From phase to micro-phase separation in flocking models

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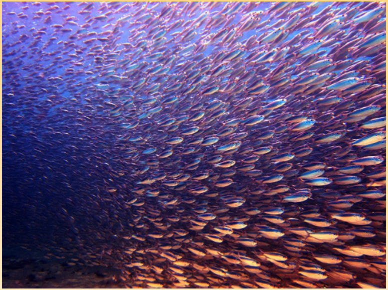
Advances in Nonequilibrium Statistical Mechanics



- Energy consumption at the microscopic scale $\longrightarrow$ Self-propulsion
- Aligning interactions

$\longrightarrow$ Collective motion (with long range-order?)

The Vicsek model [Vicsek et al. PRL 75, 1226 (1995)]

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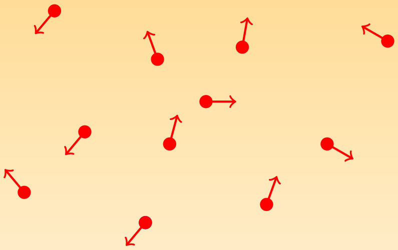

- N self-propelled particles off-lattice
- Local alignment rule

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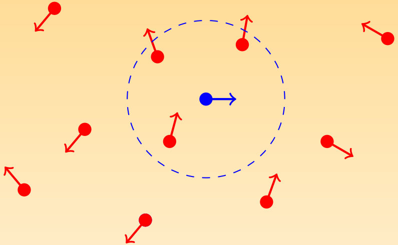

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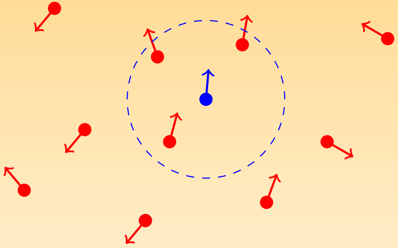

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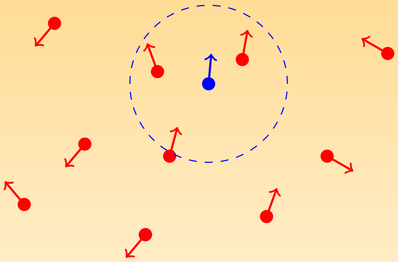

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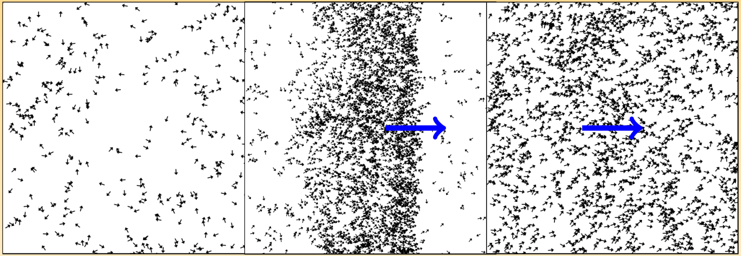
Flocking transition [Grégoire, Chaté, PRL (2004)]



Disordered | Inhomogeneous | Fluctuating flocking state

noise ↘ or density ↗

- Non-equilibrium transition to long-range order in $d = 2$

A long-standing debate

- Simulations are simple but strong finite size effects
- 2^{nd}-order (1995) vs 1^{st}-order (2004) [Gregoire and Chate, PRL 2004]
  $\longrightarrow$ hard to study numerically

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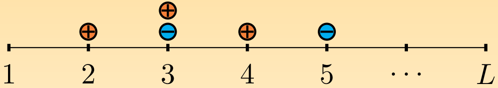
- Use a much simpler model: active Ising spins
  $\longrightarrow$ on lattice
  $\longrightarrow$ discrete symmetry

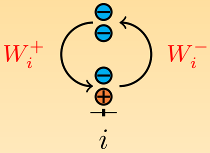
Active Ising model

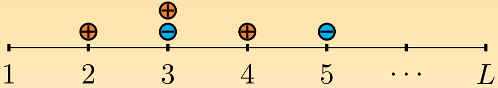


- Density $\rho_i = n_i^+ + n_i^-$ Magnetisation $m_i = n_i^+ - n_i^-$

Active Ising model

Spin-flip





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  $\leftrightarrow$ Fully connected Ising models on each site

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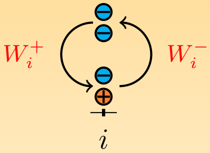
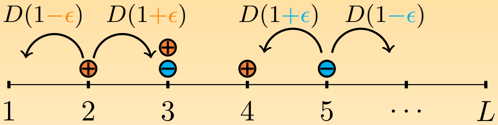


Biased diffusion



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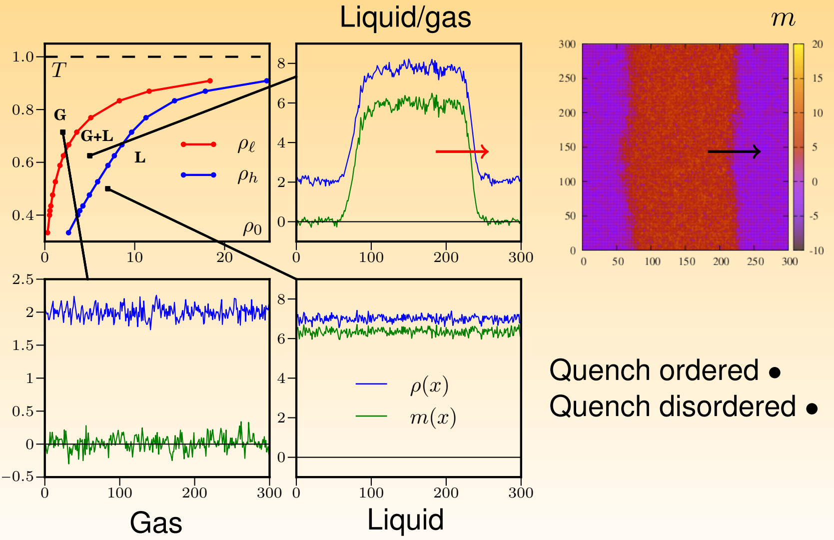
  $\leftrightarrow$ Fully connected Ising models on each site
- Self-propulsion $\leftrightarrow$ Diffusion biased by the spins for $\epsilon \neq 0$

Phase diagram in 2d



Quench ordered •
Quench disordered •

Mean-field and beyond

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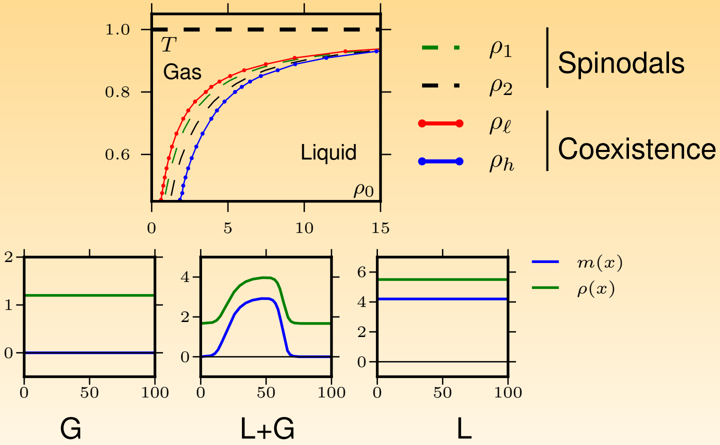
- Finite density: fluctuations $\longrightarrow$ $\beta_c = 1 + r/\rho$

- MF only valid at $\rho = \infty$ $\longrightarrow$ Refined-Mean-Field-Model (RMFM)

Simulations of the RMFM



- Same phenomenology as microscopic model

Hysteresis loops

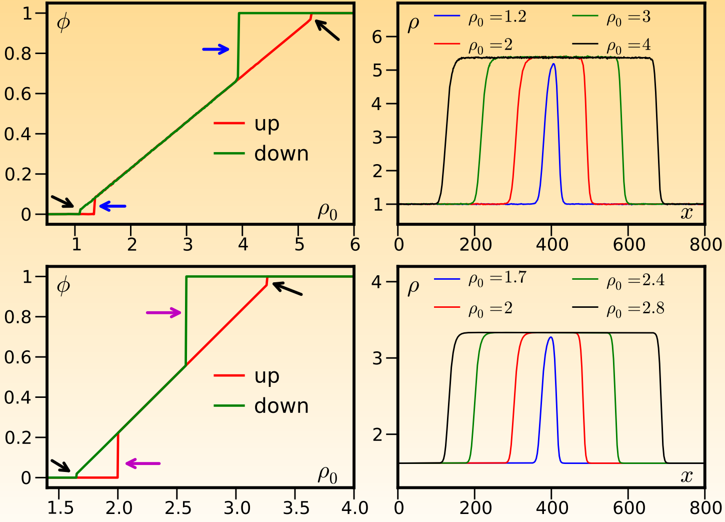
Micro 2d

interface effect

nucleation

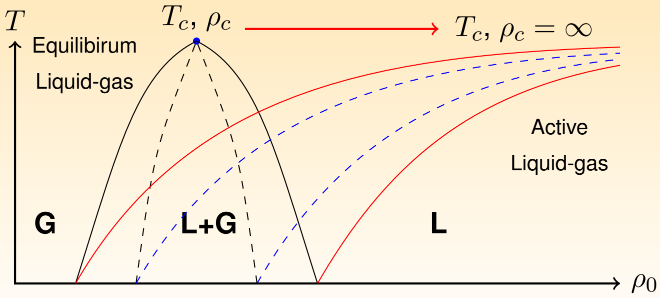
spinodal decomposition

RMFM



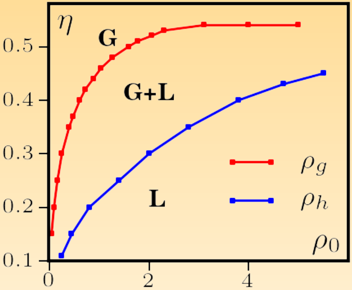
Active spins – summary

- New flocking model with discrete symmetry using active spins
- Flocking trans. $\longrightarrow$ Liquid-gas transition in canonical ensemble
- Symmetry of the liquid phase $\longrightarrow \rho_c = \infty$



Back to the Vicsek model (with H. Chaté)

- Phase diagram: liquid-gas picture seems ok



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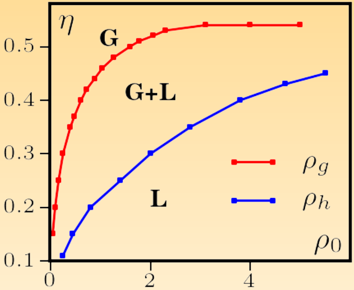
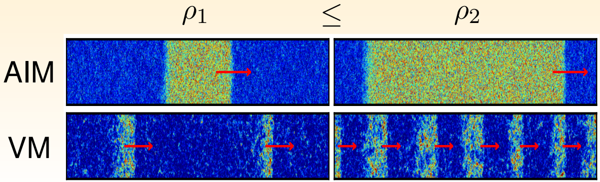


- But phase separation ⟶ micro-phase separation



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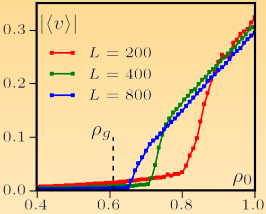

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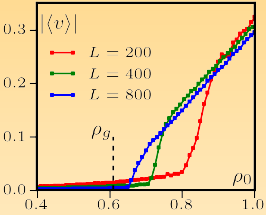
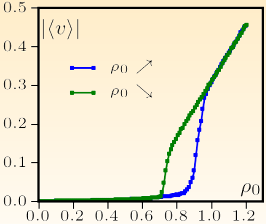
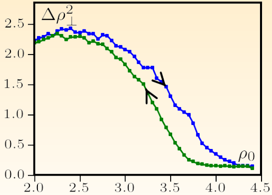


- Hysteresis:





Hydrodynamics: phase vs micro-phase separation

- Both type of propagative solutions exist for generic hydrodynamic descriptions [Caussin et al., PRL 2014]

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- Scalar order parameter (Ising)

$$\partial_t \rho = -v \partial_x m$$

$$\partial_t m + \xi m \partial_x m = D \nabla^2 m - \lambda \partial_x \rho + \left[(\rho - \rho_c) - \frac{m^2}{P_0^2 \rho} \right] m$$

- Vectorial order parameter (Vicsek)

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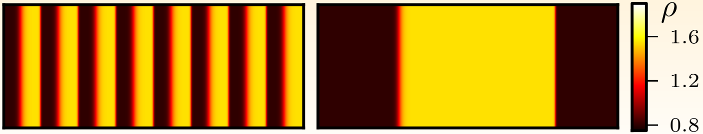

PDE Ising PDE Vicsek

Fluctuations play a crucial role

- PDEs + noises do a good job: • •

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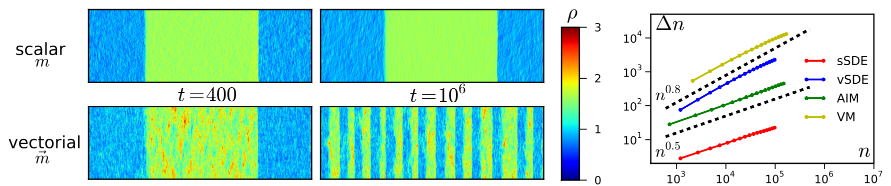

- The nature of the phase-separated states stems from the interplay between fluctuations and symmetry of the order parameter

Conclusion

- Flocking trans. $\longrightarrow$ Liquid-gas transition in canonical ensemble

- Symmetry of the liquid phase $\longrightarrow \rho_c = \infty$

- Different universality classes:
  - Ising $\longrightarrow$ phase separation
  - Vicsek $\longrightarrow$ micro-phase separation

- Active Ising Model [A. Solon, J.T., PRL **111** 078101, (2013)]

- Study of Hydrodynamic equations [JB. Caussin, A. Solon, A. Peshkov, H. Chaté , T. Dauxois, J.T., V. Vitelli, D. Bartolo et al., PRL **112** 148102, (2014)]

- AIM (follow-up) and Vicsek: hopefully next week on the arxiv !

Phase-separated profiles

- Propagating shocks between $\rho_\ell, m_\ell = 0$ and $\rho_h, m_h \neq 0$
- Stationary solutions in comoving frame of velocity c

$$D\rho'' + c\rho' - vm' = 0 \quad (1)$$

$$Dm'' + cm' - v\rho' + 2m(\beta - 1 - \frac{r}{\rho}) - \alpha\frac{m^3}{\rho^2} = 0 \quad (2)$$

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- Solvable at large densities $\rho_1 = \frac{r}{\beta-1} \gg r$
  1: Solve (1) to get $\rho = \rho_\ell + \frac{v}{c}\sum_{k=0}^{\infty}\left(-\frac{D}{c}\nabla\right)^k m$
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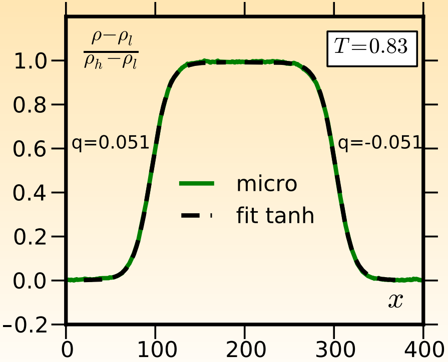
$$D(1+\frac{v^2}{c^2})m'' + [c - \frac{v^2}{c} - \frac{2Dvr}{c^2\rho_1^2}m]m' - \frac{2r(\rho_1-\rho_\ell)}{\rho_1^2}m + \frac{2rv}{c\rho_1^2}m^2 - \alpha\frac{m^3}{\rho_1^2} = 0$$

Symmetric front solutions $\beta \simeq 1$

$$m^{\pm}(x) = \frac{m_h}{2}(\tanh(q^{\pm}(x - ct)) + 1)$$

$$c = v \qquad m_h = \frac{4r}{3\alpha} \qquad q^{\pm} = \pm\frac{\beta - 1}{3\sqrt{\alpha D}}$$



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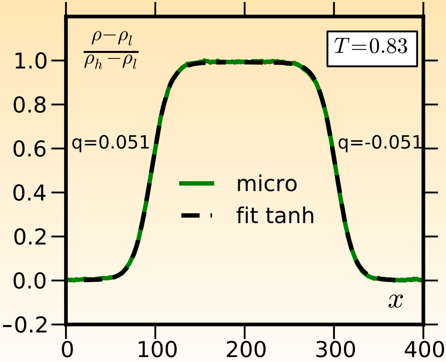
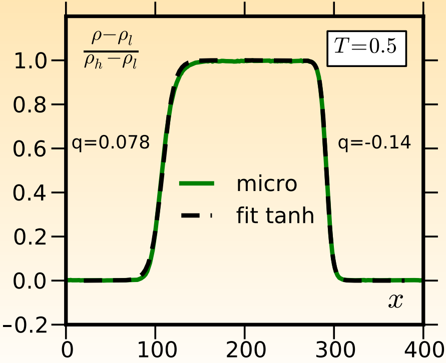


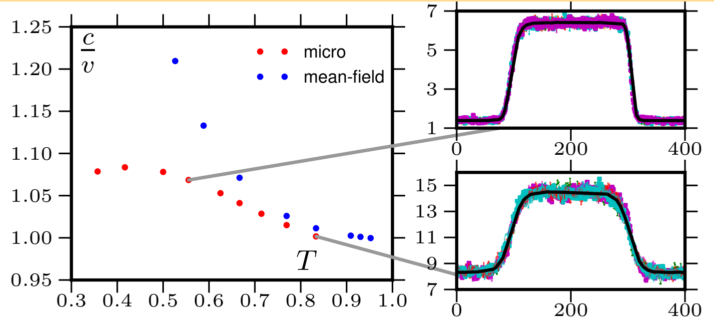
Asymmetric front solutions $\beta > 1$

$$m^{\pm}(x) = \frac{m_h}{2}(\tanh(q^{\pm}(x - ct)) + 1)$$

$$c = v + \frac{2Dr^2}{3v\alpha\rho_1^2} \quad q^{\pm} = \pm\frac{r}{3\rho_1\sqrt{\alpha D}} - \frac{r^2}{6\alpha v\rho_1^2} \quad m_h = \frac{4r}{3\alpha} - \frac{8Dr^3}{9v^2\alpha^2\rho_1^2}$$



The flock fly faster than the birds



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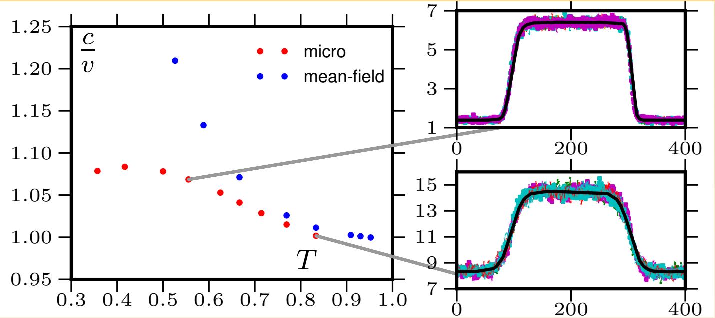

- $c = v + \frac{2Dr^2}{3v\alpha\rho_1^2}$
- v ⟶ microscopic velocities
- $\frac{2Dr^2}{3v\alpha\rho_1^2}$ ⟶ FKPP-like contribution