Long-range correlations in driven systems (II)

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Outline

Will discuss two examples where long-range correlations show up and consider some consequences

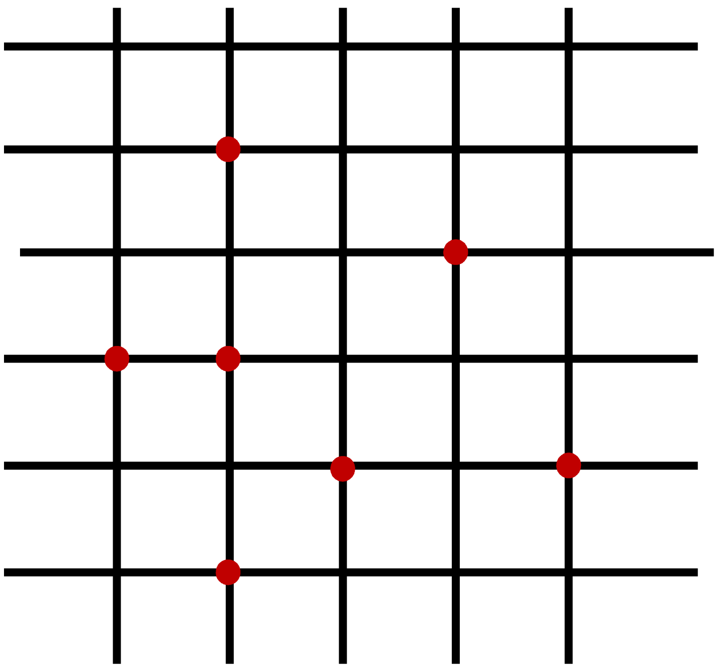
- Example I: Effect of a local drive on the steady state of a system
- Example II: Linear drive in two dimensions: spontaneous symmetry breaking

- Example I :Local drive perturbation

T. Sadhu, S. Majumdar, DM, Phys. Rev. E 84, 051136 (2011)

Local perturbation in equilibrium

Particles diffusing (with exclusion) on a grid



occupation number $n_k = 0,1$

occupation number

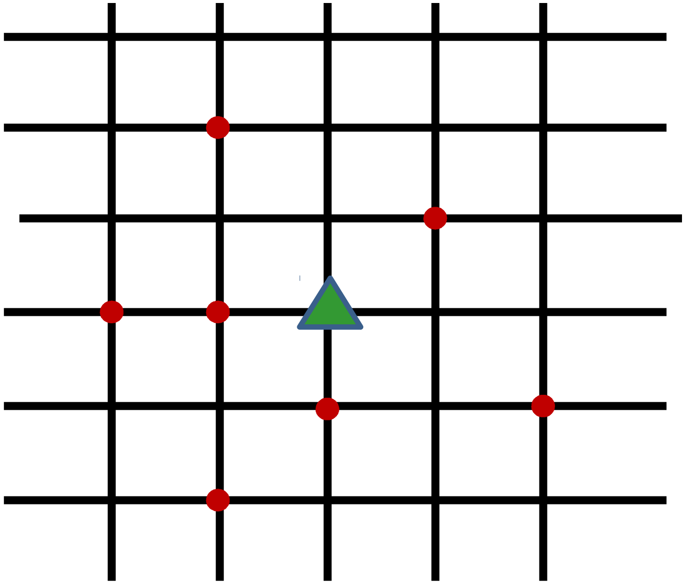
N particles
V sites

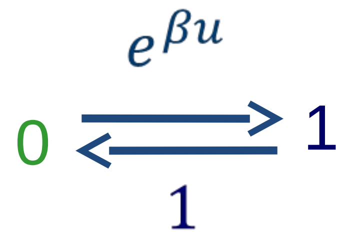
Prob. of finding a particle at site k

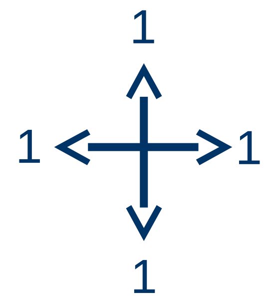
$$p(k) = \frac{N}{V}$$

Add a local potential u at site 0

N particles
V sites

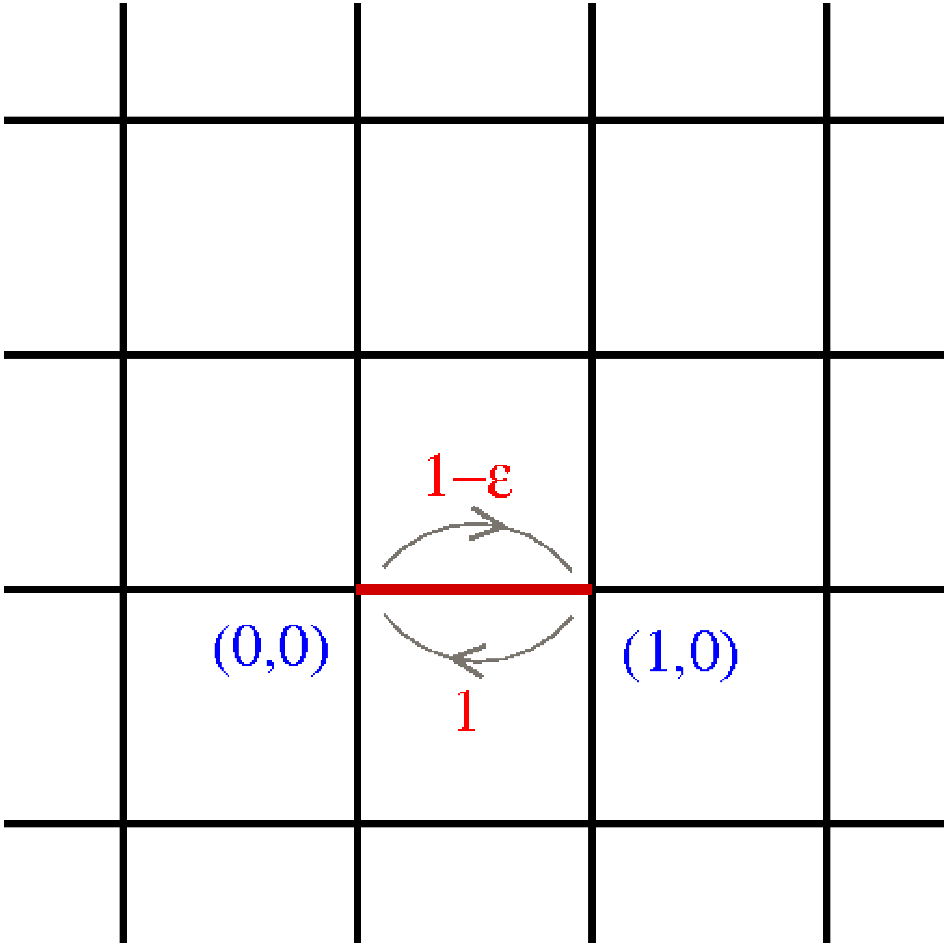






The density changes only locally.

Effect of a local drive: a single driving bond



Main Results

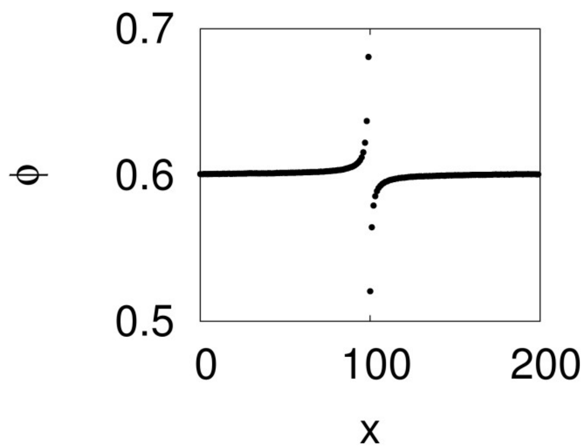
- In d ≥ 2 dimensions both the density corresponds to a potential of a dipole in d dimensions, decaying as $\phi(r) \sim 1/r^{d-1}$ for large r. The current satisfies $j(r) \sim 1/r^{d}$.

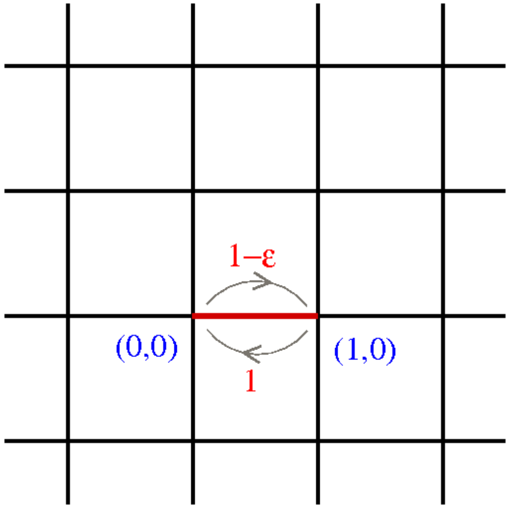
- The same is true for local arrangements of driven bonds. The power law of the decay depends on the specific configuration.

- The two-point correlation function corresponds to a quadrupole In 2d dimensions, decaying as $G(r,s) \sim 1/(r^2+s^2)^d$ for $\rho = 1/2$

- The same is true at other densities to leading order in ϵ (order ϵ^2).

Density profile (with exclusion)





The density profile The density profile

$$\phi(\vec{r}) \sim \begin{cases} 1/r^2 & \text{along the y axis} \\ 1/r & \text{in any other direction} \end{cases}$$

Non-interacting particles

- Time evolution of density:

- Time evolution of density:

$$\partial_t \phi(\vec{r},t) = \nabla^2 \phi(\vec{r},t) + \epsilon \phi\left(\vec{0},t\right)[\delta_{\vec{r},\vec{0}} - \delta_{\vec{r},\overrightarrow{e_1}}]$$

$$\nabla^2 = \phi(m+1,n) + \phi(m-1,n) + \phi(m,n+1) + \phi(m,n-1) - 4\phi(m,n)$$

The steady state equation

$$\boxed{\nabla^2 \phi(\vec{r}) = -\epsilon \phi(\vec{0})[\delta_{\vec{r},\vec{0}} - \delta_{\vec{r},\overrightarrow{e_1}}]}$$

particle density $\rightarrow$ electrostatic potential of an electric dipole

particle density electrostatic potential of an electric dipole

$$\boxed{\nabla^2\phi(\vec{r}) = -\epsilon\phi(\vec{0})[\delta_{\vec{r},\vec{0}} - \delta_{\vec{r},\vec{e_1}}]}$$

Green's function $\nabla^2 G(\vec{r},\vec{r}_o) = -\delta_{\vec{r},\vec{r}_o}$

solution $\phi(\vec{r}) = \rho + \epsilon\phi(\vec{0})[G(\vec{r},\vec{0}) - G(\vec{r},\vec{e}_1)]$

Unlike electrostatic configuration here the strength of the dipole should be determined self consistently.

Green's function of the discrete Laplace equation

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$$G(\vec{r},\vec{r}_o) \approx -\frac{1}{2\,\pi}\ln|\vec{r}-\overrightarrow{r_0}|$$

$$\phi(\vec{r}) = \rho + \epsilon\phi\left(\vec{0}\right)\left[G\left(\vec{r},\vec{0}\right) - G(\vec{r},\vec{e}_1)\right]$$

determining $\phi(\vec{0})$

determining

$$\phi(\vec{r}) = \rho + \epsilon\phi\left(\vec{0}\right)\left[G\left(\vec{r},\vec{0}\right) - G(\vec{r},\vec{e}_1)\right]$$

To find $\phi(\vec{0})$ one uses the values $G\left(\vec{0},\vec{0}\right) = 0$, $G\left(\vec{0},\vec{e}_1\right) = -\frac{1}{4}$

To find one uses the values

$$\boxed{\phi\left(\vec{0}\right) = \frac{\rho}{1 - \frac{\epsilon}{4}}}$$

at large $\vec{r}$

$$G(\vec{r},\vec{r}_o) \approx -\frac{1}{2\,\pi}\,\ln|\vec{r}-\vec{r_0}|$$

$$\phi(\vec{r}) = \rho + \epsilon\phi(\vec{0})[G(\vec{r},\vec{0}) - G(\vec{r},\vec{e}_1)]$$

$$\phi(\vec{0}) = \frac{\rho}{1-\frac{\epsilon}{4}}$$

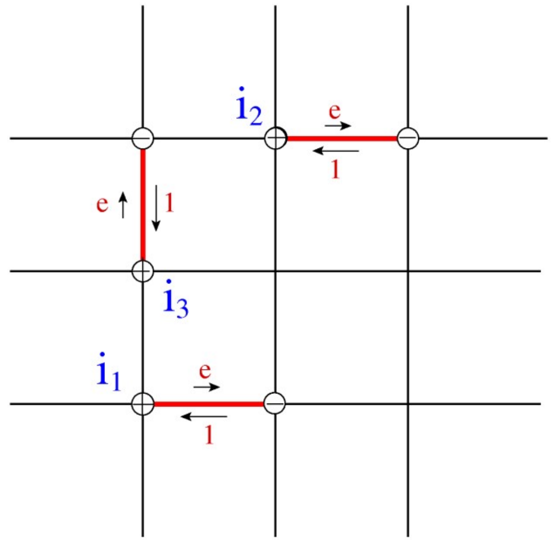
density:

$$\boxed{\phi(\vec{r}) = \rho - \frac{\epsilon\phi(\vec{0})}{2\pi}\frac{\vec{e}_1\vec{r}}{r^2} + O(\frac{1}{r^2})}$$

current:

$$\boxed{j(\vec{r}) = \frac{\epsilon\phi(\vec{0})}{2\pi}\frac{1}{r^2}[\vec{e}_1 - \frac{2(\vec{e}_1\vec{r})\vec{r}}{r^2} + O(\frac{1}{r^3})}$$

Multiple driven bonds

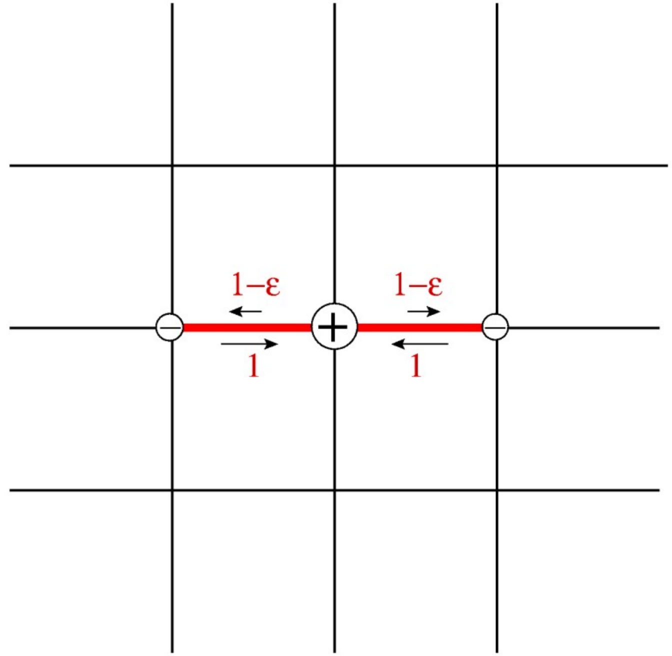


$e = 1 - \varepsilon$

$$\phi(\vec{r}) = \rho + \epsilon\phi(\vec{i}_1)[G(\vec{r},\vec{i}_1) - G(\vec{r},\vec{i}_1 + \vec{e}_1)] + \epsilon\phi(\vec{i}_2)[G(\vec{r},\vec{i}_2) - G(\vec{r},\vec{i}_2 + \vec{e}_1)] + \cdots$$

Using the Green's function one can solve for $\phi(\vec{i}_1), \phi(\vec{i}_2), \cdots$
by solving the set of linear equations for $\vec{r} = \vec{i}_1, \vec{i}_2, \cdots$

Two oppositely directed driven bonds – quadrupole field



The steady state equation: $\nabla^2\phi(\vec{r}) = -\epsilon\phi(\vec{0})[2\delta_{\vec{r},\vec{0}} - \delta_{\vec{r},\vec{e}_1} - \delta_{\vec{r},-\vec{e}_1}]$

$$\boxed{\phi(\vec{r}) = \rho - \frac{\epsilon\phi(\vec{0})}{2\pi}\left[\frac{1}{r^2} - 2\left(\frac{\vec{e}_1\vec{r}}{r^2}\right)^2\right] + O\left(\frac{1}{r^4}\right)}$$

$d \neq 2$ dimensions

dimensions

$d = 1$

$$\phi(x) = \rho - \left(\frac{\epsilon}{2}\right)\phi(0)sgn(x)$$

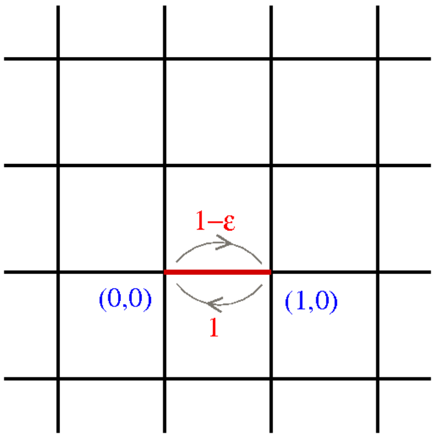
$$G(x, x_o) = -\frac{|x - x_o|}{2}$$

$d \geq 2$

$$\phi(\vec{r}) \sim \frac{1}{r^{d-1}}$$

The model of local drive with exclusion

Here the steady state measure is not known however one can determine the behavior of the density.



$$\partial_t \phi(\vec{r},t) = \nabla^2 \phi(\vec{r},t) + \epsilon \langle \tau(\vec{0})\{1-\tau(\vec{e}_1)\}\rangle [\delta_{\vec{r},\vec{0}} - \delta_{\vec{r},\vec{e_1}}]$$

$\tau = 0,1$ is the occupation variable

$$\boxed{\phi(\vec{r}) = \rho - \frac{\epsilon \langle \tau(\vec{0})\{1-\tau(\vec{e}_1)\}\rangle}{2\pi} \frac{\vec{e}_1 \vec{r}}{r^2} + O(\frac{1}{r^2})}$$

$$\boxed{\phi(\vec{r}) = \rho - \frac{\epsilon\langle\tau\left(\vec{0}\right)\{1-\tau(\vec{e}_1)\}\rangle}{2\pi}\frac{\vec{e}_1\vec{r}}{r^2} + O(\frac{1}{r^2})}$$

The density profile is that of the dipole potential with a dipole strength which can only be computed numerically.

Simulation results

Simulation on a lattice with

Simulation on a 200×200 lattice with $\rho = 0.6$

For the interacting case the strength of the dipole was measured separately.

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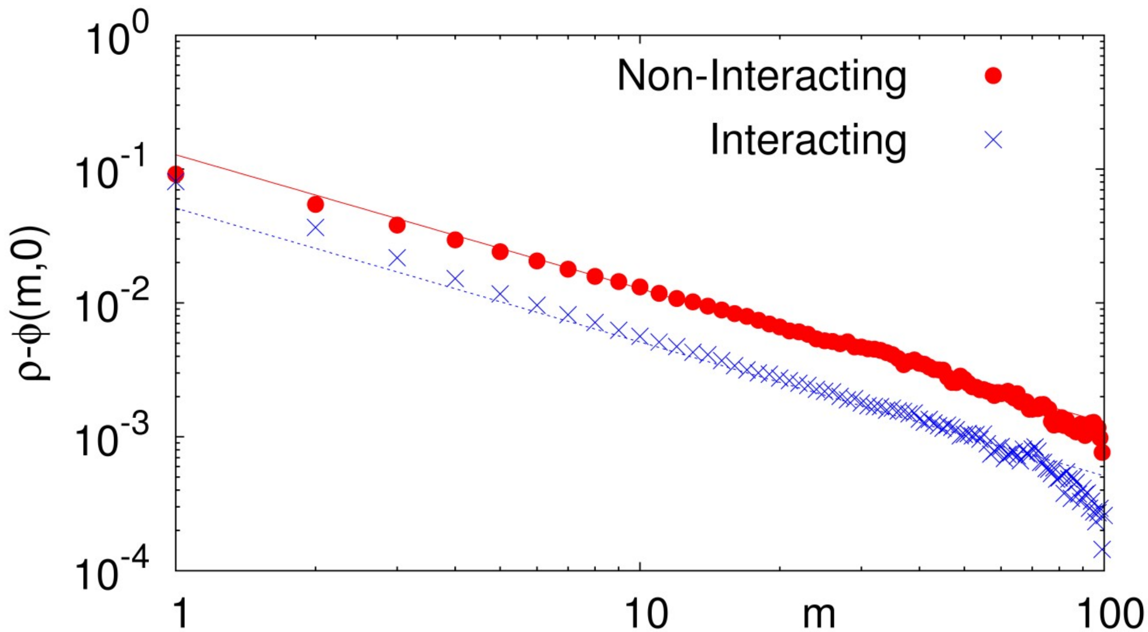
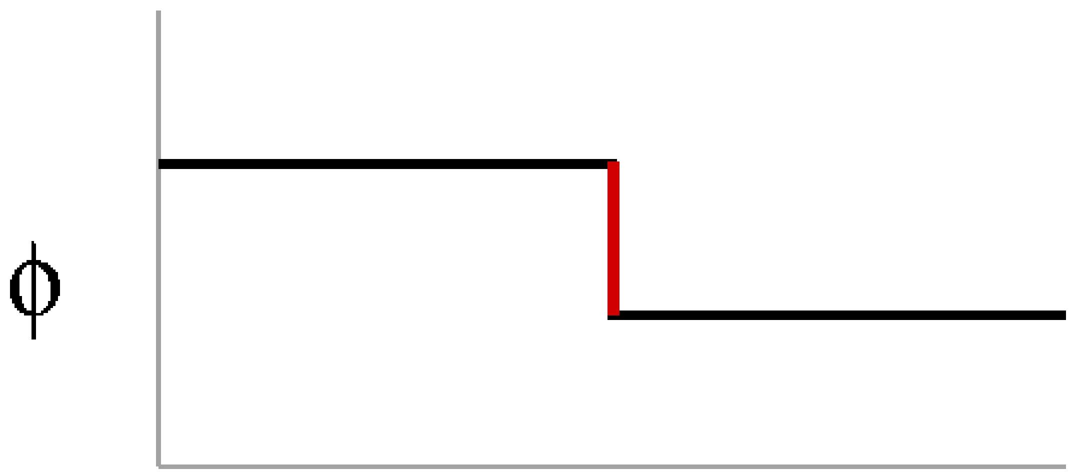


Two-point correlation function

$$C(r,s) = < \tau(r)\tau(s) > - \phi(r)\,\phi(s)$$

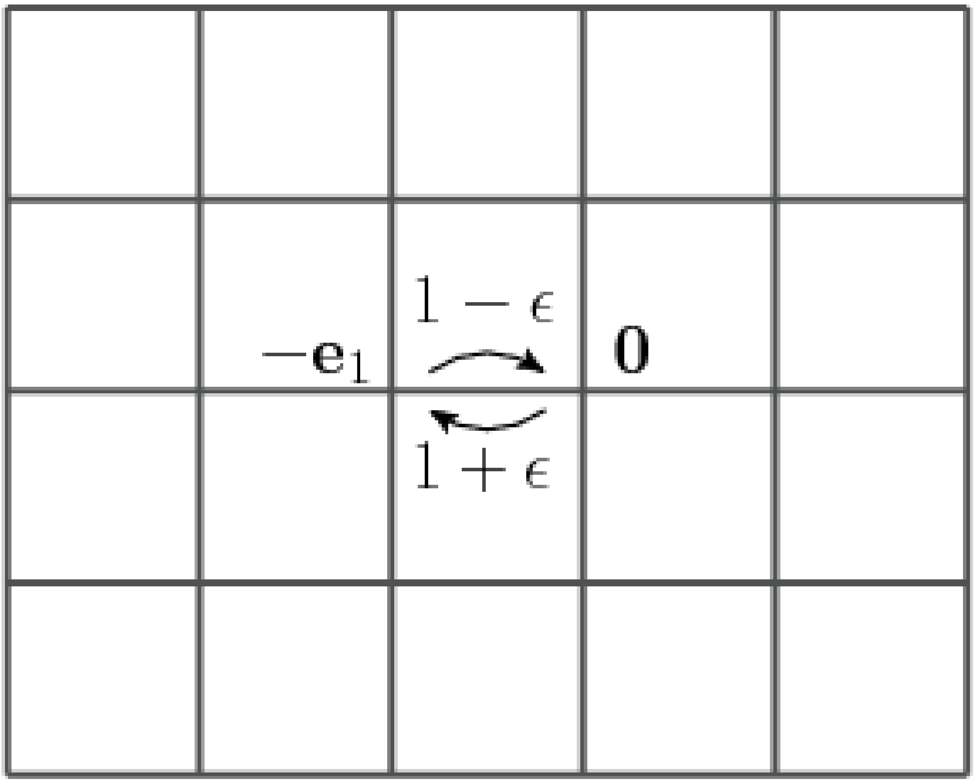
In d=1 dimension, in the hydrodynamic limit

$$g_L(i,j) = \frac{1}{L} g\left(\frac{i}{L}, \frac{j}{L}\right)$$



T. Bodineau, B. Derrida, J.L. Lebowitz, JSP, 140 648 (2010).

In higher dimensions local currents do not vanish for large L and the correlation function does not vanish in this limit.



T. Sadhu, S. Majumdar, DM, in progress

Symmetry of the correlation function:

$$C(r,s) =< \tau(r)\tau(s) > - \phi(r)\,\phi(s)$$

inversion $\quad C_\epsilon(\mathrm{r},\mathrm{s}) = \mathrm{C}_{-\epsilon}(-r,-s)$

particle-hole $\quad C_{\epsilon,\rho}(\mathrm{r},\mathrm{s}) = \mathrm{C}_{-\epsilon,1-\rho}(\mathrm{r},\mathrm{s})$

at $\rho = 1/2$ $\quad C_\epsilon(r,s) = C_{-\epsilon}(r,s)$

$$\int d^d r\, C(r,s) = 0$$

$$(\Delta_r + \Delta_s) C(r,s) = \sigma \quad (r,s)$$

$C(r,s)$ corresponds to an electrostatic potential in 2d induced by σ

corresponds to an electrostatic potential in induced by

Consequences of the symmetry:

- The net charge =0
- At $\rho = 0.5$ $C_\epsilon(r,s)$ is even in ϵ
- Thus the charge cannot support a dipole and the leading contribution in multipole expansion is that of a quadrupole (in 2d dimensions).

$$\boxed{C(r,s) \sim 1/(r^2+s^2)^d}$$

For $\rho \neq 0.5$ one can expand σ in powers of ϵ

One finds:

The leading contribution to σ is of order ϵ^2 implying no dipolar contribution, with the correlation decaying as

$$C(r, s) \sim 1/(r^2 + s^2)^d$$

$$C(r,s) = \epsilon\alpha_1(r,s) + \epsilon^2\alpha_2(r,s) + \cdots$$

$$\alpha_{2p-1}(-r,-s) = -\alpha_{2p-1}(r,s)$$

$$\alpha_{2p}(-r,-s) = \alpha_{2p}(r,s)$$

Since $\alpha_1 = 0$ (no dipole) and the net charge is zero
the leading contribution is quadrupolar

Since (no dipole) and the net charge is zero
the leading contribution is quadrupolar

$$(\Delta_r + \Delta_s)C(r,s) = \sigma_1(r,s) + \sigma_2(r,s) + \sigma_3(r,s)$$

$$\sigma_1(r,s) = \epsilon\langle Q\,\tilde{n}(r)\rangle\big(\delta_{s,0} - \delta_{s+e_1,0}\big)\big(1 - \delta_{r,0} - \delta_{r+e_1,0}\big)$$
$$+\ \epsilon\langle Q\,\tilde{n}(s)\rangle\big(\delta_{r,0} - \delta_{r+e_1,0}\big)\big(1 - \delta_{s,0} - \delta_{s+e_1,0}\big)$$

$$\sigma_2(r,s) = -\frac{1}{d}\epsilon^2\langle Q\rangle^2\delta_{r,-e_1}\delta_{s,0} + \sum_\nu\big(\phi(r+e_\nu) - \phi(r)\big)^2\delta_{s,r+e_\nu}$$
$$-\frac{1}{d}\epsilon^2\langle Q\rangle^2\delta_{s,-e_1}\delta_{r,0} + \sum_\nu\big(\phi(s+e_\nu) - \phi(s)\big)^2\delta_{s,r-e_\nu}$$

$$\sigma_3(r,s) = \sum_\nu\left[C(r+e_\nu,s) - 2C(r,s) + C(r,s-e_\nu)\right]\big(\delta_{s,r+e_\nu} + \delta_{s,r}\big)$$
$$+\sum_\nu\left[C(r,s+e_\nu) - 2C(r,s) + C(r-e_\nu,s)\right]\big(\delta_{s,r-e_\nu} + \delta_{s,r}\big)$$

$$Q = n(0)\big(1 - n(-e_1)\big) + n(-e_1)\big(1 - n(0)\big)$$

Summary

Local drive in $d > 1$ dimensions results in:

- Density profile corresponds to a dipole in d dimensions

$$\phi(r) \sim 1/r^{d-1}$$

- Two-point correlation function corresponds to a quadrupole in 2d dimensions

$$C(r,s) \sim 1/(r^2 + s^2)^d$$

At density $\rho = 0.5$ to all orders in ϵ

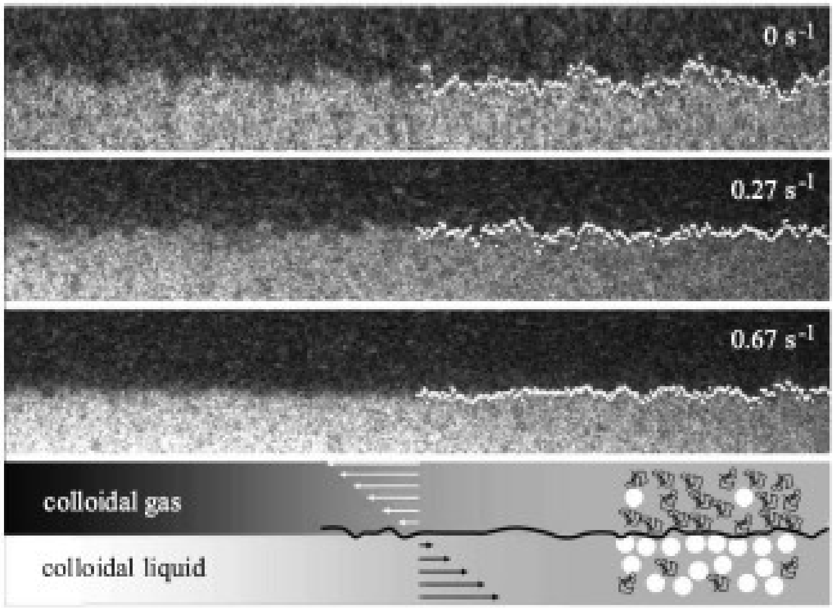
At other densities to leading (ϵ^2) order

- Example II: a two dimensional model with a driven line

The effect of a drive on a fluctuating interface

T. Sadhu, Z. Shapira, DM PRL 109, 130601 (2012)

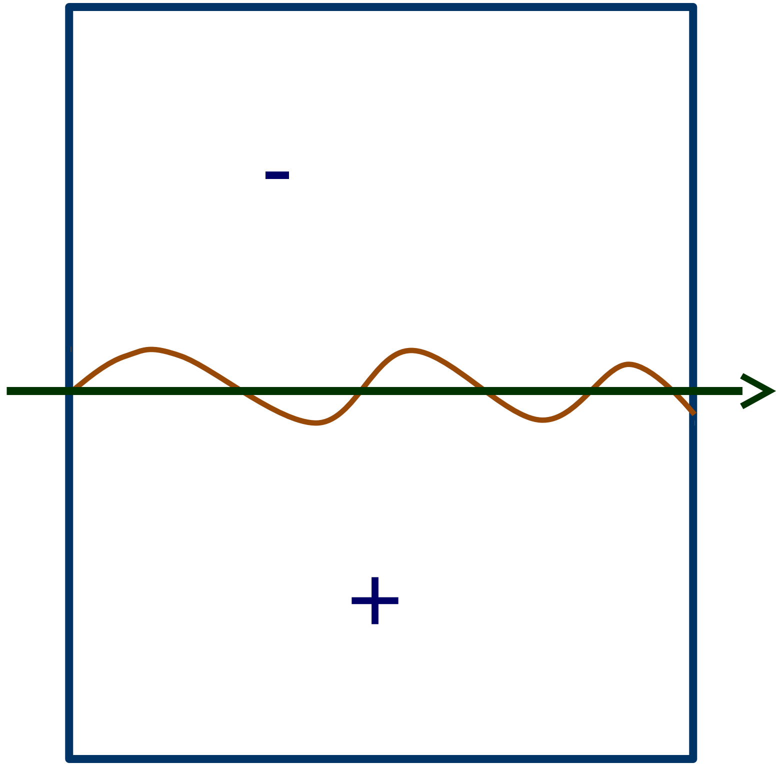
Motivated by an experimental study of the effect of shear on colloidal liquid-gas interface.



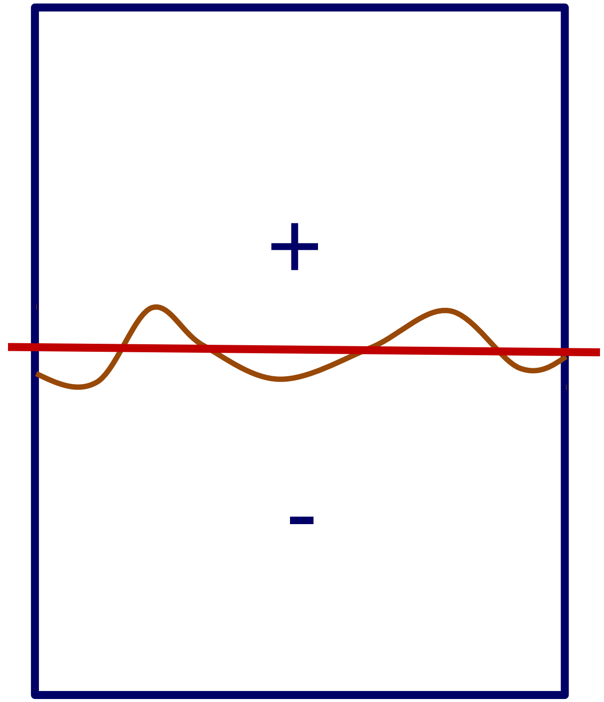
D. Derks, D. G. A. L. Aarts, D. Bonn, H. N. W. Lekkerkerker, A. Imhof, PRL 97, 038301 (2006).

T.H.R. Smith, O. Vasilyev, D.B. Abraham, A. Maciolek, M. Schmidt, PRL 101, 067203 (2008).

?What is the effect of a driving line on an interface



In equilibrium- under local attractive potential



Local potential localizes the interface at any temperature $T < T_c$

Transfer matrix: 1d quantum particle in a local attractive potential, the wave-function is localized.

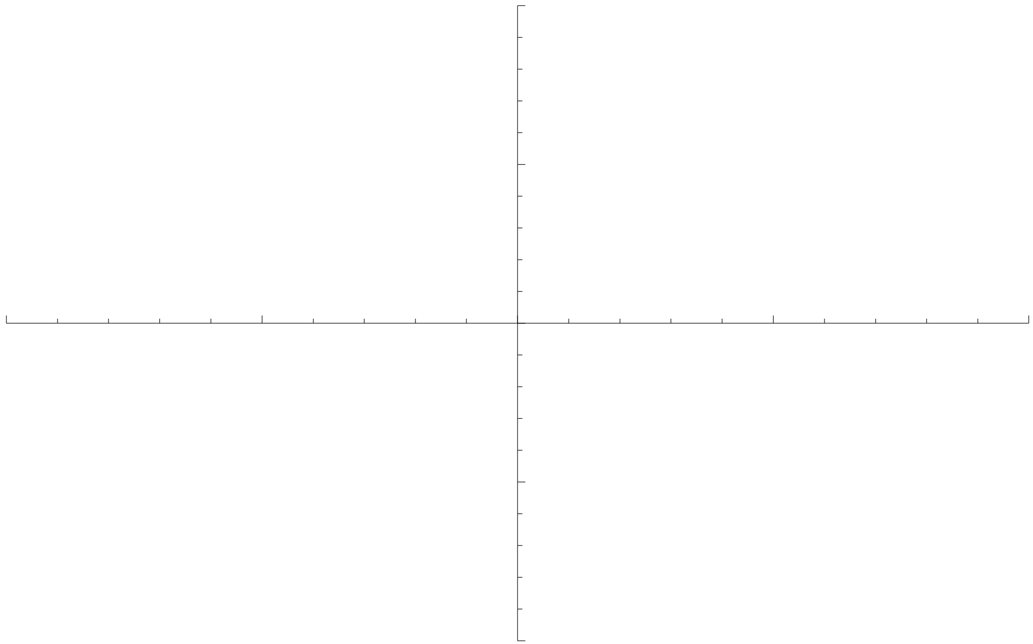
Transfer matrix: 1d quantum particle in a local attractive potential, the wave-function is localized.

no localizing potential: $< \Delta h^2 > \sim L$

with localizing potential: $< \Delta h^2 > \sim const$

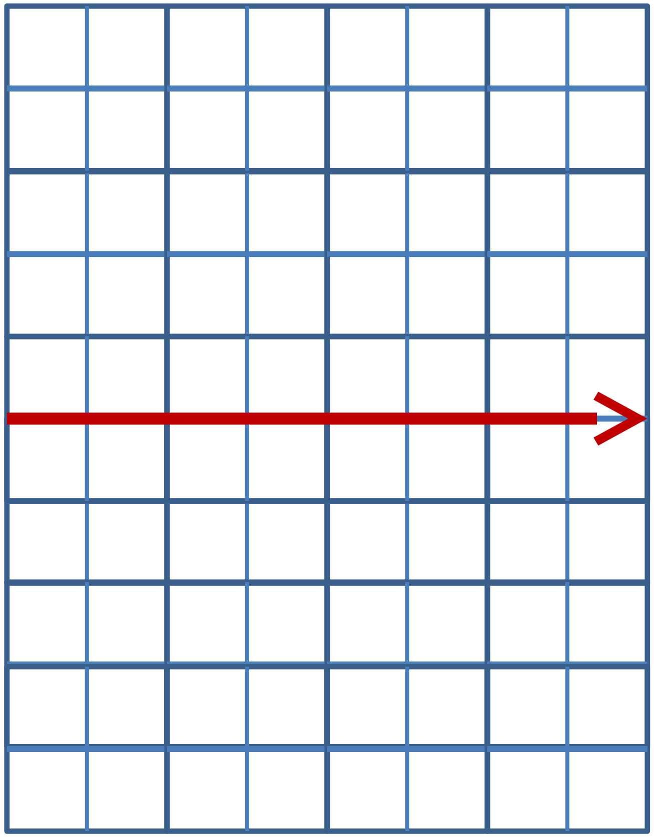
no localizing potential:
with localizing potential:

Schematic magnetization profile



The magnetization profile is antisymmetric with respect to the zero line with $m(y = 0) = 0$

Consider now a driving line



$+ - \longrightarrow - +$ with rate $\min(1, e^{-\beta\Delta H + \beta E})$

$- + \longrightarrow + -$ with rate $\min(1, e^{-\beta\Delta H - \beta E})$

$$H = -J \sum_{<ij>} S_i S_j$$

Ising model with Kawasaki dynamics which is biased on the middle row

Main results

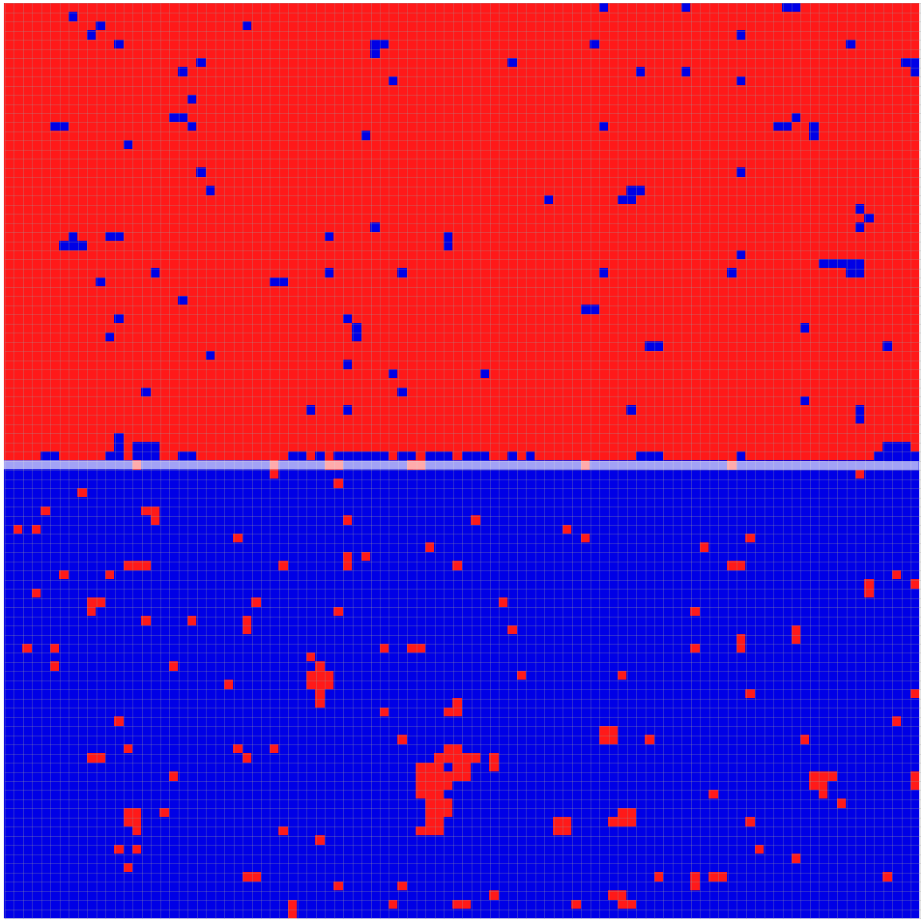
- The interface width is finite (localized)

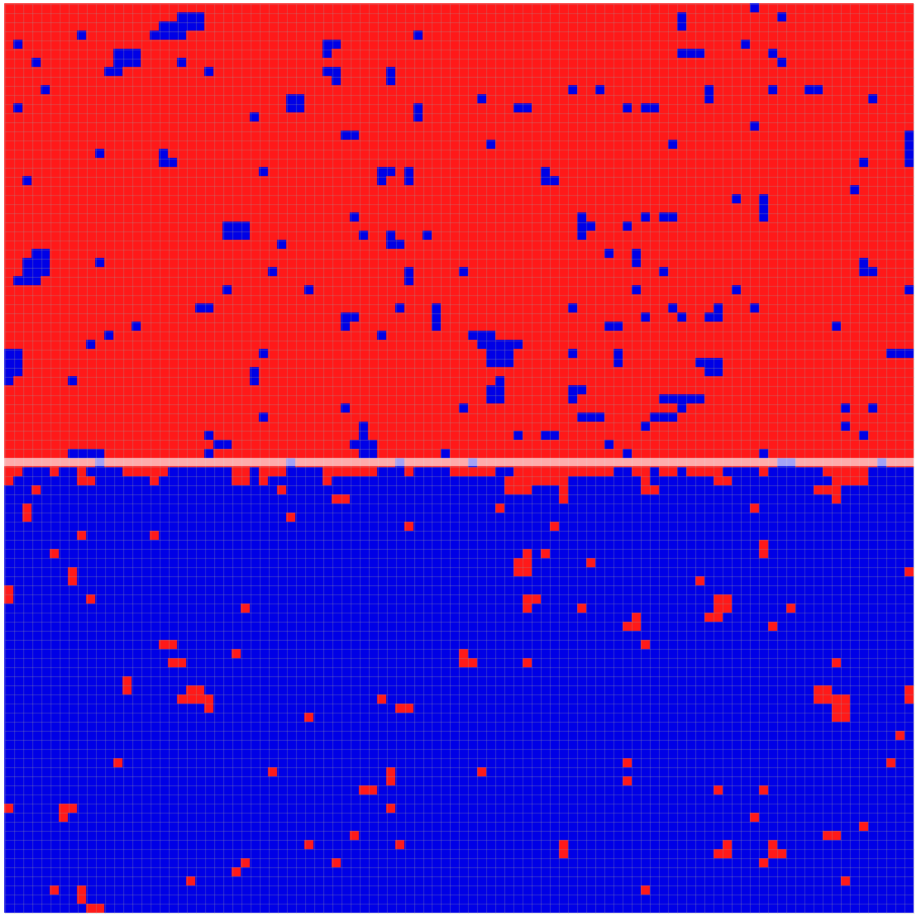
- A spontaneous symmetry breaking takes place by which the magnetization of the driven line is non-zero and the magnetization profile is not symmetric.

- The fluctuation of the interface are not symmetric around the driven line.

- These results can be demonstrated analytically in certain limit.

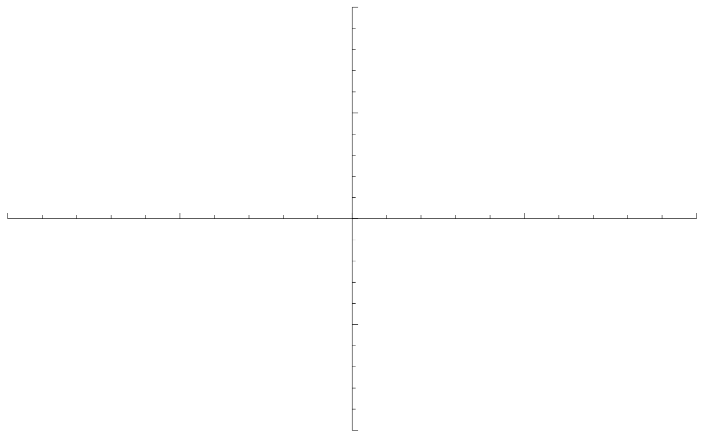
Results of numerical simulations

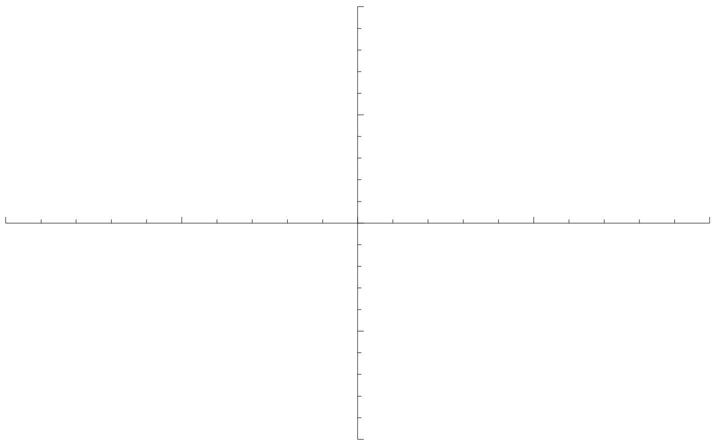




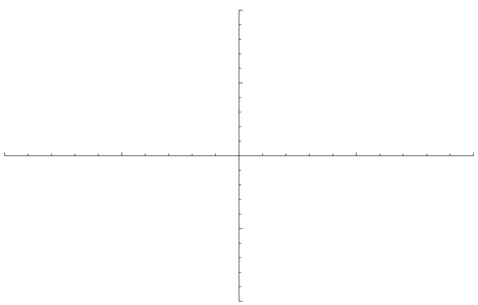
Example of configurations in the two mesoscopic states for a 100X101 with fixed boundary at T=0.85Tc

Schematic magnetization profiles

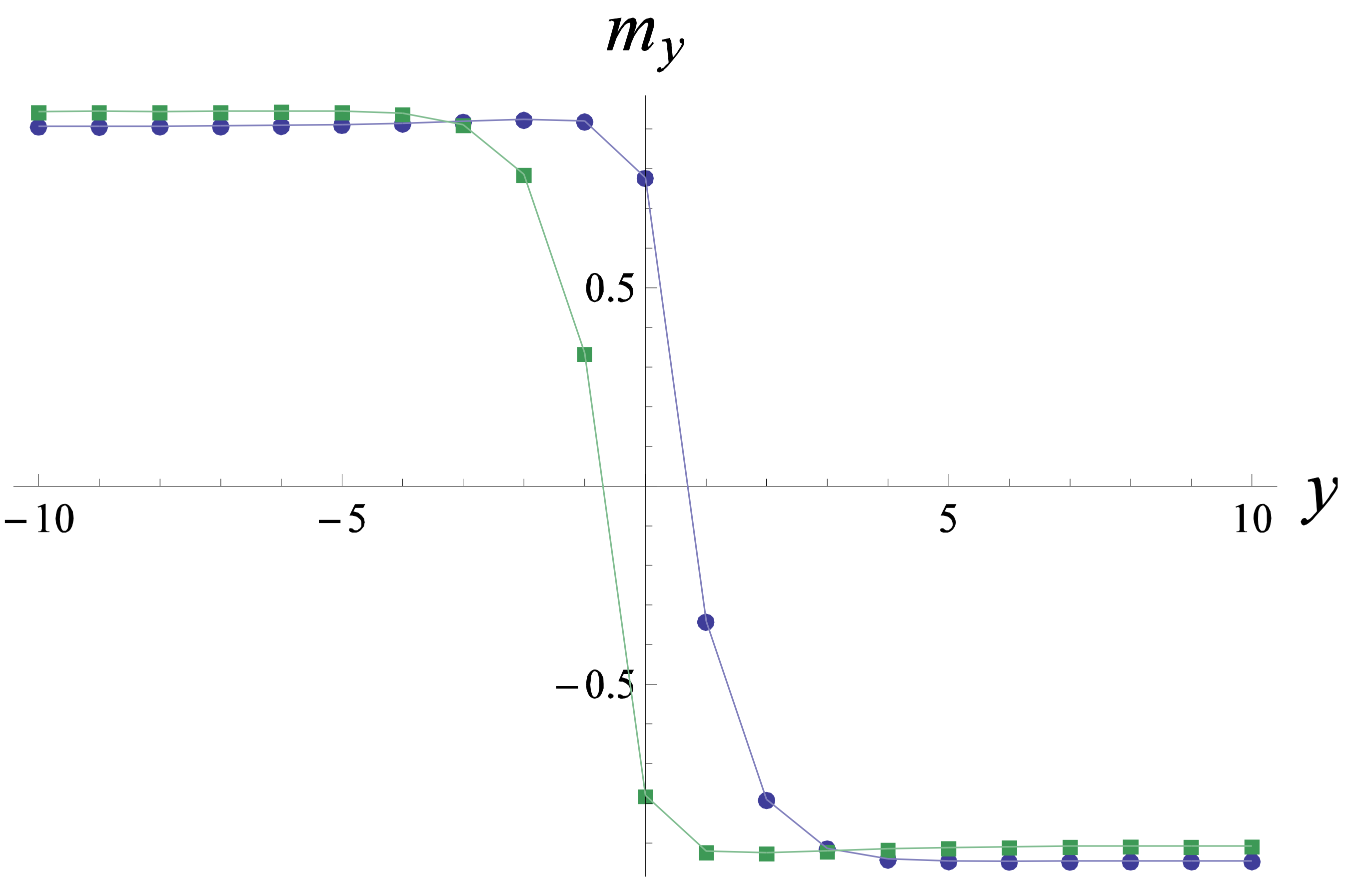




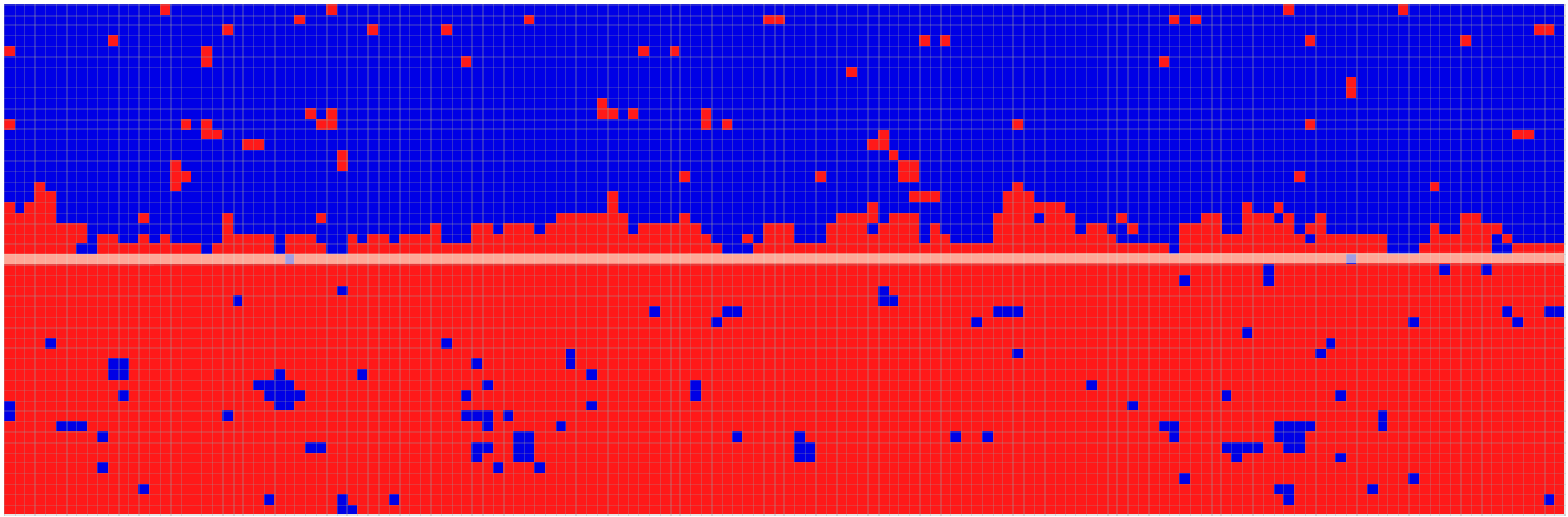
unlike the equilibrium antisymmetric profile

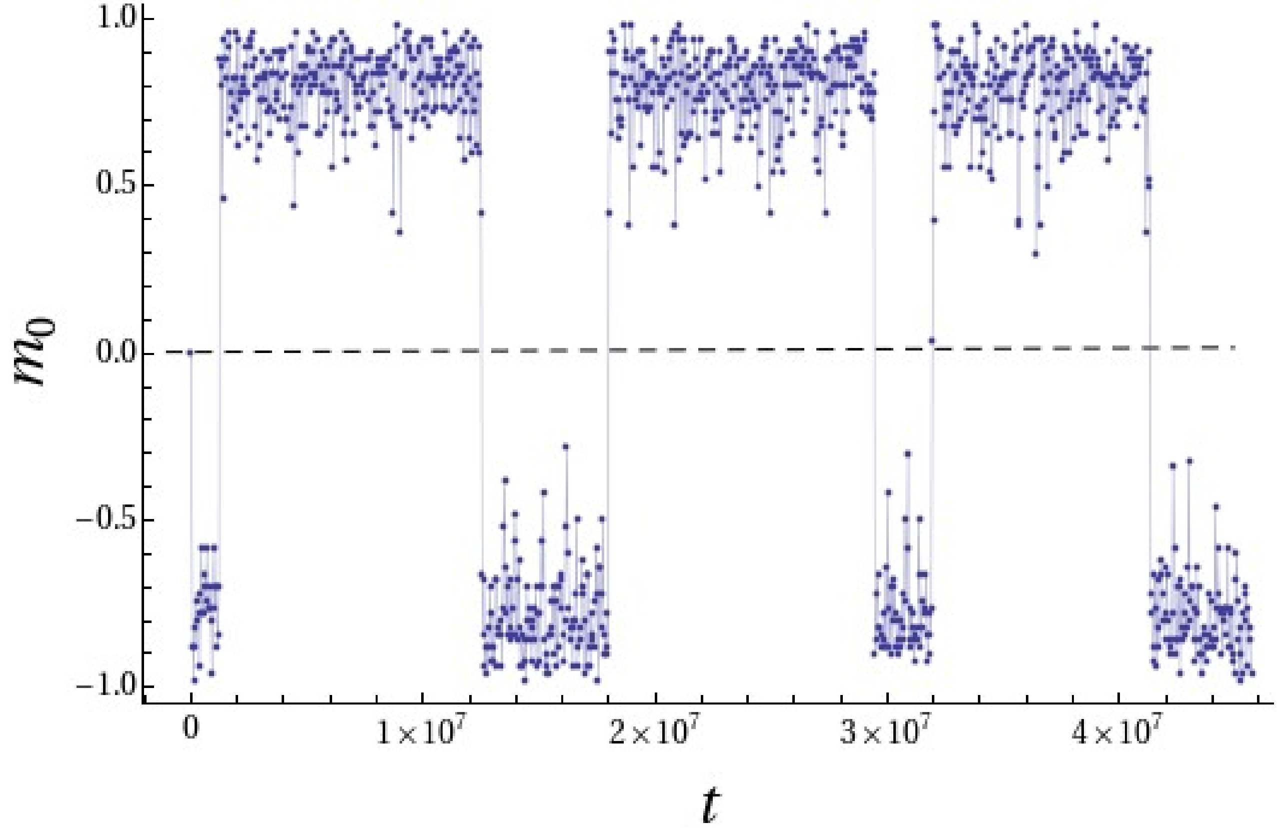


Averaged magnetization profile in the two states

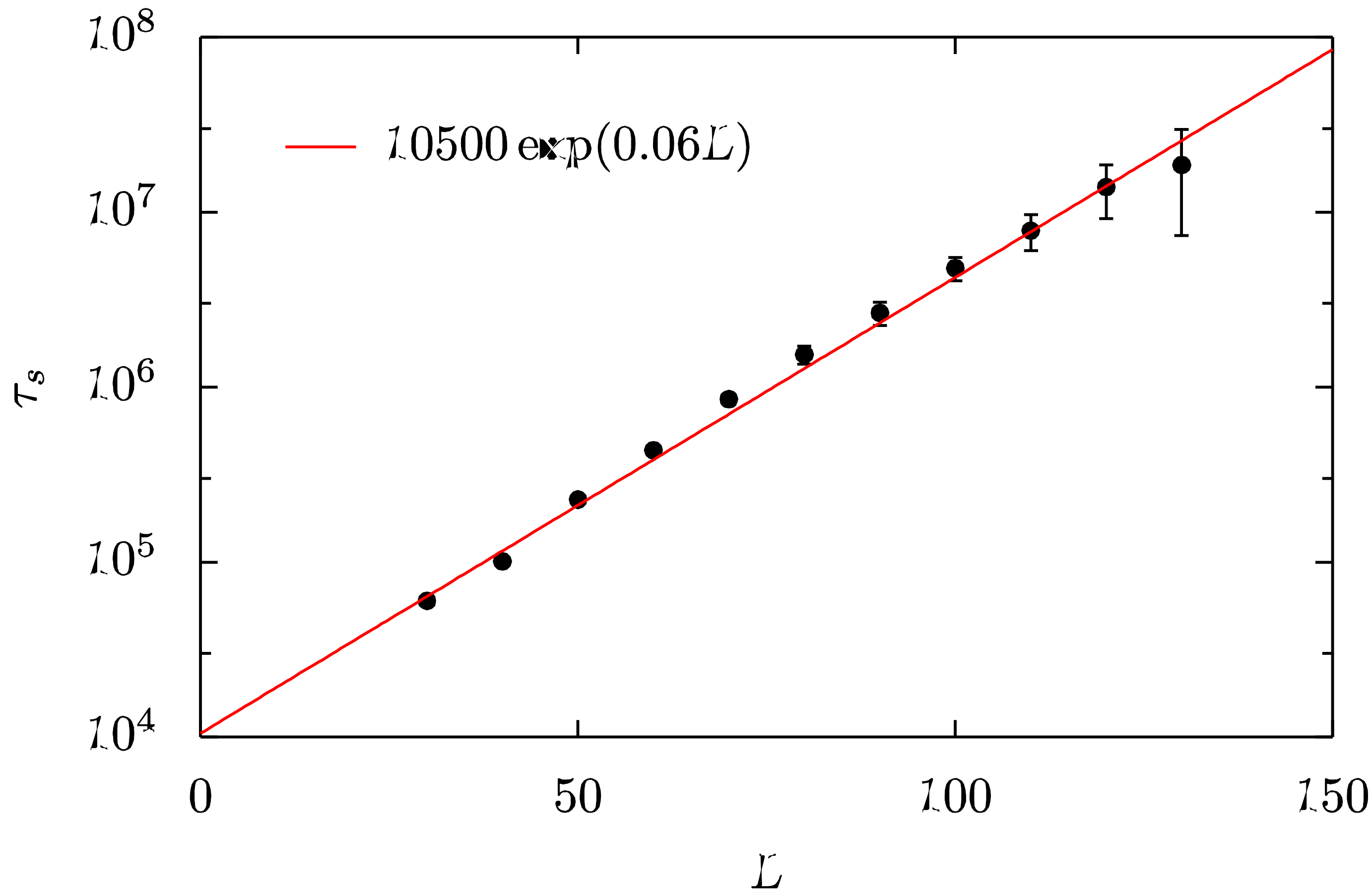


L=100 T=0.85Tc





Time series of Magnetization of driven lane for a 100X101 lattice at T= 0.6Tc.



Switching time on a square LX(L+1) lattice with Fixed boundary at T=0.6Tc.

Analytical approach

In general one cannot calculate the steady state measure of this system. However in a certain limit, the steady state distribution (the large deviations function) of the magnetization of the driven line can be calculated.

Typically one is interested in calculating $-F\big(m(x,y)\big)$
the large deviation function of a magnetization profile

$$\boxed{P(m(x,y)) \sim e^{-L^2 F(m(x,y))}}$$

We show that in some limit a restricted large deviation function, that of the driven line magnetization, m_0, can be computed

$$\boxed{P(m_0) = e^{-LU(m_0)}}$$

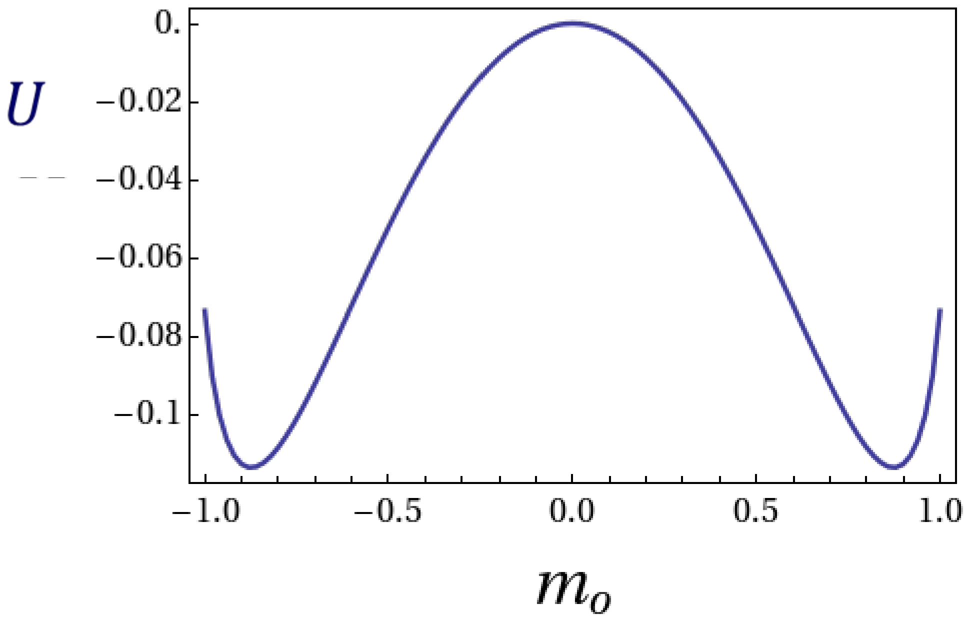
The following limit is considered

- Slow exchange rate between the driven line and the rest of the system
- Large driving field $E \gg J$
Large driving field
- Low temperature

In this limit the probability distribution of m_0 is $P(m_0) = e^{-LU(m_0)}$
where the potential (large deviations function) $U(m_0)$ can be computed.

In this limit the probability distribution of is
where the potential (large deviations function) can be computed.

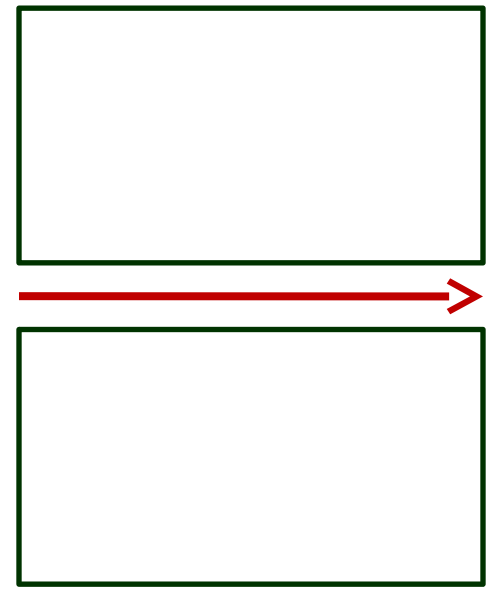
The large deviations function



$$P(m_0) = e^{-LU(m_0)}$$

- Slow exchange between the line and the rest of the system

$$+ \; - \; \underset{rw(-\Delta H)}{\overset{rw(\Delta H)}{\leftrightarrows}} \; - \; + \qquad r < O(\frac{1}{L^3})$$



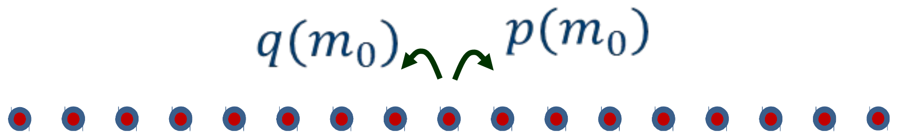
$$w(\Delta H) = \min(1, e^{-\beta\Delta H})$$

In between exchange processes the systems is composed of 3 sub-systems evolving independently

- Fast drive $E \gg J$
  - the coupling J within the lane can be ignored. As a result the spins on the driven lane become uncorrelated and they are randomly distributed (TASEP)
  - The driven lane applies a boundary field Jm_0 on the two other parts
  - Due to the slow exchange rate with the bulk, the two bulk sub-systems reach the equilibrium distribution of an Ising model with a boundary field Jm_0
- Low temperature limit
  - In this limit the steady state of the bulk sub systems can be expanded in T and the exchange rate with the driven line can be computed.

$$m_o \longrightarrow m_o + \frac{2}{L} \quad \text{with rate} \quad p(m_o)$$

$$m_o \longrightarrow m_o - \frac{2}{L} \quad \text{with rate} \quad q(m_o)$$



m_o performs a random walk with a rate which depends on m_o

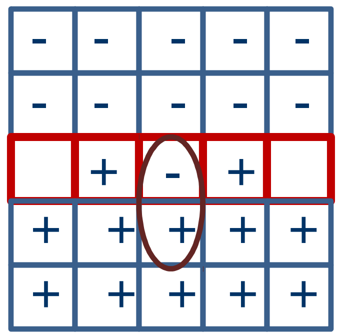
$$\boxed{P\left(m_o = \frac{2k}{L}\right) = \frac{p(0)\cdots\cdots p(\frac{2(k-1)}{L})}{q(\frac{2}{L})\cdots\cdots q(\frac{2k}{L})} \equiv e^{-U(m_o)}}$$

$$\boxed{U(m_o) = -\sum_{k=0}^{\frac{m_oL}{2}-1} \ln p\left(\frac{2k}{L}\right) + \sum_{k=1}^{\frac{m_oL}{2}} q(\frac{2k}{L})}$$

Calculate $p(m_a)$ at low temperature

Calculate p at low temperature

-	-	-	-	-
-	-	-	-	-
	+	-	+	
+	+	+	+	+
+	+	+	+	+



contribution to p(m_o): $\frac{1}{8}(1-m_o)(1+m_o)^2 e^{-2\beta J} e^{-2\beta J_1}$

contribution to p

J_1 is the exchange rate between the driven line and the adjacent lines

is the exchange rate between the driven line and the adjacent lines

The magnetization of the driven lane m changes in steps of $2/L$

The magnetization of the driven lane changes in steps of

Expression for rate of increase, $p(m_o)$

Expression for rate of increase, p

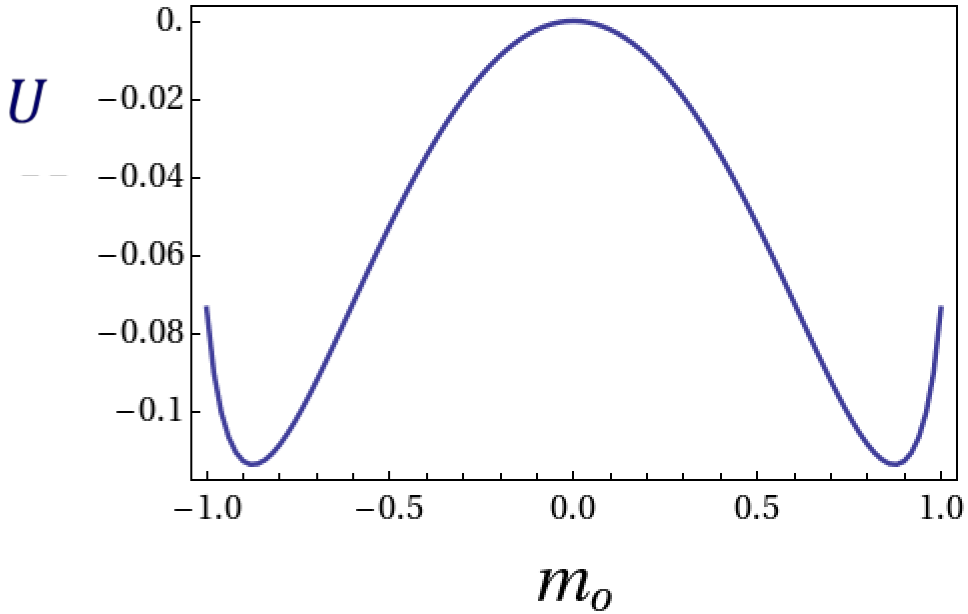
$$p(m_0) = \frac{1}{8}(1+\mathrm{m}_0)^2(1-\mathrm{m}_0)e^{-2\beta(J+J_1)}$$

$$+\frac{1}{8}[2(1+m_0)(1-m_0)^2\left(e^{-2\beta J_1}+e^{2\beta J_1 m_0}\right)$$

$$+(1+m_0)^2(1-m_0)e^{2\beta J_1\, m_0}+(1-m_0)^3 e^{2\beta J_1 m_0}]e^{-6\beta J}+O(e^{-8\beta J})$$

$$q(m_o) = p(-m_o)$$

$$U(m) = -\int_0^m \ln p(k)\, dk + \int_0^m \ln q(k) dk$$



This form of the large deviation function demonstrates the spontaneous symmetry breaking. It also yield the exponential flipping time at finite L. $(T = 0.6T_c\ , J_1 = J)$

$$P(m_0) = e^{-LU(m_0)}$$

$$< m_o > 1 - O(e^{-4\beta J})$$

Summary

- Simple examples of the effect of long range correlations in driven models have been presented.

- A limit of slow exchange rate is discussed which enables the evaluation of some large deviation functions far from equilibrium.