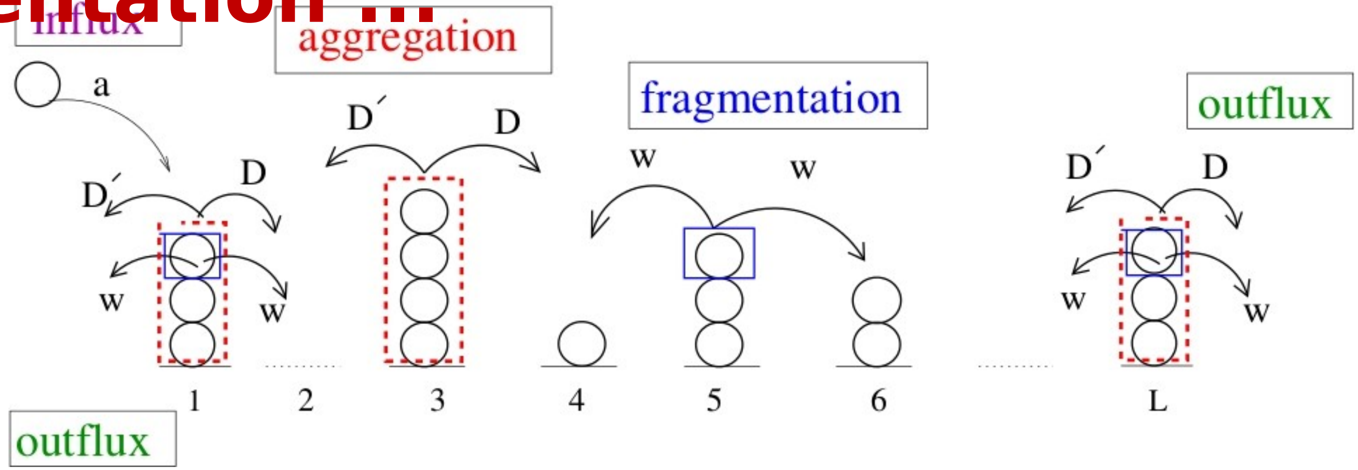
# Phase Transitions and Intermittency in an Aggregation-Fragmentation Model

Mustansir Barma

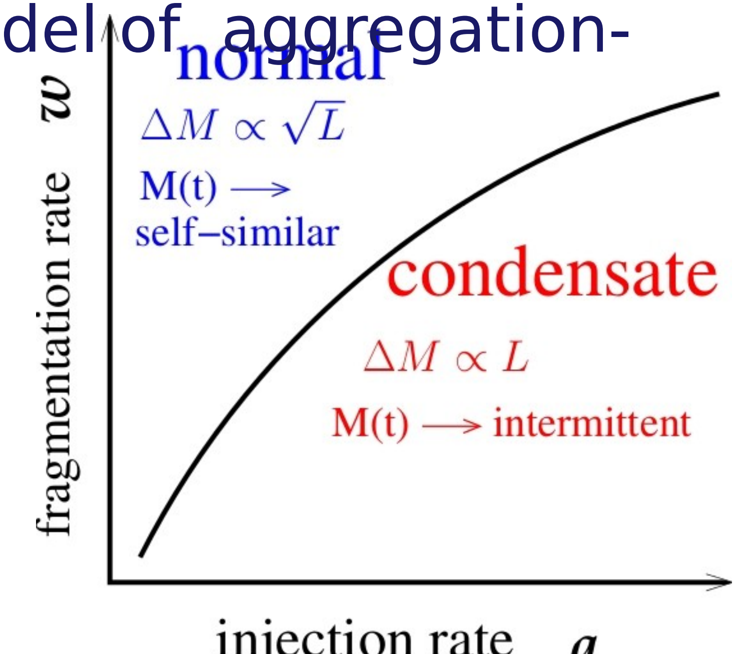
# Stochastic Model of Diffusion, Aggregation, Fragmentation ...




•Limiting case of a model of biomolecular movement and processing

•Generalization of well-studied model of aggregation-fragmentation in a closed system

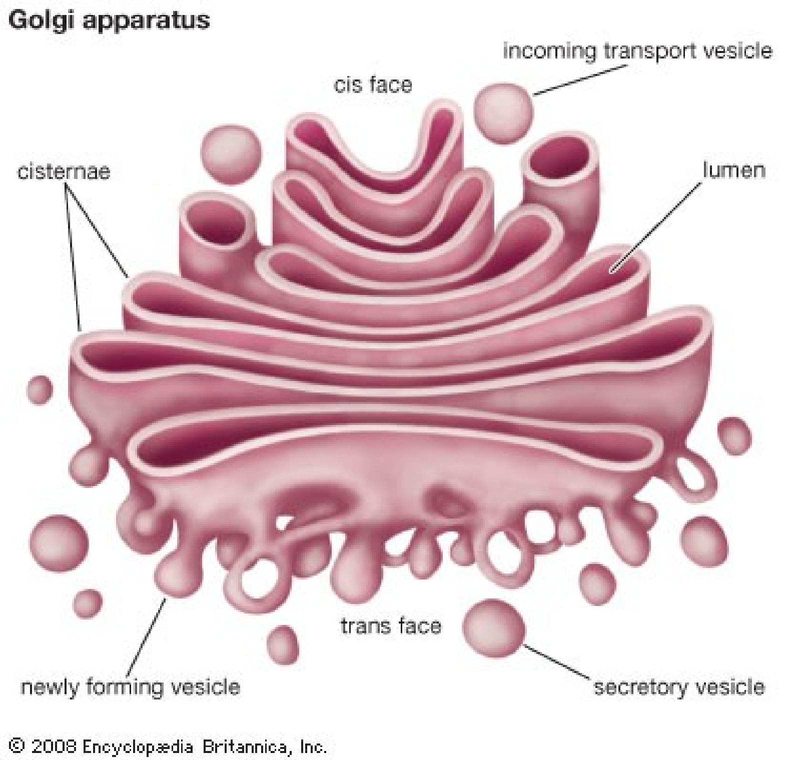
Shows a transition to a phase with Giant number fluctuations and Intermittency in dynamics

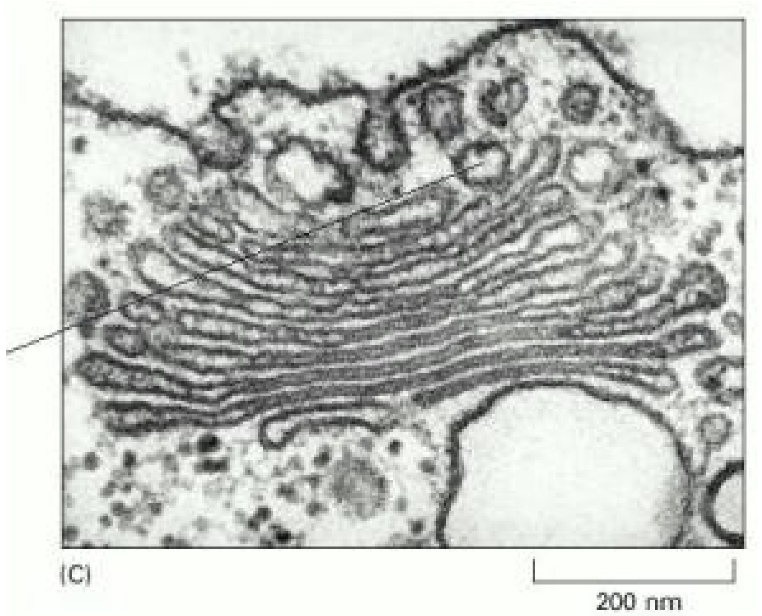



**H. Sachdeva, M. Barma , Madan Rao,**
**Phys. Rev. Lett. (2013)**

**H. Sachdeva, M. Barma, J. Stat. Phys. (2014)**

# Golgi apparatus




Protein vesicles arrive at one end; leave at other end, after processing

**Two scenarios** **[B Glick et al (1998), E Losev et al (2006), G.H. Patterson et al (2008)]**

**Vesicular transport:** *Biomolecules shuttle between compartments*

**Controversy** **Do biomolecules move singly, or in a bunch?**

'It is likely that the transport through the Golgi ... involves eleme
of both'

**Essentials of Molecular Trafficking**

( **Molecular Biology of the Cell,** **B Alberts, A Johnson, J Lewis, New Yor
Garland ; 2002. )**

•**Localized injection** of vesicles containing unprocessed biomolecules

•**Transport** By chipping of single vesicles, or movement of aggregates

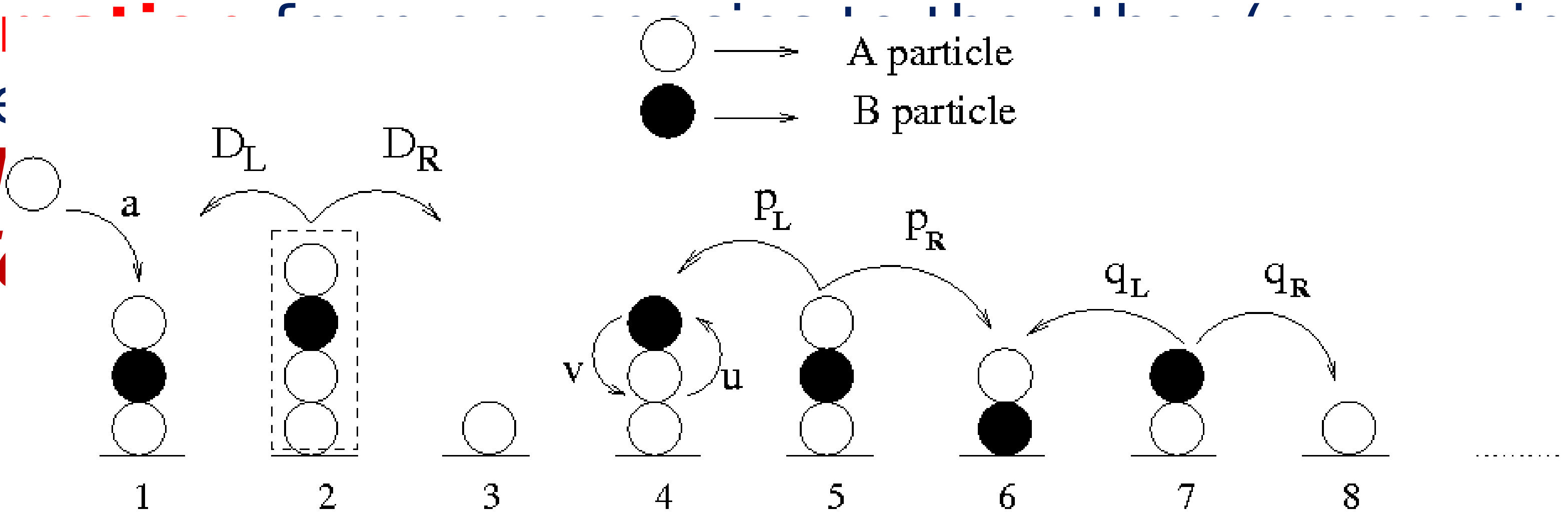
•**Transfor**

g

by enzyme

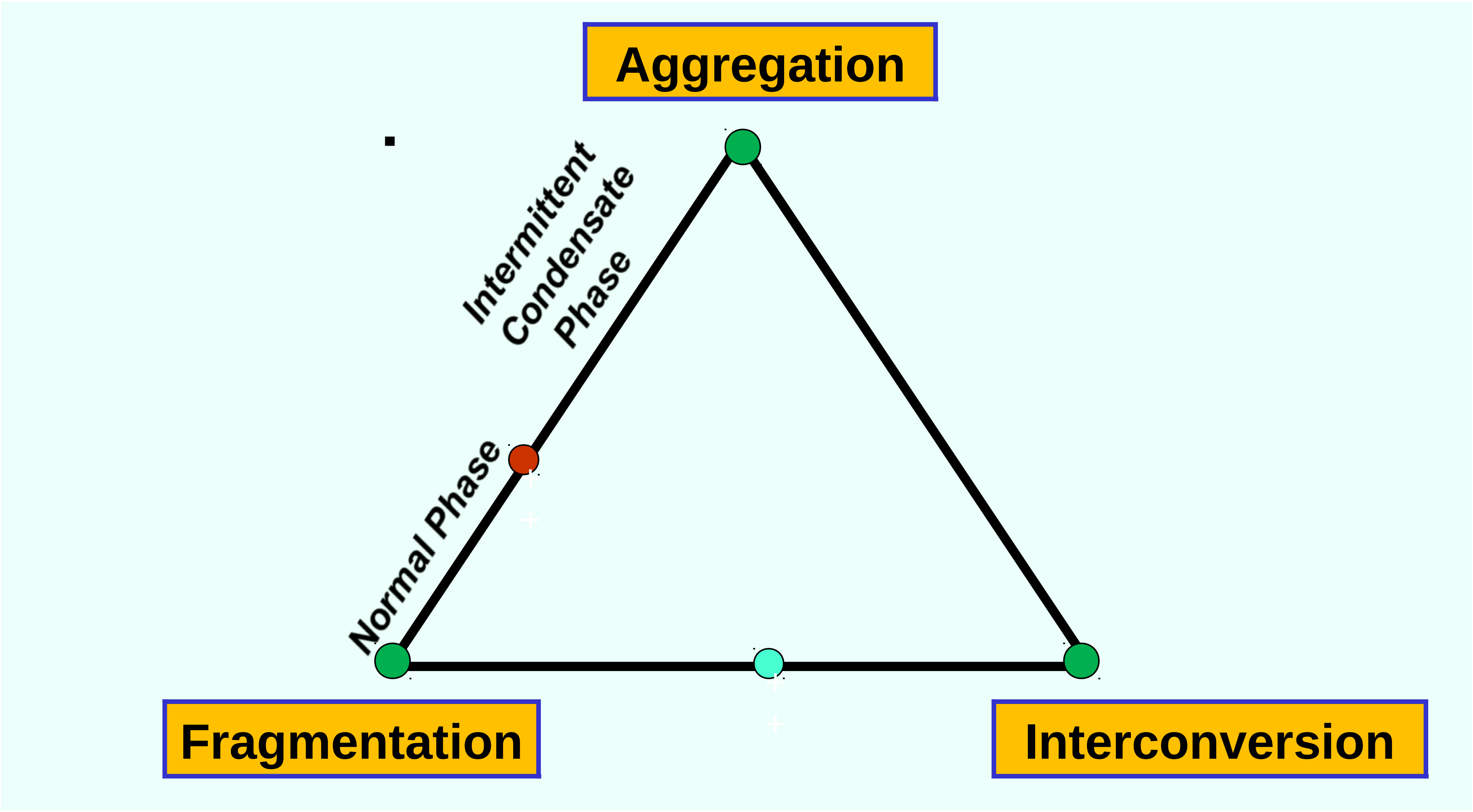
***Anal***

***these f***



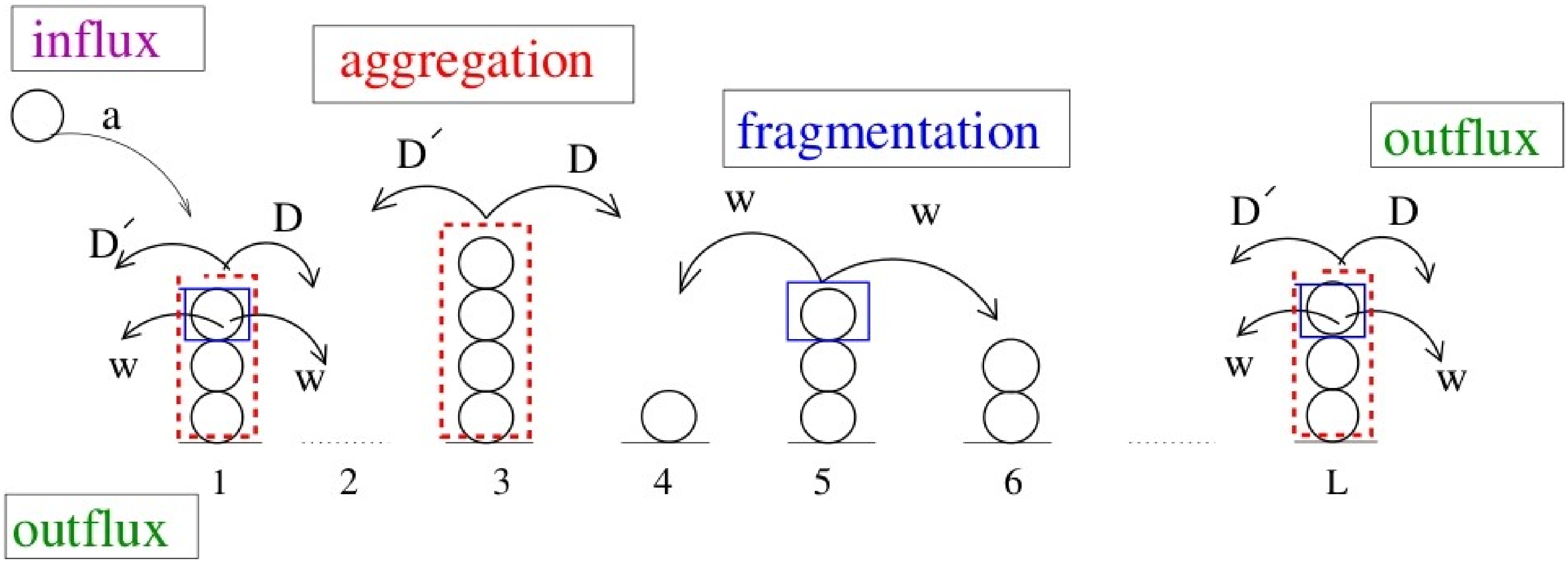

**H. Sachdeva, M. Barma , Madan Rao, Phys. Rev. E (2011)**

# Limiting Cases



# Aggregation-Fragmentation Model

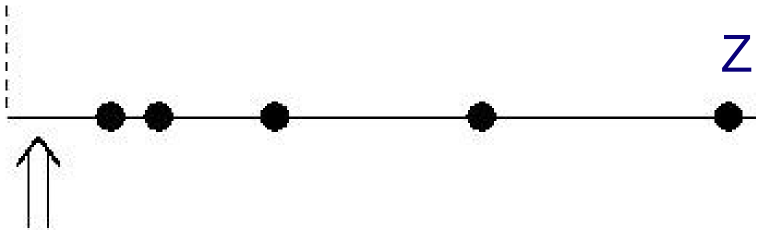
**Consider the limit of zero interconversion rate : only**




- **Influx** of unit mass with rate a at site 1.
- **Diffusion** of full stack at rate D or D'. **Aggregation** o contact.
- **Chipping** of unit mass with symmetric rate w.
- **Outflux** at site 1 or site L by exit of either the full stack or single particles.

# Related earlier work

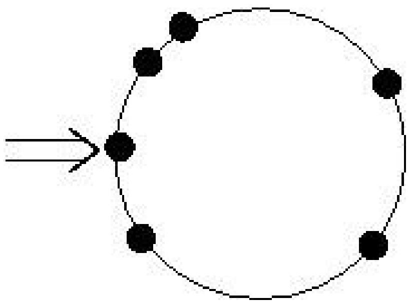
## A + A → A   (no chipping)



Z Cheng, S Redner, F Leyvraz, PRL (1989)

Input from leftmost point, No egress from left

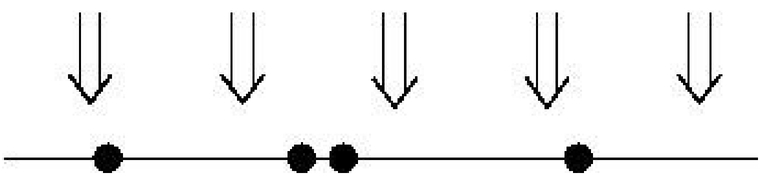
$P(m,r) \sim m^{-3/2} F(m/r^2)$



B Derrida, V Hakim, V Pasquier, PRL (1995)

Origin always occupied

→ Persistence exponent

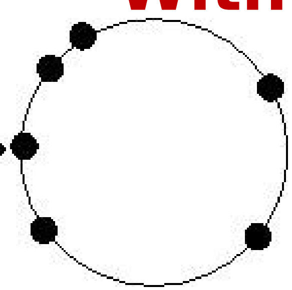


H Takayasu, I Nishikawa, H Tasaki,  PRA (1988)

Uniform input at all lattice sites

Power law mass distribution

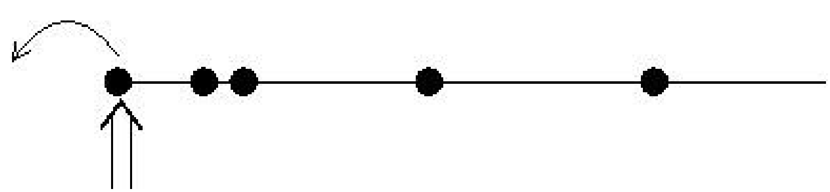
## With chipping



S Majumdar, S Krishnamurthy, M Barma, PRL (1998)

Periodic boundary conditions

On increasing density, phase transition to a state with a macroscopic `condensate’



## Model under study

# Condensation Phenomena in Closed Systems

## Zero Range Process (ZRP)

**[M R Evans, T Hanney, J Phys A (2005)]**

## Aggregation-Fragmentation on a Ring

**[S N Majumdar, S Krishnamurthy, M Barma, PRL (1998) ; J Stat Phys (2000) ]**

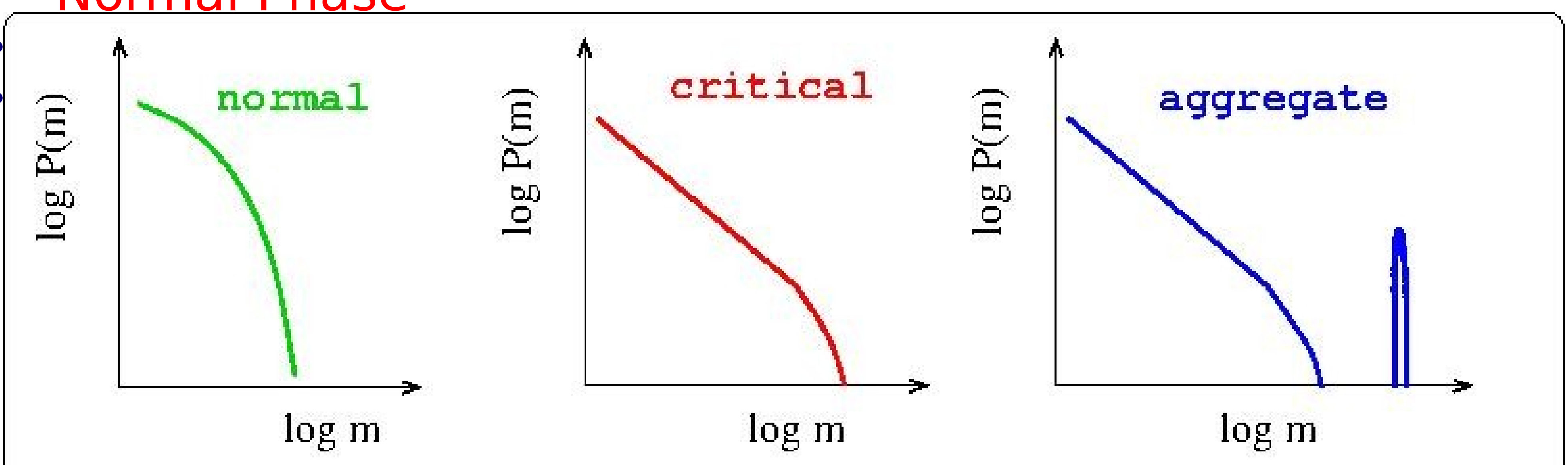
The model shows a condensate peak above a critical mass density

### Condensate Phase

•Single site mass distribution P(m) shows a power-law + Aggregate peak

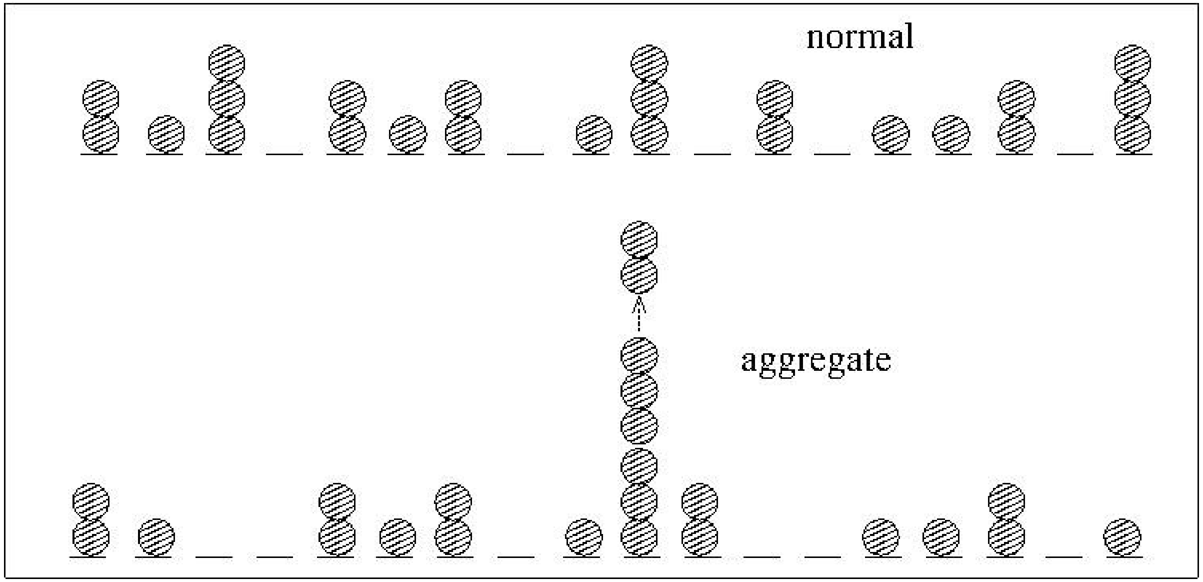
•Finite fraction of the mass in the aggregate; akin to Bose-Einstein condensation

### Normal Phase



# Related work

The mean-field analysis → Phase boundary in the w-density plane.




- **Phase boundary is exact** in all dimensions, despite correlations
  [ R. Rajesh and S.N. Majumdar , Phys Rev E (2001) ]

- **Directed chipping :** Condensate lost
  [ R.Rajesh and S. Krishnamurthy, Phys Rev E (2002) ]

- **If** $D \sim m^{-\alpha}$, Condensate curbed, but significant effect in finite system
  [ R. Rajesh, D. Das, B. Chakraborty and M. Barma, Phys Rev E (2002) ]

# Condensation in Open Systems?

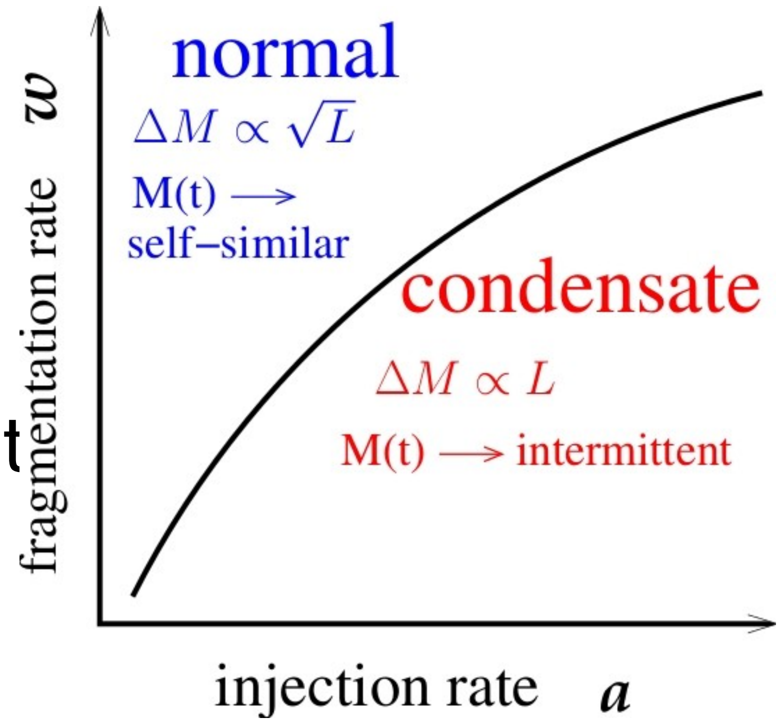
- In a closed system with conserved mass, find 'real-space Bose-Einstein condensation'

- The open system has strong mass fluctuations

  Does condensation occur?

The answer is yes.

But the condensate is very different in charact from the closed case.



# Condensation in the Open System
## Unbiased Movement (D=D’)

**Steady state and dynamical properties** Very different in the two phases.

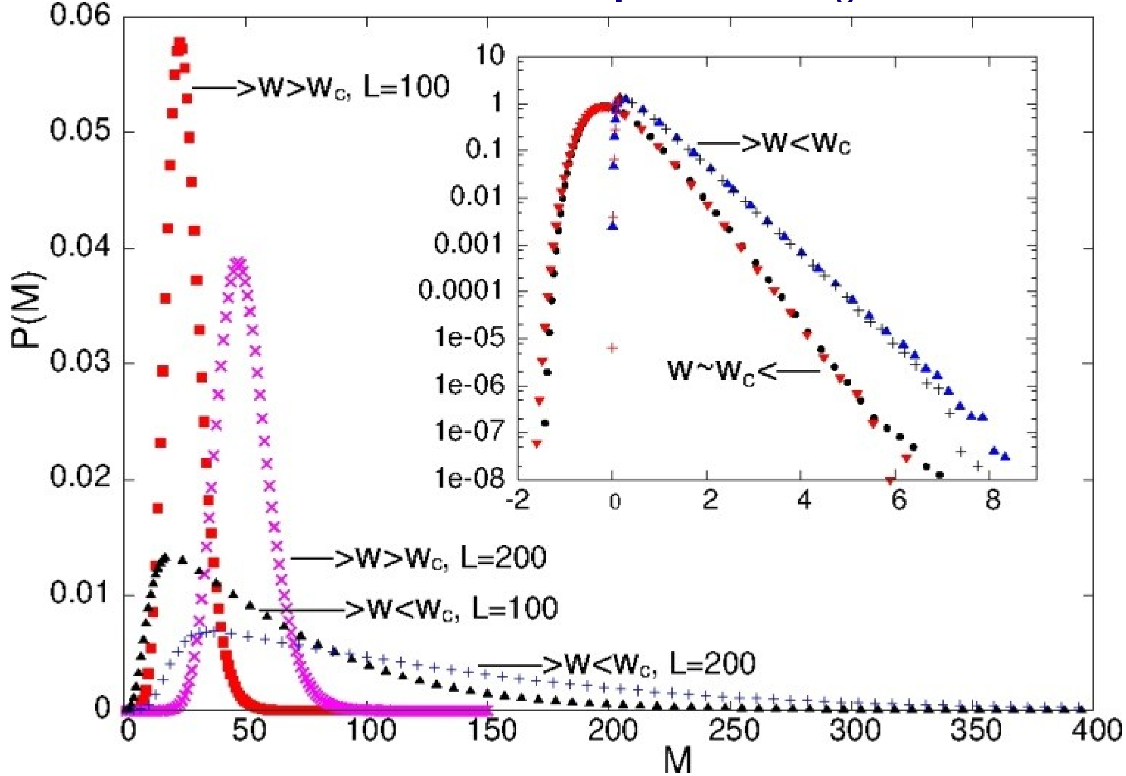
**Condensate phase**

•P(m) :  Long ‘Condensate tail’ … P(M)  ≈ A exp($-M/M_0$) at large M

$M_0 \sim L$

•Giant number fluctuations

$\Delta M \sim L$




**Normal (large w) phase**

•P(m) :  Gaussian tail

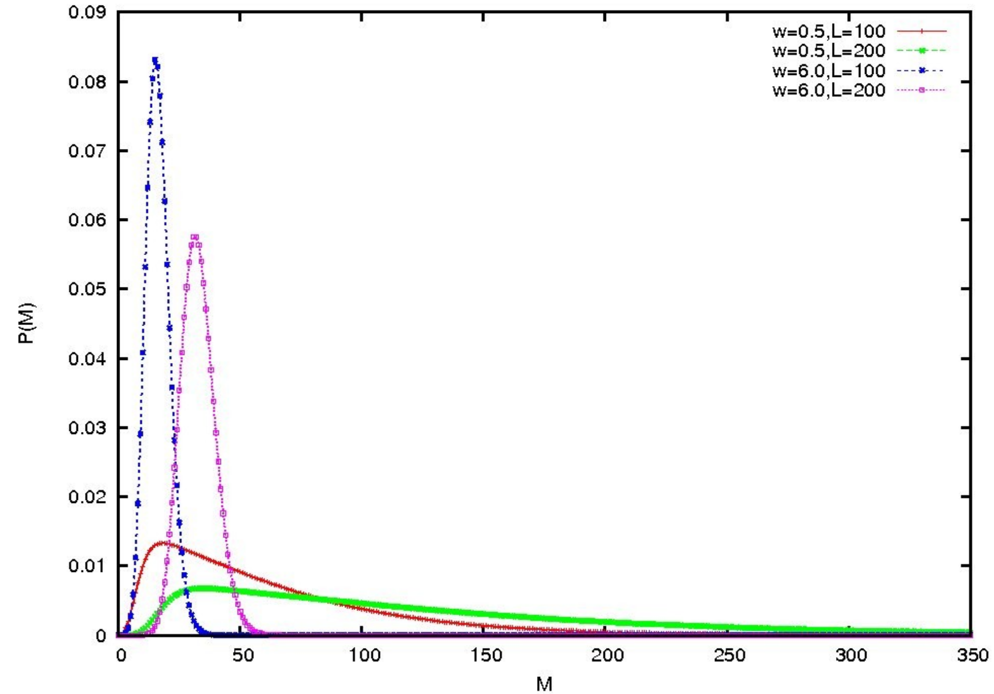
•Number fluctuations normal

# Mass Fluctuations: Size dependence

$\Delta M \sim L$ in Aggregate Phase

$\Delta M \sim L^{2/3}$ at Criticality

$\Delta M \sim L^{1/2}$ in Normal Phase

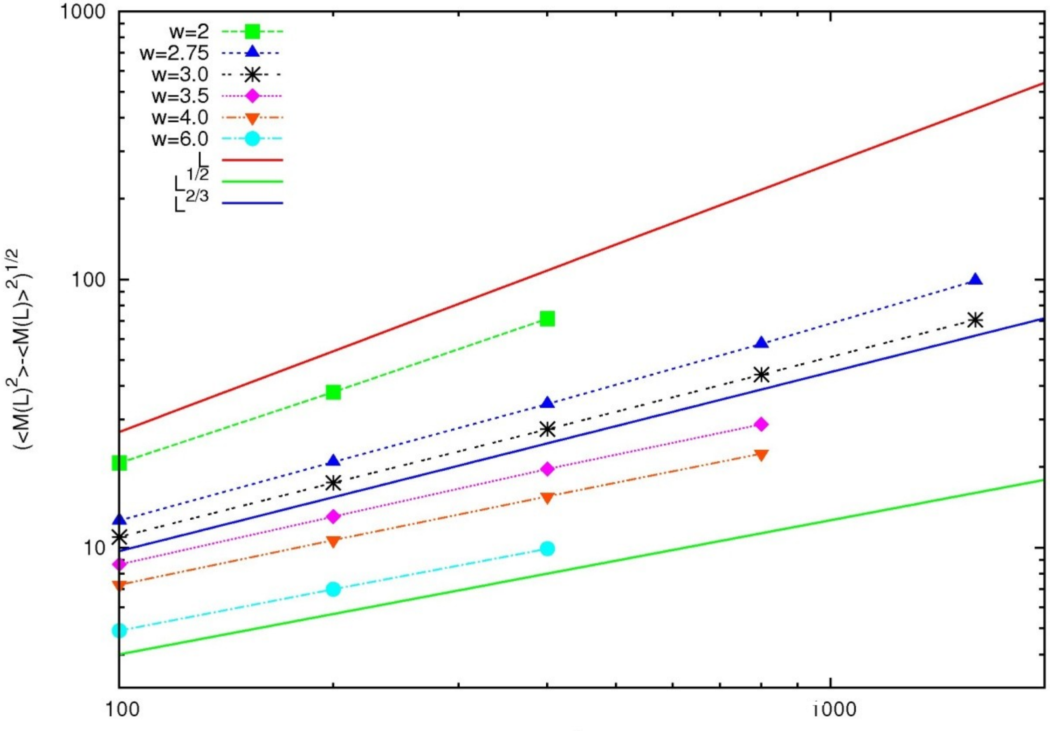



**Probability distribution**

(for p=6 and 0.5, L=100 and 200)

**Condensate tail**

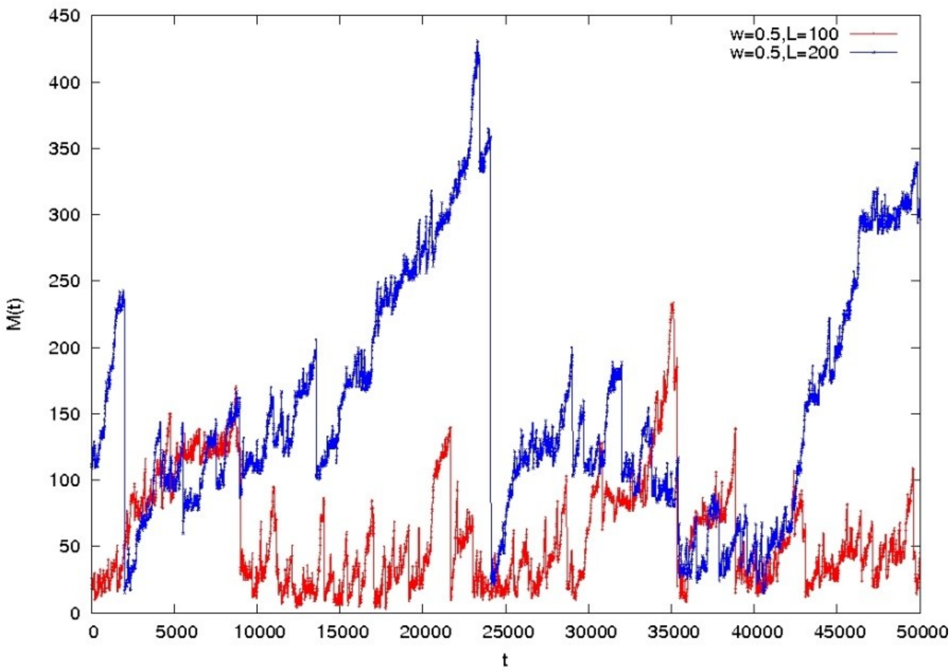
$\exp(-M/M_0)$ with $M_0 \propto L$

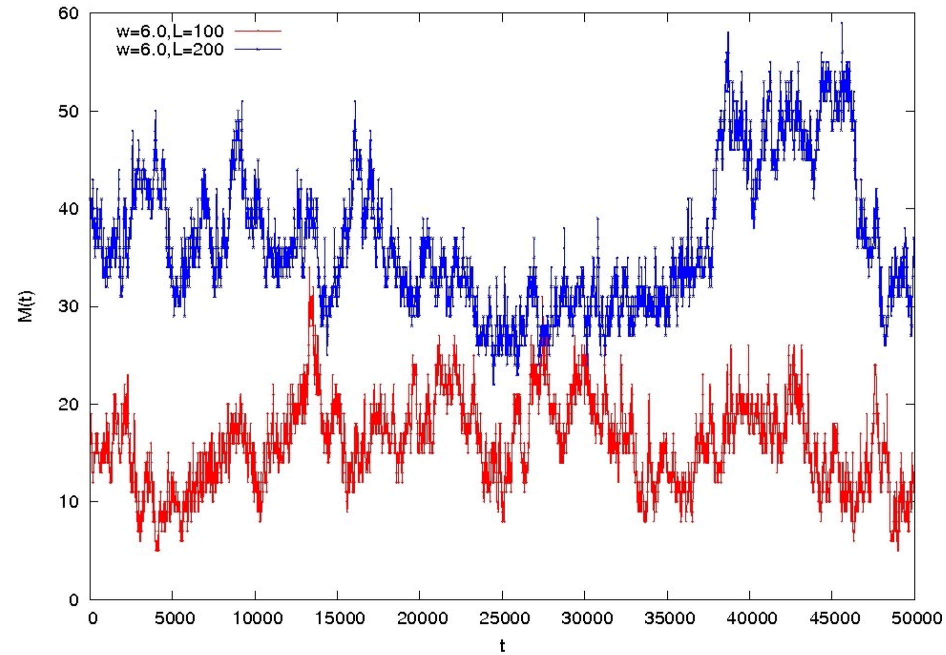



**Size dependence of second moment**

**(for w between 2 and 6)**

# Total mass M: Dynamics




## Condensate Phase

Extreme Fluctuations in time

Intermittent, not self-similar

## Normal Phase

Fluctuations are self-similar

# Self-similarity vs. Intermittency

Self-similarity: $\Delta M(t) = M(t) - M(0)$ has same statistical properties for all t

Intermittency: $\Delta M(t)$ depends strongly on t
[Distribution of M(t) is heavy-tailed: extreme events dominate]

Define structure functions in time: $u_n(t) = \langle ( M(t) – M(0) )^n \rangle$
[Analogous to structure functions of velocity field in fluid turbulence]

Self-similar signal: $u_n(t) \propto t^{\gamma n}$ as $t / \tau \rightarrow 0$
[τ is the lifetime of the largest structures]

Intermittent signal: Deviation from $u_n(t) \propto t^{\gamma n}$ at small t
Useful measures of intermittency:

Flatness: $\kappa(t) = u_4(t) / (u_2(t)^2)$

# Temporal Intermittency in the Aggregate Phase

Time dependence of Flatness

$\kappa(t) = u_4(t) / (u_2(t)^2)$ with $u_n(t) = < ( M(t) - M(0) )^n >$

For intermittent signals, $\kappa(t)$ diverges as $t/\tau \rightarrow 0$

In Normal Phase

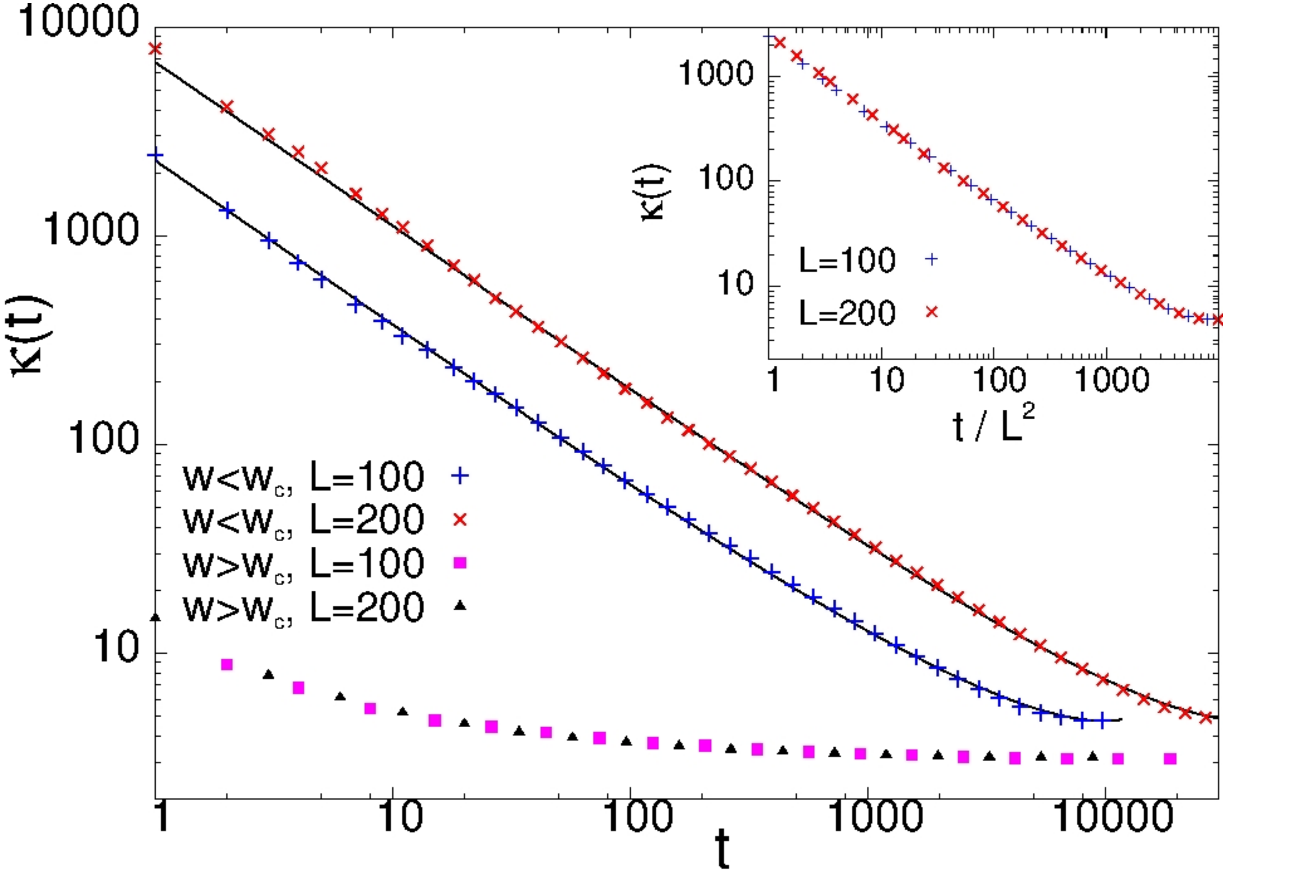
$\kappa(t) \rightarrow$ const as $t \rightarrow 0$

No L dependence

In Aggregate Phase

$\kappa(t) \approx At^{-1}$ with log corrections

Strong L



# Analytic results:  Pure aggregation limit

- Moments     $u_n(t) = \langle ( M(t) - M(0) )^n \rangle$
- Define generalized autocorrelation function

  $H_{i,j}(t) = \langle M_{i,j}(t)\, M_{0,L}(0) \rangle - \langle M_{i,j}(t) \rangle \langle M_{0,L}(0) \rangle$

  where $M_{i,j}$ is the mass between sites i and j

- Write time evolution equation for $H_{i,j}(t)$

  Take continuum limit to convert recursions to PDE for H(x,y,t)

  Can be solved by 'folding' triangle to square

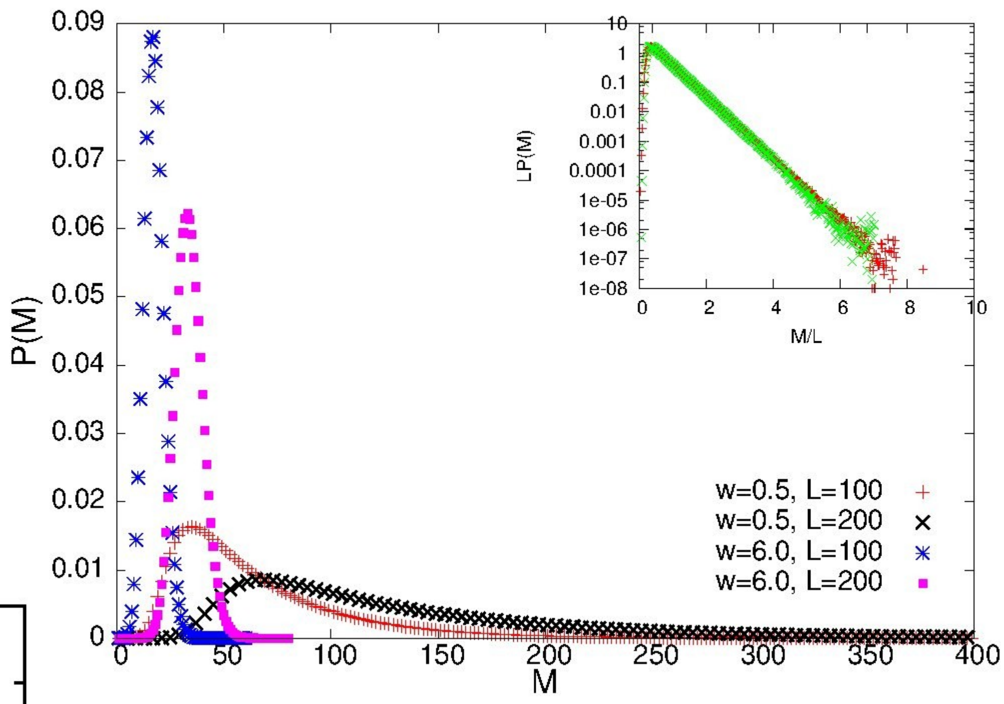
**Result:**

$u_2(t) \sim - A_0\, t \log (A_1\, Dt/L^2)$          $A_0$ , $A_1$ are constants,    $Dt << L^2$

$u_{2n}(t) \sim - L^{2n-2}\, t\; g_{2n} \log (Dt/L^2)$

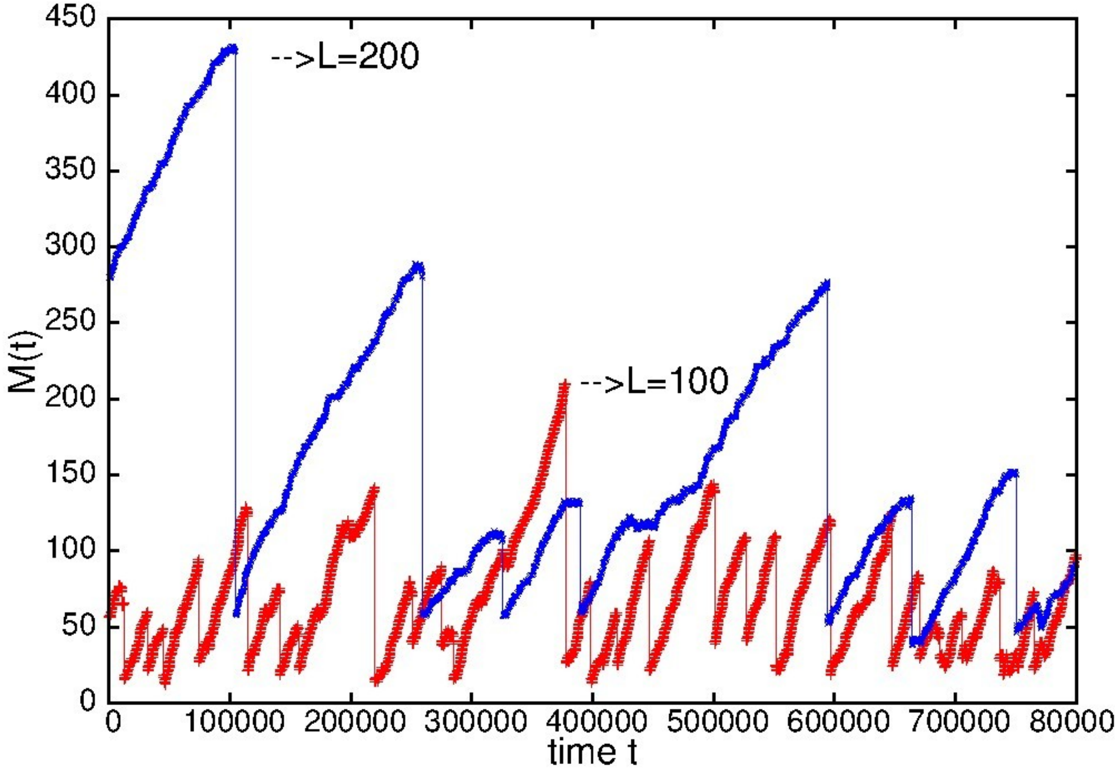
| | **Condensate Phase** ($w < w_c$) | **Normal Phase** ($w > w_c$) | **Critical Point** ($w = w_c$) |
|---|---|---|---|
| **Statics** | $P(M) \rightarrow$ Condensate tail<br><br>Giant Fluctuations:<br><br>$\Delta M \propto L$ | $P(M) \rightarrow$ Gaussian tail<br><br>Normal Fluctuations:<br><br>$\Delta M \propto \sqrt{L}$ | $P(M) \rightarrow$ Non-Gaussian tail<br><br>Large Fluctuations:<br><br>$\Delta M \propto L^{2/3}$ |
| **Dynamics** | $M(t) \rightarrow$ Strongly intermittent<br>Flatness diverges as $t / L^2 \rightarrow 0$ | $M(t) \rightarrow$ Not intermittent<br>No divergence of Flatness. | $M(t) \rightarrow$ Intermittent<br>Flatness diverges at small $t$. |

# Reflecting:  No Exit at Left




**Require:**

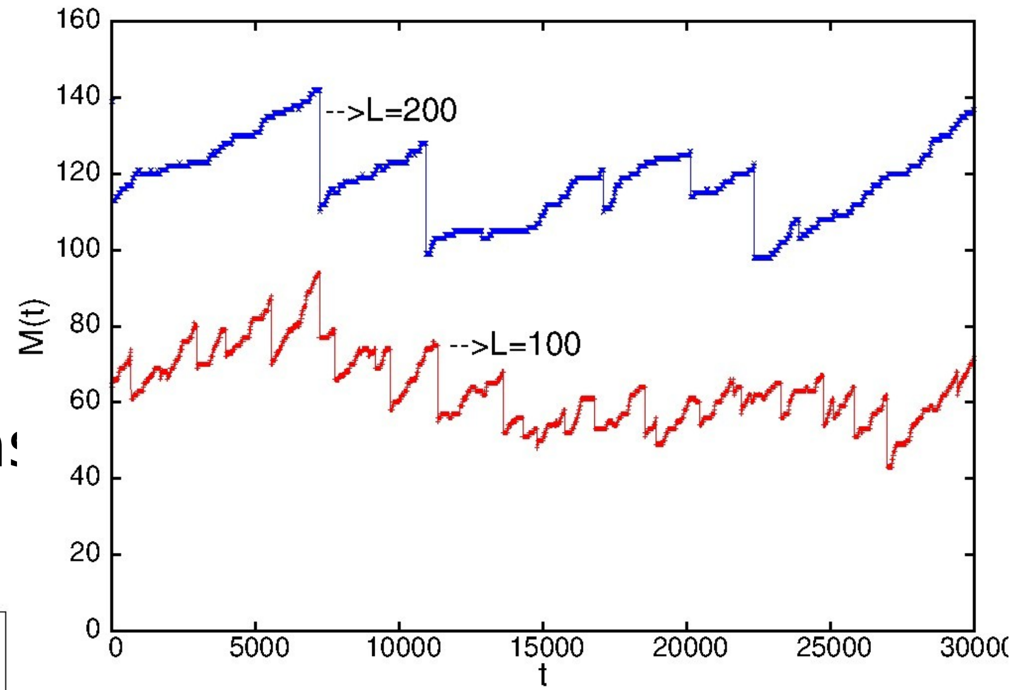
Injection rate a → a/L in order to have <M> of order L




**Find:**

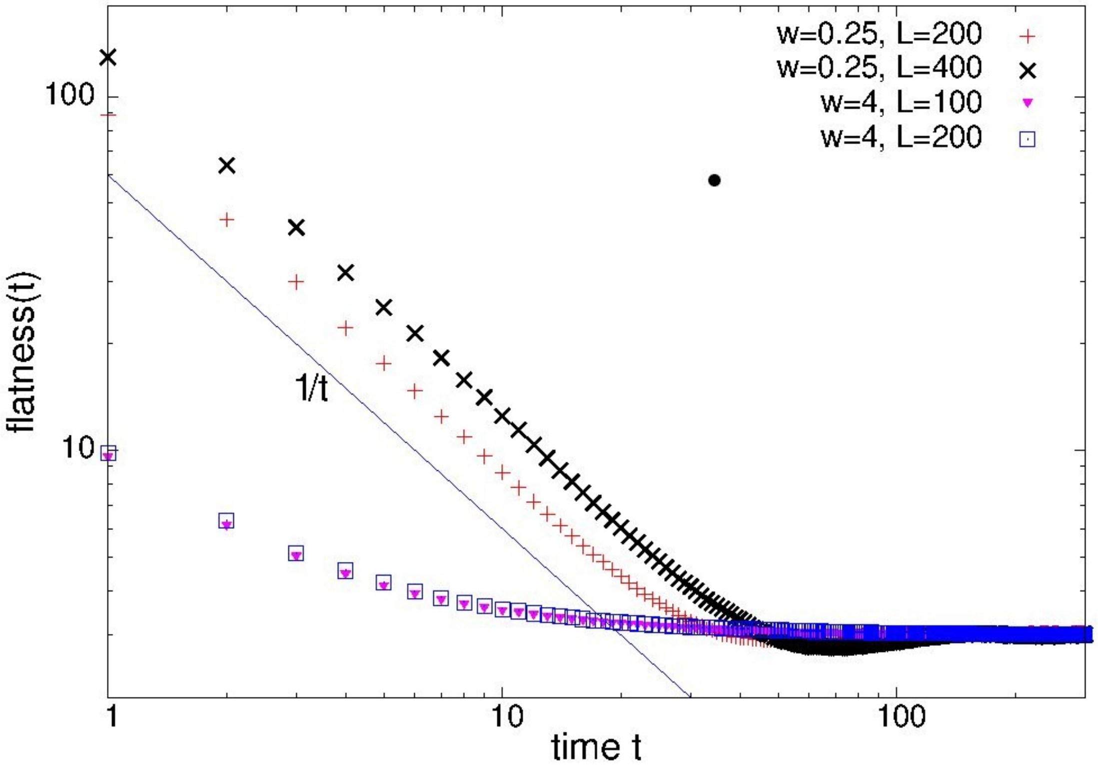
Normal phase for small D + Intermittent aggregate phase at large D

# Directed Stack Hopping




**Find:**

Phase transition from Normal to Intermittent Aggregate Phas




**Difference:**

The aggregate spends less time (O(L)) in the system, hence mass gathered is O(√

# Conclusion

**Condensation phase transition** in open system, with no mass conservation

**Key signature:** Fluctuations

- Giant number fluctuations in the condensate
- Total mass shows temporal intermittency

**Related phase transitions**

- With reflecting boundary conditions
- With directed motion of masses

**Open question**

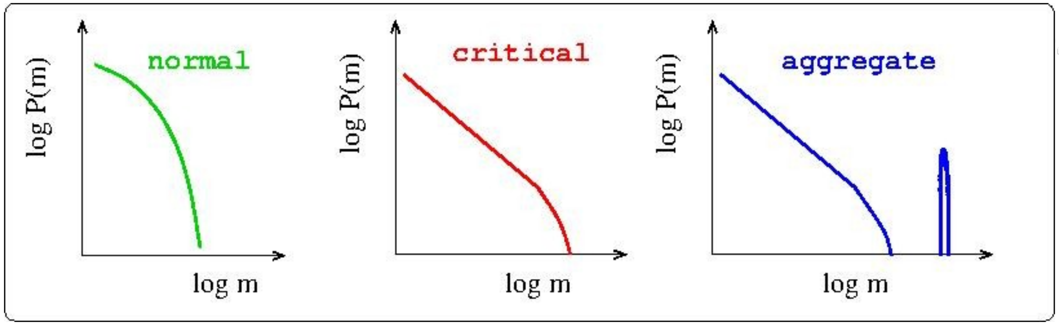
Do other systems which show clustering and giant fluctuations also exhibit temporal intermittency?

**Analysed** by Monte Carlo simulations and by solving for P(m) assuming factorizability: $P(m_1,m_2) = P(m_1)P(m_2)$

$$\frac{\partial P_m}{\partial t} = -(D+w)[1+s]P_m + wP_{m+1} + wsP_{m-1} + D\sum_{n=1}^{m} P_n P_{m-n}$$

where $s = 1 - P_0$ is the probability that the site is occupied

**Find:** Phase transition as the density is increased



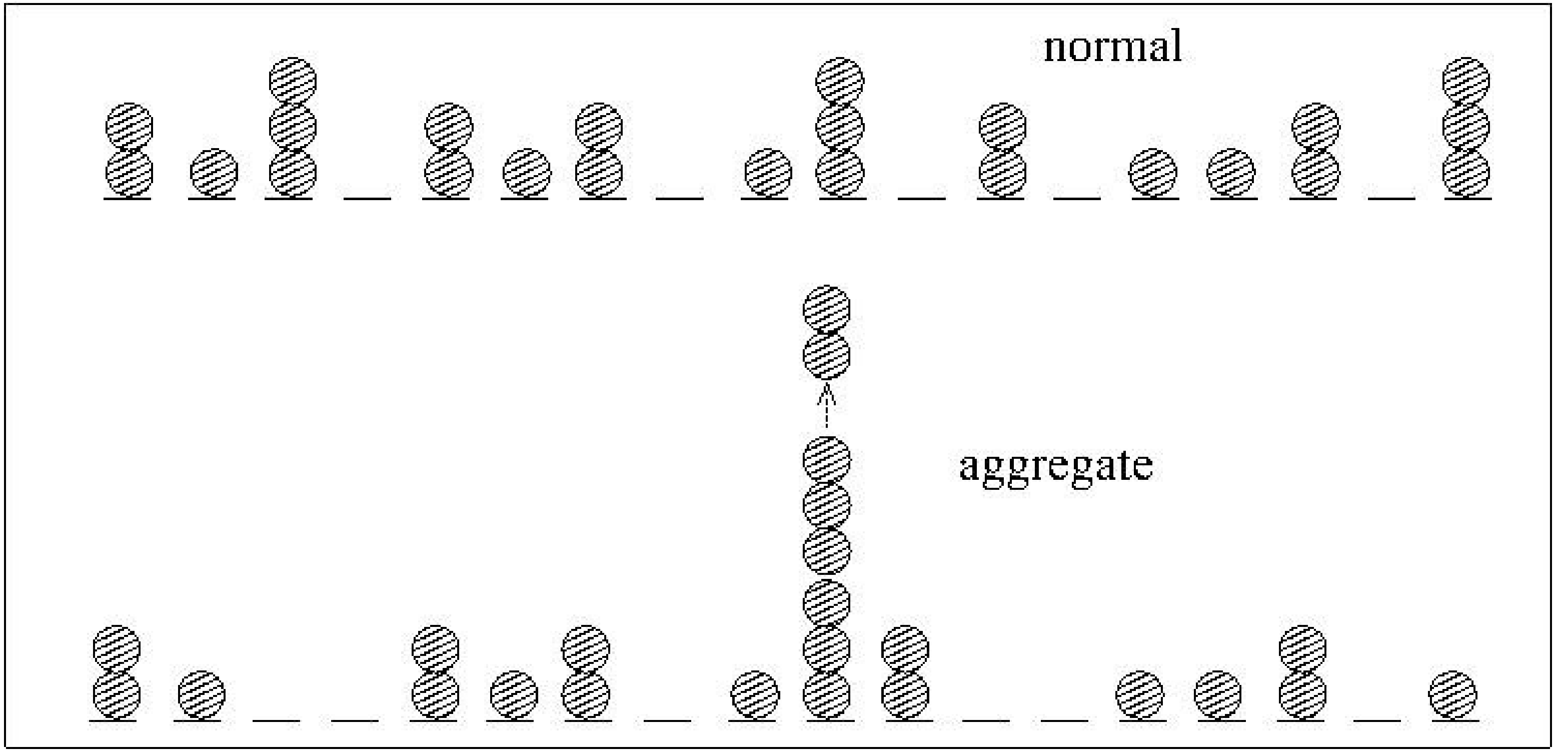

- In the normal phase, $P_m \sim e^{-m/m_0}$
- At the critical point, $P_m \sim m^{-\tau}$ ; $\tau = 5/2$
- Beyond the critical point, $P_m \sim m^{-\tau} + Condensate$

  A single site holds a finite fraction of particles ---
  Bose condensation, but in real space

**[ S. N. Majumdar, S. Krishnamurthy, M. Barma, J Stat Phys (2000) ]**

**Analysed** by Monte Carlo simulations and by solving for P(m) assuming factorizability: $P(m_1,m_2) = P(m_1)\, P(m_2)$

**Find:** In Aggregate phase, a single site holds a finite fraction o
--- akin to Bose condensation, but in real space

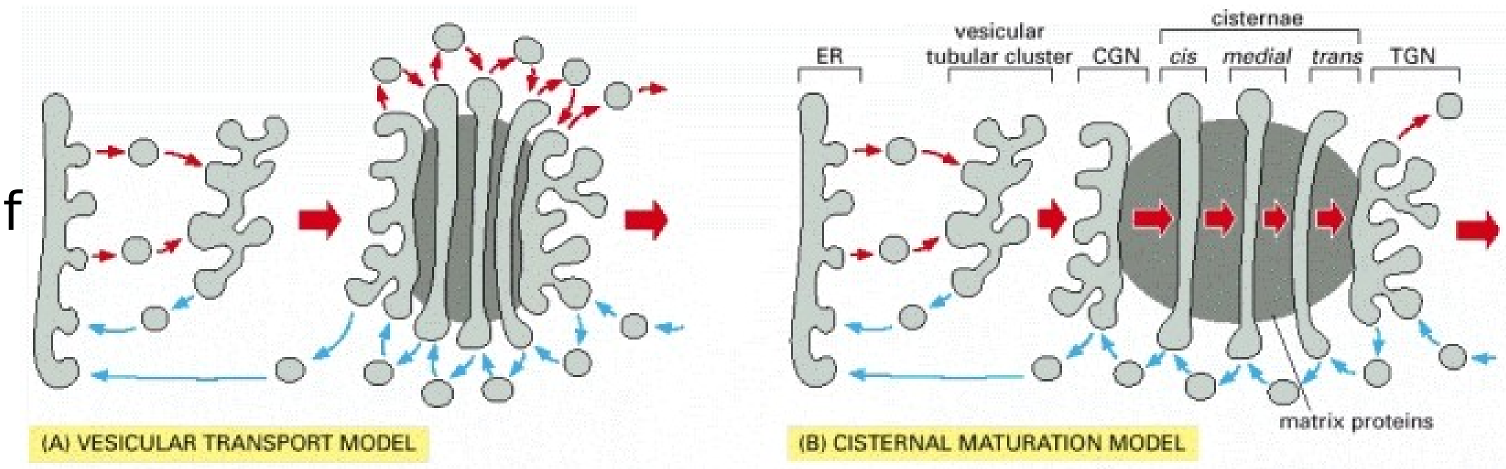



**R. Rajesh and S. Majumdar ; R. Rajesh and S. Krishnamurthy**
Phase boundary found through factorizability is exact [Phys Rev E (2001)]

Given the rules of molecular trafficking,
can one model some aspects of processes within the cell?

(e.g. motion and processing of biomolecules in the Golgi)

Caricature of biological process




( **Molecular Biology of the Cell,** **B Alberts, A Johnson, J Lewis, New York: Garland ; 2002. )**

Statistical Physics model