Strongly Coupled Gauge Theories and Strings

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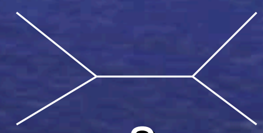
The Beginnings

String Theory was born out of attempts to understand the Strong Interactions!

Empirical evidence for a string-like structure of hadrons comes from arranging mesons and baryons into Regge trajectories.

Dolen-Horn-Schmid duality conjecture: in meson scattering, sum over s-channel exchanges



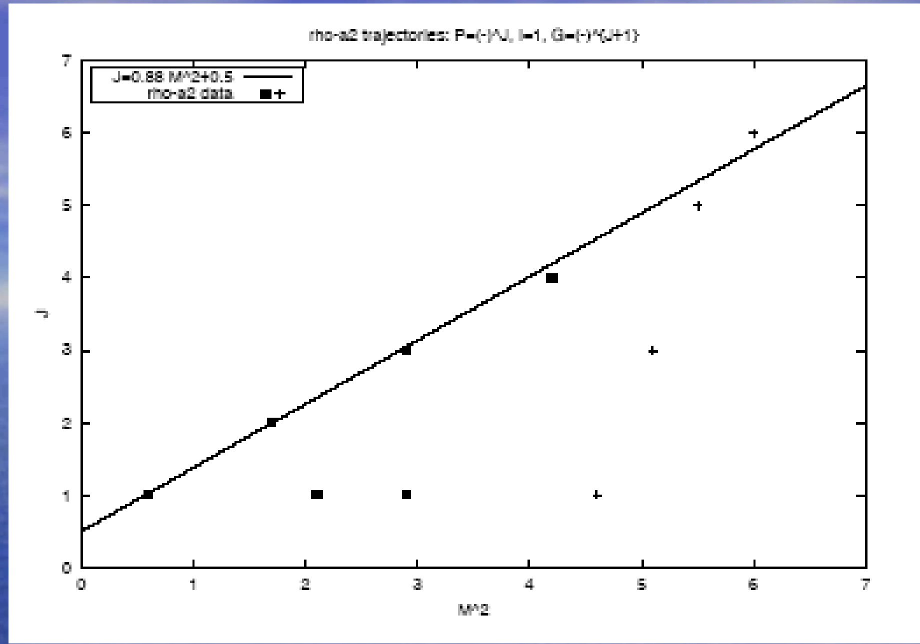
s

equals sum over t-channel exchanges

t

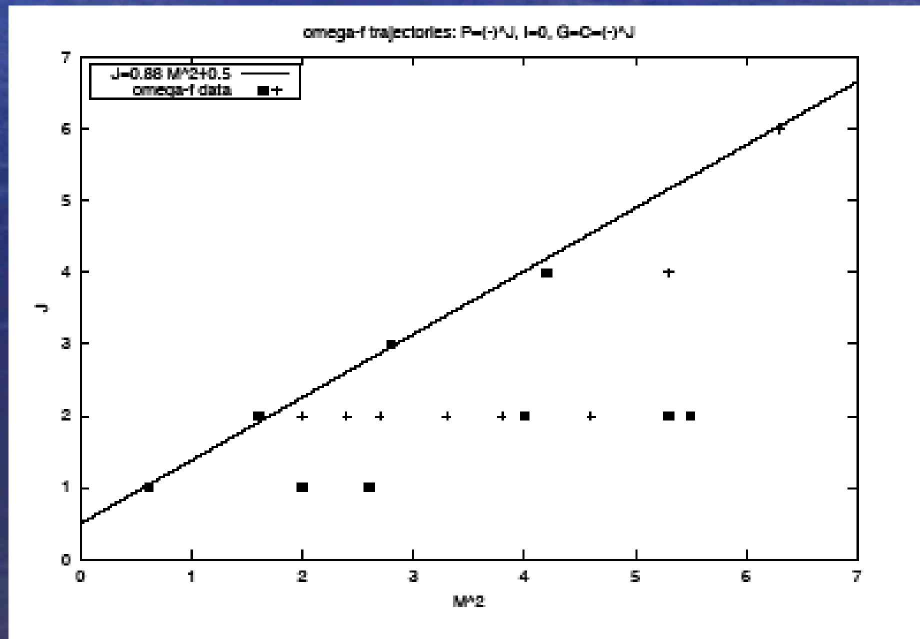
- The I=1 leading meson Regge trajectory

  (ρ, a_2 ...)



- The I=0 leading meson Regge trajectory

  (ω, f_2 ...)

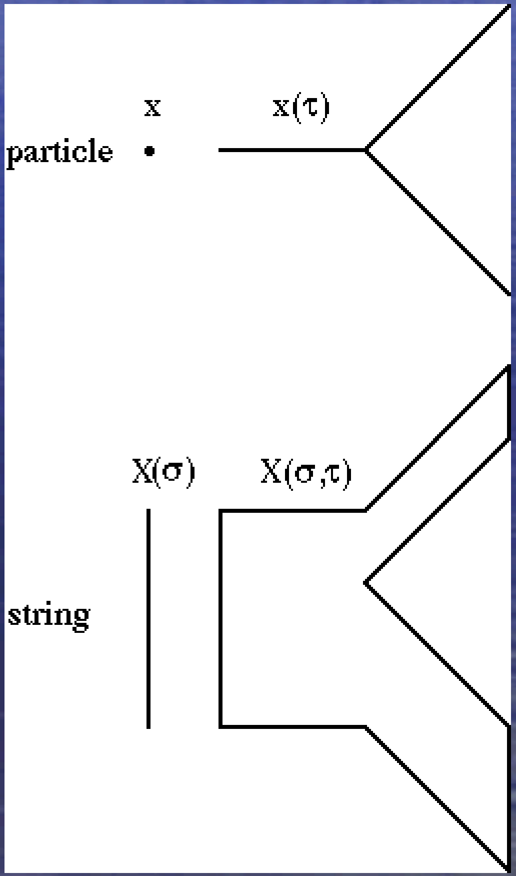


- Veneziano proposed a manifestly dual amplitude for elastic pion scattering:

$$A(s,t) \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

with linear Regge trajectory

$$\alpha(s) = \alpha(0) + \alpha' s$$



- Nambu, Nielsen and Susskind independently proposed its open string interpretation

- The string world sheet dynamics is governed by the Nambu-Goto area action

$$S_{\rm NG} = -T \int d\sigma d\tau \sqrt{-\det\ \partial_a X^\mu \partial_b X_\mu}$$

- The string tension is related to the Regge slope through

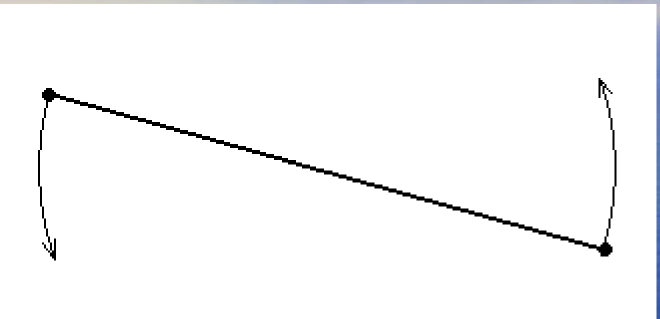
$$T = \frac{1}{2\pi\alpha'}$$

- The quantum consistency of the Veneziano model requires that the Regge intercept is $\alpha(0) = 1$ so that the spin 1 state is massless but the spin 0 is a tachyon.

- Calculation of the string zero-point energy gives $\alpha(0) = \frac{d-2}{24}$

- Hence the model has to be defined in 26 space-time dimensions.

Spinning Open String Picture of Mesons



$$J = \alpha' m^2 + \alpha(0)$$

- The linear relation between angular momentum and mass-squared is provided by a semi-classical spinning relativistic string with massless quark and anti-quark at its endpoints.

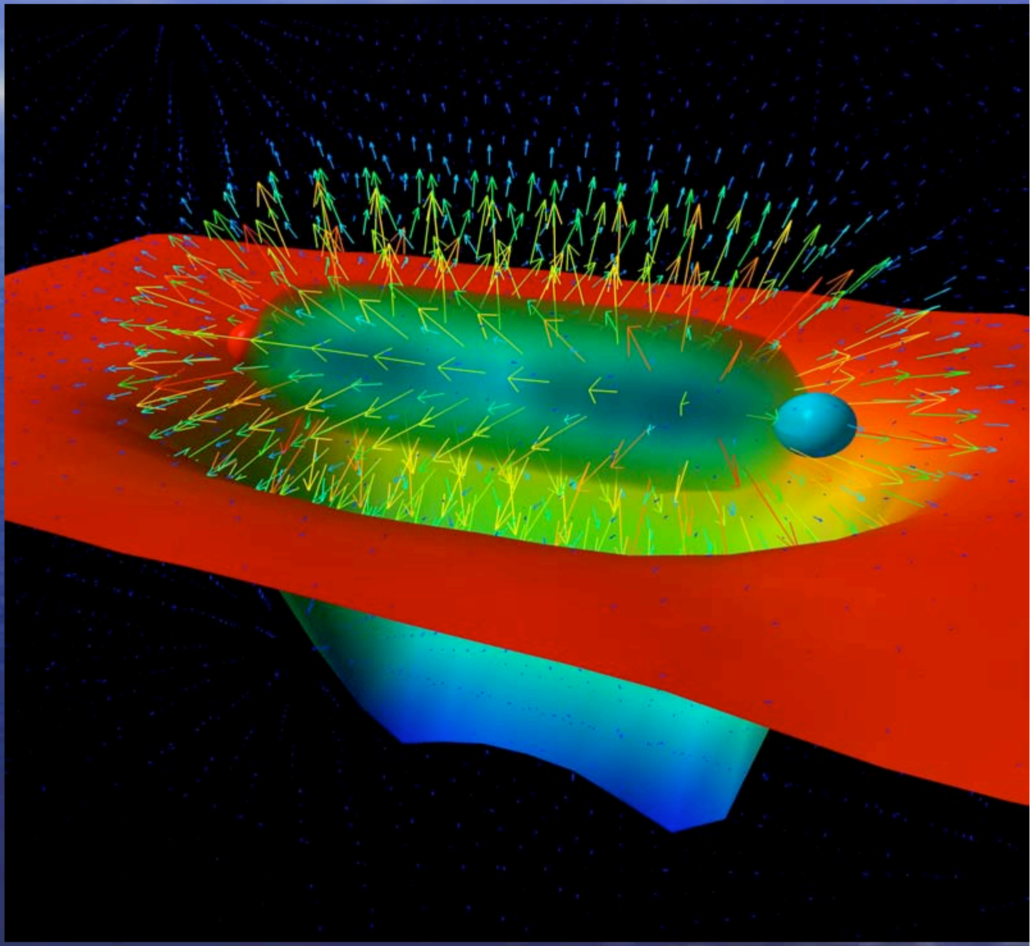
Crossroads in the 1970's

- Attempts to quantize such a string model in 3+1 dimensions lead to tachyons, problems with unitarity.
- Consistent supersymmetric string theories were discovered in 9+1 dimensions, but their relation to strong interaction was initially completely unclear.
- Most importantly, the Asymptotic Freedom of strong interactions was discovered by Gross, Wilczek; Politzer in 1973. This singled out the Quantum Chromodynamics (QCD) as the exact theory of strong interactions.

- Most physicists gave up on strings as a description of strong interactions. Instead, string theory emerged as the leading hope for unifying quantum gravity with other forces (the graviton appears in the closed string spectrum). Scherk, Schwarz; Yoneya

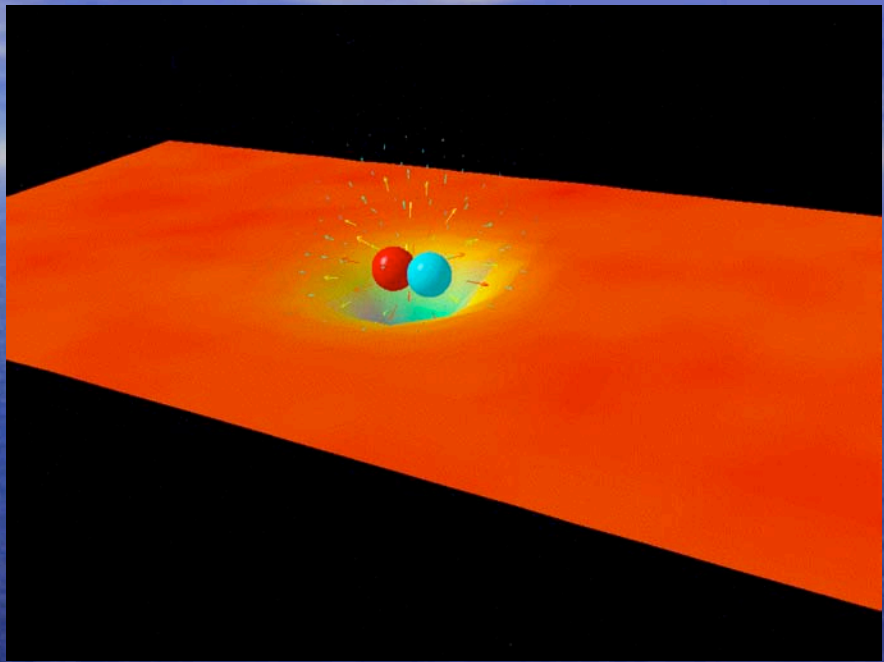
QCD gives strings a chance!

- At short distances, must smaller than 1 fermi, the quark-antiquark potential is Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.



Flux Tubes in QCD

- These objects may be approximately described by the Nambu strings
  (animation from lattice work by D. Leinweber et al, Univ. of Adelaide)

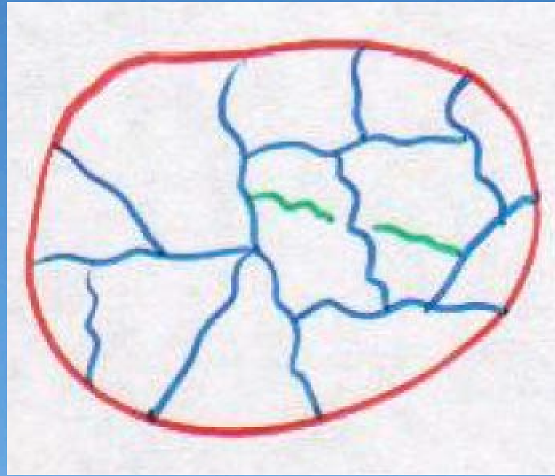


- The tubes are widely used, for example, in jet hadronization algorithms (the Lund String Model) where they snap through quark-antiquark creation.

Large N Gauge Theories

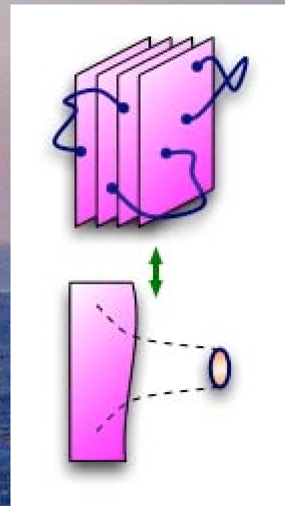
- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling $\lambda = g_{\rm YM}^2 N$ fixed.
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

- In the large N limit the gauge theory simplifies (only planar diagrams contribute). Adding a non-planar line (green) causes a $1/N^2$ suppression.



- But it is still very difficult!
- Between mid-70's and mid-90's many theorists gave up hope of finding an exact gauge/string duality.
- An important exception is Polyakov, who already in 1981 proposed that the string theory dual to a 4-d gauge theory should have a 5-th hidden dimension, and later argued that the 5-d space must be "warped".

Breaking the Ice



- Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.
- A stack of N Dirichlet 3-branes realizes N=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

- Successful matching of graviton absorption by D3-branes, related to 2-point function of stress-energy tensor in the SYM theory, with a gravity calculation in the 3-brane metric (IK; Gubser, IK, Tseytlin) was a precursor of the AdS/CFT correspondence.

The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the N=4 SYM theory this compact space is a 5-d sphere.
- The SO(2,4) geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The d-dimensional AdS space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g^2_{\mathrm{YM}} N}$

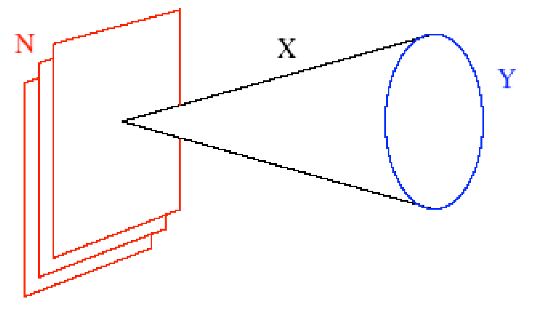
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

Could the closed string side of the duality exhibit a simplification?

- My recent work with Dymarsky and Roiban reconsiders gauge theory on a stack of D3-branes at the tip of a cone R^6/Γ where the orbifold group Γ breaks all the supersymmetry.
- At first sight, the gauge theory seems conformal because the beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$.

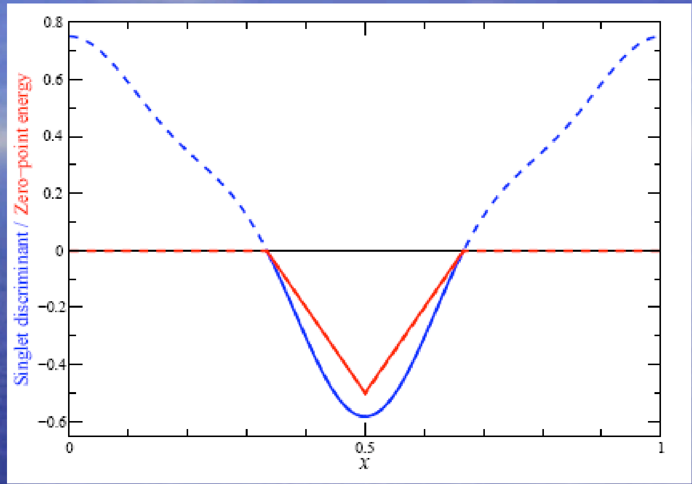
  Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson



- However, double-trace operators made out of twisted single-trace ones, $f\, O_n\, O_{-n}$, are induced at one-loop. Their beta-functions have the form

$$\beta_f = a\,\lambda^2 + 2\,\gamma\, f\,\lambda + f^2$$

- If $D=\gamma^2 - a < 0$, then there is no real fixed point for f.
- Here is a plot of a one-loop $SU(N)^k$ gauge theory quantity, D, and of the ground state closed string m^2 on the cone without the D-branes. $n=1, \ldots, k-1$ labels the twisted sector for a class of Z_k orbifolds that are freely acting on the 5-sphere, and $x=n/k$.



- The one-loop beta functions destroy the conformal invariance precisely in those twisted sectors where there exist closed-string tachyons localized at the tip of R^6/Γ. Thus, a very simple correspondence between perturbative gauge theory and free closed string on an orbifold emerges. Why?

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimension is determined by the mass of the dual field; e.g. for scalar operators

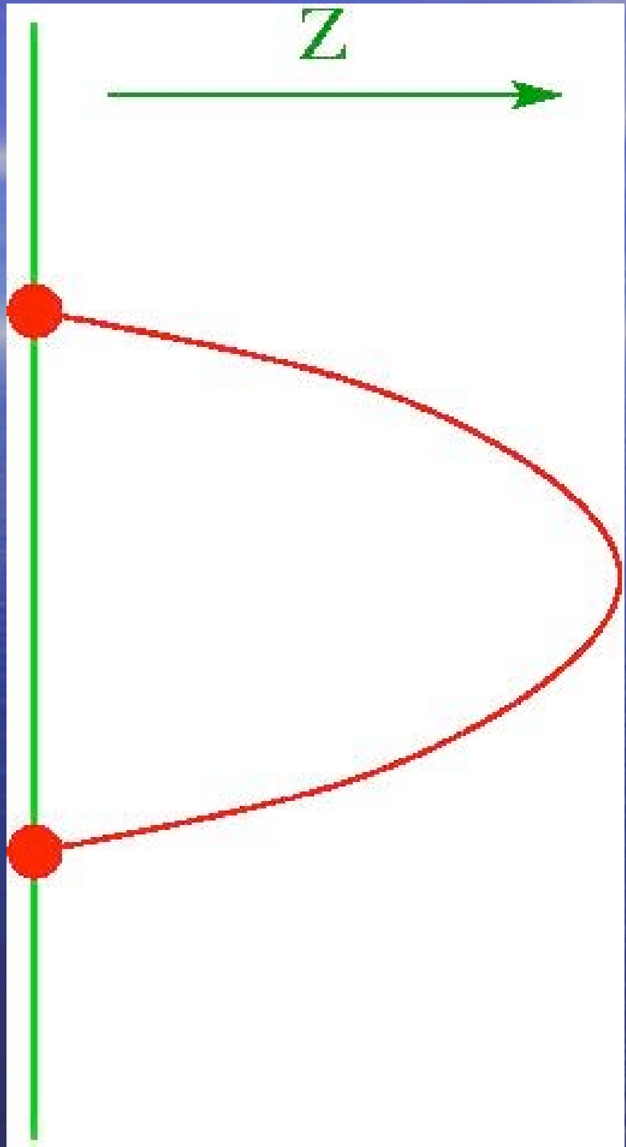
$$\Delta_{\pm} = 2 \pm \sqrt{4 + m^2 L^2}$$

- Correlation functions are calculated from the dependence of string theory path integral on boundary conditions ϕ_0 in AdS_5, imposed near z=0:

$$\langle \exp \int d^4 x \phi_0 \mathcal{O} \rangle = Z_{\rm string}[\phi_0]$$

- In the large N limit the path integral is found from the classical string action: $Z_{\rm string}[\phi_0] \sim \exp(-I[\phi_0])$

- The z-direction is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- In a pleasant surprise, because of the 5-th dimension z, the string picture applies even to theories that are conformal (not confining!). The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)



Application: entropy of thermal supersymmetric SU(N) theory

- Thermal CFT is described by a black hole in AdS_5

$$ds^2_{BH} = \frac{L^2}{z^2}\left(\frac{dz^2}{1 - z^4/z_h^4} - (1 - z^4/z_h^4)(dx^0)^2 + \sum_{i=1}^{3}(dx^i)^2\right)$$

- The CFT temperature is identified with the Hawking T of the horizon located at z_h
- Any event horizon contains Bekenstein-Hawking entropy $S_{BH} = \frac{2\pi A_h}{\kappa^2}$
- A brief calculation gives the entropy density $s = \frac{\pi^2}{2}N^2T^3$

Gubser, IK, Peet

- This is interpreted as the strong coupling limit of

$$s = \frac{2\pi^2}{3} f(\lambda) N^2 T^3$$

- For small `t Hooft coupling, Feynman graph calculations in the N=4 SYM theory give

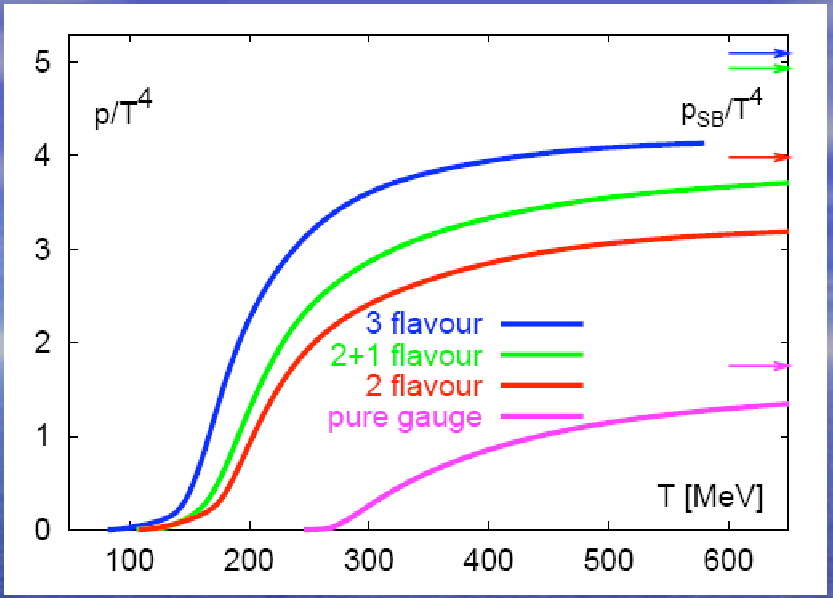
$$f(\lambda) = 1 - \frac{3}{2\pi^2}\lambda + \frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2} + \ldots$$

- We deduce from AdS/CFT duality that

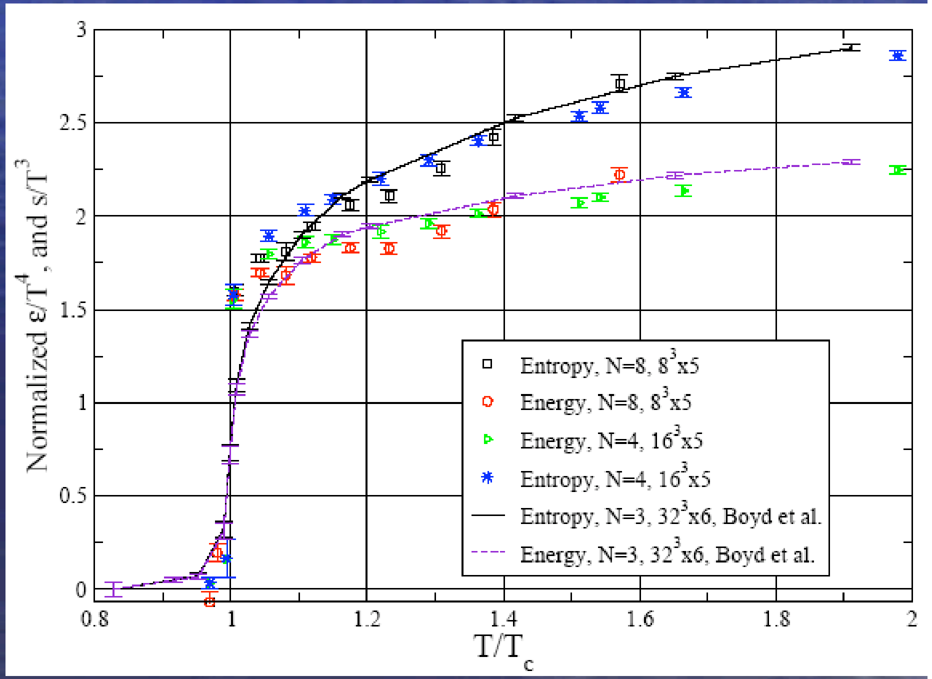
$$\lim_{\lambda\to\infty} f(\lambda) = \frac{3}{4}$$

- The entropy density is multiplied only by _ as the coupling changes from zero to infinity. Gubser, IK, Tseytlin

- A similar effect is observed in lattice simulations of non-supersymmetric gauge theories for N=3: the arrows show free field values.
  Karsch (hep-lat/0106019).



- N-dependence in the pure glue theory enters largely through the overall normalization.
  Bringoltz and Teper (hep-lat/0506034)



Shear Viscosity η of the Plasma

- In a comoving frame, $$T_{ij} = \delta_{ij} p - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right)$$
- Can be also determined through the Kubo formula
$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0,0)] \rangle$$
- For the N=4 supersymmetric YM theory this 2-point function may be computed from graviton absorption by the 3-brane metric.
- At very strong coupling, Policastro, Son and Starinets found $$\eta = \frac{\pi}{8} N^2 T^3$$

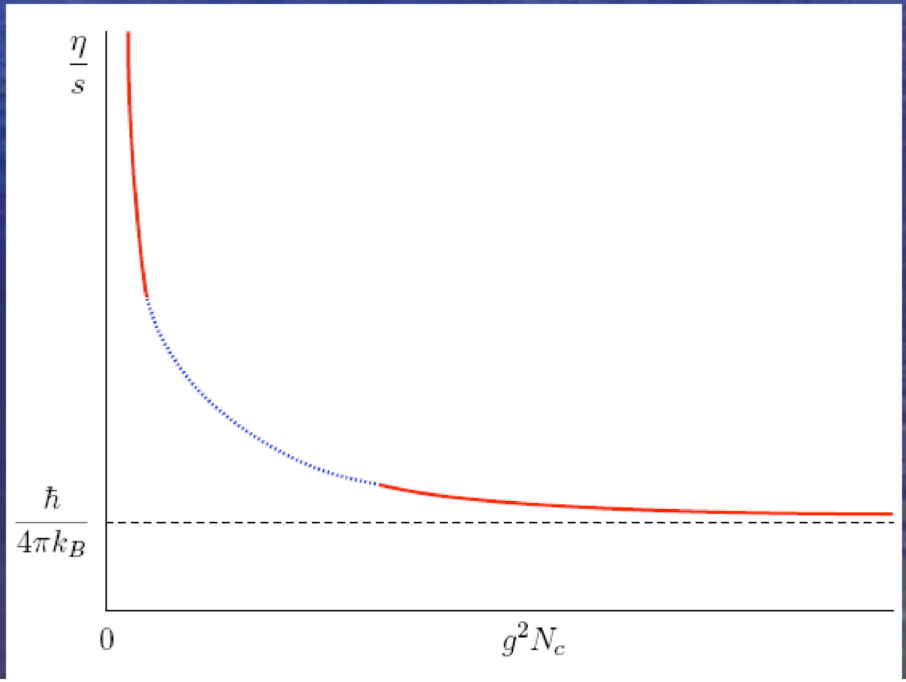
Viscosity/entropy lower bound?

Kovtun, Son, Starinets

- In the SYM theory at very strong coupling $\frac{\eta}{s} = \frac{\hbar}{4\pi}$
- This is reasonable on general grounds. The shear viscosity η ~ energy density times quasiparticle mean free time τ. So, η/s ~ quasiparticle energy x τ, which is bounded from below by the uncertainty principle.

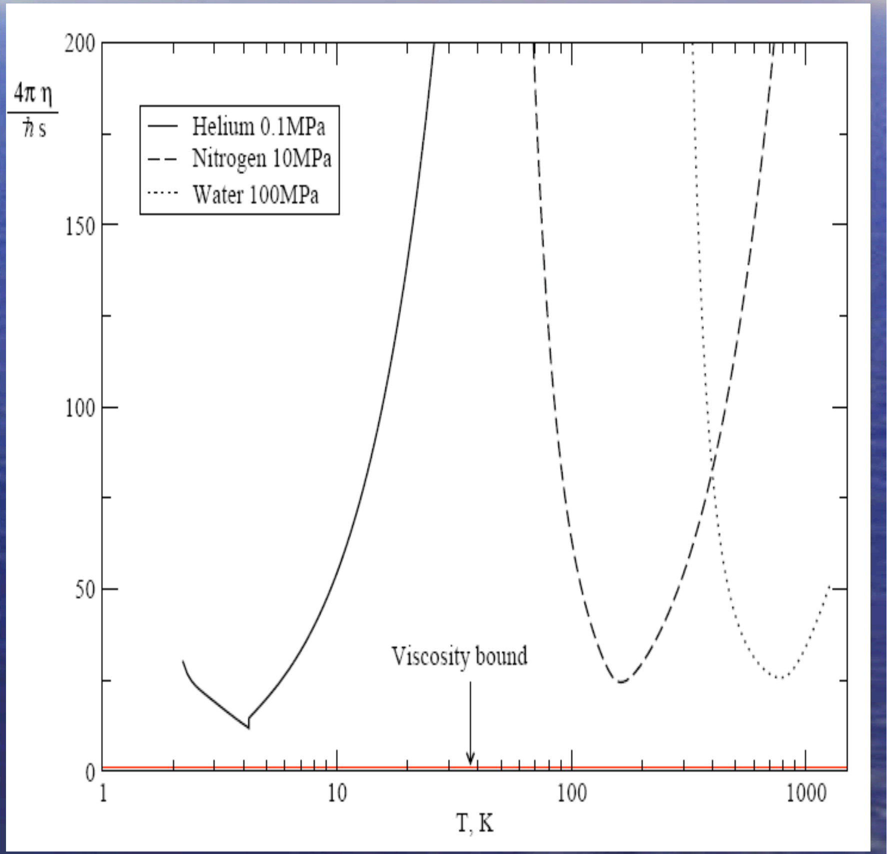
- At weak coupling η/s is large. There is evidence it decreases monotonically.

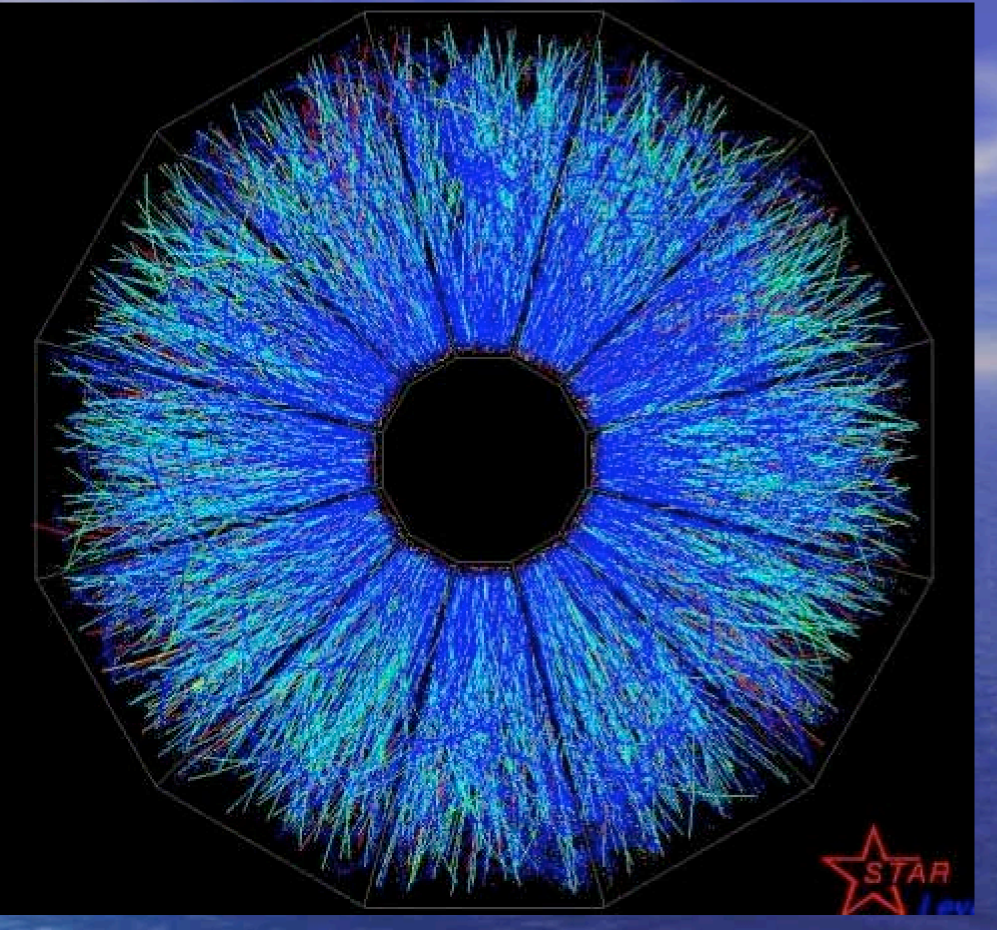
Buchel, Liu, Starinets



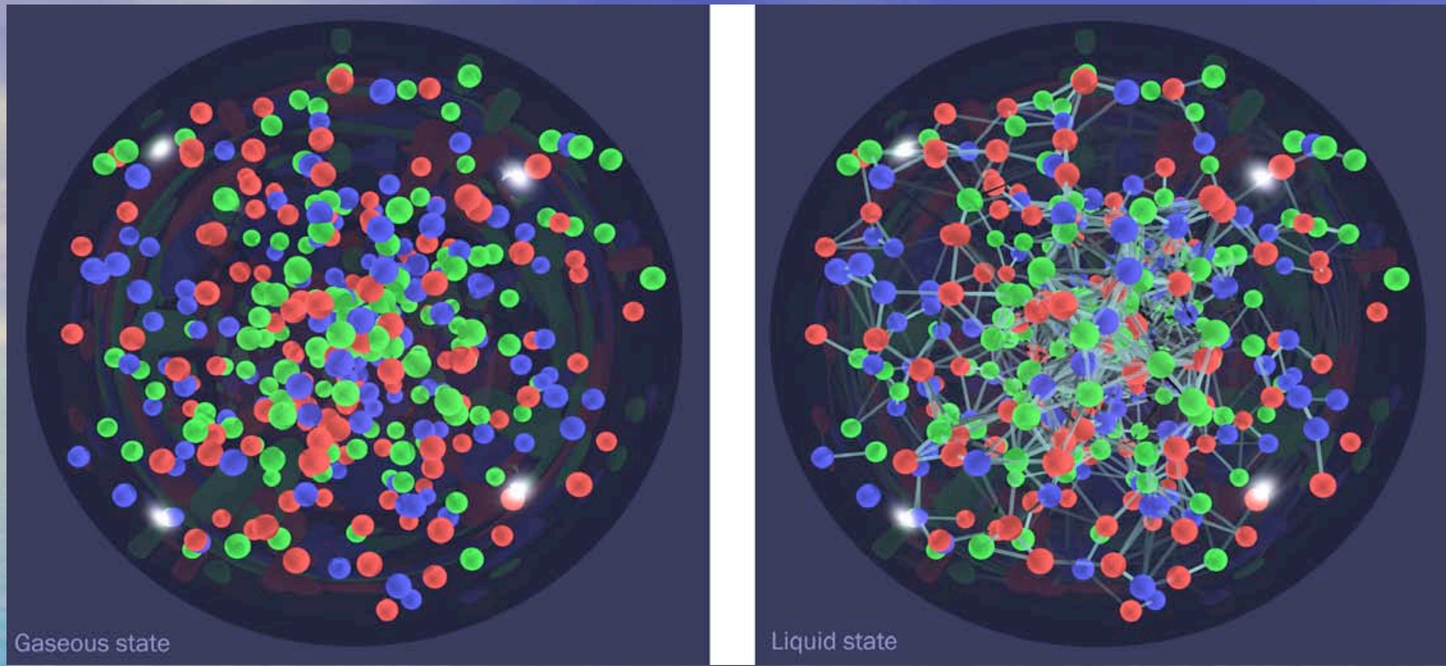
Is very strongly coupled SYM the most perfect fluid?

- For known fluids (e.g. helium, nitrogen, water) η/s is considerably higher.
- The quark-gluon plasma produced at RHIC is believed to be strongly coupled and to have very low viscosity, within a factor of 2 of the bound.
  Shuryak, Teaney, Gyulassy, McLerran, Hirano, ...
- A new term has been coined, sQGP, to describe the state observed at RHIC. Could it be approximated by a strongly coupled CFT?





A high-energy collision of gold ions at BNL's Relativistic Heavy Ion Collider (RHIC) produces a fireworks display of particle tracks, as recorded by the STAR detector. In central collisions, about 7500 hadrons are produced.



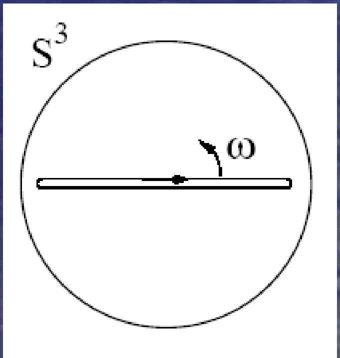
"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science. "The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating. String theory seeks to unify the two great intellectual achievements of twentieth-century physics, general relativity and quantum mechanics, and it may well have a profound impact on the physics of the twenty-first century." (from a BNL press release, April 2005)

Spinning Strings vs. Long Operators

- Vibrating closed strings with large angular momentum on the 5-sphere are dual to SYM operators with large R-charge. Berenstein, Maldacena, Nastase
- Generally, semi-classical spinning strings are dual to long operators, e.g. the dual of a high spin operator

$$\mathrm{Tr}\, F_{+\mu} D_+^{J-2} F_+{}^{\mu}$$

is a folded string spinning around the center of AdS_5. Gubser, IK, Polyakov



- The anomalous dimension of such a high spin twist-2 operator is $\Delta - (J+2) \to f(\lambda) \ln J$
- AdS/CFT predicts that at strong coupling

$$f(\lambda) \to \frac{\sqrt{\lambda}}{\pi}$$

- A 3-loop perturbative N=4 SYM calculation gives Kotikov, Lipatov, Onishchenko, Velizhanin

$$f(\lambda) = \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{11\lambda^3}{11520} + O(\lambda^4)\right)$$

- An approximate extrapolation formula works with about 10% accuracy:

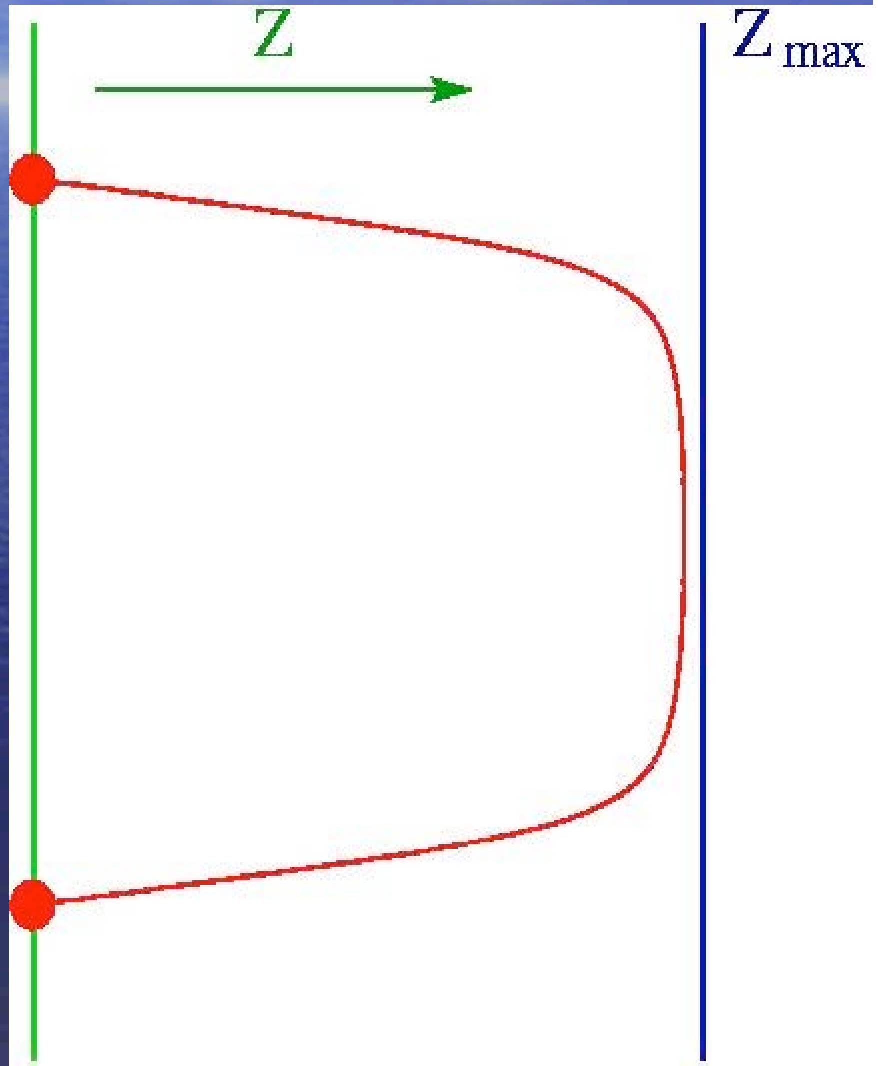
$$\tilde{f}(\lambda) = \frac{12}{\pi^2}\left(-1 + \sqrt{1+\lambda/12}\right) = \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{\lambda^3}{1152} + O(\lambda^4)\right)$$

String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A "cartoon" of the necessary metric is

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

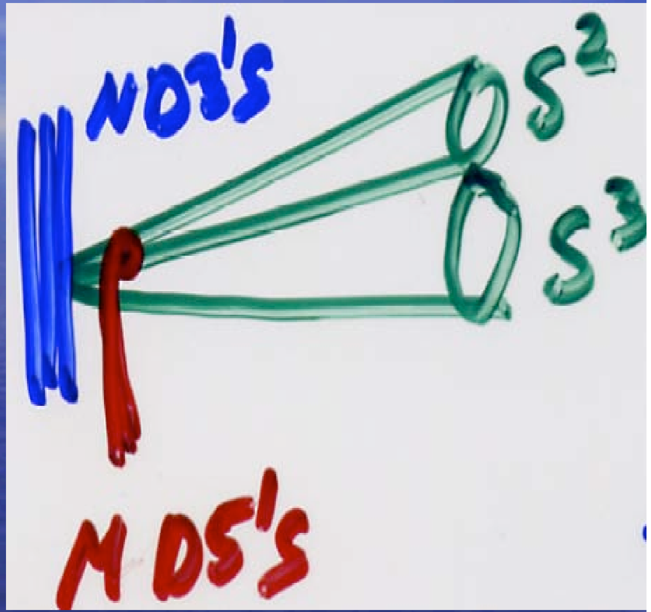
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\max})}{2\pi\alpha'}$



- Several 10-dimensional backgrounds with these qualitative properties are known (the compact space is actually “mixed” with the 5-d space).
- Witten (1998) constructed a background in the universality class of non-supersymmetric pure glue gauge theory. While there is no asymptotic freedom in the UV, hence no dimensional transmutation, the background serves as a simple model of confinement.
- Many infrared observables may be calculated from this background using classical supergravity. The lightest glueball masses are found from normalizable fluctuations around the supergravity solution. Their spectrum is discrete, and resembles qualitatively the results of lattice simulations in the pure glue theory.

Confinement in SYM theories

- Introduction of minimal supersymmetry (N=1) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)

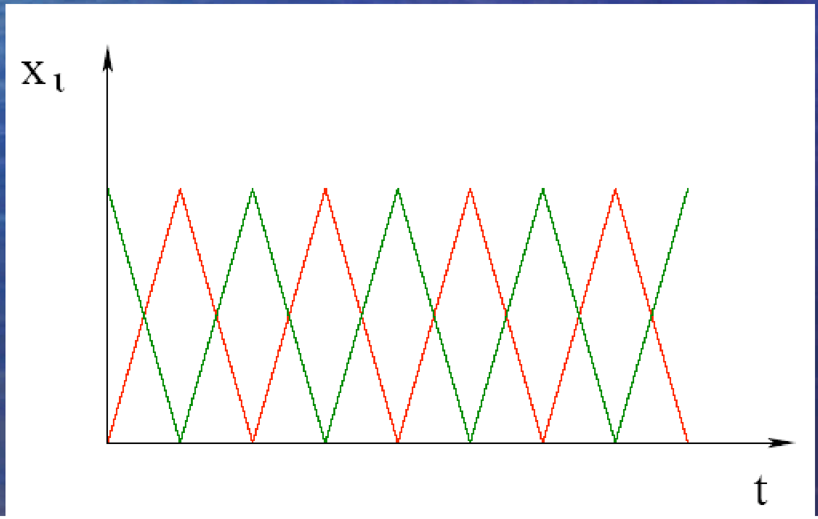


$$ds_{10}^2 = h^{-1/2}(\tau)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(\tau)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- Both the metric of the deformed conifold, and the warp factor are known in terms of hyperbolic functions, which allows for many explicit calculations.
- In the UV the background exhibits logarithmic running of the couplings in the dual SU(M(p+1))xSU(Mp) gauge theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2. A novel phenomenon, called a duality cascade, takes place, where p repeatedly changes by 1.



The two inverse gauge couplings $x_{i=1,2} = \frac{1}{g_i^2}$

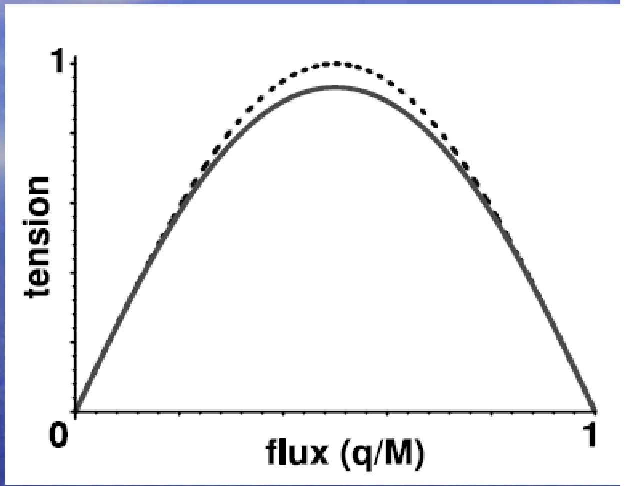
- Dimensional transmutation takes place. The dynamically generated confinement scale is $\sim \varepsilon^{2/3}$

  In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f=N_c$.
- The baryon and anti-baryon operators

$$\mathcal{B} \sim \epsilon_{\alpha_1\alpha_2\ldots\alpha_{2M}} (A_1)_1^{\alpha_1} (A_1)_2^{\alpha_2} \ldots (A_1)_M^{\alpha_M} (A_2)_1^{\alpha_{M+1}} (A_2)_2^{\alpha_{M+2}} \ldots (A_2)_M^{\alpha_{2M}}$$
$$\bar{\mathcal{B}} \sim \epsilon^{\alpha_1\alpha_2\ldots\alpha_{2M}} (B_1)_{\alpha_1}^1 (B_1)_{\alpha_2}^2 \ldots (B_1)_{\alpha_M}^M (B_2)_{\alpha_{M+1}}^1 (B_2)_{\alpha_{M+2}}^2 \ldots (B_2)_{\alpha_{2M}}^M$$

  acquire expectation values. Hence, we observe confinement without a mass gap: due to $U(1)_B$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.
- The warped deformed conifold is part of a continuous family (moduli space) of confining backgrounds which interpolate towards another similar background, the Maldacena-Nunez solution. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- This family of solutions is dual to the baryonic branch in the gauge theory: $\mathcal{B} = i\xi\Lambda_{2M}^{2M}\,, \qquad \bar{\mathcal{B}} = \frac{i}{\xi}\Lambda_{2M}^{2M}$

- An interesting observable is the tension of a composite string connecting q quarks with q anti-quarks. In any SU(M) gauge theory it must be symmetric under q -> M-q. This is achieved through a stringy effect: q strings blow up into a wrapped D3-brane. Herzog, IK

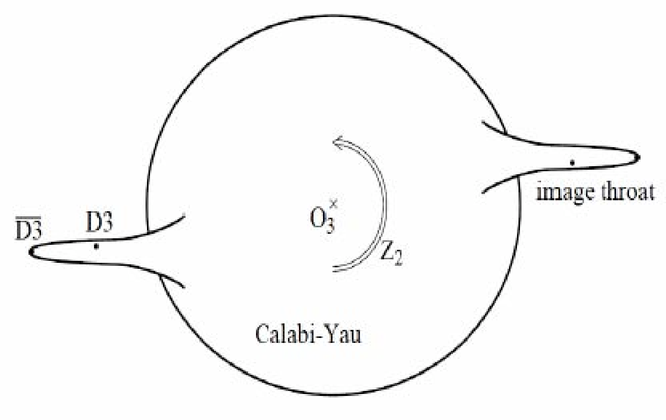


- Dashed line refers to the MN background where

$$T(q) \sim \sin(\pi q/M)$$

- More generally, the availability of string duals of confining backgrounds makes it possible to study deep-inelastic and hadron-hadron scattering. Polchinski, Strassler

Embedding in Flux Compactifications



- A long warped throat embedded into a compactification with NS-NS and R-R fluxes leads to a small ratio between the IR scale at the bottom of the throat and the string scale.

  Randall, Sundrum; Verlinde; IK, Strassler; Giddings, Kachru, Polchinski; KKLT; KKLMMT

- In the dual cascading gauge theory the IR scale is the confinement scale: confinement stabilizes the hierarchy between the Planck scale and the SM or the inflationary scale.

- Cascading gauge theories dual to "standard model throats" may offer interesting possibilities for new physics beyond the standard model. Cascales, Franco, Hanany, Saad, Uranga, ...

Connection with Cosmic strings

Copeland, Myers, Polchinski

- A fundamental string at the bottom of the warped deformed conifold is dual to a confining string. A D-string is dual to a solitonic string which couples to the Goldstone mode.
- Upon embedding of the warped throat into a flux compactification, these objects can be used to model cosmic strings.
- This throat is not the “standard model throat” but another throat, like the “inflationary throat,” dual to some hidden sector cascading gauge theory.

- Upon compactification, global symmetries become gauged. On the baryonic branch $U(1)_B$ is broken through a SUSY Higgs mechanism. We find an N=1 massive vector multiplet containing a massive vector degenerate with a Higgs scalar. The baryonic branch is lifted by the D-term potential $\sim g^2_{U(1)} \left(|\mathcal{B}|^2 - |\bar{\mathcal{B}}|^2 \right)^2$

- Hence, only the Z_2 symmetric warped deformed conifold solution is allowed.
- A D-string at the bottom of the throat, which has been used to model a cosmic string, should be dual to a solitonic ANO string in the cascading gauge theory, described at low energies by an effective field theory.

Gubser, Herzog, IK

Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions
- The AdS/CFT correspondence makes this connection precise. It makes a multitude of dynamical statements about strongly coupled conformal (non-confining) gauge theories.
- Its extensions to confining theories provide a new geometrical view of such important phenomena as dimensional transmutation and chiral symmetry breaking. This allows for calculations of glueball and meson spectra and of hadron scattering in model theories.

- This recent progress offers new tantalizing hopes that an analytic approximation to QCD may be achieved along this route, at least for a large number of colors.
- But there is much work to be done if this hope is to become a reality. Understanding the string duals of weakly coupled gauge theories remains an important open problem.
- Embedding gauge/string dualities into string compactifications offers new possibilities for physics beyond the SM, and for modeling inflation and cosmic strings.