Experimental investigation of two dimensional Anderson localization of light in the presence of a nonlocal nonlinearity

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Two directions

- Dissipative (gain and losses):
  - Random lasers

- Hamiltonian case:
  - Transverse localization
    - Introduction
    - Effect of nonlinearity
    - Disordered fiber experiments
    - Action at a distance
• Dep Physics Sapienza Rome
  • Marco Leonetti (IPCF-CNR)
  • Viola Folli (IPCF-CNR)

• University of Wisconsin-Milwaukee
  • Salman Karbasi & Arash Mafi
Above a certain amount of disorder no transport is possible "Anderson localization"

The reason: localized states due to disorder
Literature

• Observation of Anderson localization in
  • Nonlinear Optics
    – Y. Lahini et al. PRL 100, 013806 (2008)
  • Bose-Einstein condensation
  • Linear disordered media (optics)
1D Bosons (BEC)

- Billy et al. Nature 2008

Localization length versus strength of disorder

Also Roati et al. Nature 2008
3D Fermions (BEC)

- Kondov et al. Science 2011

Fig. 1. (A) Ultracold gas expanding into an optical speckle field (green) and separating into localized (blue) and mobile (red) components. (B) The measured optical depth, proportional to the atomic density integrated through $y$, is shown in false color. The image depicts a 480-nK gas that has expanded for 20 ms through the disordered potential with $\Delta = k_B \times 240$ nK. All images shown in this manuscript are averaged over at least five experimental realizations. Slices are shown through the image along $x$ (C) and $z$ (D). The filled curves are fits to independent mobile (red) and localized (blue) components.

Localization length Versus disorder
Figure 4 | Inverse of the mean-square width $\sigma^2$ of the plateau versus $kt^*$ for different samples. As can be seen, the width (corresponding to the localization length) diverges at $t^* \approx 4.5$, indicating the transition from a localized to a non-localized state. The increase of the localization length approaching the critical turbidity can also be used to estimate the critical exponent. All error bars correspond to systematic errors.
TRANSVERSE Anderson Loc


INDEX CONTRAST 0.0001
PROPAGATION 1 cm
The effect of nonlinearity on the 2D Anderson localization profile

The simplest model
The model

- One-dimensional NLS with a random potential

\[ i\psi_t = -\psi_{xx} + V(x)\psi - \chi |\psi|^2\psi \]

<table>
<thead>
<tr>
<th>( \chi )</th>
<th>Linear</th>
<th>Focusing</th>
<th>Defocusing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>+1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>-1</td>
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</table>

\[ \langle V(x)V(x') \rangle = V_0^2 \delta(x - x') \]

\[ P = \int |\psi|^2 dx \]
Nonlinear Anderson localization

- Bound state equation

\[ \psi = \varphi \exp(-iEt) \]

\[ -\varphi_{xx} + V(x)\varphi - \chi \varphi^3 = E\varphi, \]

- This is solved numerically by a pseudospectral Newton-Raphson algorithm
The simplest Anderson localization

\[-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,\]

\[\chi = 0\]

- One dimensional \textbf{LINEAR} Schroedinger equation with random potential
  - Specific case:
    - a Gaussianly distributed random potential
  - Known issues:
    - Existence of exponentially localized states (negative eigenvalues)
    - Distribution of eigenvalues
    - Localization length
Linearly localized states

- Gaussian potential
- Negative eigenvalues
- Decays as $\exp(-\sqrt{-E}|x|)$
- Link between localization length and eigenvalue

\[-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,\]

$E = -5$

$V_0 = 4$
The statistical distribution of eigenvalues

- There is a tail of negative energies corresponding to exponentially highly localized states

\[ \langle V(x)V(x') \rangle = V_0^2 \delta(x - x') \]

\[ \bar{E}_L \approx -V_0^{4/3}/3 \]

The localization length decreases as the inverse square root of the absolute value of the energy, hence the localization length decreases with the amount of disorder (as observed experimentally)
Localization length $l$

- It is calculated by the inverse participation ratio

\[
l = \frac{\left| \int \varphi^2 dx \right|^2}{\int \varphi^4 dx} = \frac{P^2}{\int \varphi^4 dx}
\]

\[
P = \int |\psi|^2 dx
\]

- For an exponentially localized state

\[
\varphi_e = \frac{e^{-2|x|/l}}{\sqrt{l/2}}
\]
Link between localization length and eigenvalue in the **LINEAR** case

- The localization length scales as inverse squares root of the eigenvalue

\[ l = \frac{3}{\sqrt{-E}} \]

- The lower the negative energy, the more localized
Parameters for the nonlinear case

- **INPUT POWER**
  \[ P = \int |\psi|^2 dx \]
  - Controls the amount of nonlinearity
  - What happens when increasing nonlinearity?

- In the presence of nonlinearity we have
  - **POWER DEPENDENT EIGENVALUE**
    \[ E = E(P) \]
  - **POWER DEPENDENT LOCALIZATION**
    \[ l = l(P) \]
Two regimes

- Strong perturbation regime (soliton for focusing)
  
  **HIGH POWER, LARGE P**

  \[ i\psi_t = -\psi_{xx} + V(y)\psi - \chi|\psi|^2\psi \]

- Weak perturbation regime (Anderson localization)
  
  **LOW POWER, SMALL P**

  \[ i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi \]
STRONG PERTURBATION (SOLITON)
Strong perturbation theory

• A simple multiple scale approach on the NLS shows that the random potential becomes negligible when increasing power.

$$\varphi = P\eta(Px)$$  
$$x_P \equiv Px$$

High P expansion

$$\frac{d\eta^2}{dx^2_P} + \chi\eta^3 = E_P\eta,$$

In this regime the only supported localization is the bright soliton.

$$\varphi = \sqrt{-2E}/\cosh(\sqrt{-Ex})$$

$$E = E_S = -P^2/16$$  
$$l = l_S = 12/P.$$
Solitons

- Features in common with Anderson localization
  - Location (they can be located anywhere in space)
  - Exponential localization
  - Negative (nonlinear) eigenvalue
  - Link between localization length and the eigenvalue

\[ l = \frac{3}{\sqrt{-E}} \]
Calculated exact profiles

- The linear fundamental state is numerically prolonged to high power
- Profiles for different powers $-\varphi_{xx} + V(x)\varphi - \chi\varphi^3 = E\varphi$, 

![Graph showing nonlinear Anderson state and soliton fraction for different powers.](image)
WEAK PERTURBATION
(Anderson states)
Perturbation of the Anderson state

- It is possible to develop a perturbation theory in terms of the power $P$
- We derive expressions for the localization length and for the eigenvalue valid at small $P$

\[ \psi = \sqrt{P}(\psi_0 + P\psi^{(1)} + P^2\psi^{(2)} + \ldots) \]

The lowest order term is the Anderson state with the smallest negative energy.
Perturbation of the Anderson state

- Eigenvalue ($E < 0$)

$$E = E_0 - \chi \frac{P}{l_0}$$

- In the **defocusing case** there is a power at which the eigenvalue becomes positive.
Perturbation of the Anderson state

- Localization length
  
  \[ l = l_0 \left( 1 - \chi \frac{P}{P_0} \right) \]

\[ \frac{1}{P_0} = 4l_0 \sum_{n>0} \frac{(\varphi_n, \varphi_0^3)^2}{E_n - E_0} \]

- In the **FOCUSING CASE** there is power at which the **localization length** becomes negative
Focusing Vs Defocusing case (weak perturbation theory results)

- In the **defocusing** case the energy increases
  - The wave delocalizes with P
  - There is a power at which the eigenvalue changes sign $P = |E_0| l_0$

- In the **focusing case** the energy decreases
  - $|E|$ increases with P
  - The wave becomes more localized
  - There is a power at which the localization length becomes zero ($P = P_0$)
TWO critical powers!

In the defocusing case for delocalization

\[ P_{\text{defocusing}} = l_0 |E_0| \]

In the focusing case for solitonization

\[ P_{\text{soliton}} = P_0 \]
Comparing the weak expansion with the numerical results

- Localization length $l(P)$

FOCUSING
Statistical distribution of the critical power in the focusing case

- Critical power to become a soliton

\[ \frac{1}{P_0} = 4l_0 \sum_{n>0} \frac{(\phi_n, \phi_0^3)^2}{E_n - E_0} \]
NON PERTURBATIVE APPROACH
(disorder averaged variational ansatz)
Results from the variational approach

- Final exact expression for the nonlinear Anderson state features

\[ E = E(P) \]

\[ E_C = -\frac{P^2}{16} \left(1 + \frac{P_C}{P}\right)^2 \]

Nonlinear eigenvalue

\[ l = l(P) \]

\[ l_C = \frac{12/P}{(1 + P_C/P)} \]

Localization length

One single parameter

\[ P_C = 4V_0^{2/3}/\sqrt{3}. \]
Strong and weak limits

- As \( P \) grows

\[
l_C = \frac{12/P}{1 + P_C/P} \quad \Rightarrow \quad l_S = \frac{12}{P}.
\]

- As \( P \) grows

\[
E_C = -\frac{P^2}{16} \left( 1 + \frac{P_C}{P} \right)^2 \quad \Rightarrow \quad E_S = -\frac{P^2}{16}
\]

- Also the weak limit provides the correct result, and \( P_C \) turns out to be a good approx for \( P_0 \)

- The found expressions correctly reproduce the two perturbative limits (strong and weak)!
Numerical localization length

- compare
Distribution of critical power

- $P_c$ gives the peak of the distribution

$P_C = 4V_0^{2/3}/\sqrt{3}$. 
Transverse localization in 2D fibers

Our experiments on transverse localization in two dimensional fibers
40000 pieces of PMMA and 40000 pieces of PS randomly mixed and fused together
\[ n(PS) = 1.59 \]
\[ n(PMMA) = 1.49 \]

**Mixture of PS and PPMA**
Index contrast 0.1
Propagation \( > 7 \text{ cm} \)

**Observation of transverse Anderson localization in an optical fiber**

Salman Karbasi, Craig R. Mirr, Parisa Gandomkar Yarandi, Ryan J. Frazier, Karl W. Koch, and Arash Mafi
Absence of diffusion

(a) 60 μm

(b) 60 μm

66 μm

(c)

homogeneous fiber

28 μm

disordered fiber

28 μm

(d)

(e)
Multicolor transverse Anderson-localization

- we excite several localizations at different wavelengths simultaneously
At any spatial location there are several localized modes at different frequencies.
Nonlinear regime

- at any wavelength we study the localization profile Vs power
Measurement of critical power

Homogeneous fiber

Disordered fiber

Pc=14mW
We observe focalization of any of the localized mode when increasing power.

Mode at 820nm

Mode profile at 820nm

Mode at 835nm

We observe focalization of any of the localized mode when increasing power.
2D SELF-FOCUSING of Anderson localizations

Experiments

Localization length Versus Intensity (50 modes)

Numerically calculated bound states of the 2D-NLS with Gaussian disorder

Theory from the variational approach
Folli, Conti, OL 2011
Conti, PRA, 2012
Which the origin of the observed nonlinear focusing?

- it's thermal!
Timescale is compatible with thermal effects
(PMMA and PS absorb the infrared light)
Action at a distance between Anderson localizations in nonlinear nonlocal media

- thermal nonlinearity is nonlocal!
MODIFIED SETUP
Probe Anderson mode (532nm)

Pump Anderson Mode (800nm)

20 microns
The size of the probe changes with the pump power!

Probes Anderson mode (532nm)

<table>
<thead>
<tr>
<th>Power (mW)</th>
<th>Image</th>
</tr>
</thead>
<tbody>
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<td>10</td>
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<tr>
<td>47</td>
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</tr>
<tr>
<td>83</td>
<td><img src="83mW.png" alt="Image" /></td>
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</tbody>
</table>

DISPLACEMENT

\[
D \text{(µm)} \quad \Omega \text{(µm)}
\]

\[
\begin{array}{cccccc}
0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 \\
0 & 2 & 4 & 6 & 8 & 10 & 0 & 2 & 4 & 6
\end{array}
\]
The migration of the multicolor Anderson localization

A form of transport in the Anderson regime
Density map of localizations

- We count the states in any spatial location

FIBER OUTPUT

Here localizations

300 microns

25 microns

7 mW
Density map of locs Vs power

- 7 mW
- 14 mW
- 20 mW
- 28 mW

- 36 mW
- 45 mW
- 51 mW
- 54 mW

25 microns
Model with nonlocal nonlinearity

\[ 2ik \frac{\partial A}{\partial z} + \nabla_{x,y} A + 2k^2 \frac{\Delta n}{n_0} A = 0, \]

\[ \Delta n = n_{\text{PS}} - n_{\text{PMMA}} = \Delta n_{R} + \Delta n_{NL} \]

\( \Delta n_{R}(x, y) \) due to the disorder

\[ \Delta n_{NL} = \int K(x - x', y - y')|A|^2(x', y')dx'dy'. \]

\[ \Delta n_{NL} \cong K(x, y) \int |A|^2 d\mathbf{r} \cong P \left( \Delta n_{1} + \frac{r^2}{2} \Delta n_{2} \right). \]
Collective coordinates

\[ P_p \frac{d^2 \mathbf{r}_p}{dz^2} = \int I_p(\mathbf{r} - \mathbf{r}_p) \nabla_{x,y} \frac{\Delta n_{NL}}{n} \, d\mathbf{r}, \]

\[ \Delta n_{NL} = \sum_{q=1}^{N} \Delta n_{NL,q} \equiv \sum_{q=1}^{N} \frac{P_q \Delta n_2}{2} (\mathbf{r} - \mathbf{r}_q)^2. \]

\[ P_p \frac{d^2 \mathbf{r}_p}{dz^2} = -\nabla_{x_p,y_p} \sum_{q=1}^{N} \frac{|\Delta n_2| P_q P_p}{2n_0} |\mathbf{r}_p - \mathbf{r}_q|^2. \]
Action at a distance for two states

\[ D(z) = D(0) \left(1 - \frac{|\Delta n_2| z^2}{2n_0} P_{\text{pump}} \right). \]
Conclusions

- Nonlinearity and nonlocality in 2D disorder fibers
- Action at a distance
- Transport in the Anderson regime
- Incoherent Anderson states and interactive focusing (see poster)
- Variational theoretical approaches

THANKS!

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