New Heterotic GUT and Standard Model Vacua

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hep-th/0603015

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Motivation

Mainly two kinds of semi-realistic compactifications:

- Compactifications with intersecting D-branes

  (see talk by M.Cvetic)

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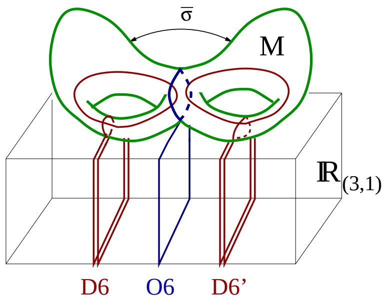

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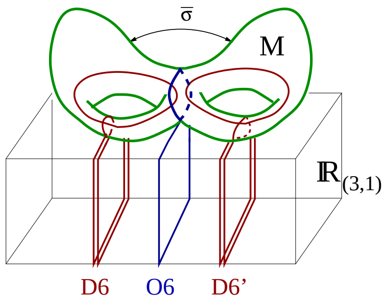

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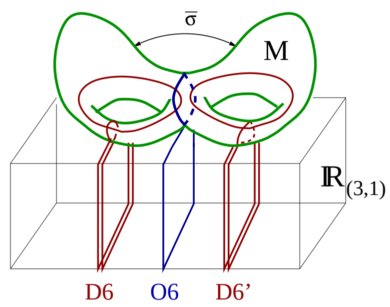

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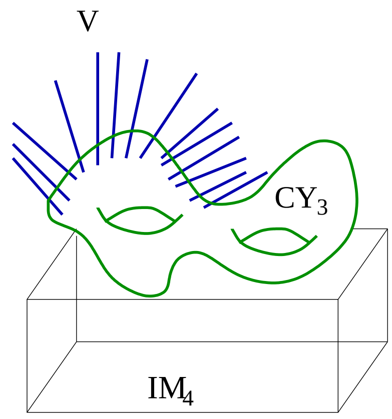

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Alternatively:

- Consider the $E_8 \times E_8$ heterotic string equipped with the specific class of bundles

$$W = V \oplus L$$

  with structure group $G = SU(4) \times U(1)$.
- Embedding this structure group into one of the E_8 factors leads to the breaking t $H = SU(5) \times U(1)_X$, where the adjoint of E_8 decomposes as follows into $G \times H$ representations.

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$$\mathbf{248} \longrightarrow \left\{ \begin{array}{c} (\mathbf{15},\mathbf{1})_0 \\ (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{10})_4 + (\mathbf{1},\overline{\mathbf{10}})_{-4} + (\mathbf{1},\mathbf{24})_0 \\ (\mathbf{4},\mathbf{1})_{-5} + (\mathbf{4},\overline{\mathbf{5}})_3 + (\mathbf{4},\mathbf{10})_{-1} \\ (\overline{\mathbf{4}},\mathbf{1})_5 + (\overline{\mathbf{4}},\mathbf{5})_{-3} + (\overline{\mathbf{4}},\overline{\mathbf{10}})_1 \\ (\mathbf{6},\mathbf{5})_2 + (\mathbf{6},\overline{\mathbf{5}})_{-2} \end{array} \right\} .$$

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reps.	Cohomology
$\mathbf{10}_{-1}$	$H^*(\mathcal{M}, V \otimes L^{-1})$
$\mathbf{10}_{4}$	$H^*(\mathcal{M}, L^{4})$
$\overline{\mathbf{5}}_{3}$	$H^*(\mathcal{M}, V \otimes L^{3})$
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Table 1: Massless spectrum of $H = SU(5) \times U(1)_X$ models.

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Table 1: Massless spectrum of $H = SU(5) \times U(1)_X$ models.

Candidate for a flipped $SU(5)$ model $\rightarrow$ need to understand structure of $E_8 \times E_8$ compactification with $U(N)$ bundles.

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$SU(3) \times SU(2) \times U(1)_Y$	Cohom.
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$(\mathbf{3}, \mathbf{2})_{-\frac{5}{3}}$	$H^*(L^{-1})$
$(\overline{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}}$	$H^*(\bigwedge^2 V)$
$(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}}$	$H^*(V \otimes L^{-1})$
$(\mathbf{1}, \mathbf{2})_{-1}$	$H^*(\bigwedge^2 V \otimes L^{-1})$
$(\mathbf{1}, \mathbf{1})_{2}$	$H^*(V \otimes L)$
$(\mathbf{1}, \mathbf{1})_{1}$	$H^*(L^{-1})$

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We choose

$$W_i = V_{N_i} \oplus \bigoplus_{m_i=1}^{M_i} L_{m_i}$$

with $U(N_i)$ bundle V_{N_i} and the complex line bundles L_{m_i}.

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with $U(N_i)$ bundle V_{N_i} and the complex line bundles L_{m_i}.

$$c_1(W_i) = c_1(V_{N_i}) + \sum_{m_i=1}^{M_i} c_1(L_{m_i}) = 0.$$

W can be embedded into an $SU(N_i + M_i) \subset E_8$.

Tadpole cancellation

Tadpole cancellation

- The Bianchi identity for the three-form H implies the tadpole cancellation condition

$$0 = \frac{1}{4(2\pi)^2}\left(\mathrm{tr}(\overline{F}_1^2) + \mathrm{tr}(\overline{F}_2^2) - \mathrm{tr}(\overline{R}^2)\right) - \sum_a N_a \overline{\gamma}_a,$$

to be satisfied in cohomology. Here $\overline{\gamma}_a$ are Poincare dual to two-cycles Γ_a wrapped by the N_a M5-branes.

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This can be written as

$$\sum_{i=1}^{2}\left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2}\sum_{m_i=1}^{M_i} c_1^2(L_{m_i})\right) - \sum_a N_a \overline{\gamma}_a = -c_2(T).$$

Massless spectrum

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$$H^*(X, W),$$

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- The net-number of chiral matter multiplets is given by the Euler characteristic of the respective bundle $\mathcal{W}$

$$\chi(X, \mathcal{W}) \;=\; \int_X \left[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12}\, c_2(T_X)\, c_1(\mathcal{W}) \right].$$

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- All non-abelian cubic gauge anomalies do cancel, whereas the mixed abelian-nonabelian, the mixed abelian-gravitational and the cubic abelian ones do not. They need to be cancelled by a generalised Green-Schwarz mechanism involving the terms

$$S_{GS} = \frac{1}{24\,(2\pi)^5\,\alpha'} \int B \wedge X_8,$$

and

$$S_{kin} = -\frac{1}{4\kappa_{10}^2} \int e^{-2\phi_{10}}\, H \wedge \star_{10} H.$$

(Lukas, Stelle, hep-th/9911156), (R.B., Honecker, Weigand, hep-th/0504232)

Hermitian Yang-Mills equation

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- At string tree level, the connection of the vector bundle has to satisfy the hermitian Yang-Mills equations

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  F has to be a holomorphic vector bundle.

- A necessary condition is the so-called Donaldson-Uhlenbeck-Yau (DUY) condition,

$$\int_X J \wedge J \wedge c_1(V_{N_i}) = 0, \qquad \int_X J \wedge J \wedge c_1(L_{m_i}) = 0,$$

  to be satisfied for all n_i, m. If so, a theorem by Uhlenbeck-Yau guarantees a unique solution provided each term is μ-$stable$.

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There exists a corresponding stringy one-loop correction to the HYM equation of the form

$$\star_6 \Bigg[J \wedge J \wedge F_i^{ab} - \frac{\ell_s^4}{4(2\pi)^2}\, e^{2\phi_{10}}\; F_i^{ab} \wedge \Big(\mathrm{tr}_{E_{8i}}(F_i \wedge F_i) - \frac{1}{2}\mathrm{tr}(R \wedge R) \Big) + \ell_s^4 e^{2\phi_{10}} \sum_a N_a \left(\frac{1}{2} \mp \lambda_a \right)^2 F_i^{ab} \wedge \overline{\gamma}_a \Bigg] + (\mathrm{non-pert.\ terms}) = 0..$$

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There exists a unique solution, once the bundle satisfies the corresponding integrability condition and the bundle is Λ-stable with respect to the slope

$$\Lambda(\mathcal{F}) = \frac{1}{\mathrm{rk}(\mathcal{F})}\Bigg[\int_X J \wedge J \wedge c_1(\mathcal{F}) - \ell_s^4\, g_s^2 \int_X c_1(\mathcal{F}) \wedge$$

$$\left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2} \sum_{n_i=1}^{M_i} c_1^2(L_{n_i}) + \frac{1}{2}\, c_2(T)\right) + \ (\mathrm{npt}).$$

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If, as for $SU(N)$ Bundles

$$\lambda(V) = \mu(V),$$

we can immediately conclude that a μ-stable bundle is also λ-stable for sufficiently small string coupling g_s.

Flipped $SU(5)$ vacua

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Consider heterotic string on a Calabi-Yau manifold X with bundle

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  (Tatar, Watari, hep-th/0602238), (Andreas, Curio, hep-th/0602247)
- Embed a second line bundle into the other E_8, such that a linear combination of the two observable $U(1)$'s remains massless .

Flipped $SU(5)$ vacua

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- Concretely, we embed the line bundle L also in the second E_8, where it leads to the breaking $E_8 \to E_7 \times U(1)_2$ and the decomposition

$$\mathbf{248} \stackrel{E_7 \times U(1)}{\longrightarrow} \left\{ \ (\mathbf{133})_0 + (\mathbf{1})_0 + (\mathbf{56})_1 + (\mathbf{1})_2 + c.c. \ \right\}.$$

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- More general breakings are possible.

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- The linear combination

$$U(1)_X = -\frac{1}{2}\left(U(1)_1 - \frac{5}{2}\,U(1)_2\right)$$

remains massless if the following conditions are satisfied

$$\int_X c_1(L) \wedge c_2(V) = 0, \; \int_{\Gamma_a} c_1(L) = 0 \quad \text{for all M5 branes.}$$

Flipped $SU(5)$ vacua: spectrum

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reps.	bundle	SM part.
$(\mathbf{10},\mathbf{1})_{\frac{1}{2}}$	$\chi(V)=g$	$(q_L,d_R^c,\nu_R^c)+[H_{10}]$
$(\mathbf{10},\mathbf{1})_{-2}$	$\chi(L^{-1})=0$	$-$
$(\overline{\mathbf{5}},\mathbf{1})_{-\frac{3}{2}}$	$\chi(V\otimes L^{-1})=g$	(u_R^c,l_L)
$(\overline{\mathbf{5}},\mathbf{1})_{1}$	$\chi(\bigwedge^2 V)=0$	$[(h_3,h_2)+(\overline{h}_3,\overline{h}_2)]$
$(\mathbf{1},\mathbf{1})_{\frac{5}{2}}$	$\chi(V\otimes L)+\chi(L^{-2})=g$	e_R^c
$(\mathbf{1},\mathbf{56})_{\frac{5}{4}}$	$\chi(L^{-1})=0$	$-$

Table 2: Massless spectrum of $H=SU(5)\times U(1)_X\times E_7$ models with $g=\frac{1}{2}\int_X c_3(V)$.

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- GUT breaking via $H_{10} + \overline{H}_{10}$ leads to a natural solution of the doublet-triplet splitting problem via a missing partner mechanism in the superpotential coupling

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- Gauge invariant Yukawa couplings

$$\mathbf{10}^{i}_{\frac{1}{2}}\,\mathbf{10}^{j}_{\frac{1}{2}}\,\mathbf{5}_{-1}, \quad \mathbf{10}^{i}_{\frac{1}{2}}\,\overline{\mathbf{5}}^{j}_{-\frac{3}{2}}\,\overline{\mathbf{5}}_{1}, \quad \overline{\mathbf{5}}^{i}_{-\frac{3}{2}}\,\mathbf{1}^{j}_{\frac{5}{2}}\,\mathbf{5}_{-1},$$

lead to Dirac mass-terms for the d, (u,ν) and e quarks and leptons after electroweak symmetry breaking.

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- Since the electroweak Higgs carries different quantum numbers than the lepton doublet, the dangerous dimension-four proton decay operators

$$\mathbf{l\,l\,e} \quad \in \quad \overline{\mathbf{5}}^i_{-\frac{3}{2}}\,\mathbf{1}^j_{\frac{5}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}},\ \mathbf{q\,d\,l}, \quad \mathbf{u\,d\,d} \quad \in \quad \mathbf{10}^i_{\frac{1}{2}}\,\mathbf{10}^j_{\frac{1}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}}$$

  are not gauge invariant.

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- Breaking a stringy $SU(5)$ or $SO(10)$ GUT model via discrete Wilson lines, the Standard Model tree level gauge couplings satisfy

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at the string scale.

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- Since the $U(1)_X$ has a contribution from the second E_8, this relation gets modified to

$$\alpha_3 = \alpha_2 = \frac{8}{3}\alpha_Y = \alpha_{GUT}$$

Bundles on elliptically fibered CYs

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Elliptically fibered Calabi-Yau manifold X

$$\pi : X \to B$$

with the property that the fiber over each point is an elliptic curve E_b and that there exist a section σ.

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- Friedman, Morgan and Witten have defined stable $SU(N)$ bundles on such spaces via the so-called spectral cover construction. (Friedman, Morgan, Witten, hep-th/9701162)

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Mathematically, such a prescription is realized by the Fourier-Mukai transform

$$V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P}_B)$$

with

$$\begin{tikzcd}
 & \Big(X \times_B C,\, \mathcal{P}_B \otimes \pi_2^* \mathcal{N}\Big) \arrow[dl, "\pi_1"'] \arrow[dr, "\pi_2"] & \\
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Cohomology classes

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- Determine cohomologies of line bundles over complete intersections of divisors in X $\rightarrow$ Koszul sequences allow one relate them eventually to line bundles on B.

Cohomology classes

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The cohomology classes of the anti-symmetric and symmetric tensor products are more involved but can be computed by similar methods.

Outlook

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Using bundle extensions

$$0 \rightarrow V_1 \rightarrow V \rightarrow V_2 \rightarrow 0$$

we have so far found concrete flipped $SU(5)$ models with just three generations of MSSM quarks and leptons plus one vector-like GUT Higgs, i.e.

$$H^i(X, V) = (0, 1, 4, 0).$$

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The number of weak Higgses and the stability of these extensions are still under investigation.

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- Heterotic Landscape?

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Sorry, but if our construction is correct then it follows for the LHC,

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The straightforward proof is left to the audience.
Experimental results are expected July 9, 2006.