$$T_K = \Delta(\Delta/\omega_c)^{\alpha/(1-\alpha)}$$

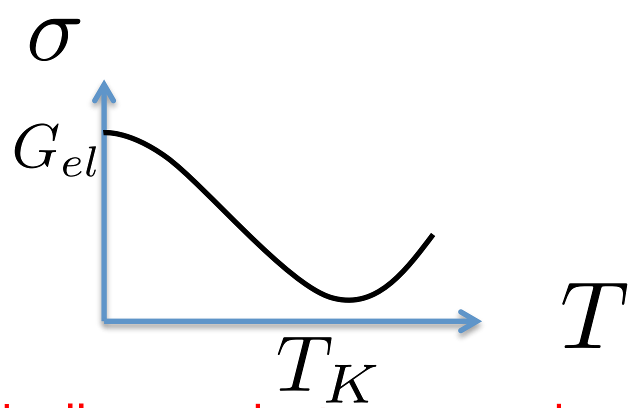




3. Transport arising from Kondo Physics



◇ Enhancement of conductance



◇ Finally conodunctance reaches conductance quantum

$$\sigma \longrightarrow G_{el} \quad T \ll T_K$$

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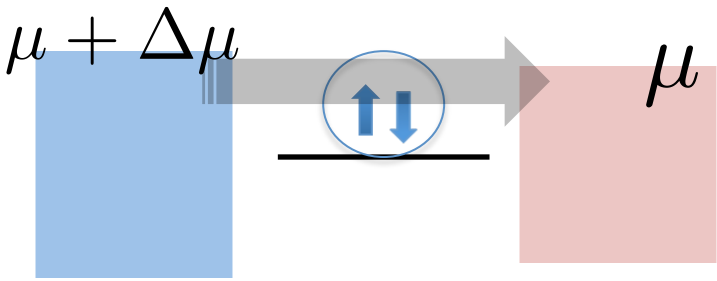

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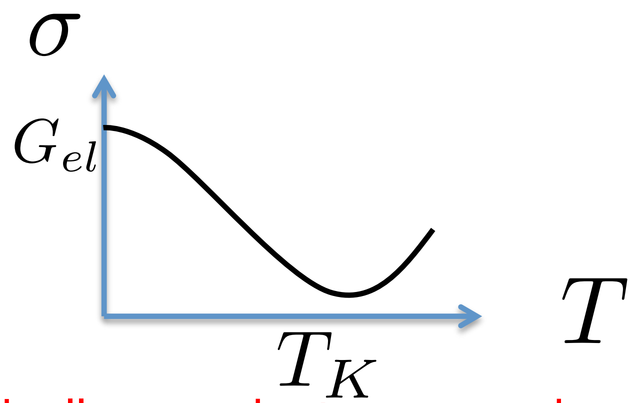

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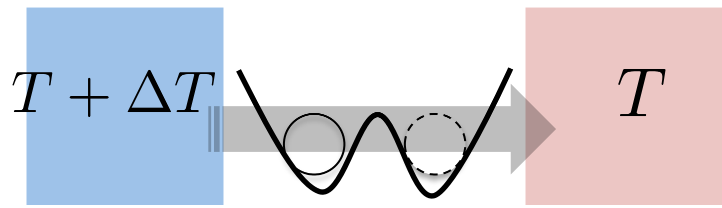


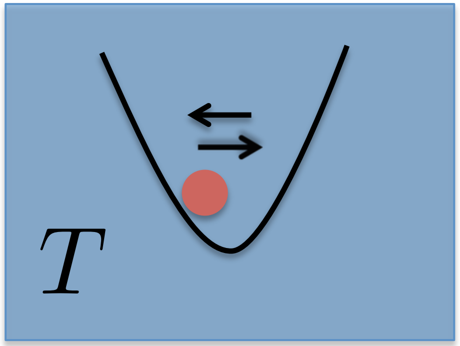
What is an effect of Kondo physics

In zero-dimensional heat conduction

?

Method : Connection to nonequilibrium statistical physics

◇ Harada-Sasa's relation for FDT violation (PRL 2005)



Harada Sasa PRL (2005)
KS EPL (2008)

Equilibrium: FDT holds $$C(\omega) = 2k_B T\chi'(\omega)$$

$C(t) := \langle v(t)v \rangle$ $\chi'(t)$ Response function

Method : Connection to nonequilibrium statistical physics

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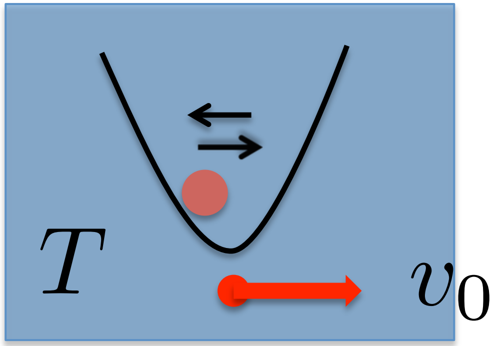

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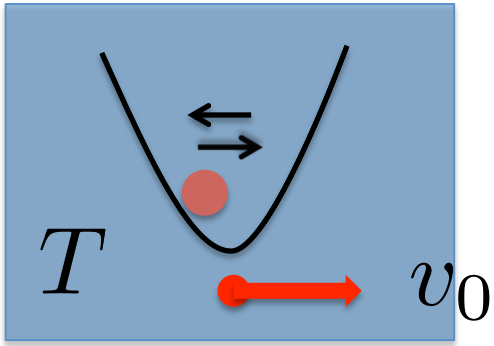

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Equilibrium: FDT holds $C(\omega) = 2k_B T \chi'(\omega)$

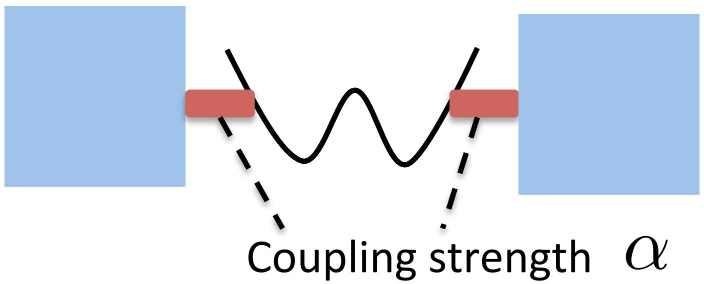
$C(t) := \langle v(t) v \rangle$ $\quad \chi'(t)$ Response function

Nonequilibrium: Violation of FDT yields energy flow to thermal environment

$$I = \gamma \left\{ v_s^2 + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left[C(\omega) - 2k_{\mathrm{B}} T \chi'(\omega) \right] \right\}$$

◇ Extension this to quantum regime and two thermal reservoirs

+ Zero-dimensionality



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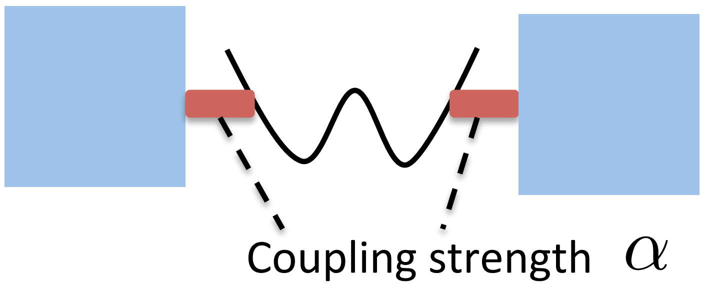


Exact formula for thermal current

$$I = \frac{\hbar^2 \alpha}{4} \int_0^{\omega_c} d\omega \, \omega \chi''(\omega) \left[n_L(\omega) - n_R(\omega) \right]$$

$$\chi(t,t') = i\hbar^{-1} \theta(t-t') \langle [\sigma_z(t), \sigma_z(t')] \rangle \, .$$

$$n_\nu(\omega) = 1/[e^{\beta_\nu \hbar\omega} - 1] \quad (\nu = L, R)$$

Transport property in the linear response regime

◇ Exact formula of thermal conductance

$$\kappa = \frac{dI}{dT} = \frac{k_B \hbar \alpha}{4} \int_0^{\omega_c} d\omega\, \chi''(\omega)\, \omega \left[\frac{\beta\hbar\omega/2}{\sinh(\beta\hbar\omega/2)} \right]^2,$$

$$\chi(t,t') = i\hbar^{-1}\theta(t-t')\langle[\sigma_z(t), \sigma_z(t')]\rangle \,.$$

◇ Monte-Calro simulation for $\chi''(\omega)$

Matsubara's relation

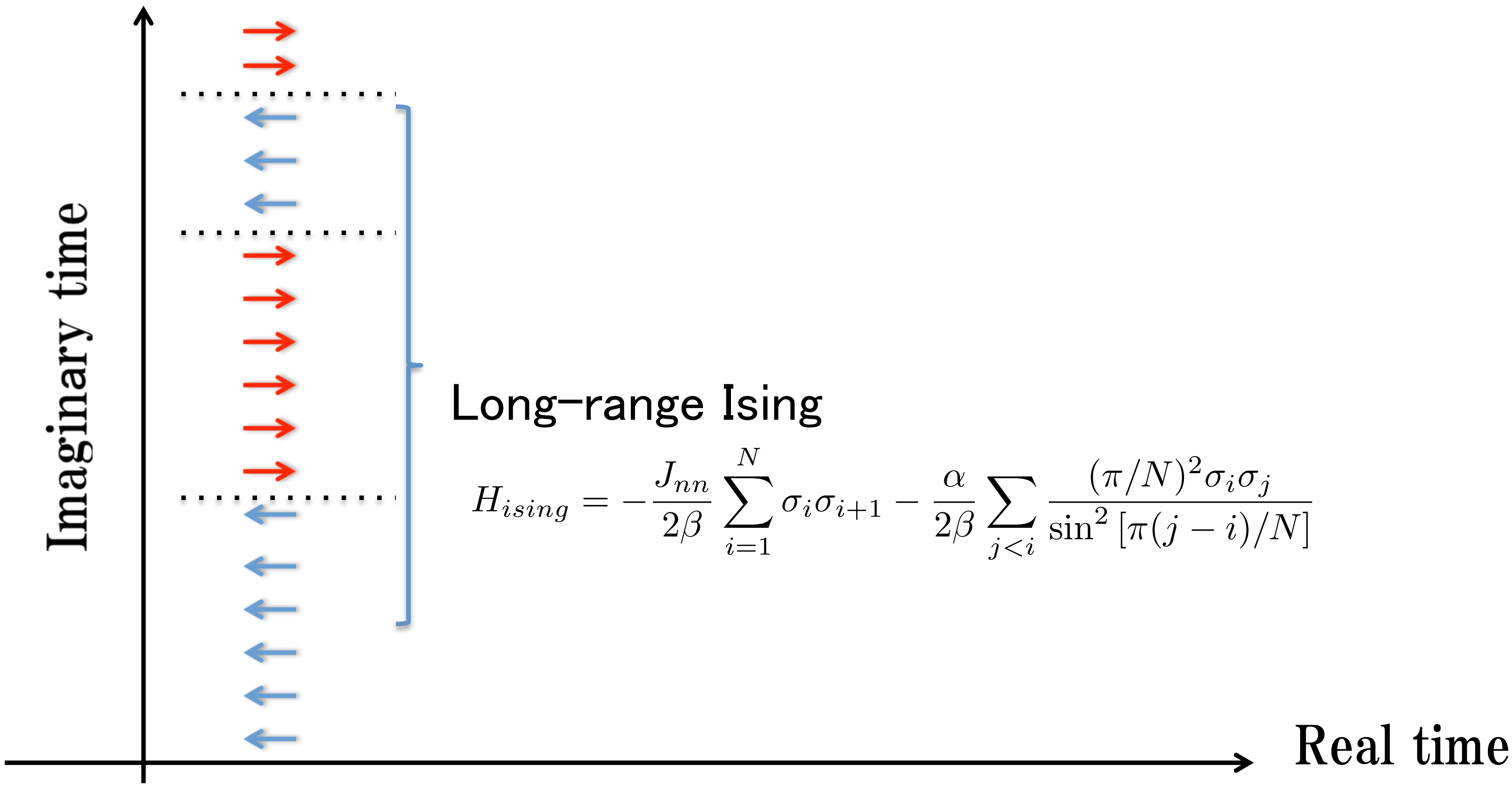
$$\mathcal{G}(i\omega + i\delta) \longrightarrow \chi(\omega)$$

$$\mathcal{G}(u) = \langle e^{uH}\sigma^z e^{-uH}\sigma^z e^{-\beta H}\rangle / Z$$

Mapped onto Long-range interacting system

◇Partition Function: path-integral expression

$$Z = \mathrm{Tr} e^{-\beta H}$$



$$H_{ising} = -\frac{J_{nn}}{2\beta}\sum_{i=1}^{N}\sigma_i\sigma_{i+1} - \frac{\alpha}{2\beta}\sum_{j<i}\frac{(\pi/N)^2\sigma_i\sigma_j}{\sin^2\left[\pi(j-i)/N\right]}$$

Mapped onto Long-range interacting system

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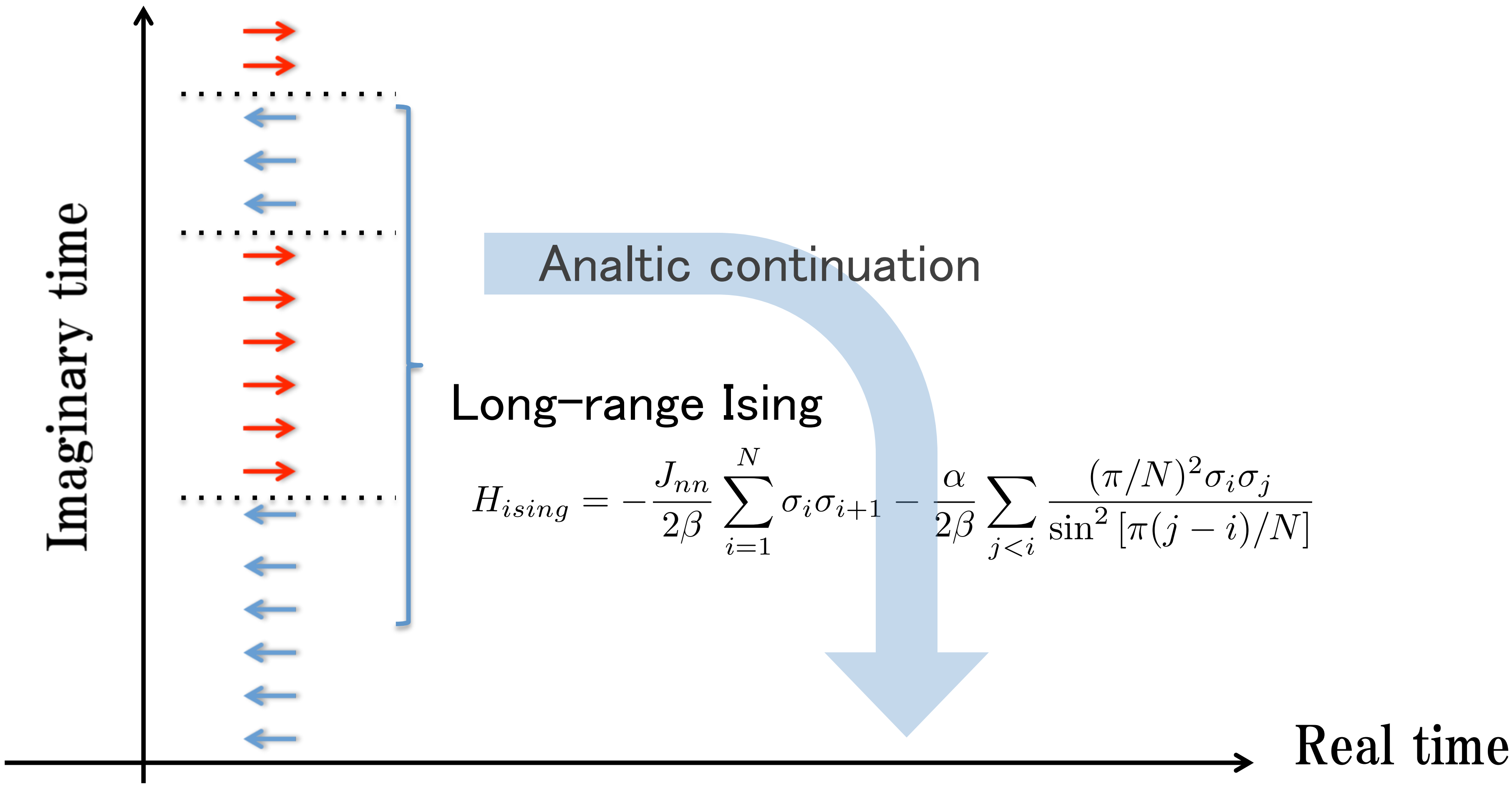

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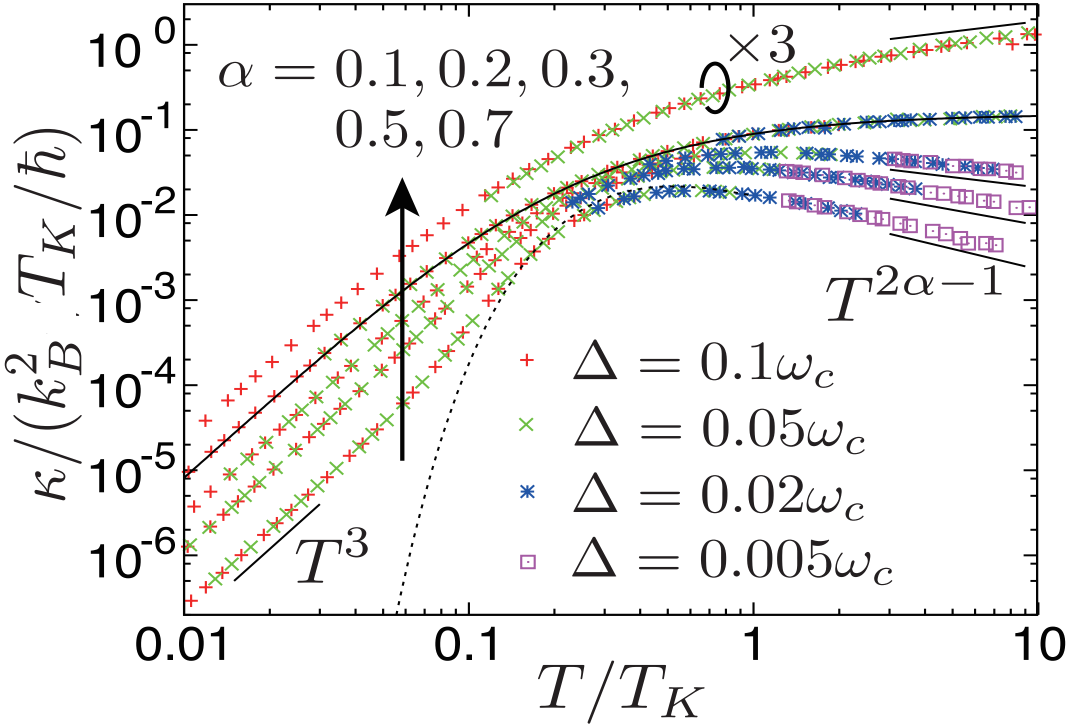
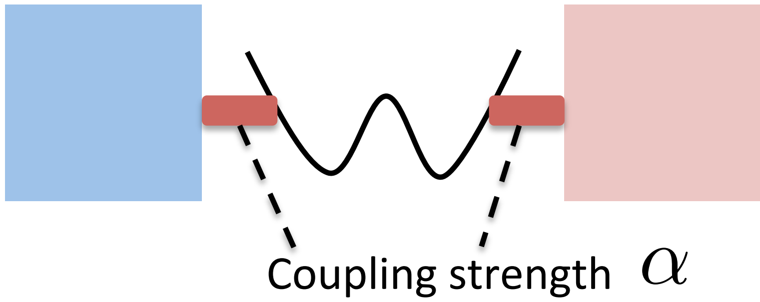
Results 1. Temperature dependence of conductance



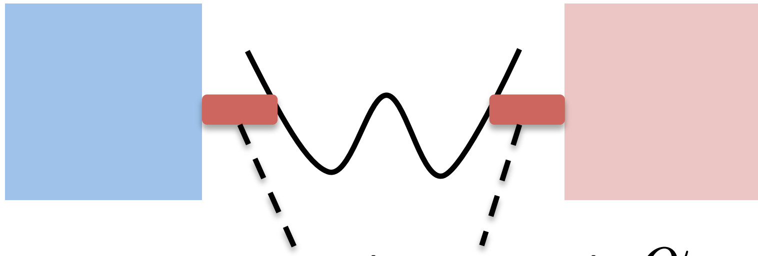


◇ Scaling form $\kappa(T) \sim (k_B^2 T_K/\hbar)\, f(\alpha, T/T_K)$

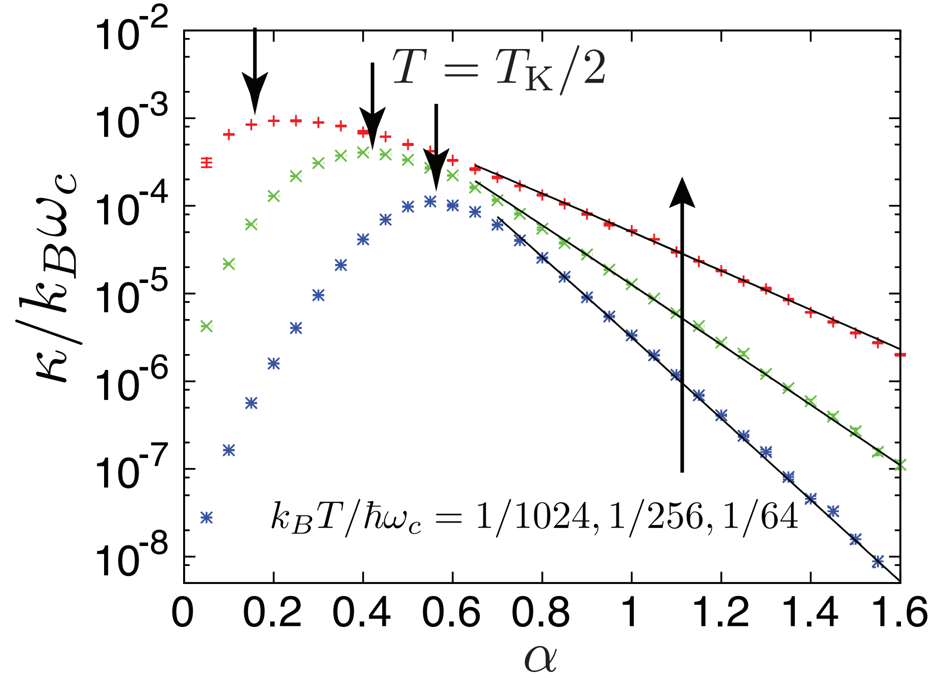
◇ Universal T^3 law $\kappa(T) \propto T^3$ $T \ll T_K$

$\kappa \not\to G_{th}$

◇ $\kappa(T) \propto T^{2\alpha-1}$ $T \gg T_K$



2. Dependence on the coupling strength



◇ Exponential suppression of conductance

Conclusion: Zero-dimensional transport properties

◇ Electric transport
(quantum-dot)

1. Enhancement of conductance

2. $\sigma \to G_0$ $\qquad T \ll T_K$

3. High-temperature
Coulomb blockade effect

◇ heat transport
(spin-boson)

0. current formula using stochastic thermodynamics

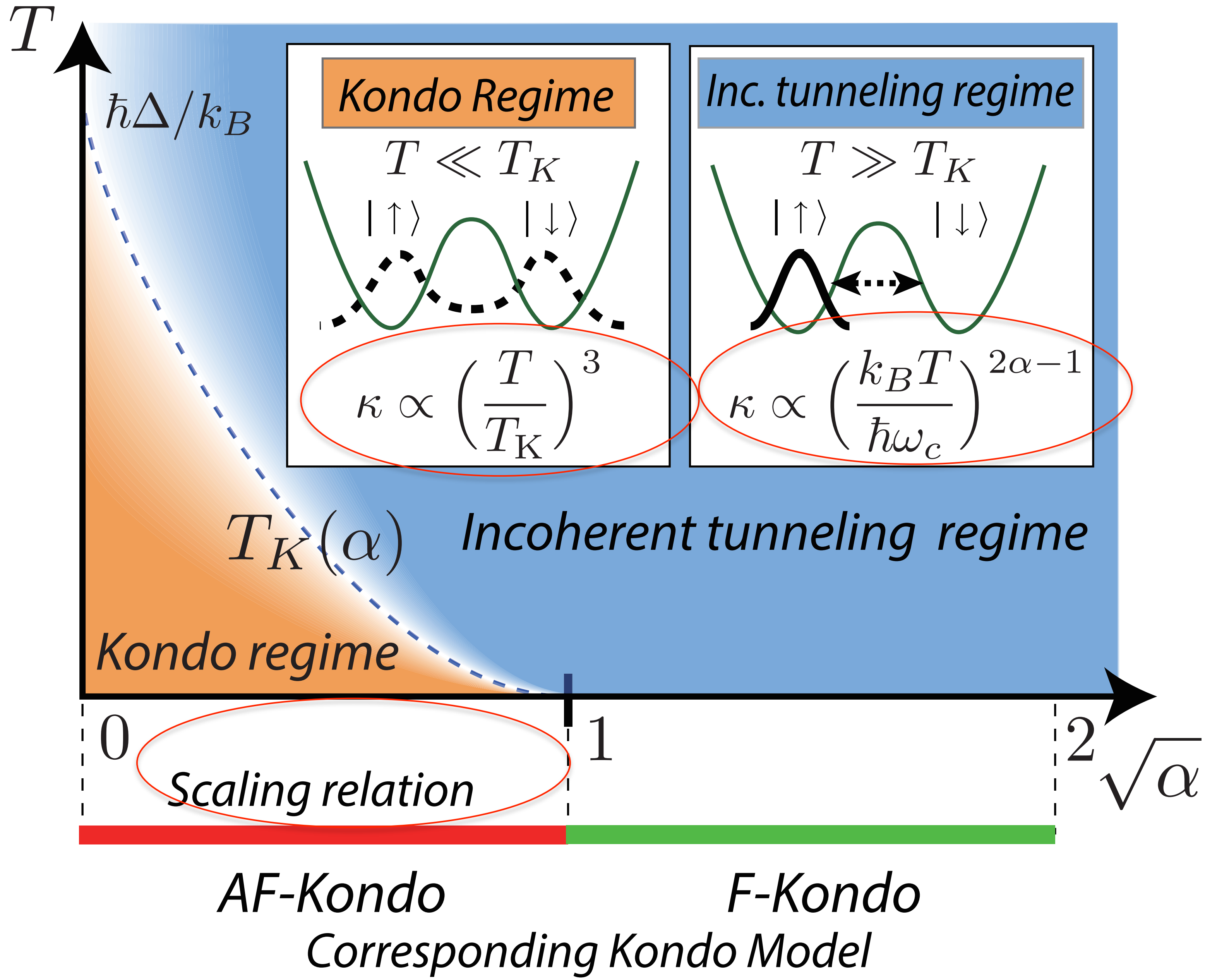
1. Enhancement of conductance

2. $\kappa(T) \propto T^3$ $\qquad T \ll T_K$
$\kappa \not\to G_{th}$

3. High-temperature $\qquad \kappa(T) \propto T^{2\alpha-1}$

4. Scaling law $\quad \kappa(T) \sim (k_B^2 T_K/\hbar)\, f(\alpha, T/T_K)$

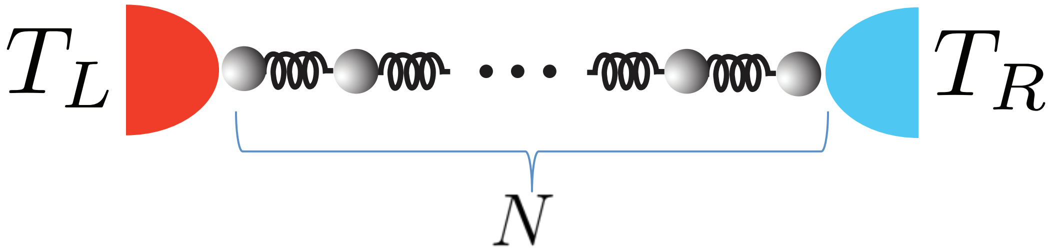
5. Exponential reduction for strong coupling



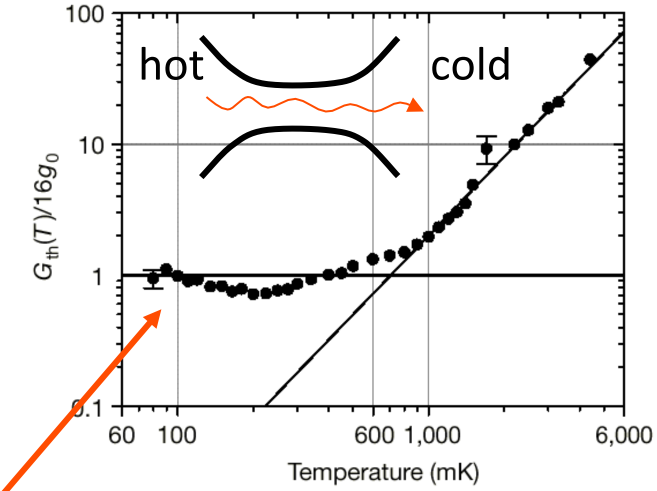
Thank you for your attention

KS and Kato PRL vol111, 214301 (2013)

Quantum of Thermal Conductance



$$H = \sum_{\ell=1}^{N} \frac{p_\ell^2}{2m} + \sum_{\ell=1}^{N-1} \frac{k}{2}(x_{\ell+1} - x_\ell)^2$$

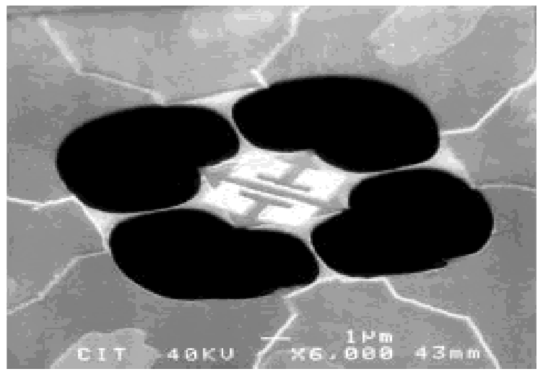


K.Schwab et al, Nature (2000)

◆ Quantum of thermal conductance at low temperatures

$$\langle \mathcal{I} \rangle = \int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T}(\omega)\left(f_{hot}(\omega) - f_{cold}(\omega)\right)$$

$$\boxed{\frac{d\langle \mathcal{I} \rangle}{dT} \to g_0 = \frac{\pi^2 k_B^2 T}{3h}}$$



Kondo regime

$$\kappa = \frac{dI}{dT} = \frac{k_B \hbar \alpha}{4} \int_0^{\omega_c} d\omega \, \chi''(\omega) \, \omega \Big[\frac{\beta \hbar \omega / 2}{\sinh(\beta \hbar \omega / 2)} \Big]^2,$$

Shiba's relation $$\chi''/\omega \Big|_{T \to 0} \to \chi_m$$

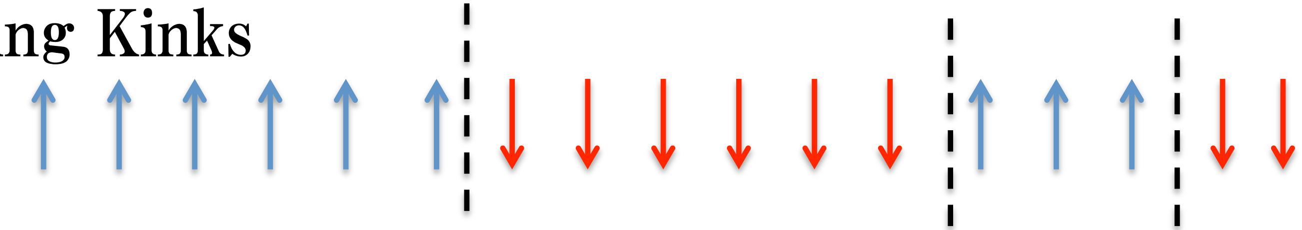
$$\kappa \sim \frac{k_B \hbar \alpha \chi_m}{4} \int_0^{\infty} d\omega \, \omega^2 \Big[\frac{\beta \hbar \omega / 2}{\sinh(\beta \hbar \omega / 2)} \Big]^2 \propto (T/T_K)^3$$

◇ Partition function of Ising model.

J. Cardy, J. Phys. A 14, 1407 (1981)

$$H_I = -\sum_{j<i} V(i-j)\sigma_i\sigma_j - h\sum_j \sigma_j$$

Counting Kinks



$$Z = \sum_{n=0}^{\infty}\int_0^{\beta}\frac{d\tau_1}{a}\int_0^{\tau_1-a}\frac{d\tau_2}{a}\cdots\int_0^{\tau_{2n}-a}\frac{d\tau_{2n}}{a}$$

$$\times\exp\{\sum_{j<i}(-1)^{i-j}4U(\frac{\tau_i-\tau_j}{a}) + 2h\sum_j(-1)^j\left(\frac{\tau_j}{a}\right)\}$$

◇ Comaparison this with the spin-Boson

$$H_{spin-boson} = -\frac{J_{nn}}{2\beta}\sum_{i=1}^{N}\sigma_i\sigma_{i+1} - \frac{\alpha}{2\beta}\sum_{j<i}\frac{(\pi/N)^2\sigma_i\sigma_j}{\sin^2\left[\pi(j-i)/N\right]}$$

$$J_{nn} = -\alpha(1+\gamma) - \ln(\Delta\tau_c/2)$$

◇Partition Function: path-integral expression

$$Z = \mathrm{Tr} e^{-\beta H} = Z_+ + Z_-$$

$$= \sum_{n=0}^{\infty} \mathrm{Tr}_{\mathrm{boson}} \Big\{ \langle + | e^{-\beta H_z} \int_0^{\beta} d\tau_1 \cdots \int_0^{\tau_{2n-1}} d\tau_{2n} \left(\frac{\Delta}{2}\right)^{2n} \tilde{\sigma}_x(\tau_1) \cdots \tilde{\sigma}_x(\tau_{2n}) | + \rangle \Big\}$$

$$= Z_0 \sum_{n=0}^{\infty} \left(\frac{\Delta \tau_c}{2}\right)^{2n} \int_0^{\beta} \frac{d\tau_1}{\tau_c} \int_0^{\tau_1 - \tau_c} \frac{d\tau_1}{\tau_c} \cdots \int_0^{\tau_{2n-1} - \tau_c} \frac{d\tau_{2n}}{\tau_c} \exp\Big\{ 2\alpha \sum_{i<j} (-1)^{i+j} \ln \Big| \frac{\beta}{\pi \tau_c} \sin(\pi(\tau_j - \tau_i)/\beta) \Big| \Big\}$$

◇Mapping Long-range Ising model

$$H_{spin-boson} = -\frac{J_{nn}}{2\beta} \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - \frac{\alpha}{2\beta} \sum_{j<i} \frac{(\pi/N)^2 \sigma_i \sigma_j}{\sin^2 [\pi(j-i)/N]}$$

$$J_{nn} = -\alpha(1+\gamma) - \ln(\Delta \tau_c / 2)$$

◇ Matsubara Green function

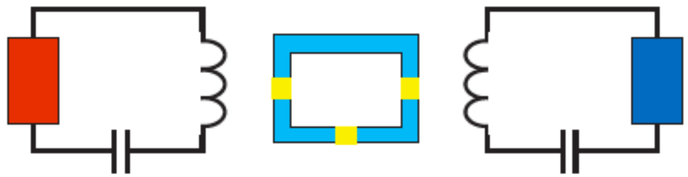
$$\mathcal{G}(j) = \langle \sigma_j^z \sigma_0^z \rangle$$

Realization

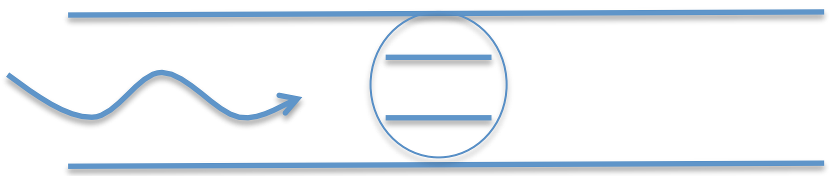
◇ Examples of realization

Molecularjunction

Superconducting circuit

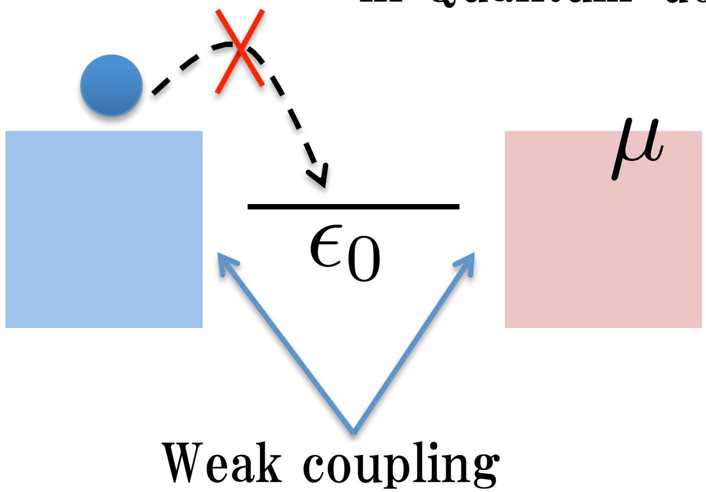


Light in wave guide



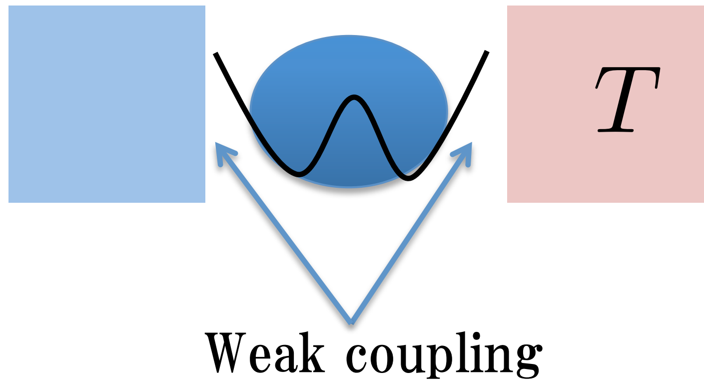
Results by Master equation approach in the weak coupling limit

◇Coulomb Blockade effect
in Quantum-dot



◇Effect of Shotcky specific heat
in two-level system

$$H_S = \Delta \sigma_x$$



◇ Exponential reduction of
electric condutance

$$\sigma \propto e^{-\beta|\epsilon_0 - \mu|}$$

◇ Exponential reduction of
thermal condutance

$$\kappa \propto \frac{\Delta}{2n(\Delta)+1}\left[\frac{\beta\hbar\Delta/2}{\sinh(\beta\hbar\Delta/2)}\right]^2$$

$$\propto \beta^2 e^{-2\beta\hbar\Delta}$$

Segal & Nitzan, PRL (2005)

◇ Note that Kondo physics is a

nonpertubative effect, which can not be

captured by the master equation

◇ Nonperturbative approach is necessary

◇ Exponential reduction of
electric condutance

to see Kondo at extremely low-temperatures

Remark：Weak coupling approximation

◇ Weak coupling approaximation $\alpha \to 0$

$$\kappa_{\mathrm{WC}} = \frac{k_B \alpha_L \alpha_R}{2\alpha} \frac{\pi\Delta}{2n(\Delta)+1} \left[\frac{\beta\hbar\Delta/2}{\sinh(\beta\hbar\Delta/2)} \right]^2$$

Exponential reduction at low temperature !

Analogous to strong suppression of electric conductance in the Coulomb Blockade regime

◇ Spin-spin correlation

$$\mathcal{G}(u) = \langle e^{uH} \sigma^z e^{-uH} \sigma^z e^{-\beta H} \rangle / Z$$

◇ Matsubara relation

$$\mathcal{G}(i\omega + i\delta) \rightarrow G(\omega)$$