Dynamical spin properties in helical Luttinger liquids

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in collaboration with

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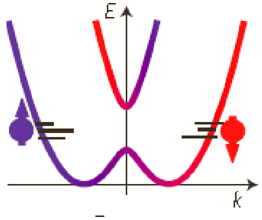
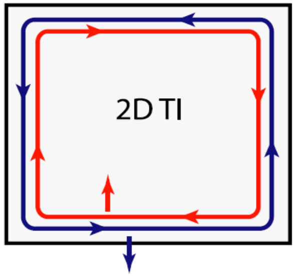
Outline

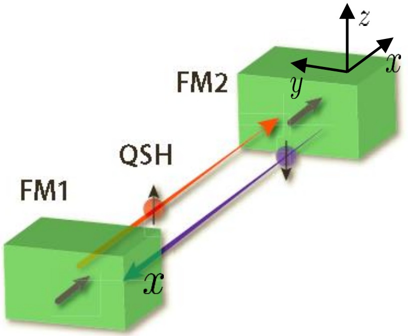
Introduction to helical liquids



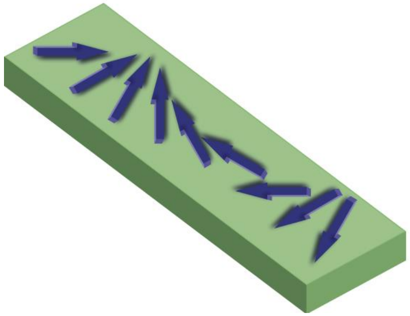
- Rashba quantum wires
- Topological insulators



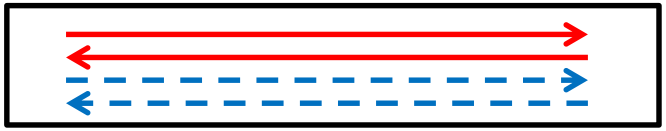
Spin properties in helical finite systems



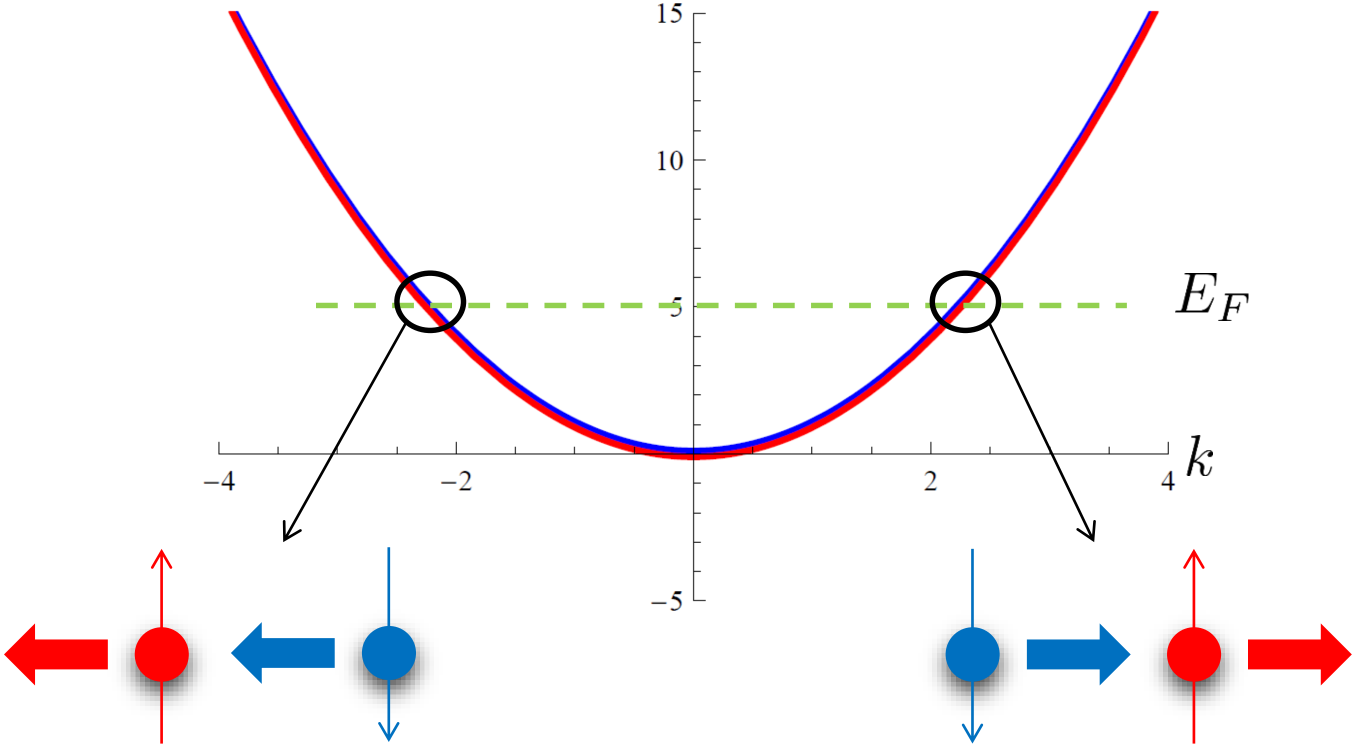
- Confinement: helical 1D quantum dot
- Peculiar spin textures



Spinfull electrons in a 1D quantum wire



$$\epsilon_\uparrow(k) = \epsilon_\downarrow(k) = \frac{k^2}{2m}$$



Helical 1D system

Helical 1D system

spin-momentum locking!

right-moving spin up; left-moving spin down

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spin-momentum locking!

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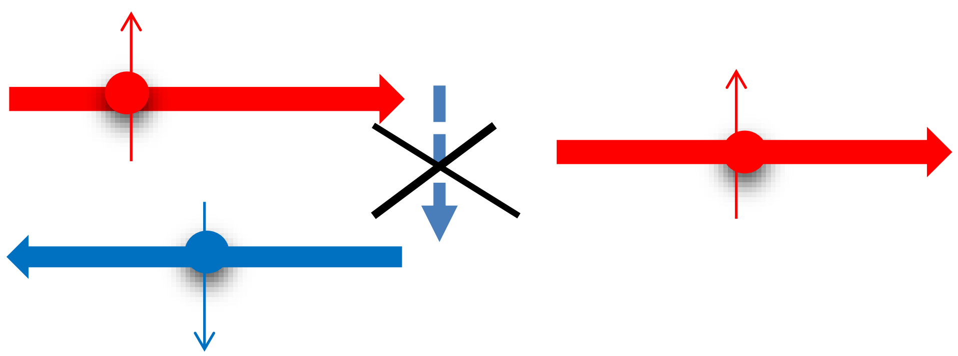
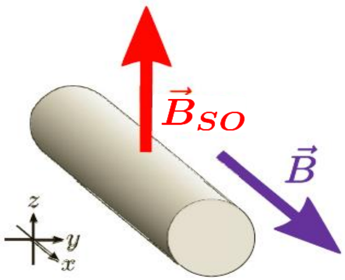
Topological protection from backscattering in the presence of TRS



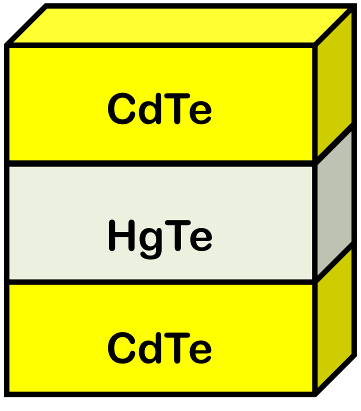
T= 1

Realizations

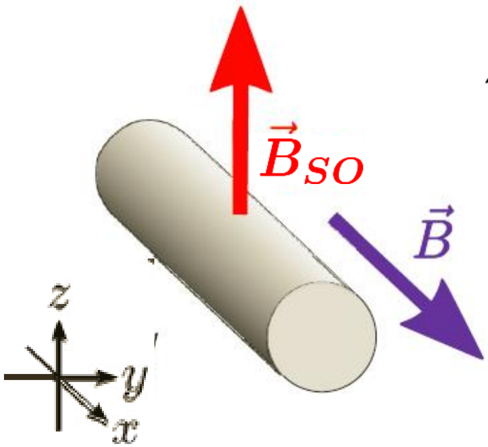
Spin-orbit coupled quantum wires in magnetic fields



Edge states in two-dimensional topological insulators



Spin-orbit interaction in semiconductor quantum wire (InSb, InAs, GaAs/AlGaAs)



$$\mathcal{H}_{so} = i\alpha\sigma_z\partial_x \propto \vec{B}_{SO}\vec{\sigma}$$ Rashba spin-orbit interaction

$$\mathcal{H}_B = \frac{g^*\mu_B}{2}\vec{B}\cdot\vec{\sigma}$$ Zeeman coupling

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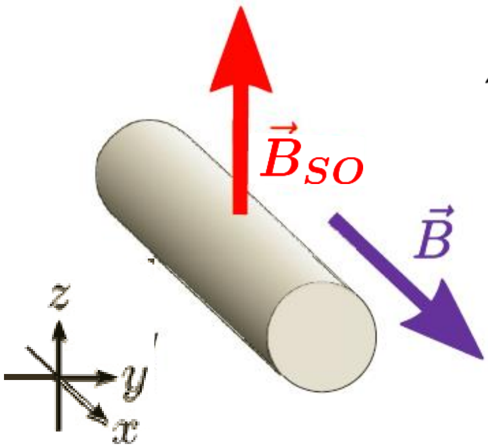

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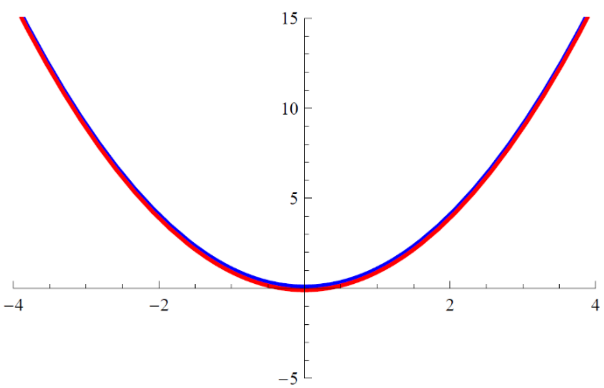
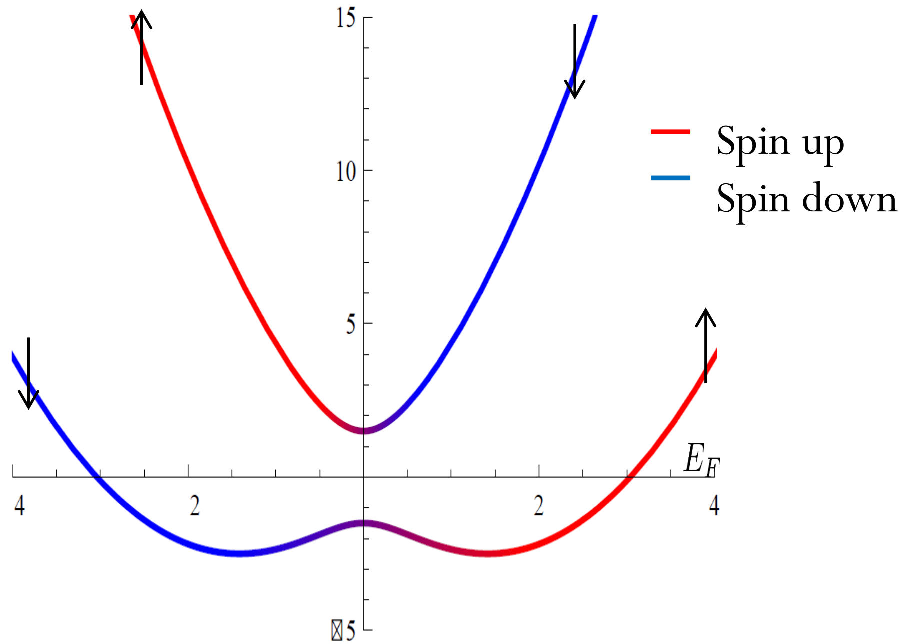
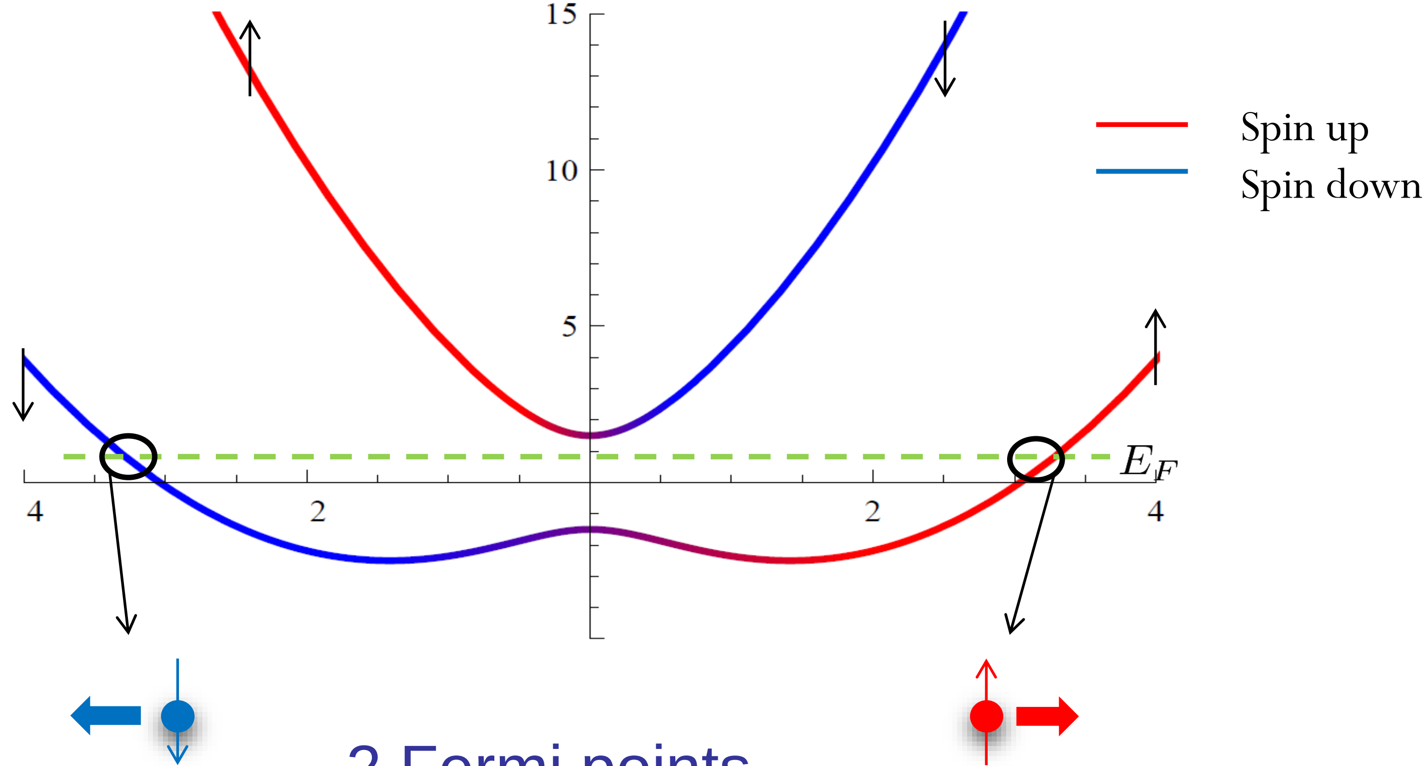




[Streda, Seba PRL (2003); Pershin Nesteroff, Privman PRB (2004); Zhang et al. PRB (2006); Quay *et* al., Nature Physics (2010); Meng Loss PRB (2013)…….]

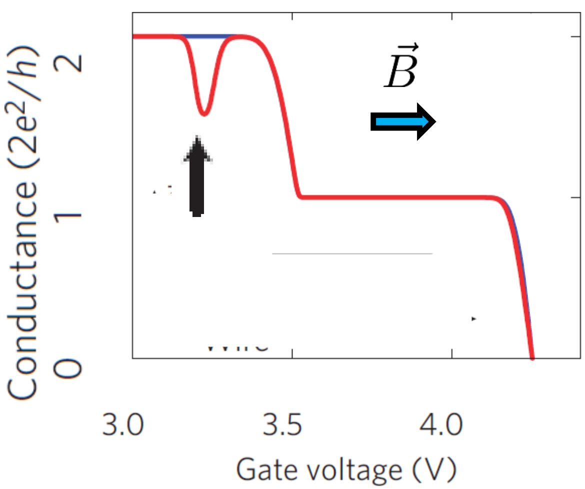


2 Fermi points

right moving spin up, left moving spin down: **helical liquid!**

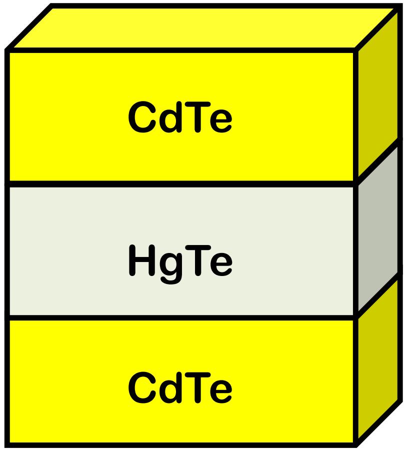
Experimental detection

[C. H. L. Quay *et* al., Nature Physics (2010)]



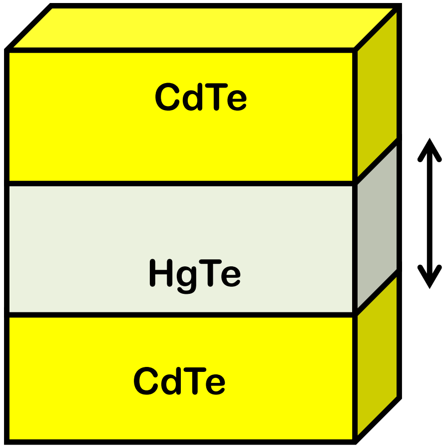
Edge states of 2D topological insulators

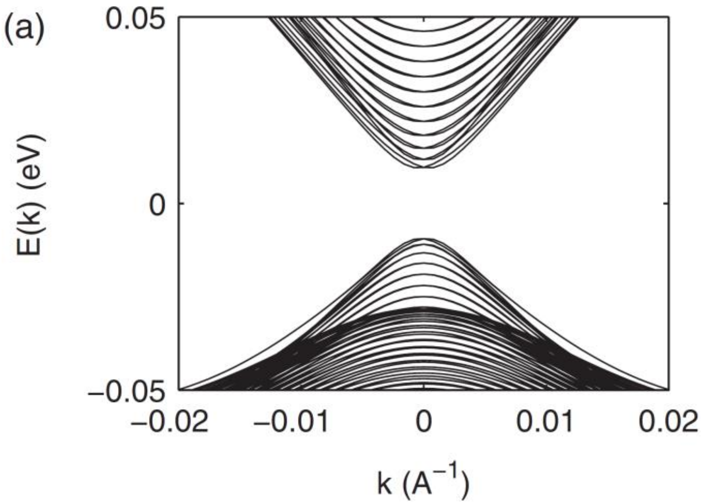
HgTe/CdTe QWs



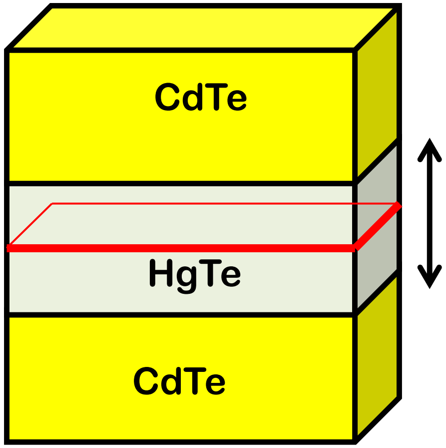
[B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006)]

Two-dimensional topological insulators

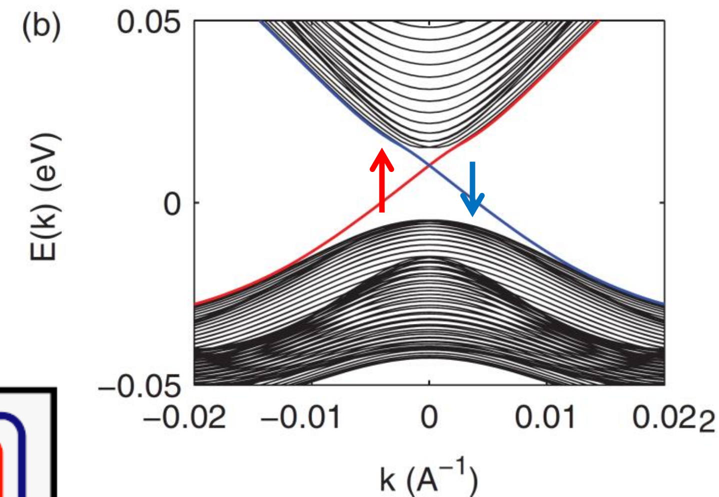


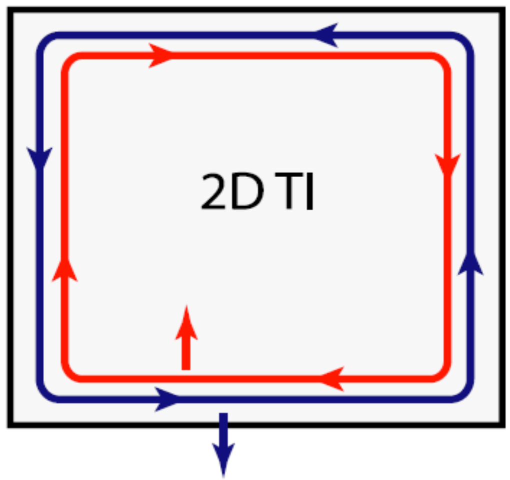


Two-dimensional topological insulators



$$d > d_c \approx 6.3\text{nm}$$

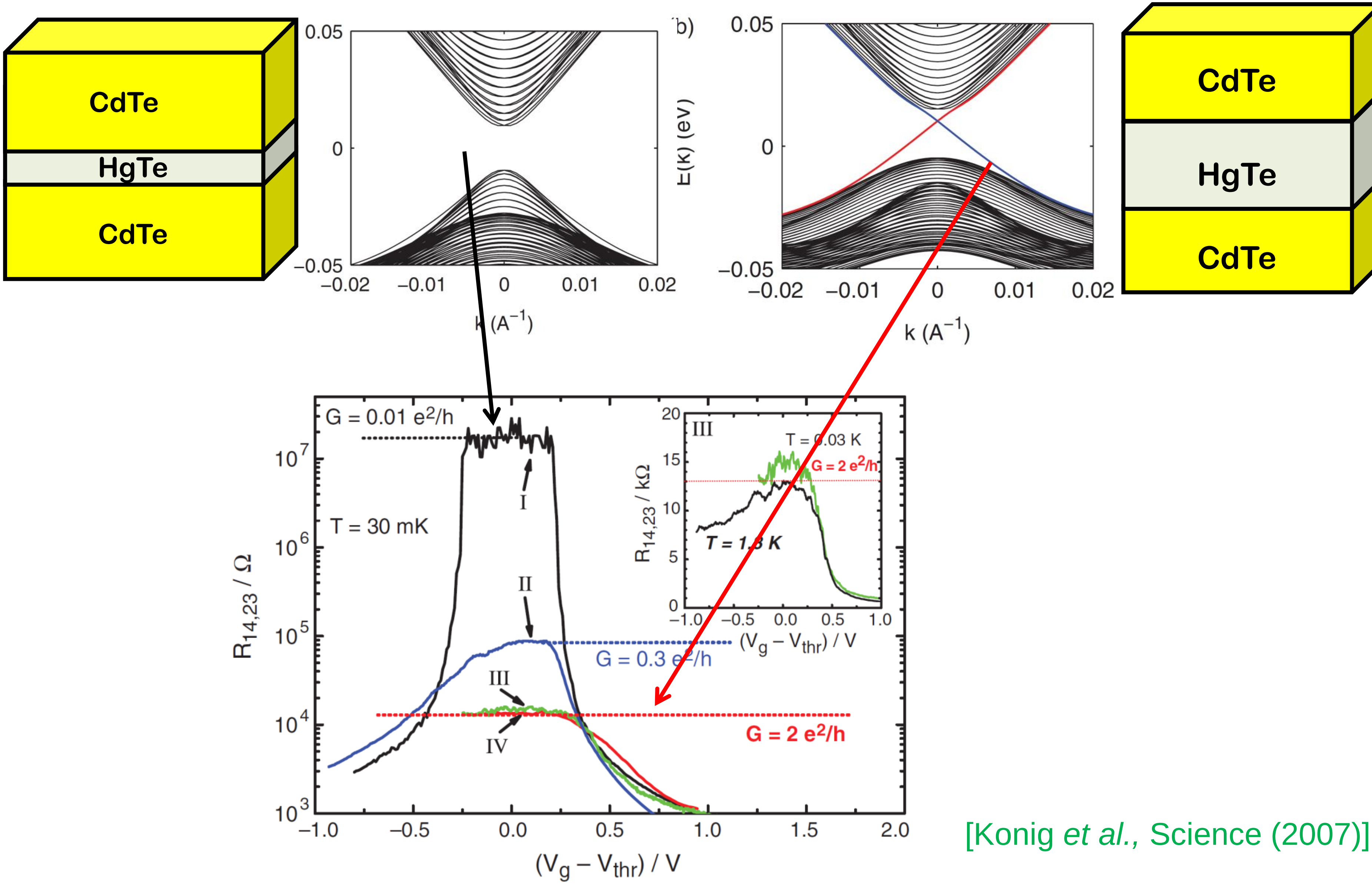




Insulating bulk states, gapless counterpropagating edge states connected by time reversal symmetry with spin-momentum locked

[B.A.Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006); B.A. Bernevig, SC Zhang PRL (2006); L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011)]

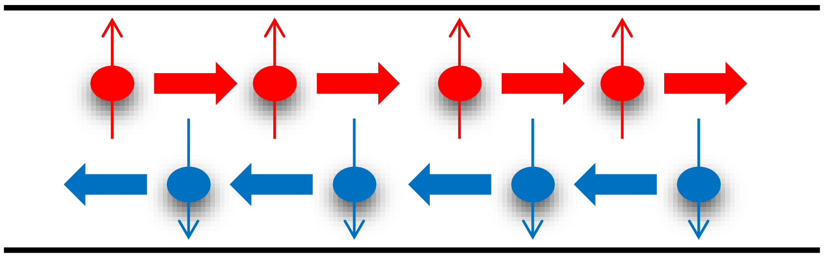
Experimental observation of the QSH effect



[Konig *et al.*, Science (2007)]

Why are helical liquids useful?

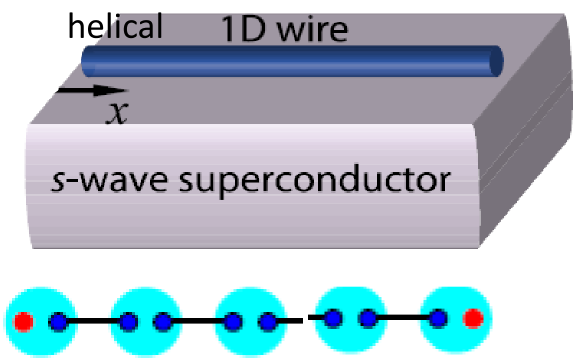
- **Spintronics**: spin manipulation, spin current generation



$$I_C = I_\uparrow + I_\downarrow = 0$$
$$I_S = I_\uparrow - I_\downarrow \neq 0$$

[Roth et al. Science (2009); Dolcini PRB (2011); Citro, Romeo, Andrei PRB (2001); Sukhanov, Sablikov J.Phys Cond Matt (2012); Dolcetto et al. PRB (2013); Michetti Trauzettel APL (2013); Ferraro et al PRB (2013)……]

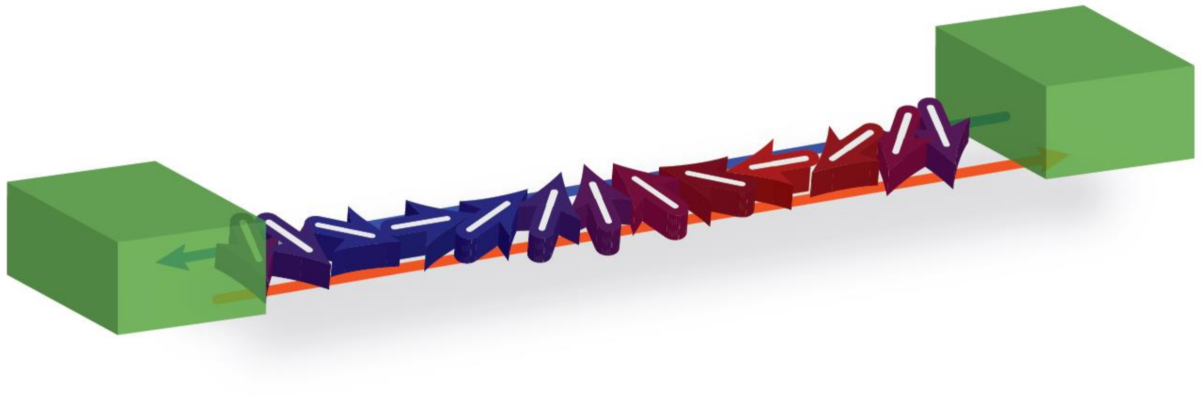
- **Realizations of Majorana fermions** in solid state devices



unpaired Majorana fermions on opposite edges of the helical wire

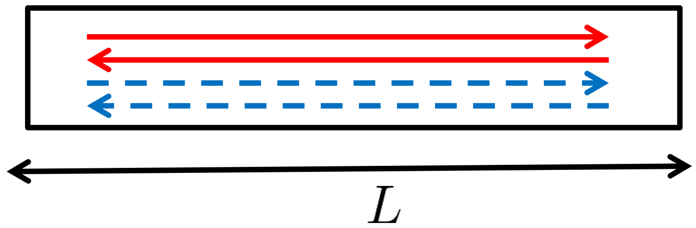
[Kitaev Phys. Usp (2001); Oreg, Refael, von Oppen, PRL (2010); Lutchyn, Sau, Das Sarma PRL (2010); Alicea Rep Prog Phys (2012) (review); Mourik et al. Science (2012) (exp)……..]

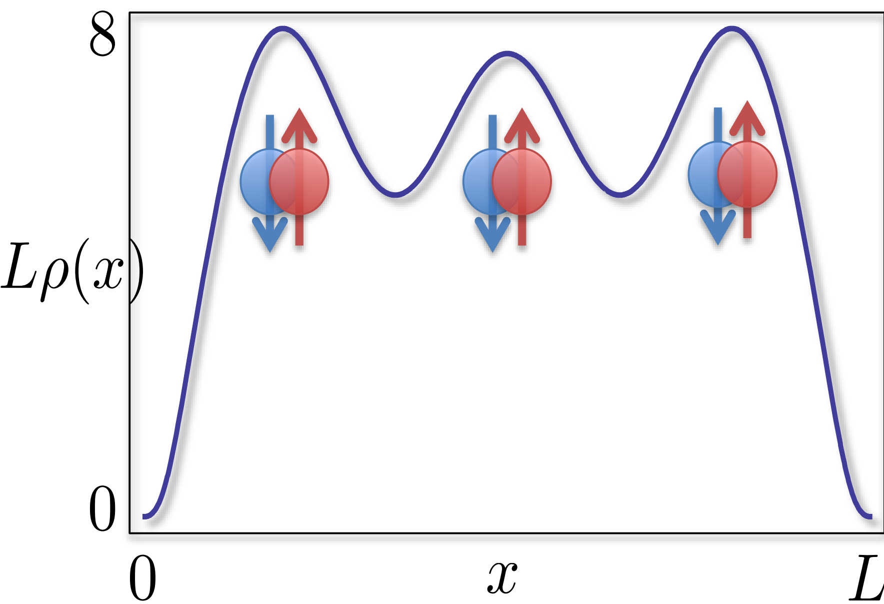
Spin textures in finite size helical systems

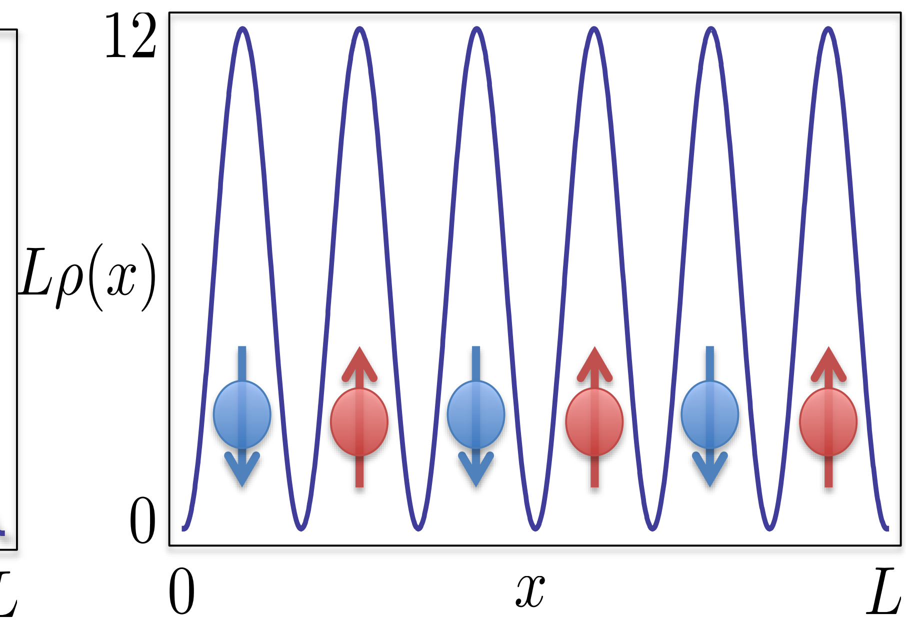


G. Dolcetto, N. Traverso, M. Biggio, F. Cavaliere, M.S. PRB 87, 235423 (2013);
G. Dolcetto, N. Traverso, M. Biggio, F. Cavaliere, M.S. RRL 7, 1059 (2013);
G. Dolcetto, F. Cavaliere, M.S. PRB 89, 125419 (2014)

Wigner density oscillations in finite 1D spinfull wires







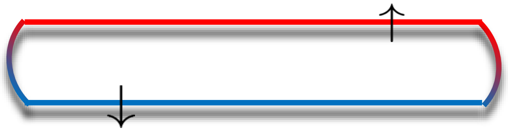
Weak interactions
Finite size Friedel oscillations

N=6

Strong interactions
Wigner cristallization

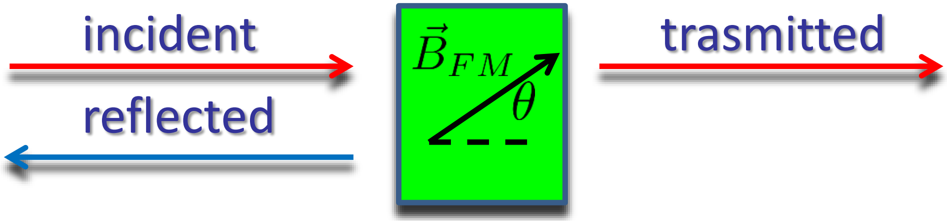
K. Jauregui et al., EPL(1993); S.H. Abendipour et al. .PRA (2007); G.A. Fiete RMP (2007); A. Secchi, M. Rontani PRB (2009) (2010); N. Traverso, F. Cavaliere, M. S., PRB (2011), PRB (2012), NJ.P (2013)

Finite size helical edge

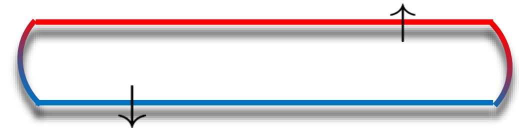


Confinement with magnetic barriers

breaking time reversal simmetry → possible spin flip processes



Finite size helical edge



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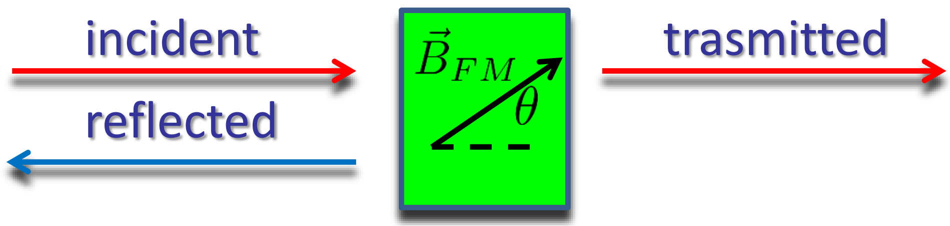
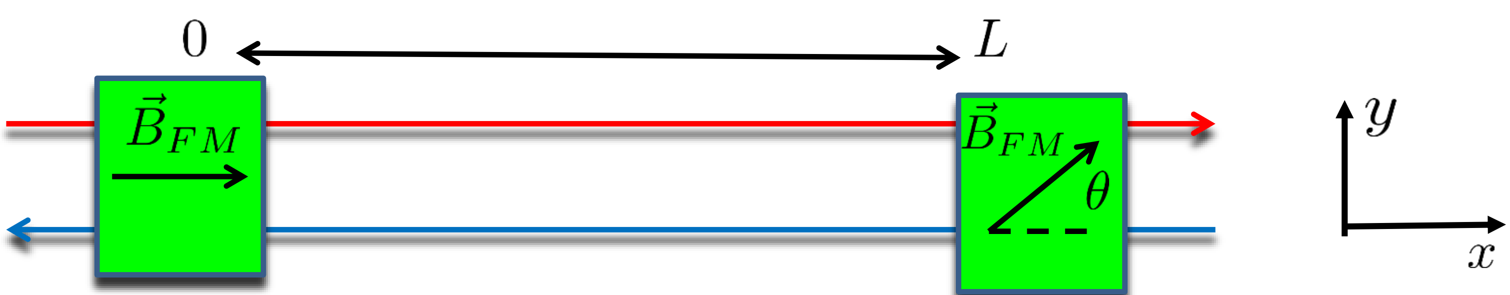


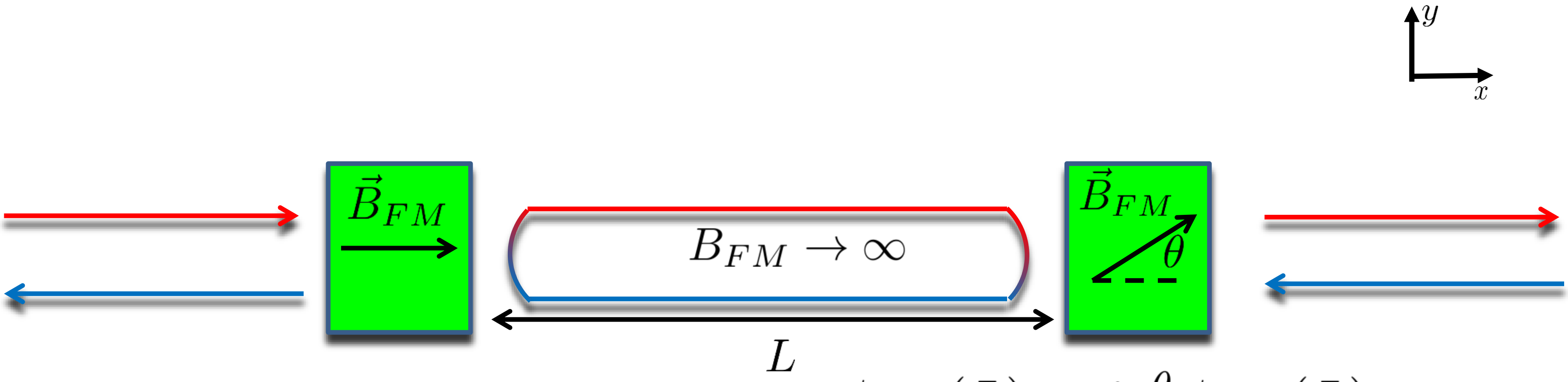
$$\mathcal{H}_{FM} = g\mu_B \int_{-\infty}^{\infty} dx \vec{B}_{FM}(x)\vec{S}(x)$$



$B_{FM}(1,0,0)\mathcal{L}_{FM}\delta(x)$

$B_{FM}(\cos\theta, \sin\theta, 0)\mathcal{L}_{FM}\delta(x-L)$

confined helical 1D quantum island

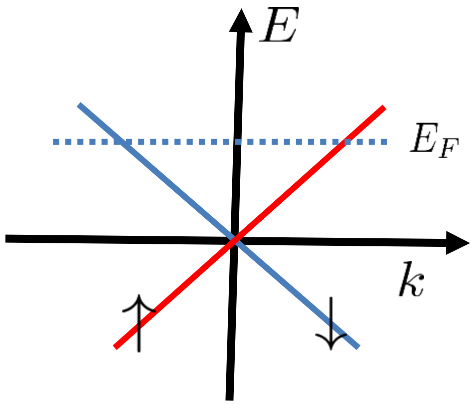


$$\psi_{L,\downarrow}(0) = -i\psi_{R,\uparrow}(0) \qquad \psi_{L,\downarrow}(L) = ie^{\theta}\psi_{R,\uparrow}(L)$$

θ - dependent boundary conditions

$$\psi_{R,\uparrow}(-L) = e^{i\pi(1-\theta/\pi)}\psi_{R,\uparrow}(L) \qquad \psi_{L,\downarrow}(x) = -i\psi_{R,\uparrow}(-x)$$

Finite size helical 1D Luttinger liquid



$$\Psi(x) = \begin{pmatrix} \psi_{R,\uparrow}(x) \\ \psi_{L,\downarrow}(x) \end{pmatrix} = \begin{pmatrix} \psi_{R,\uparrow}(x) \\ -i\psi_{R,\uparrow}(-x) \end{pmatrix}$$

$$\mathcal{H}_0 = -iv_F \int_0^L dx \left[\psi^\dagger_{R,\uparrow} \partial_x \psi_{R,\uparrow} - \psi^\dagger_{L,\downarrow} \partial_x \psi_{L,\downarrow} \right]$$

Interaction term

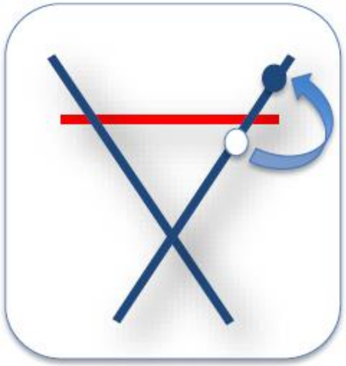
$$\mathcal{H}_{int} = \frac{1}{2} \int_0^L dx \int_0^L dx' V(x - x') \rho(x) \rho(x')$$

$$\rho(x) = \Psi^\dagger(x) \Psi(x)$$

Diagonalization

$$\mathcal{H} = v_\rho \sum_{k>0} k\hat{a}_k^\dagger \hat{a}_k + \frac{\pi v_F}{2g^2 L}(N - N_0 + \frac{\theta}{2\pi})^2$$

$$\mathcal{H} = \mathcal{H}_p + \mathcal{H}_N$$



plasmon modes with velocity $v_\rho = \frac{v_F}{g}$

electron interaction parameter $g = \frac{1}{\sqrt{1+\frac{V(q\to 0)}{\pi v_F}}} \leq 1$

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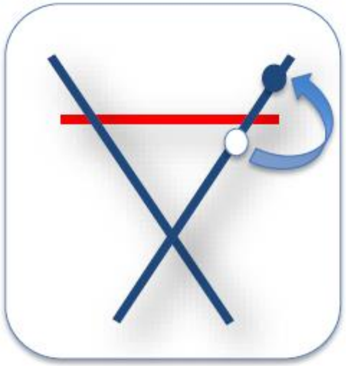


plasmon modes with velocity $v_\rho = \frac{v_F}{g}$

electron interaction parameter $g = \frac{1}{\sqrt{1+\frac{V(q\to 0)}{\pi v_F}}} \leq 1$

Electron field bosonization

$$\psi_{R,\uparrow}(x) = \sqrt{\frac{N}{2L}}\mathcal{F}e^{i\pi\frac{x}{L}(N-\frac{1}{2}+\frac{\theta}{2\pi})}e^{-i\phi(-x)}$$

$$\phi(x) = -\sum_{k>0}\sqrt{\frac{\pi}{kL}}\left[\sqrt{g}\cos(kx)(\hat{a}_k + \hat{a}_k^\dagger) + i\frac{\sin(kx)}{\sqrt{g}}(\hat{a}_k - \hat{a}_k^\dagger)\right]$$

Spin density components

$$\vec{S}(x) = \Psi^\dagger(x)\frac{\vec{\sigma}}{2}\Psi(x)$$

$$\Psi(x) = \begin{pmatrix} \psi_{R,\uparrow}(x) \\ \psi_{L,\downarrow}(x) \end{pmatrix} = \begin{pmatrix} \psi_{R,\uparrow}(x) \\ -i\psi_{R,\uparrow}(-x) \end{pmatrix}$$

$$S_x(x) = \frac{1}{2}[-i\psi_R^\dagger(x)\psi_R(-x) + h.c.]$$

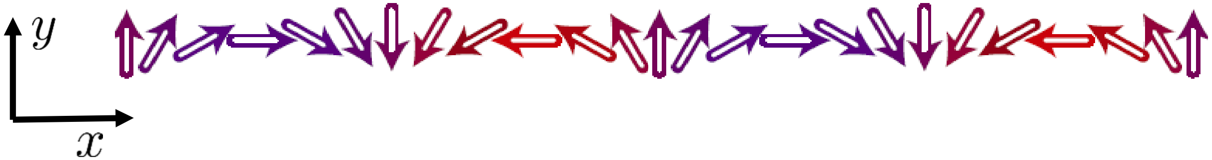
$$S_y(x) = \frac{1}{2}[\psi_R^\dagger(x)\psi_R(-x) + h.c.]$$

$$S_z(x) = \frac{1}{2}[\psi_R^\dagger(x)\psi_R(x) - \psi_R^\dagger(-x)\psi_R(-x)]$$

thermal averages

$$\langle S_i(x)\rangle = \langle N|Tr\{\frac{1}{Z_p}e^{-\beta\mathcal{H}_p}S_i(x)\}|N\rangle$$

Exact evaluation of spin averages via bosonization technique



$$\langle S_z(x)\rangle = 0$$

$$\langle S_x(x)\rangle = \sin\left[\frac{2\pi x}{L}\left(N - \frac{1}{2} + \frac{\theta}{2\pi}\right)\right]\cdot F(x)$$

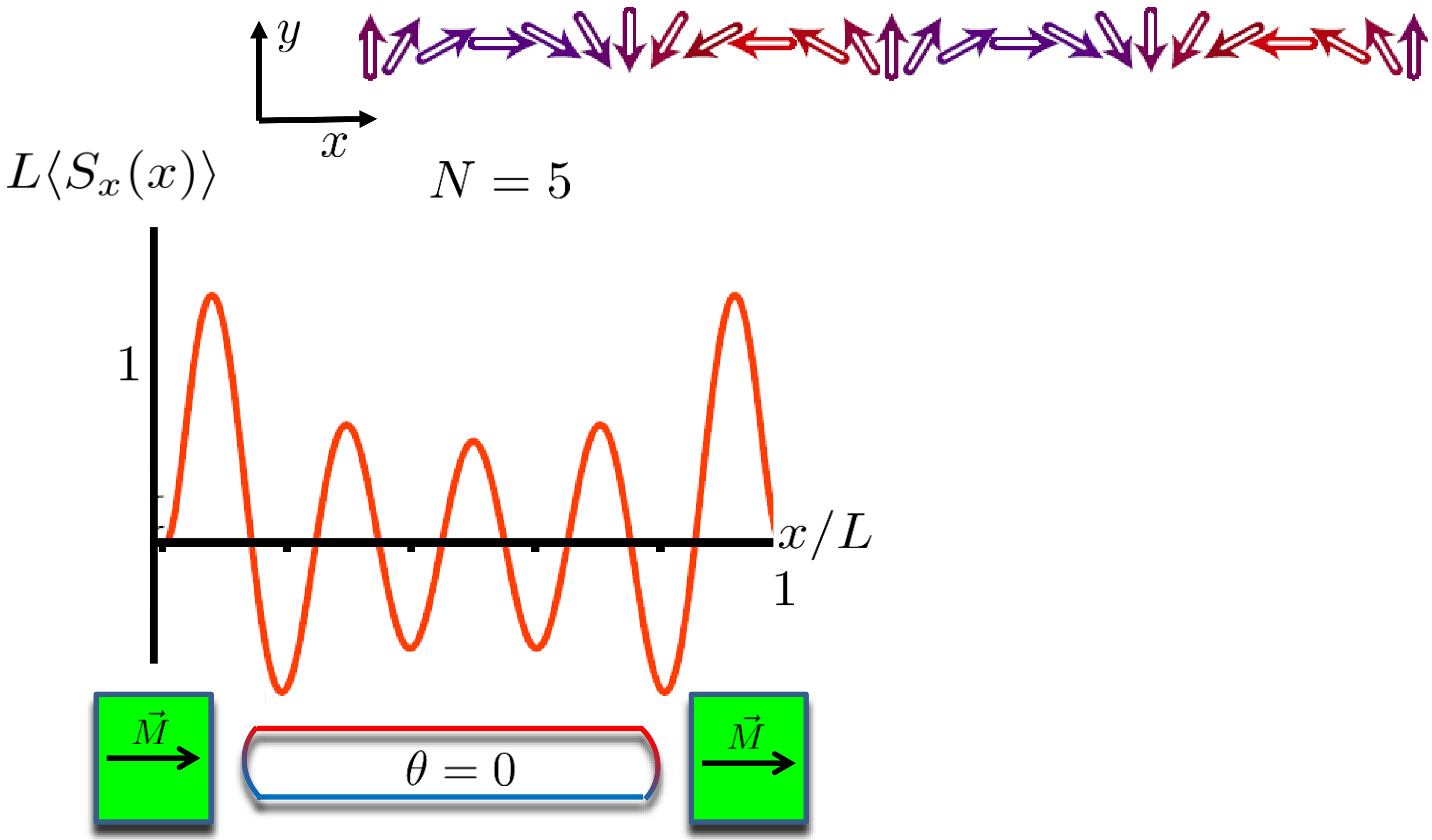
$$\langle S_y(y)\rangle = \cos\left[\frac{2\pi x}{L}\left(N - \frac{1}{2} + \frac{\theta}{2\pi}\right)\right]\cdot F(x)$$

Oscillating term related to the number of particles inside the island

LL power-law envelope function

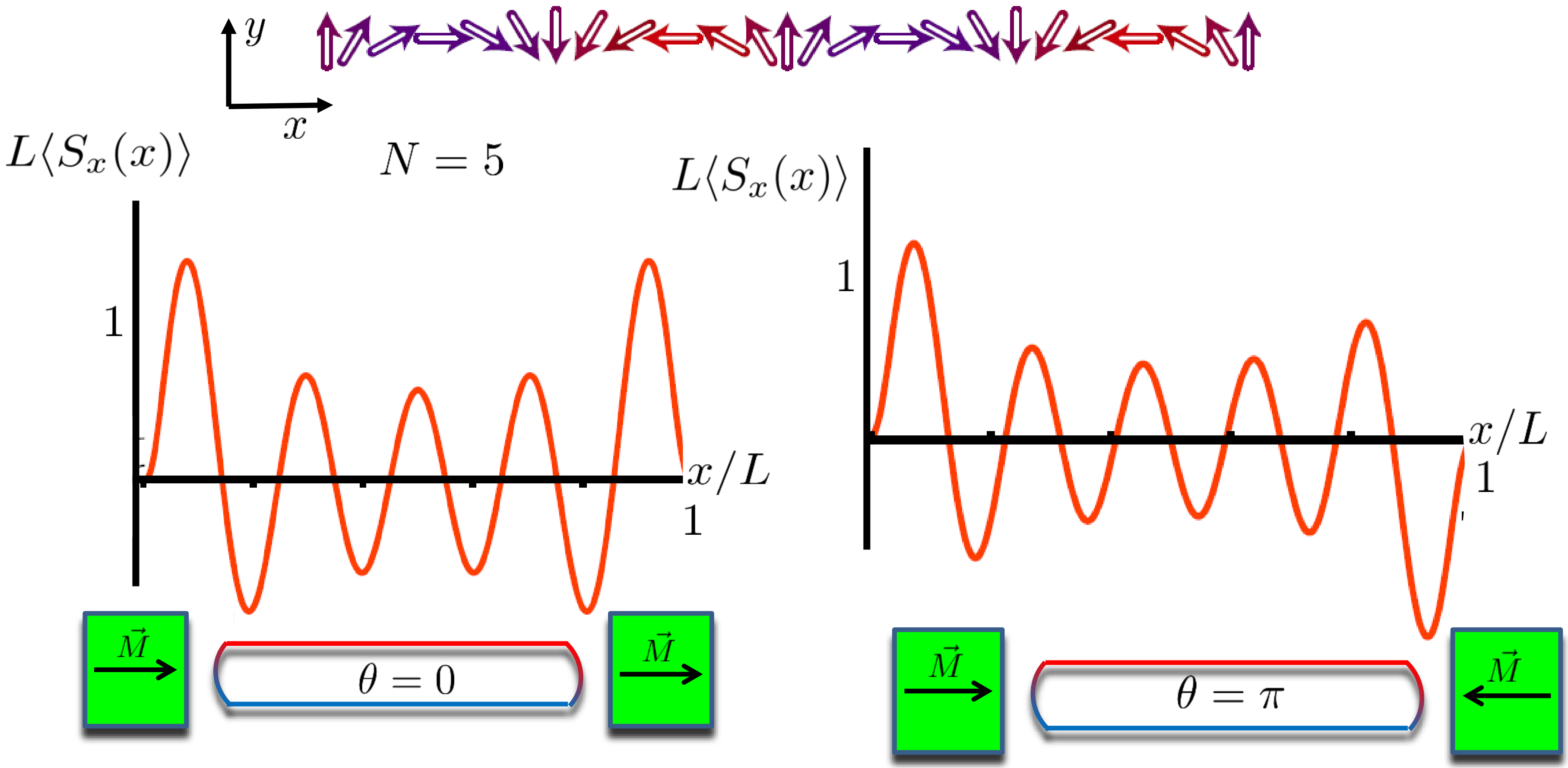
$$F(x) = \frac{N}{L}\left(\frac{\sinh(1/2N)}{\sqrt{\sinh^2(1/2N) + \sin^2(\pi x/L)}}\right)^g$$

Peculiar spin-density oscillating patterns driven by the magnetization angle



$T = 0, g = 0.7$

Peculiar spin-density oscillating patterns driven by the magnetization angle



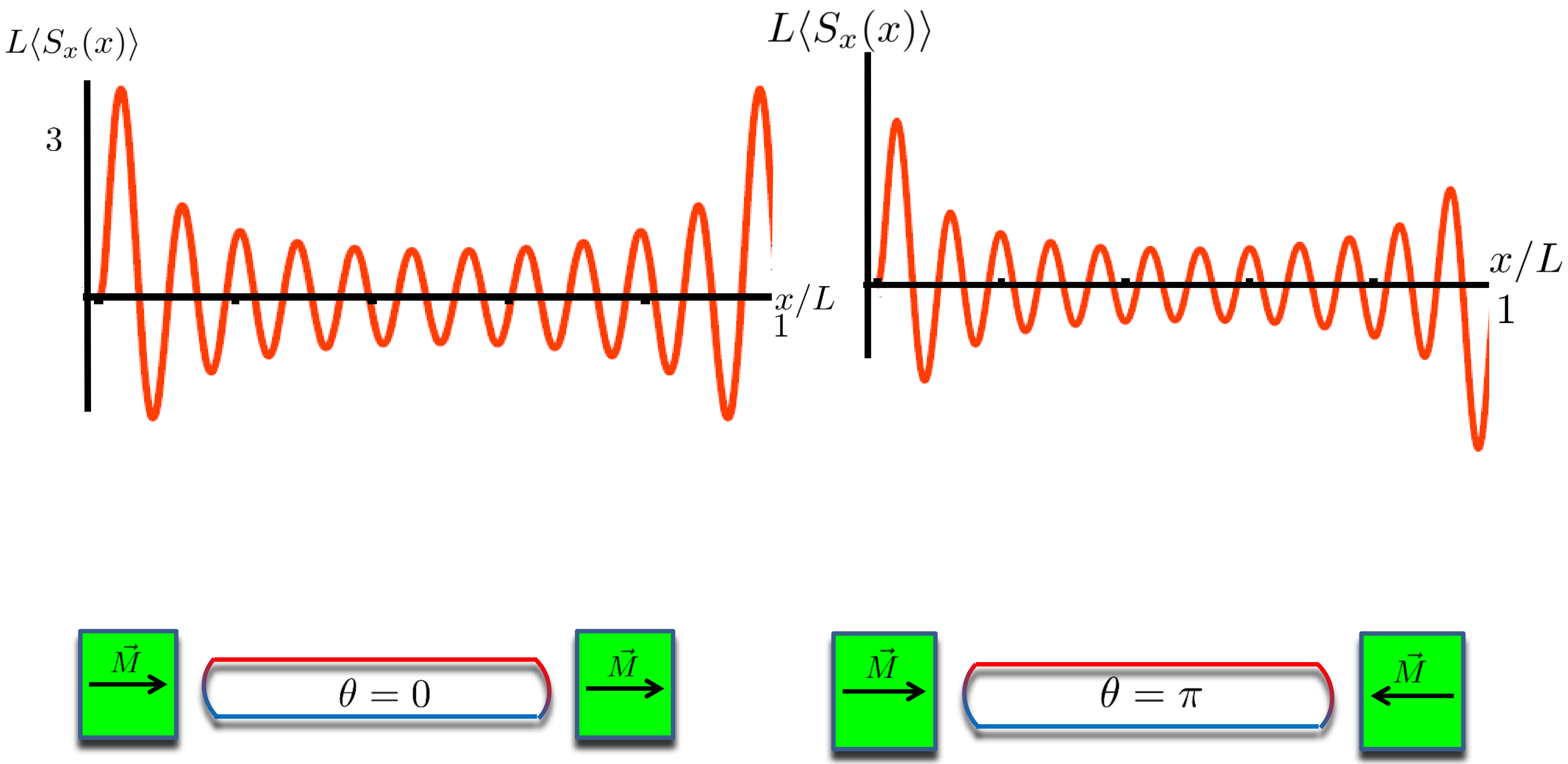
$T = 0, g = 0.7$

Difference of half an oscillation!

e/2 topological background charge inside the helical island

[Qi *et al.*, Nat Phys 08;Meng *et al.*, PRL 12]14

$$N = 12$$

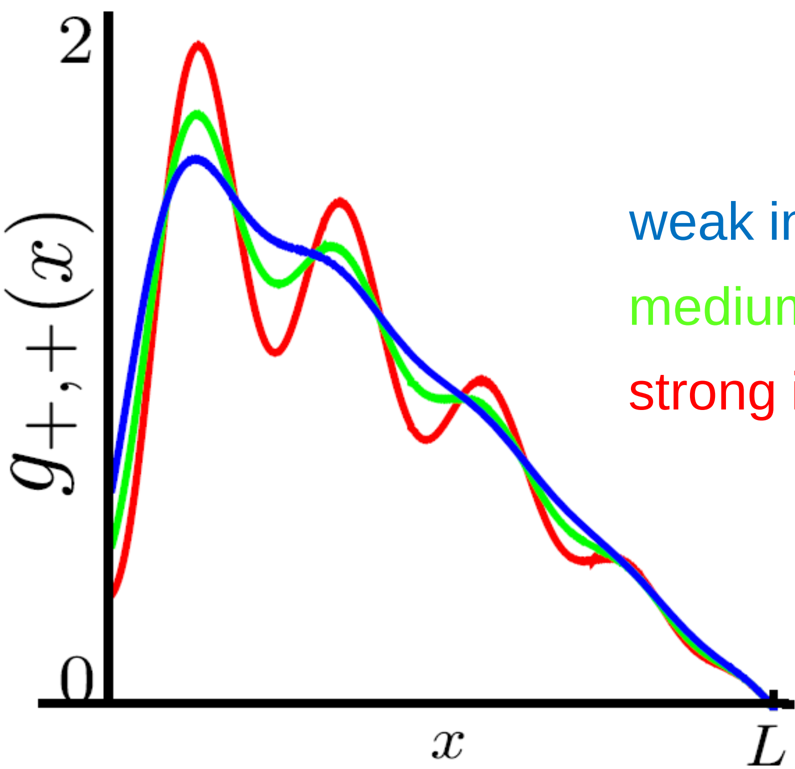


G. Dolcetto, N. Traverso, M. Biggio, F. Cavaliere, M.S. PRB 87, 235423 (2013)

Spin-resolved correlation functions for spin states

Probability of founding two electrons with spin + in the x- direction at relative distance x

$$g_{+,+}(x) = \frac{1}{N(N-1)} \int dy \langle \Psi^\dagger_+(y+\frac{x}{2})\Psi^\dagger_+(y-\frac{x}{2})\Psi_+(y-\frac{x}{2})\Psi_+(y+\frac{x}{2})\rangle$$



weak interactions: uncorrelated liquid state g=0.9

medium interactions g=0.55

strong interactions g=0.25 spin correlated state

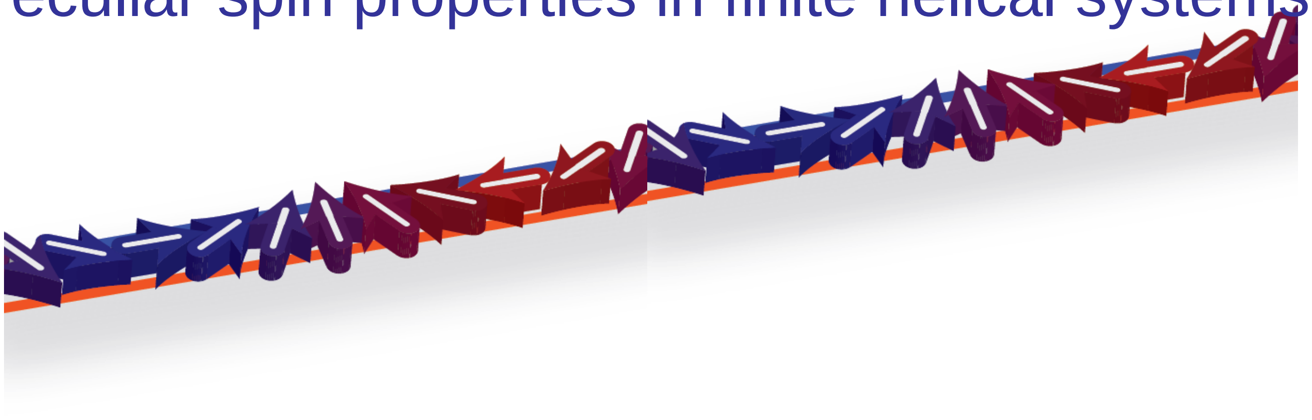
$N=5$ $T=0$

Marked peaks for strong interactions → tendency towards spin ordering

G. Dolcetto, N. Traverso, M. Biggio, F. Cavaliere, M.S. RRL (2013)

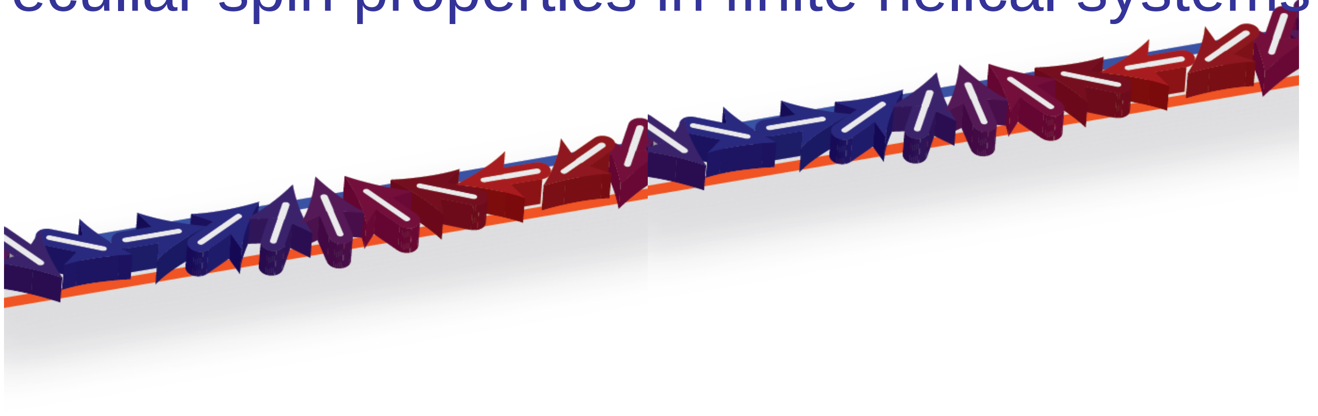
Conclusions

Peculiar spin properties in finite helical systems



Conclusions

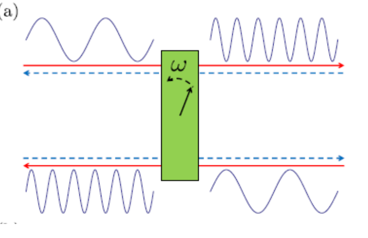
Peculiar spin properties in finite helical systems



Non-equilibrium spin properties (not shown)

- Magnetic AC control of the spin textures in a helical LL

  [G. Dolcetto, F. Cavaliere, M.S, PRB 89, 125419 (2014)]



- Spin pumping in helical systems

  [Ferraro, Dolcetto, Romeo, Citro, M.S. PRB 87, 085425 (2013)]