Gauge/String Duality and D-Brane Inflation

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The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The SO(2,4) geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The d-dimensional AdS space is a hyperboloid

$$(X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2 \, .$$

- Its metric is

$$ds^2 = \frac{L^2}{z^2} \left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2} (dx^i)^2 \right)$$

- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\mathrm{YM}}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$

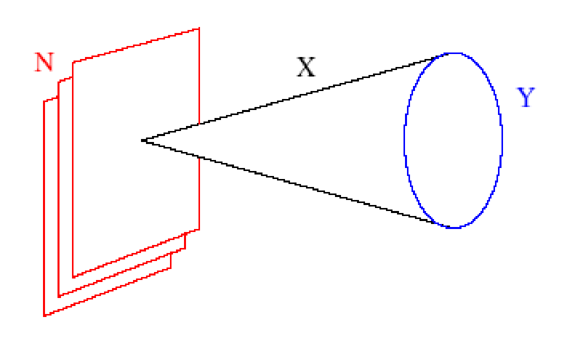
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

Could the closed string side of the duality exhibit a simplification?

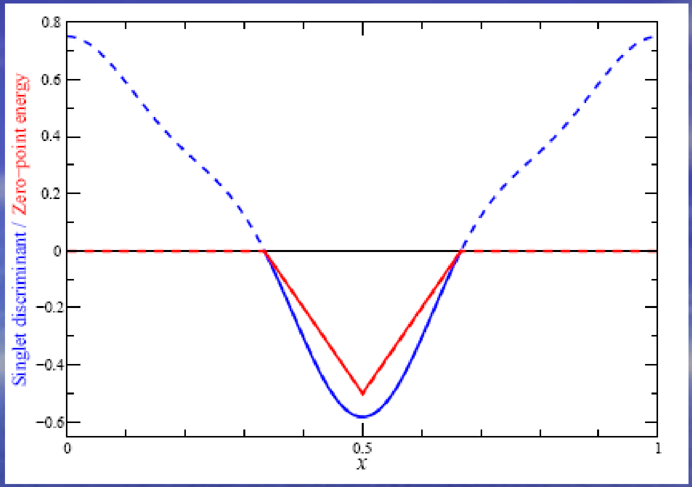
- My recent work with Dymarsky and Roiban reconsiders gauge theory on a stack of D3-branes at the tip of a cone R^6/Γ where the orbifold group Γ breaks all the supersymmetry.
- At first sight, the gauge theory seems conformal because the planar beta functions for all single-trace operators vanish. The candidate string dual is $AdS_5 \times S^5/\Gamma$. Kachru, Silverstein; Lawrence, Nekrasov, Vafa; Bershadsky, Johanson
- However, dimension 4 double-trace operators made out of twisted single-trace ones, $f\, O_n O_{-n}$, are induced at one-loop. Their planar beta-functions have the form

$$\beta_f = a\,\lambda^2 + 2\,\gamma\, f\,\lambda + f^2$$

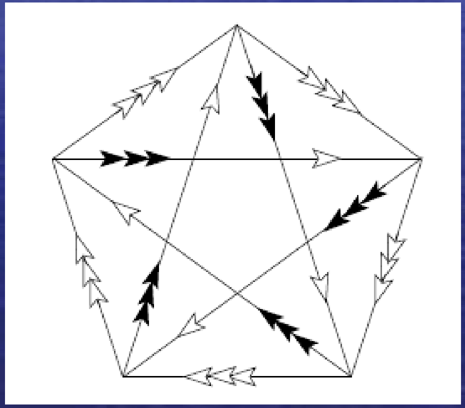
$$\beta_\lambda = 0$$



- If $D=\gamma^2 - a < 0$, then there is no real fixed point for f.
- Here is a plot of a one-loop $SU(N)^k$ gauge theory discriminant, D, and of the ground state closed string m^2 on the cone without the D-branes. n=1, ..., k-1 labels the twisted sector for a class of Z_k orbifolds with global SU(3) symmetry that are freely acting on the 5-sphere, and x=n/k.



- The simplest freely acting non-susy example is Z_5 where there are four induced double-trace couplings



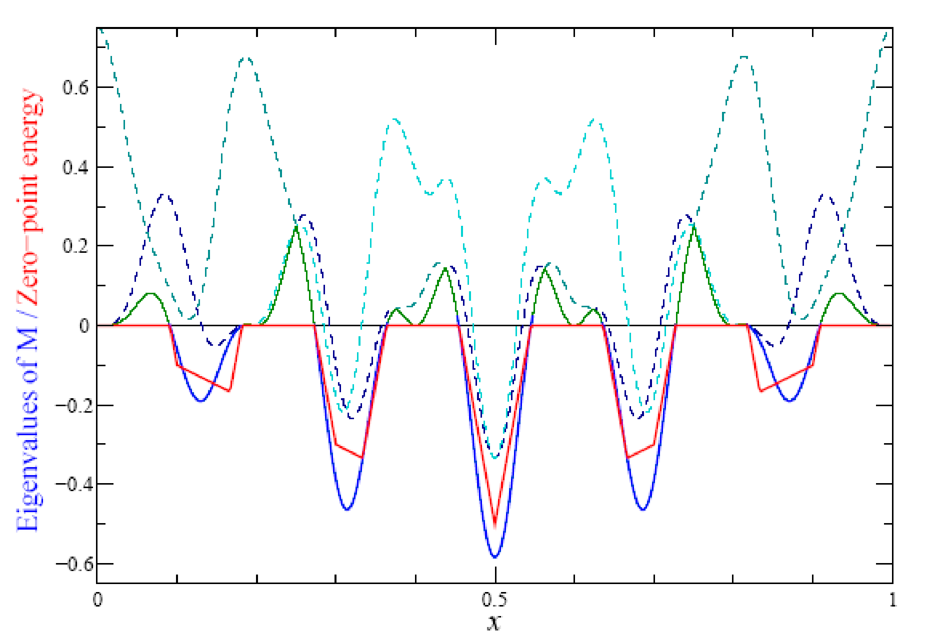
$$\delta_{2\ \mathrm{trace}}\mathcal{L} = f_{8,1} O_1^{\langle i\bar{j}\rangle} O_{-1}^{\langle j\bar{i}\rangle} + f_{8,2} O_2^{\langle i\bar{j}\rangle} O_{-2}^{\langle j\bar{i}\rangle} + f_{1,1} O_1 O_{-1} + f_{1,2} O_2 O_{-2}$$

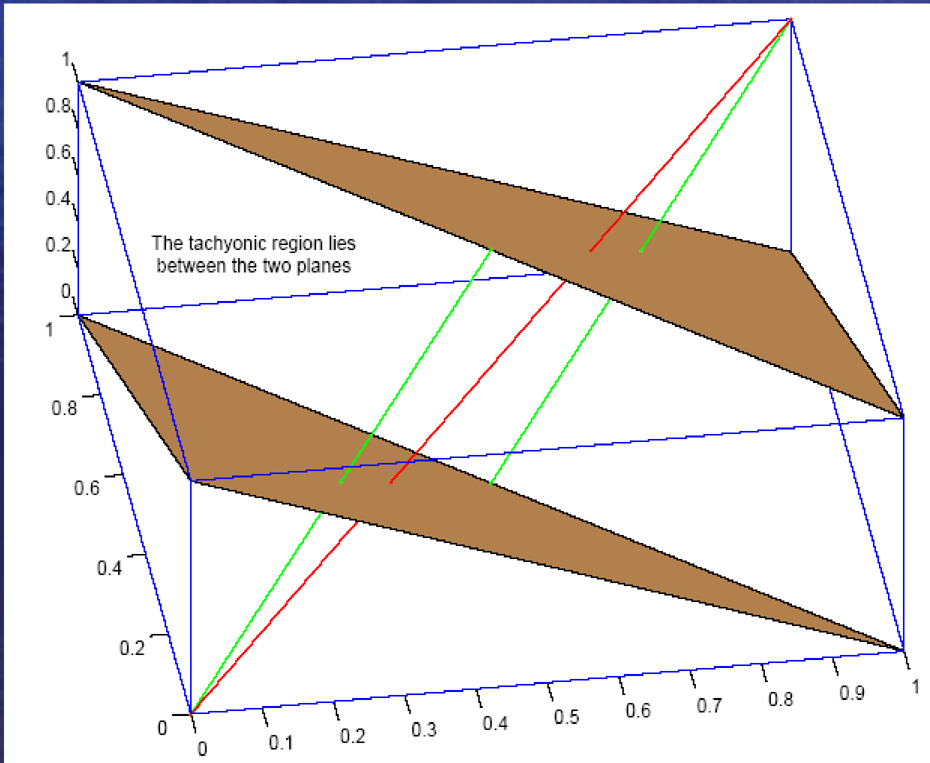
- For example, the SU(3) adjoints

$O_n^{\langle i\bar{j}\rangle}$ are $\displaystyle\sum_{k=1}^{5} \left(\Phi^i_{k,k+2} \Phi^{\bar{j}}_{k+2,k} - \frac{1}{3} \eta^{i\bar{j}} \Phi^l_{k,k+2} \Phi^{\bar{l}}_{k+2,k} \right) e^{in\alpha(k-1)}$

($\alpha=2\pi/5$)

- For more complicated orbifolds, crossing of eigenvalues of the discriminant matrix becomes important. The agreement holds.
- Generally, there are three twists that define a cube. The stability/instability regions agree between one-loop gauge theory and string theory.

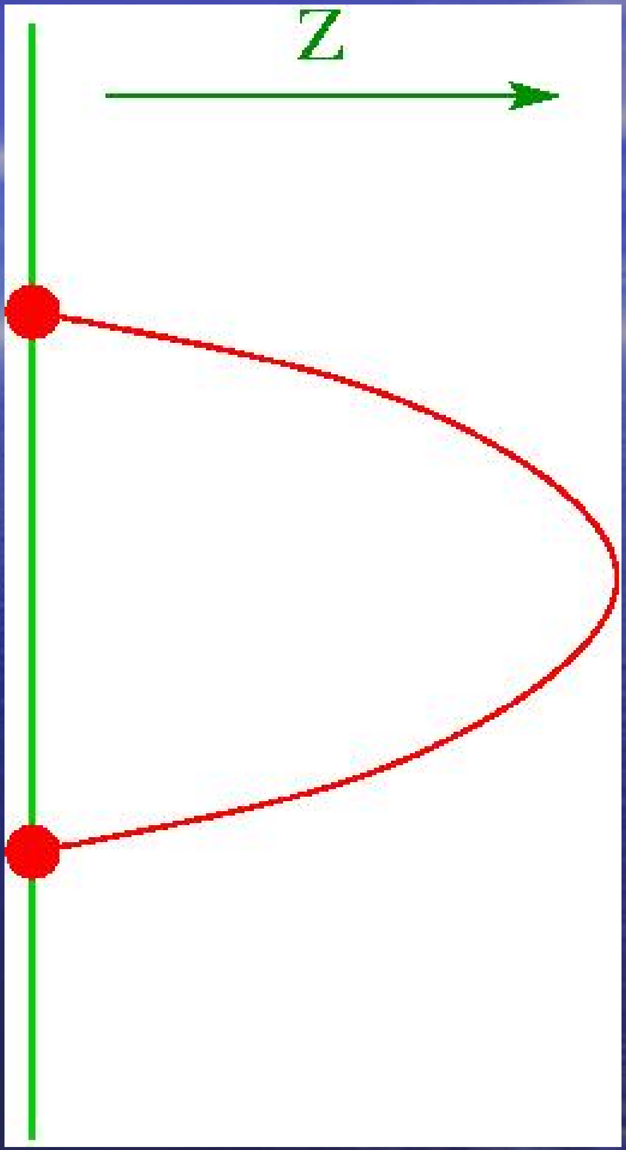




- Any non-SUSY abelian orbifold contains unstable operators. This appears to remove all such orbifolds from a list of large N perturbatively conformal gauge theories.
- The one-loop beta functions destroy the conformal invariance precisely in those twisted sectors where there exist closed-string tachyons localized at the tip of R^6/Γ. Thus, a very simple correspondence emerges between perturbative gauge theory and free closed string on an orbifold. Why? In the presence of tachyons, the standard AdS/CFT decoupling argument probably fails.

Quark Anti-Quark Potential

- The z-direction is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- In a pleasant surprise, because of the 5-th dimension z, the string picture applies even to theories that are conformal (not confining!). The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)



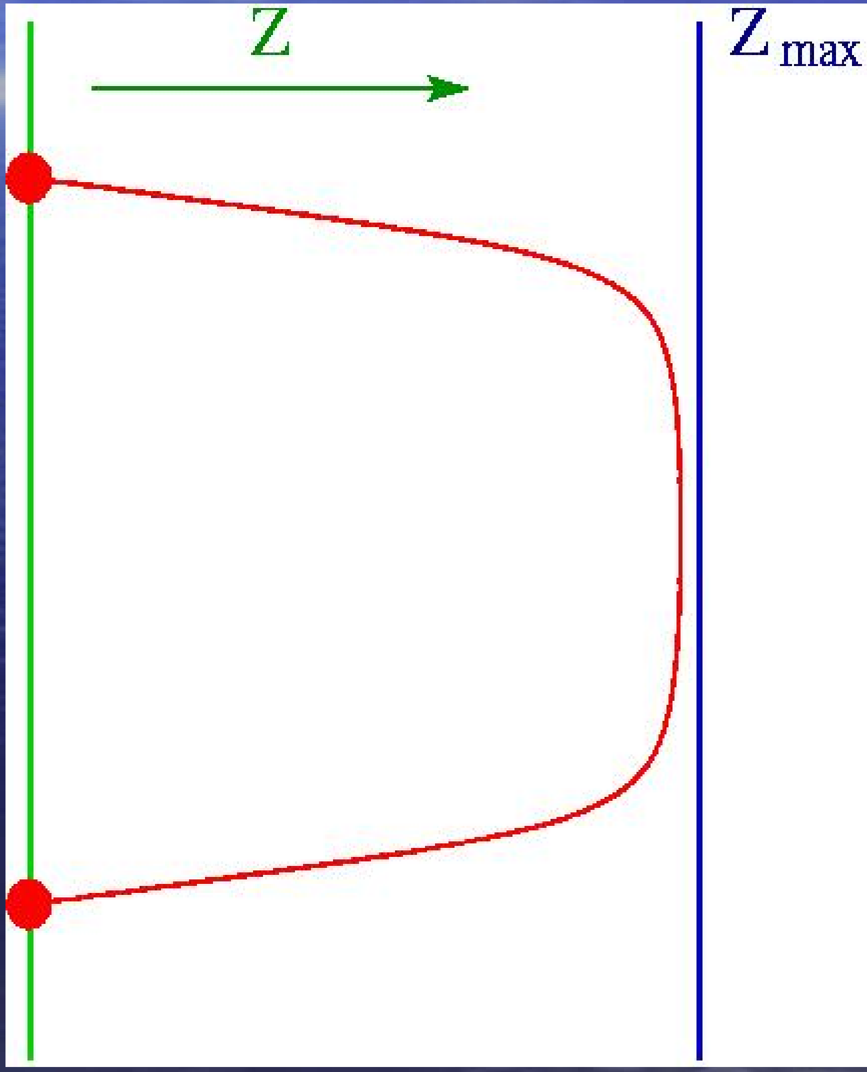
$$E_0(L) = -c\frac{\sqrt{g^2 N}}{L}$$

String Theoretic Approach to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- A "cartoon" of the necessary metric is

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\left(-(dx^0)^2 + (dx^i)^2\right)$$

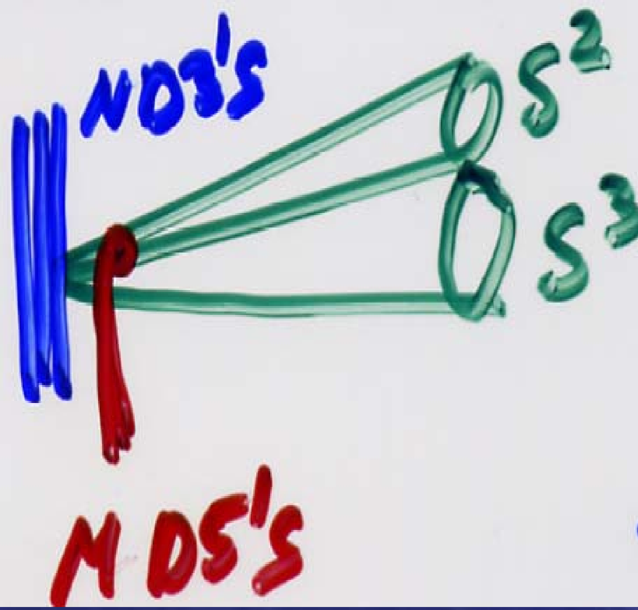
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\rm max})}{2\pi\alpha'}$



- Several 10-dimensional backgrounds with these qualitative properties are known (the compact space is actually “mixed” with the 5-d space).
- Witten (1998) constructed a background in the universality class of non-supersymmetric pure glue gauge theory. While there is no asymptotic freedom in the UV, hence no dimensional transmutation, the background serves as a simple model of confinement.

Confinement in SYM theories

- Introduction of minimal supersymmetry ($\mathcal{N}$=1) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



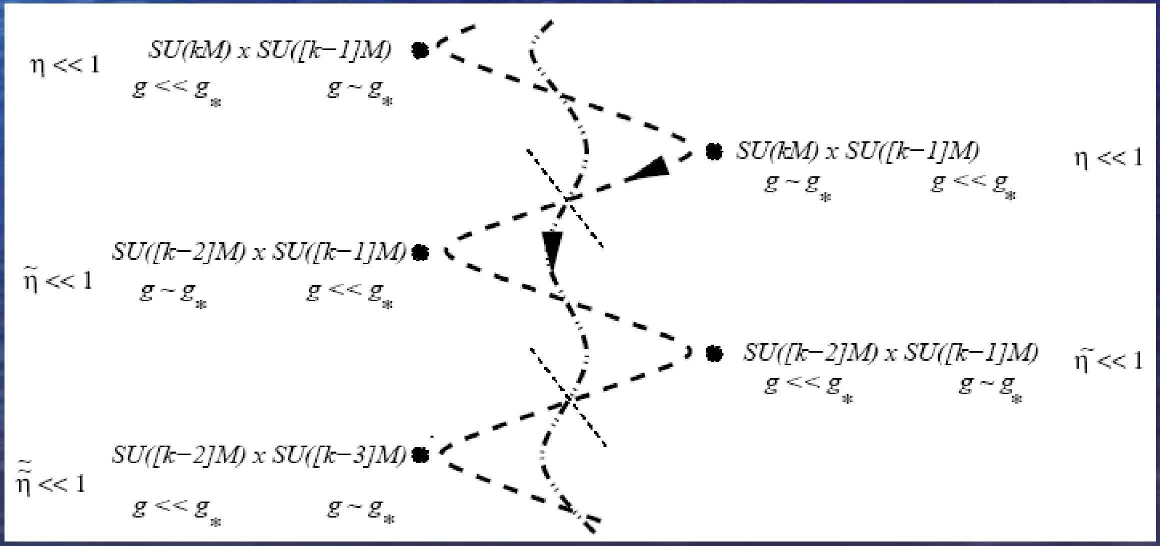
$$ds_{10}^2 = h^{-1/2}(t)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(t)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- In the UV there is a logarithmic running of the gauge couplings. Surprisingly, the 5-form flux, dual to N, also changes logarithmically with the RG scale. IK, Tseytlin
- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler

(diagram of RG flows from a review by M. Strassler)



- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: $Z_{2M} \to Z_2$.
- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f = N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \ldots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which $A_k \to e^{ia} A_k$; $B_l \to e^{-ia} B_l$. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.

- The KS solution is part of a moduli space of confining SUGRA backgrounds, **resolved warped deformed conifolds**. Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- To look for them we need to use the PT ansatz:

$$ds_{10}^2 = H^{-1/2} dx_m dx_m + e^x ds_6^2,$$

$$ds_6^2 = (e^g + a^2 e^{-g})(e_1^2 + e_2^2) + e^{-g} \sum_{i=1}^{2} \left(\epsilon_i^2 - 2a e_i \epsilon_i\right) + v^{-1}(\tilde{\epsilon}_3^2 + dt^2)$$

- H, x, g, a, v, and the dilaton are functions of the radial variable t. The asymptotic near-AdS radial variable is $r \sim \varepsilon^{2/3} e^{t/3}$
- Additional radial functions enter into the ansatz for the 3-form field strengths. The PT ansatz preserves the SO(4) but breaks a Z_2 charge conjugation symetry, except at the KS point.

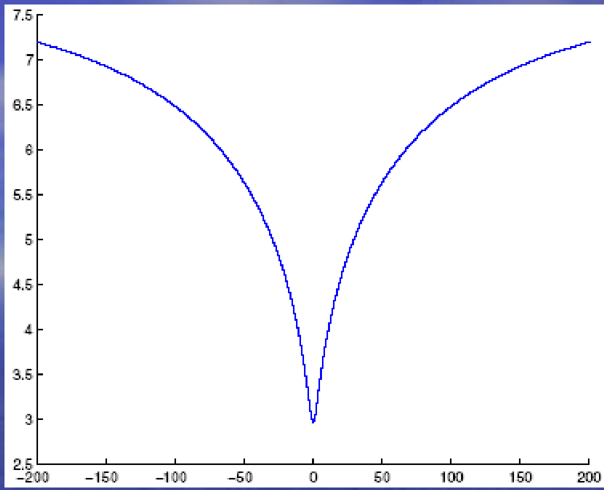
- BGMPZ used the method of SU(3) structures to derive the complete set of coupled first-order equations.
- A simplification is that the warp factor and the dilaton are related: $H(t) = \tilde{H}\left(e^{-2\phi(t)} - 1\right)$

  Dymarsky, IK, Seiberg
- The integration constant determines the `modulus' U: $\tilde{H} = \gamma U^{-2}$ where $\gamma = 2^{10/3}(g_s M\alpha')^2\varepsilon^{-8/3}$
- At large t the solution approaches the KT `cascade asymptotics', e.g. $a(t) = -2e^{-t} + Ue^{-5t/3}(-t+1) + \ldots$

$$\gamma^{-1}H(t) = \frac{3}{32}e^{-4t/3}(4t-1) - \frac{3}{32\cdot 512}U^2(256t^3 - 864t^2 + 1752t - 847)e^{-8t/3} + O\left(e^{-10t/3}\right)$$

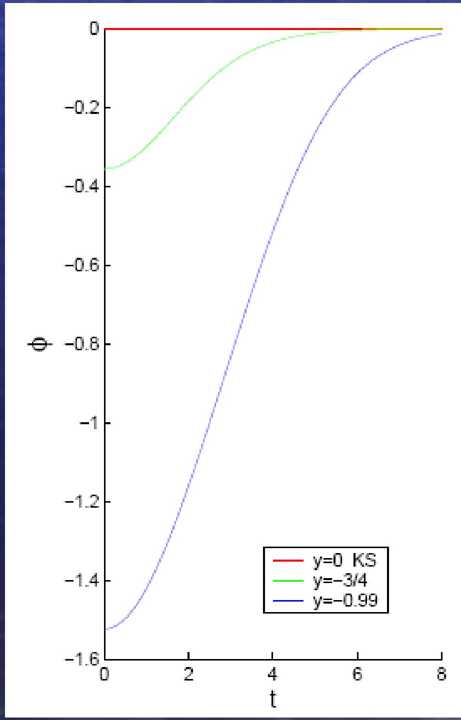
- The resolution parameter U is proportional to the VEV of the operator

$$\mathcal{U} = \mathrm{Tr}\left(\sum_{\alpha} A_\alpha A_\alpha^\dagger - \sum_{\dot\alpha} B_{\dot\alpha}^\dagger B_{\dot\alpha} \right)$$



- This family of resolved warped deformed conifolds is dual to the \`baryonic branch' in the gauge theory (the quantum deformed moduli space):

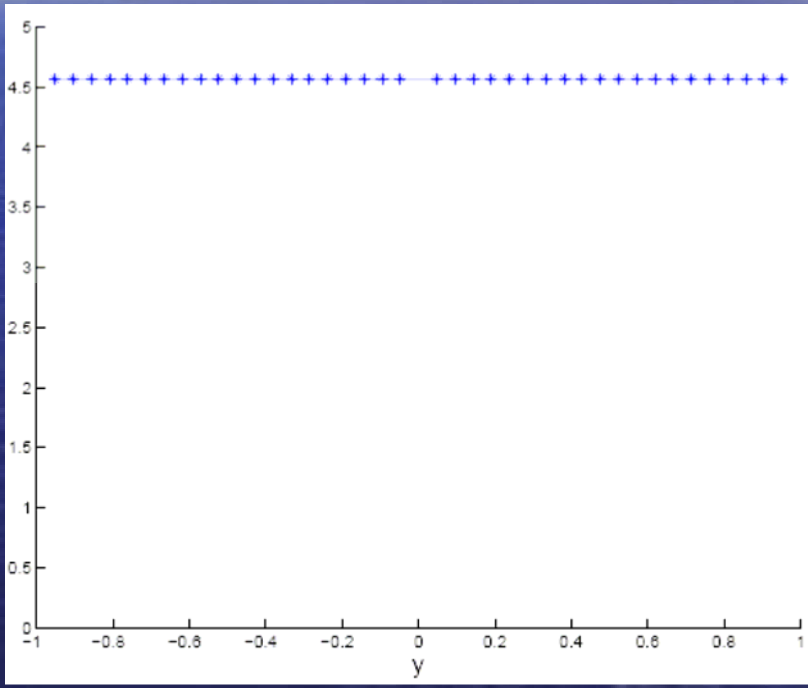
$$\mathcal{A} = i\Lambda_1^{2M}\zeta\ , \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$



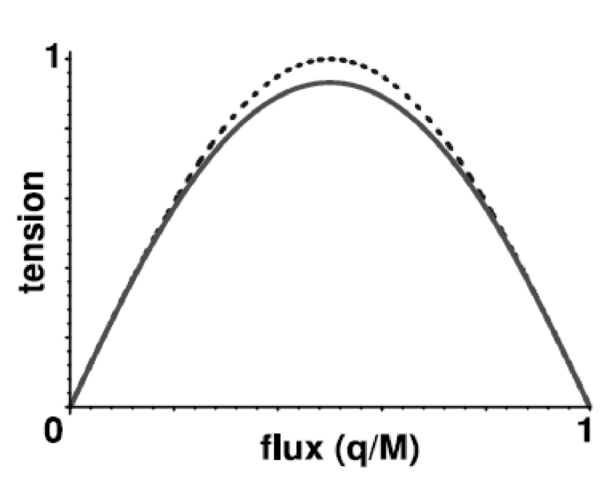
- Various quantities have been calculated as a function of the modulus U=ln |ζ|.
- Here are plots of the string tension (a fundamental string at the bottom of the throat is dual to an \`emergent' chromo-electric flux tube) and of the dilaton profiles Dymarsky, IK, Seiberg

BPS Domain Walls

- A D5-brane wrapped over the 3-sphere at the bottom of the throat is the domain wall separating two adjacent vacua of the theory.
- Since it is BPS saturated, its tension cannot depend on the baryonic branch modulus. This is indeed the case, which provides a check on the choice of the UV boundary conditions, and on the numerical integration procedure necessary away from the KS point.



- An interesting observable is the tension of a composite string connecting q quarks with q anti-quarks. In any SU(M) gauge theory it must be symmetric under q -> M-q. This is achieved through a stringy effect: q strings blow up into a wrapped D3-brane. Herzog, IK



- Dashed line refers to being far along the baryonic branch (near the MN limit) where

$$T(q) \sim \sin(\pi q/M)$$

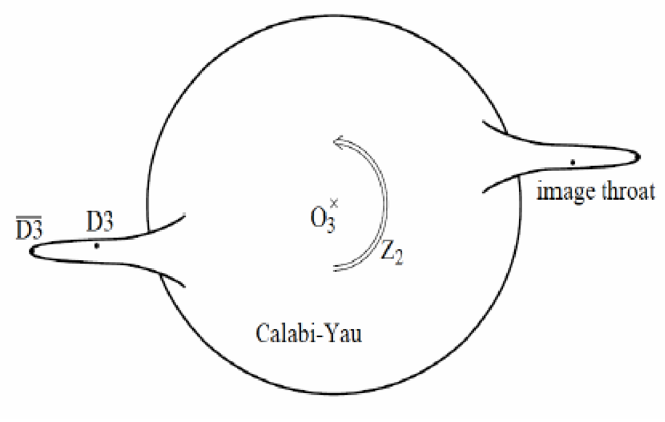
- Solid line refers to the KS background

- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for testing various ideas.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez, ...

Embedding in Flux Compactifications

- A long warped throat embedded into a compactification with NS-NS and R-R fluxes leads to a small ratio between the IR scale at the bottom of the throat and the string scale.

  Randall, Sundrum; Verlinde; IK, Strassler; Giddings, Kachru, Polchinski; KKLT; KKLMMT

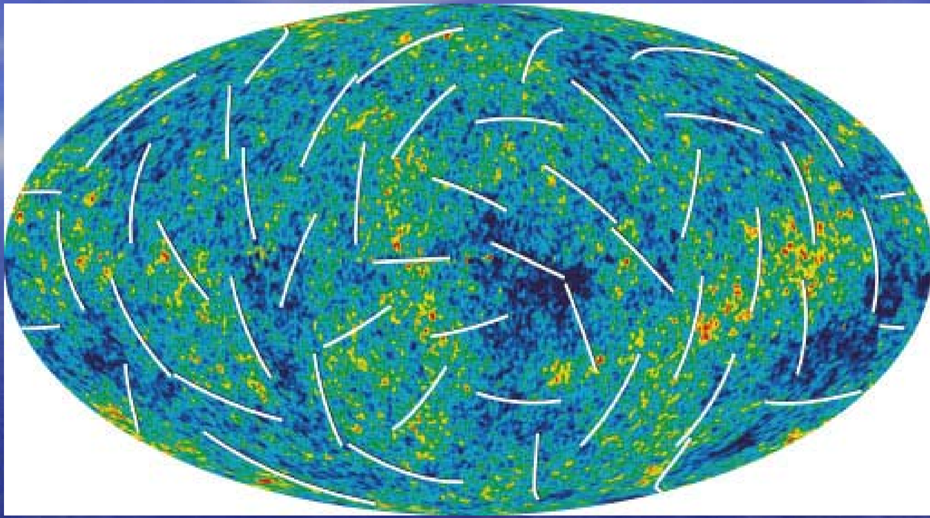


- In the dual cascading gauge theory the IR scale is the confinement scale: confinement stabilizes the hierarchy between the Planck scale and the SM or the inflationary scale.

- Cascading gauge theories dual to “standard model throats” may offer interesting possibilities for new physics beyond the standard model. Cascales, Franco, Hanany, Saad, Uranga, …

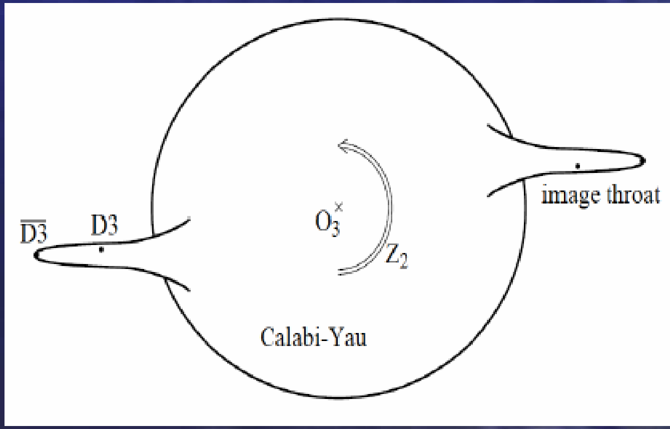
Applications to D-brane Inflation

- The Slow-Roll Inflationary Universe (Linde; Albrecht, Steinhardt) is a very promising idea for generating the CMB anisotropy spectrum observed by the WMAP.
- Finding models with very flat potentials has proven to be difficult. Recent string theory constructions use moving D-branes. Dvali, Tye, ...



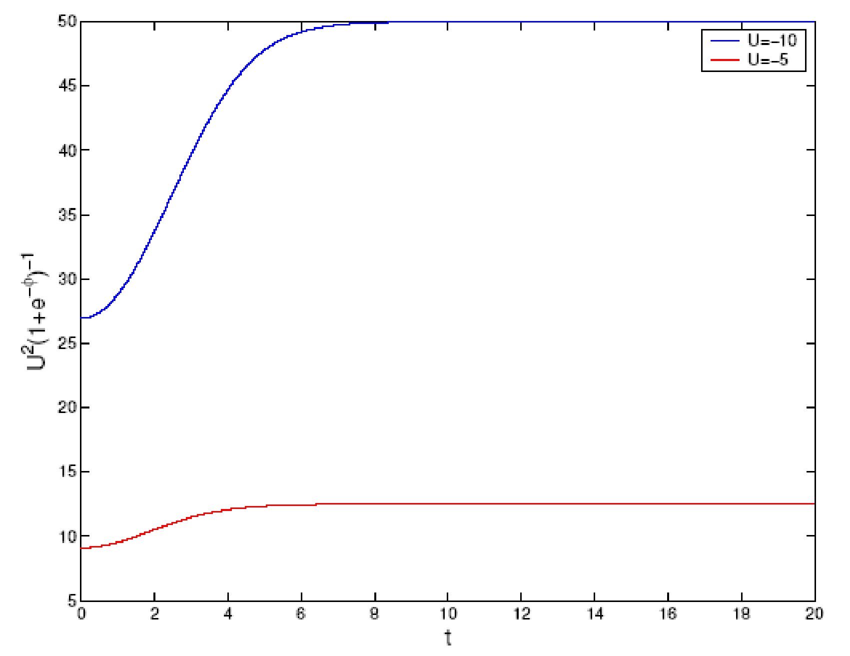
- In the KKLT/KKLMMT model, the warped deformed conifold is embedded into a string compactification. An anti-D3-brane is added at the bottom to break SUSY and generate a potential. A D3-brane rolls in the throat. Its radial coordinate plays the role of an inflaton.

Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi



A related suggestion for D-brane inflation (A. Dymarsky, IK, N. Seiberg)

- In a flux compactification, the $U(1)_{baryon}$ is gauged. Turn on a Fayet-Iliopoulos parameter ξ .
- This makes the throat a resolved warped deformed conifold.
- The probe D3-brane potential on this space is asymptotically flat, if we ignore effects of compactification. The plots are for two different values of $U \sim \xi$.
- No anti-D3 needed: in presence of the D3-brane, SUSY is broken by the D-term ξ. Related to the `D-term Inflation' Binetruy, Dvali; Halyo



Slow roll D-brane inflation?

- Effects of D7-branes and of compactification generically spoil the flatness of the potential. Some `fine-tuning' seems to be needed, as usual. This is currently under investigation with D. Baumann, A. Dymarsky, L. McAllister, A. Murugan and P. Steinhardt.

Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions
- The AdS/CFT correspondence makes this connection precise. It makes a multitude of dynamical statements about strongly coupled conformal (non-confining) gauge theories.
- Its extensions to confining theories provide a new geometrical view of such important phenomena as dimensional transmutation, chiral symmetry breaking, and quantum deformation of moduli space. They allow for calculations of glueball and meson spectra and of hadron scattering in model theories.

- This recent progress offers new tantalizing hopes that an analytic approximation to QCD may be achieved along this route, at least for a large number of colors.
- But there is much work to be done if this hope is to become a reality. Understanding the string duals of weakly coupled gauge theories remains an important open problem. Phenomenological AdS/QCD approaches to it appear to give nice results.
- Embedding gauge/string dualities into string compactifications offers new possibilities for physics beyond the SM, and for modeling inflation and cosmic strings.