SUPERSYMMETRIC STANDARD MODEL FROM THE HETEROTIC STRING

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OUTLINE

1. Local grand unification (LGU)

2. Heterotic string on a $\mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold

3. Supersymmetric standard model from LGU

4. $B - L$ symmetry and phenomenology

(1) Local grand unification

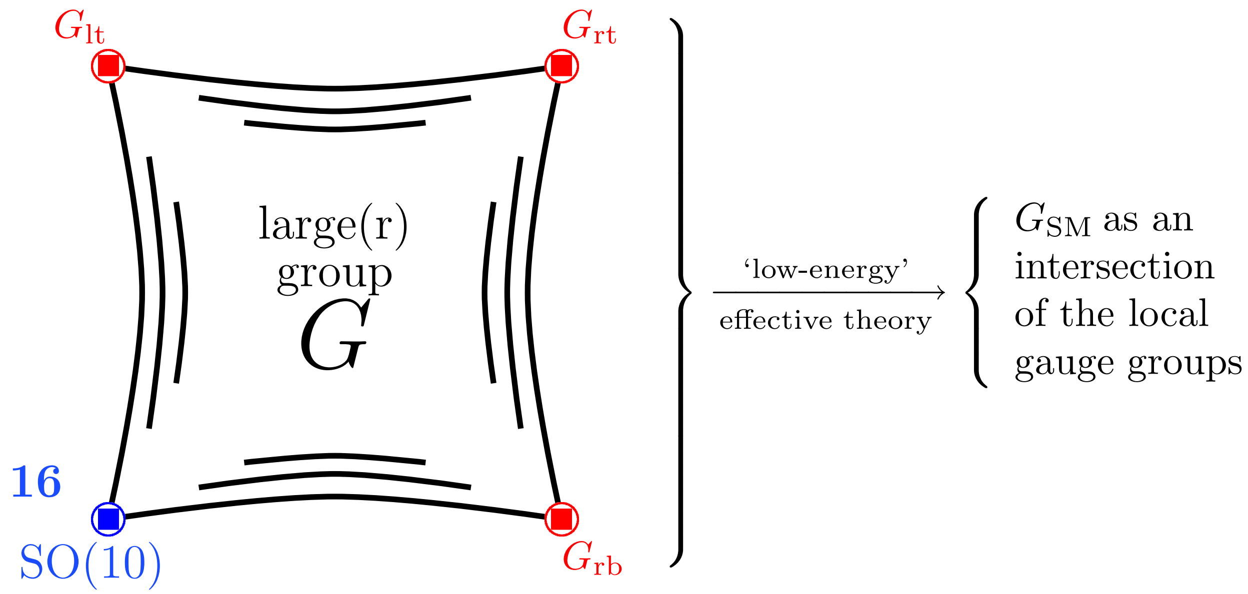
There are strong hints for grand unification: the unification of the gauge couplings, the smallness of neutrino masses (seesaw), and the symmetries and the particle content of the standard model. Quarks and leptons can be grouped into three $\mathrm{SU}(5)$ multiplets,

$$\mathbf{10} = (q_L, u_R^c, e_R^c)\ , \quad \mathbf{5}^* = (d_R^c, l_L)\ , \quad (\mathbf{1} = \nu_R)\ ,$$

which form a single **16**-plet of the GUT group $\mathrm{SO}(10)$,

$$\mathbf{16} = \mathbf{10} + \mathbf{5}^* + \mathbf{1}\ .$$

Puzzle: Higgs fields are $\mathrm{SU}(2)$ doublets, i.e., 'split multiplets'; requires large representations in 4D GUTs, not contained in adjoint of E_8. New possibilities in higher dimensions: orbifold compactifications; orbifold GUTs provide natural explanation of 'doublet-triplet splitting'.



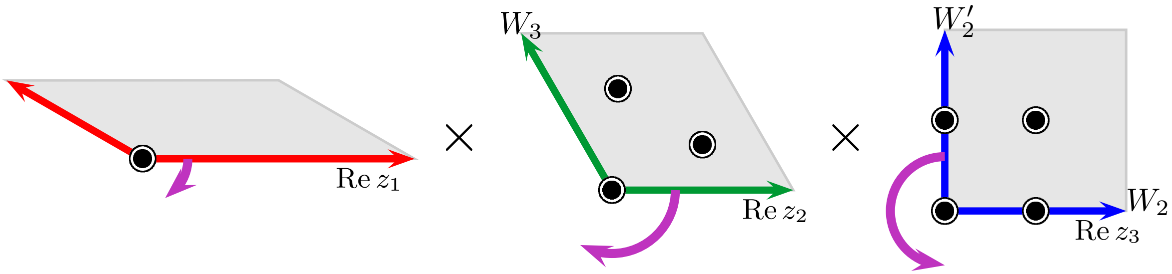
Idea of local grand unification (LGU): GUT group is realized *locally* in higher dimensions; matter fields are localized at orbifold fixed points, GUT representations survive at low energies; Higgs fields are bulk fields and therefore split multiplets $\rightarrow$ successful orbifold GUT models. Can these effective, nonrenormalizable field theories be embedded in string theory ??

('04: Kobayashi, Raby, Zhang; Förste, Nilles, Vaudrevange, Wingerter; WB, Hamaguchi, Lebedev, Ratz)

(2) Heterotic String on a $\mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold

Wanted: Orbifold which has fixed points with $\mathrm{SO}(10)$ symmetry and localized $\mathbf{16}$-plets; systematic classification is available (Katsuki et al. '89); 'minimal' example (Kobayashi, Raby, Zhang '04): $\mathcal{O} = \mathbb{R}^6/\mathbb{S}$, where the space goup $\mathbb{S}$ acts crystallographically on the root lattice Λ of the Lie algebra

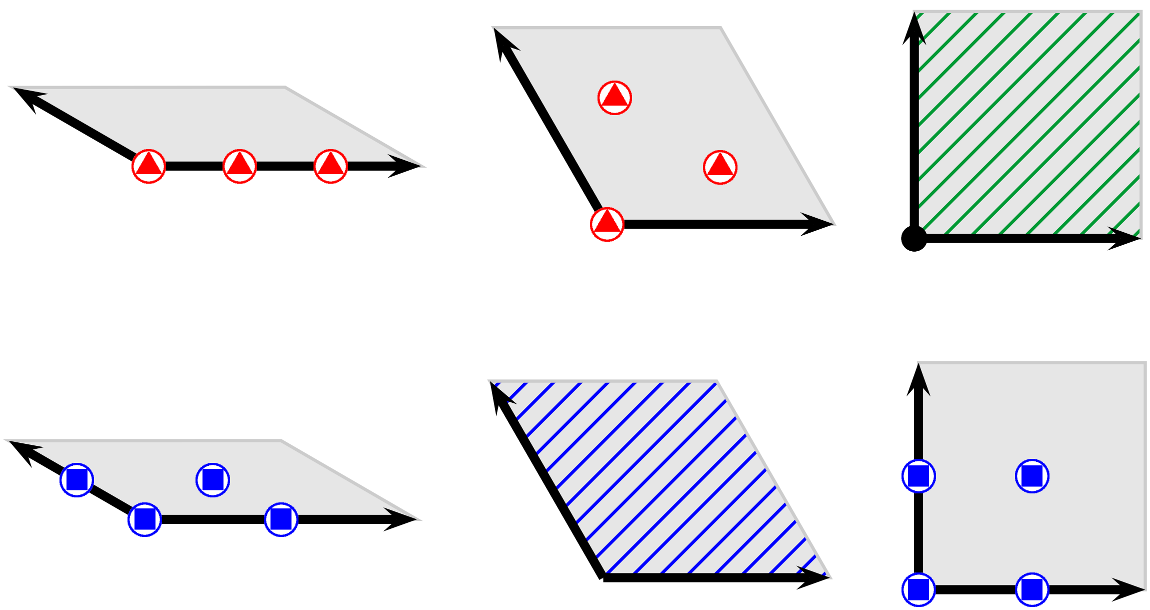
$$\mathrm{G}_2 \times \mathrm{SU}(3) \times \mathrm{SO}(4) \ .$$



Identification of points in the compact dimensions: $z \sim gz$, $g \in \mathbb{S}$.

The orbifold has $\mathbb{Z}_6$ $(k=1)$ fixed points, and $\mathbb{Z}_3$ $(k=2)$ and $\mathbb{Z}_2$ $(k=3)$ fixed points with invariant tori, defined by $(g=(\theta^k,l)\in\mathbb{S})$

$$f=(\theta^k,\ell)f=\theta^k f+2\pi\ell\ ,\quad \ell=m_a e_a\ ,\quad m_a\in\mathbb{Z}\ .$$



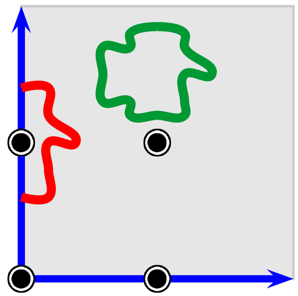
String quantisation: world-sheet fields $\phi(\tau,\sigma)$ (Z^i, Z^{*i}, H^i, X^I) with boundary conditions for twisted sectors

$$\phi(\sigma+2\pi) \;\sim\; g\phi(\sigma)\;, \quad g=\big(\theta^k,l\big)\in \mathbb{S}\;, \quad l=m_a e_a\;.$$

Embedding of space group into gauge group:

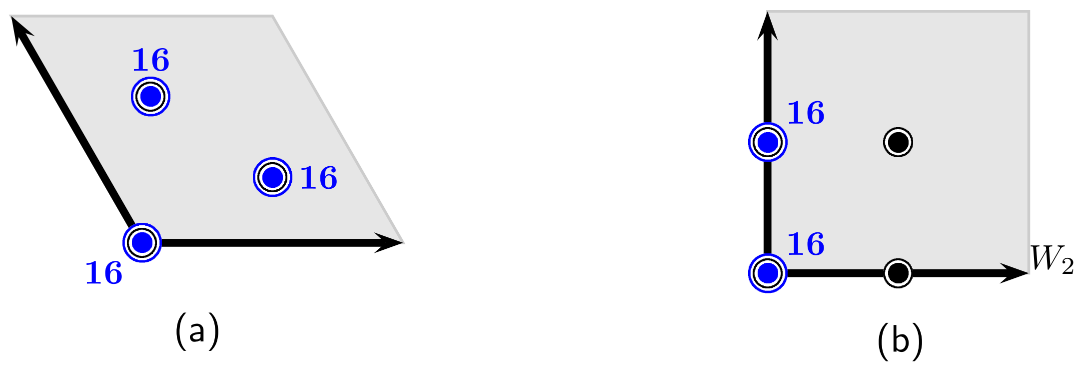
$$(\theta^k, m_a e_a) \;\longrightarrow\; (\mathbb{1}, k\,V_6^I + m_a\,W_{na}^I)\;,$$

with shift vector V_6 and discrete Wilson lines W_n (background gauge fields). Strings localised at different fixed points have different Hilbert spaces.



(3) Supersymmetric standard model from LGU

Search for MSSM based on local GUT idea: local $\mathrm{SO}(10)$ with $\mathbf{16}$–plets at fixed points. For $\mathbb{Z}_6$ orbifolds there are only two such shifts: V_6 (a), V_6' (b).



The shift V_6 (3 'sequential' families !) does not work, chiral exotics! V_6' leads to '2+1' generation models; there are $\mathcal{O}(10^4)$ models with SM gauge group and $\mathcal{O}(10^2)$ models with 3 generations and vectorlike exotic matter; we found only one model for which the exotics can be decoupled !!.

Gauge shift and Wilson lines of the model:

$$V_6 \;=\; \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, 0, 0, 0, 0, 0\right) \left(\frac{1}{3}, 0, 0, 0, 0, 0, 0, 0\right) \;,$$

$$W_2 \;=\; \left(\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0\right) \left(-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right) \;,$$

$$W_3 \;=\; \left(\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0\right) \;.$$

Gauge group and massless matter after compactification:

$$G \;=\; \mathrm{SU}(3) \times \mathrm{SU}(2) \times [\mathrm{SU}(4) \times \mathrm{SU}(2)'] \times \mathrm{U}(1)^9 \;,$$

with 3 $\mathbf{16}$-plets of chiral matter ($+$ vector-like):

$$2 \times \mathbf{16} \in T_1 \;, \quad \mathbf{16} \in U, T_2, T_4 \;.$$

The quantum numbers of the massless states w.r.t. $G_{\rm SM} \times [{\rm SU}(4) \times {\rm SU}(2)]$ (and field labels) are listed in the table; very interesting: further ${\rm U}(1)$ charges, localisation quantum numbers, etc. $\longrightarrow$ long tables in paper; there is a large number of 'extra' states, partly with exotic quantum numbers; can they be decoupled ?

name	irrep	count		name	irrep	count
q_i	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{1/6}$	3		$\bar{u}_i$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-2/3}$	3
$\bar{d}_i$	$(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{1/3}$	7		d_i	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/3}$	4
$\bar{\ell}_i$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{1/2}$	5		ℓ_i	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{-1/2}$	8
m_i	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{0}$	8		$\bar{e}_i$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1}$	3
s_i^-	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/2}$	16		s_i^+	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1/2}$	16
s_i	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0}$	69		h_i	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{0}$	14
f_i	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{0}$	4		$\bar{f}_i$	$(\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1})_{0}$	4
w_i	$(\mathbf{1},\mathbf{1};\mathbf{6},\mathbf{1})_{0}$	5				

Spontaneous gauge symmetry breaking and decoupling

is triggered by an anomalous $\mathrm{U}(1)$ which induces a FI D–term,

$$D_{\mathrm{anom}} \;=\; \sum q_{\mathrm{anom}}^{(i)}\,|\phi_i|^2 + \frac{g\,M_{\mathrm{P}}^2}{192\pi^2}\,\mathrm{tr}\,\mathsf{t}_{\mathrm{anom}}\;, \qquad \mathrm{tr}\,\mathsf{t}_{\mathrm{anom}} \;=\; 88\;.$$

Scalars s_{α_k} attain VEVs, breaking part of the gauge symmetry,

$$G \;\xrightarrow{\mathrm{VEVs}}\; G_{\mathrm{low-energy}}\;.$$

These VEVs can also provide mass terms for some of the matter states,

$$\Delta W = x_i\,\bar{x}_j \times \langle s_{\alpha_1}\ldots s_{\alpha_n}\rangle\;,$$

where x_i and $\bar{x}_j$ are vector-like states w.r.t. $G_{\mathrm{low-energy}}$.

Result: VEVs of SM singlets $s = \{s_i\}$ lead to

$$G \longrightarrow \mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y \times G_\mathrm{hidden} ,$$

with $G_\mathrm{hidden} = \mathrm{SU}(4) \times \mathrm{SU}(2)'$ and a truly hidden sector. Mass matrices of states charged under G_SM:

$$W_\mathrm{mass} = d_i \, \mathcal{M}_d^{ij}(s) \, \bar{d}_j + \bar{\ell}_i \, \mathcal{M}_\ell^{ij}(s) \, \ell_j + m_i \, \mathcal{M}_m^{ij}(s) \, m_j + s_i^+ \, \mathcal{M}_s^{ij}(s) \, s_j^- ,$$

$$\mathcal{M}_d^{ij}(s) = \begin{pmatrix} s^5 & s^5 & s^5 & s^5 & s^5 & s^3 & s^3 \\ s^1 & s^1 & s^3 & s^3 & s^3 & s^3 & s^3 \\ s^1 & s^1 & s^3 & s^3 & s^3 & s^3 & s^3 \\ s^6 & s^6 & s^6 & s^3 & s^3 & s^6 & s^6 \end{pmatrix} ,$$

$$\mathcal{M}_\ell^{ij}(s) \;=\; \begin{pmatrix} s^3 & s^4 & s^4 & s^1 & s^1 & s^1 & s^1 & s^1 \\ s^1 & s^2 & s^2 & s^5 & s^5 & s^3 & s^3 & s^3 \\ s^1 & s^2 & s^2 & s^5 & s^5 & s^3 & s^3 & s^3 \\ s^1 & s^2 & s^2 & s^5 & s^5 & s^6 & s^3 & s^3 \\ s^1 & s^6 & s^6 & s^3 & s^3 & s^6 & s^3 & s^3 \end{pmatrix} ,$$

$\mathcal{M}_m^{ij}(s)$ and $\mathcal{M}_s^{ij}(s)$ $\longrightarrow$ paper; explicit matrix element, e.g.,

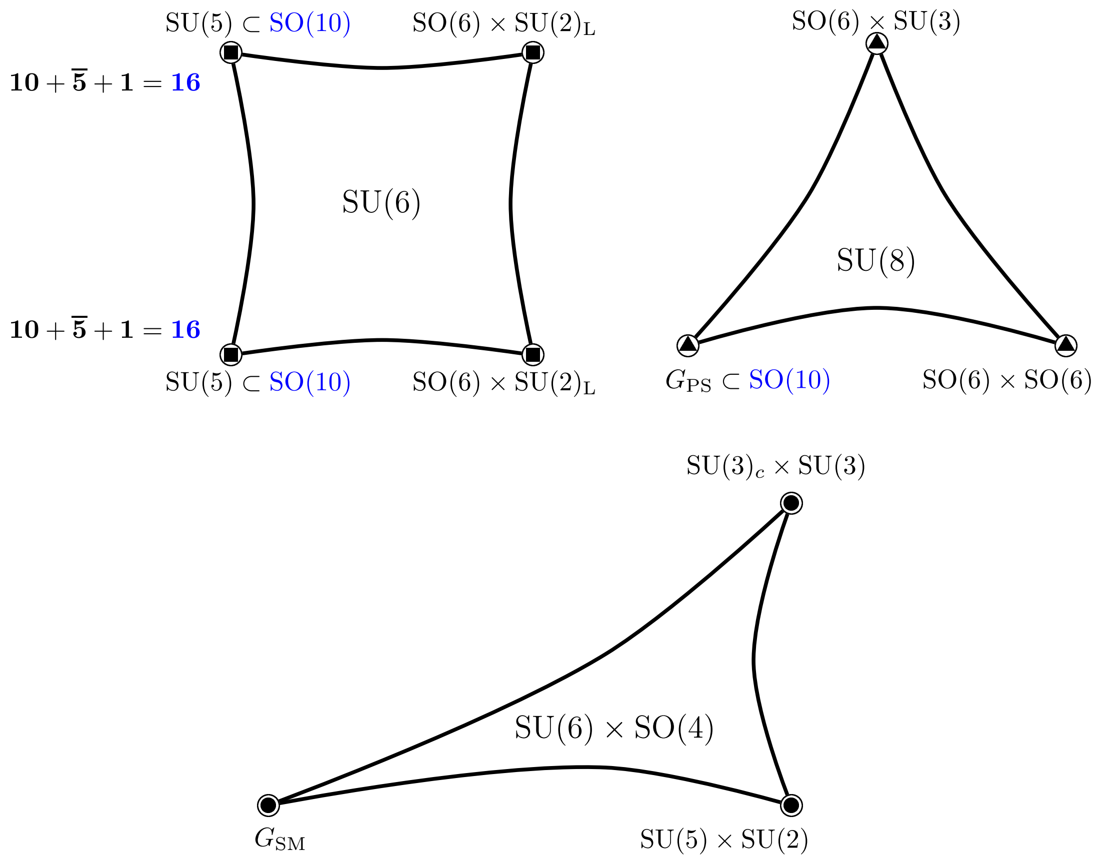
$$W_{d_1\bar{d}_1} \;=\; d_1\bar{d}_1(s_3 s_{20} s_{39} s_{44} s_{65} + s_7 s_{34} s_{35} s_{40} s_{41} + \cdots) \,.$$

Note: $\mathcal{M}_d$ is a 4×7 matrix and $\mathcal{M}_\ell$ is a 5×8 matrix, i.e., 3 $\bar{d}$ and 3 l mix with 4 heavy $\mathrm{SU}(5)$ $\bar{\mathbf{5}}+\mathbf{5}$; in addition, there is one pair of $\mathrm{SU}(2)$ 'Higgs' doublets, which form split multiplets (cf. Asaka, WB, Covi '03).

Local GUT groups and representations

For different k, n_3 and n_2 the groups are in general differently embedded into E_8. Non-Abelian singlets and $\mathrm{U}(1)$ factors are omitted.

k	n_3	$n_2 = 0$	$n_2 = 1$
1	0	$\mathrm{SO}(10) \times \mathrm{SO}(4) \times [\mathrm{SO}(14)]$ $(\mathbf{16}, \mathbf{1}, \mathbf{1}; \mathbf{1}) \oplus 2 \times (\mathbf{1}, \mathbf{2}, \mathbf{1}; \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{2}; \mathbf{1})$	$\mathrm{SO}(8) \times \mathrm{SU}(4) \times [\mathrm{SU}(7)]$ $(\mathbf{1}, \mathbf{4}; \mathbf{1})$
1	1	$\mathrm{SO}(12) \times [\mathrm{SO}(8) \times \mathrm{SU}(4)]$ $(\mathbf{1}; \overline{\mathbf{8}}, \mathbf{1}) \oplus (\mathbf{1}; \mathbf{1}, \overline{\mathbf{4}})$	$\mathrm{SO}(8) \times \mathrm{SU}(4) \times [\mathrm{SU}(7)]$ $(\mathbf{1}, \mathbf{4}; \mathbf{1})$
1	2	$\mathrm{SU}(7) \times [\mathrm{SO}(8) \times \mathrm{SU}(4)]$ $(\mathbf{1}; \mathbf{1}, \overline{\mathbf{4}})$	$\mathrm{SO}(8) \times \mathrm{SU}(4) \times [\mathrm{SO}(10) \times \mathrm{SO}(4)]$ $(\mathbf{1}, \overline{\mathbf{4}}; \mathbf{1}, \mathbf{1}, \mathbf{2})$
2	0	$\mathrm{SO}(14) \times [\mathrm{SO}(14)]$ $(\mathbf{14}; \mathbf{1}) \oplus (\mathbf{1}; \mathbf{14})$	
2	1	$\mathrm{SO}(14) \times [\mathrm{SO}(14)]$ $(\mathbf{14}; \mathbf{1}) \oplus (\mathbf{1}; \mathbf{14})$	
2	2	$\mathrm{SO}(14) \times [\mathrm{SO}(14)]$ $(\mathbf{14}; \mathbf{1}) \oplus (\mathbf{1}; \mathbf{14})$	
3	$0 \ldots 2$	$\mathrm{E}_7 \times \mathrm{SU}(2) \times [\mathrm{SO}(16)]$ $(\mathbf{1}, \mathbf{2}; \mathbf{16})$	$\mathrm{SO}(16) \times [\mathrm{E}_7 \times \mathrm{SU}(2)]$ $(\mathbf{16}; \mathbf{1}, \mathbf{2})$
4	$0 \ldots 2$	$\mathrm{SO}(14) \times [\mathrm{SO}(14)]$	



GUTs appear as intermediate effective field theories if some radii are $\mathcal{O}(1/M_\mathrm{GUT})$, i.e. larger than the string scale; 6D orbifold GUT limits: $\mathrm{SO}(4)$ plane : $\mathrm{SU}(6)$, $N = 2$, $\mathrm{SU}(3)$ plane : $\mathrm{SU}(8)$, $N = 2$, G_2 plane : $\mathrm{SU}(6) \times \mathrm{SO}(4)$, $N = 4$.

(3) $B-L$ symmetry and phenomenology

First step towards realistic phenomenology: vacuum configurations which preserve

$$G_{\mathrm{SM}} \times \mathrm{U}(1)_{B\text{–}L} \times [\mathrm{SU}(4)]\ ,$$

with hidden sector $\mathrm{SU}(4)$ for gaugino condensation; $B-L$ generators is not standard. Possible vacuum configuration (D-flat):

$$\begin{aligned} 0 &= \langle s_3, s_{34}, s_{41}, s_{48}, s_{59}, s_{62}, h_1, h_2, h_6 \rangle = 0\ , \\ 0 &\neq \langle s_1, s_2, s_5, s_7, s_9, s_{12}, s_{14}, s_{16}, s_{18}, s_{19}, s_{20}, s_{22}, s_{23}, s_{24}, s_{39}, s_{40}, \\ &\qquad s_{53}, s_{54}, s_{57}, s_{58}, s_{60}, s_{61}, s_{65}, s_{66}, h_{3-5}, h_{7-14} \rangle \equiv \langle \widetilde{s}_i \rangle\ , \end{aligned}$$

where vanishing F-terms are assumed (unbroken SUSY).

Decoupling becomes more transparent, occurs at order 11, e.g.,

$$\mathcal{M}_\ell^{ij}(\widetilde{s}) \;=\; \begin{pmatrix} \widetilde{s}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \widetilde{s} & 0 & 0 & 0 & 0 & \widetilde{s}^9 & 0 & 0 \\ \widetilde{s} & 0 & 0 & 0 & 0 & \widetilde{s}^9 & 0 & 0 \\ 0 & \widetilde{s}^8 & \widetilde{s}^8 & 0 & 0 & 0 & \widetilde{s}^9 & \widetilde{s}^9 \\ \widetilde{s} & 0 & 0 & \widetilde{s}^9 & \widetilde{s}^9 & 0 & 0 & 0 \end{pmatrix} .$$

Massless up-type Higgs is dominated by $\bar{\ell}_1$,

$$\phi_u \;\simeq\; \bar{\ell}_1 + \sum_{i=2,3} \varepsilon_i \bar{\ell}_i \;, \quad |\varepsilon_i| \ll 1 \;.$$

Consequence: large top quark Yukawa coupling,

$$W = g\; q_1\, \bar{u}_1\, \bar{\ell}_1 \;, \quad (U_1\, U_2\, U_3) \;.$$

Other Yukawa couplings,

$$W_{\text{Yukawa}} = Y_u^{ij}(\widetilde{s})\,\phi_u\,q_i\,\bar{u}_j + Y_d^{ia}(\widetilde{s})\,\phi_d\,q_i\,\bar{d}_a + Y_e^{ib}(\widetilde{s})\,\phi_d\,\bar{e}_i\,\ell_b\ ,$$

are at order $\widetilde{s}^8$, with 4 $\bar{d}d$ and 4 $\bar{l}l$ pairs integrated out,

$$Y_u(\widetilde{s}) = \begin{pmatrix} g & \widetilde{s}^8 & \widetilde{s}^8 \\ 0 & \widetilde{s}^8 & \widetilde{s}^8 \\ 0 & \widetilde{s}^8 & \widetilde{s}^8 \end{pmatrix}\ ,$$

$$Y_d(\widetilde{s}) = \begin{pmatrix} \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^2 & \widetilde{s}^2 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^1 & \widetilde{s}^1 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^1 & \widetilde{s}^1 \end{pmatrix}\ ,\ Y_e(\widetilde{s}) = \begin{pmatrix} \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^8 & \widetilde{s}^8 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^8 & \widetilde{s}^8 \end{pmatrix}\ .$$

Mass hierarchies can be obtained from different powers of $\widetilde{s}$, similar to Frogatt Nielsen mechanism; further work needed...

Summary and Outlook

Compactifications of the heterotic string with local $\mathrm{SO}(10)$ GUT symmetry can reproduce the gauge group and the particle content of the (supersymmetric) standard model; phenomenology appears promising.

Open questions include:

- 'Geometric' understanding of decoupling procedure ('blow-up' moduli)
- Vacuum degeneracy, supersymmetry breaking and moduli stabilisation !!
- Connection with Calabi-Yau/vector bundle compactifications (Braun, He, Ovrut, Pantev '05/'06; Bouchard, Donagi, Cvetic '05/'06; Blumenhagen, Moster, Weigand '06) ?
- Phenomenology: $B-L$ breaking and seesaw mechanism, R-parity, proton decay