HAUNTINGS

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Based on: work with C. Charmousis, R. Gregory, A. Padilla; and work in preparation with D. Kiley.

Overview

- Who cares?
- Chasing ghosts in DGP
  - Codimension-1 case
  - Specteral analysis: diagnostics
  - Shock therapy
- Shocking codimension-2
  - Gravity of photons = electrostatics on cones
  - Gravitational See-Saw
- Summary

The Concert of Cosmos?

- Einstein's GR: a beautiful theoretical framework for gravity and cosmology, consistent with numerous experiments and observations:
  - Solar system tests of GR ***New tests?***
  - Sub-millimeter (non)deviations from Newton's law ***New tests?***
  - Concordance Cosmology! ***Or, Dark Discords?***
- How well do we REALLY know gravity?
  - Hands-on observational tests confirm GR at scales between roughly ***0.1 mm*** and - say - about ***100 MPc;*** are we ***certain*** that GR remains valid at ***shorter*** and ***longer*** distances?

Headaches

- Changing gravity $\rightarrow$ adding new DOFs in the IR
- They can be problematic:
  - Too light and too strongly coupled $\rightarrow$ new long range forces

    Observations place bounds on these!
  - Negative mass squared or negative residue of the pole in the propagator for the new DOFs: ***tachyons*** and/or ***ghosts***

    Instabilities can render the theory nonsensical!

*Caveat emptor: this need not be a theory killer; it means that a naive perturbative description about some background is very bad. Hence one ***must*** develop a meaningful perturbative regime before surveying phenomenological issues and applications.*

DGP Braneworlds

- Brane-induced gravity (Dvali, Gabadadze, Porrati, 2000):
  - Ricci terms BOTH in the bulk and on the end-of-the-world brane, arising from e.g. wave function renormalization of the graviton by brane loops
  - May appear in string theory (Kiritsis, Tetradis, Tomaras, 2001; Corley, Lowe, Ramgoolam, 2001)
  - Related works on exploration of brane-localized radiative corrections (Collins, Holdom, 2000)

Codimension-1

- Action: for the case of codimension-1 brane,

$$S = M_5^3 \int_{\mathcal{M}} d^5x\sqrt{-g}R + 2M_5^3 \int_{\Sigma} d^4x\sqrt{-\gamma}\Delta K + \int_{\Sigma} d^4x\sqrt{-\gamma}(M_4^2\mathcal{R} - \sigma + \mathcal{L}_{matter})$$

- Assume ∞ bulk: 4D gravity has to be mimicked by the exchange of bulk DOFs!
- 5th dimension is concealed by the brane curvature enforcing momentum transfer → $1/p^2$ for $p > 1/r_c$ (DGP, 2000; Dvali, Gabadadze, 2000):

$$G(p)|_{z=0} = \frac{1}{M_4^2p^2 + 2M_5^3p}\left(\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

Strong coupling caveats

- In massive gravity, naïve linear perturbation theory in massive gravity on a flat space breaks down → idea: nonlinearities improve the theory and yield continuous limit (Vainshtein, 1972)?
- There are examples without IvDVZ discontinuity in curved backgrounds (Kogan *et al*; Karch *et al*; Porrati; 2000). (dS with a rock of salt!)
- Key: the scalar graviton is strongly coupled at a scale much bigger than the gravitational radius (a long list of people… sorry, y'all!).
- In DGP a naïve expansion around flat space also breaks down at macroscopic scales (Deffayet, Dvali, Gabadadze, Vainshtein, 2001; Luty, Porrati, Rattazi, 2003; Rubakov, 2003). Including curvature may push it down to about *~ 1 cm* (Rattazi & Nicolis, 2004).
- LPR also claim a ghost in the scalar sector on the self-accelerating branch; after some vacillation, others seem to agree (Koyama^2; Koyama, 2005; Gorbunov, Koyama, Sibiryakov, 2005).

Perturbing cosmological vacua

- Difficulty: equations are hard, perturbative treatments of **both** background and interactions subtle... Can we be more precise?
- An attempt: construct realistic backgrounds; solve

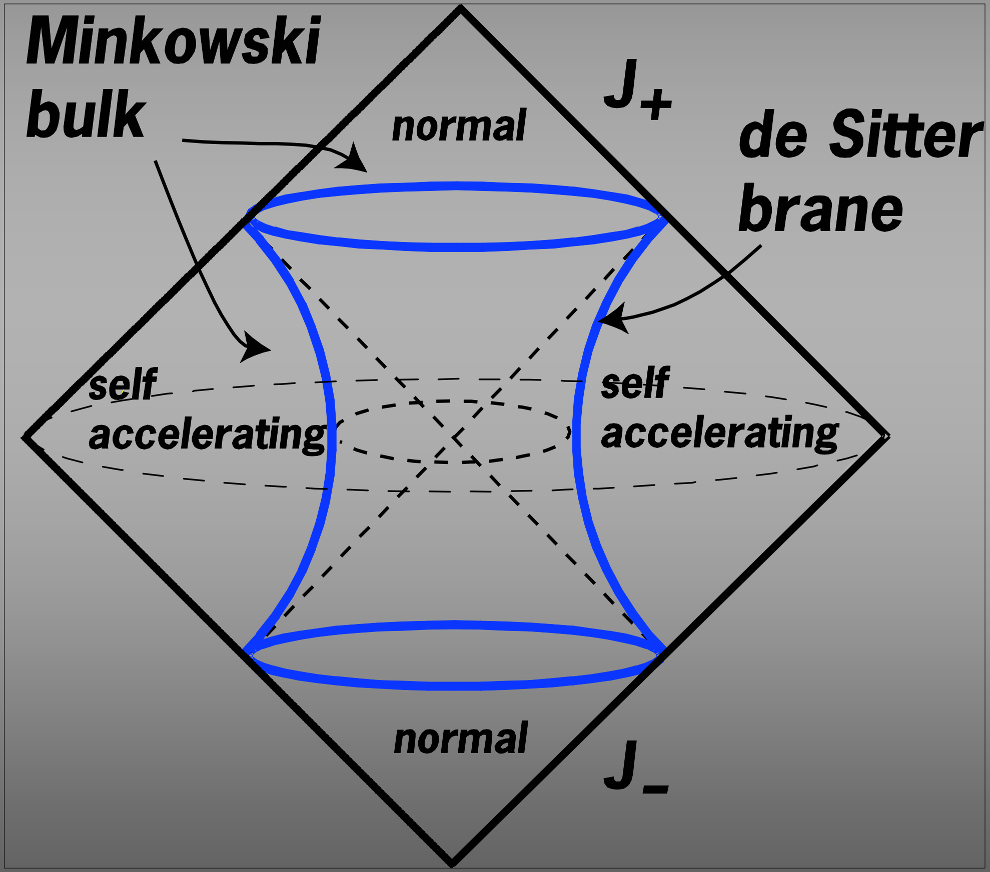
$$M_5^3 G_5{}^A{}_B + M_4^2 G_4{}^\mu{}_\nu \delta^A_\mu \delta^\nu_B \delta(w) = -T^\mu{}_\nu \delta^A_\mu \delta^\nu_B \delta(w)$$

- Look at the vacua first: $T^\mu{}_\nu = -\frac{\sigma}{2}\delta^\mu{}_\nu$
- Symmetries require (see e.g. N.K, A. Linde, 1998):

$$ds_5^2 = (1 - \epsilon H|w|)^2 ds^2_{4dS} + dw^2$$

where 4d metric is de Sitter.

Codimension-1 vacua



Normal and self-inflating branches

- The intrinsic curvature and the tension related by (N.K.; Deffayet,2000)

$$3M_4^2H^2 - 6M_5^3H = \frac{\sigma}{2} \quad \rightarrow \quad H = \frac{M_5^3}{M_4^2}\left[\epsilon + \sqrt{1 + \frac{M_4^2\sigma}{6M_5^6}}\,\right]$$

- $\varepsilon = \pm 1$ an integration constant; $\varepsilon = -1$ normal branch,

$$M_5 \gg M_4 \quad \rightarrow \quad \frac{2M_5^3}{M_4^2}H \simeq \frac{\sigma}{6M_4^2}$$

i.e. this reduces to the usual inflating brane in 5D!

- $\varepsilon = 1$ self-inflating branch, inflates even if tension vanishes!

$$\sigma = 0 \rightarrow H = 2M_5^3/M_4^2$$

Specteroscopy

- Logic: start with the cosmological vacua and perturb the bulk & brane system, allowing for brane matter as well; gravity sector is

$$ds^2 = a^2(y)(\hat{\gamma}_{ab} + a(y)^{-3/2} h_{ab}(x,y)) dx^a dx^b \,, \qquad y = F(x^\mu)$$

$$\text{perturbations} \begin{cases} h_{\mu\nu} \,, & \text{a tower of 4D tensors ;} \\ h_{y\mu} \,, & \text{a tower of 4D vectors ;} \\ h_{yy} \,, & \text{a tower of 4D scalars ;} \\ F \,, & \text{a single 4D scalar.} \end{cases}$$

- But, there are still unbroken gauge invariances of the bulk+brane system! Not all modes are physical.
- The analysis here is *linear* - think of it as a diagnostic tool. But: it reflects problems with perturbations at lengths > Vainshtein scale.

Gauge symmetry I

- Infinitesimal transformations

$$y \to y' = y + \zeta(x,y)\,, \qquad x^\mu \to x'^\mu = x^\mu + \chi^\mu(x,y)$$

- The perturbations change as

$$\begin{aligned} h'_{\mu\nu} &= h_{\mu\nu} - a^{3/2}(D_\mu\chi_\nu + D_\nu\chi_\mu + 2\epsilon H\zeta\bar{\gamma}_{\mu\nu})\,, \\ h'_{y\mu} &= h_{y\mu} - a^{3/2}(D_\mu\zeta + \partial_y\chi_\mu)\,, \\ h'_{yy} &= h_{yy} - 2a^{3/2}(\partial_y\zeta + \epsilon H\zeta)\,, \\ F' &= F + \zeta_{y=0}\,, \end{aligned}$$

- Set e.g. $h_{y\mu}$ and h_{yy} to zero; that leaves us with $h_{\mu\nu}$ and F

Gauge symmetry II

- Decomposition theorem (see CGKP, 2006) :

$$h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + D_\mu A_\nu + D_\nu A_\mu + D_\mu D_\nu \phi - \frac{1}{4}\bar{\gamma}_{\mu\nu} D^2 \phi + \frac{h}{4}\bar{\gamma}_{\mu\nu}$$

where: $h^{\rm TT}_{\mu\nu}$ a TT-tensor, $D_\mu h^{{\rm TT}\ \mu}{}_\nu = h^{{\rm TT}\ \mu}{}_\mu = 0$, with 5 components, A_μ a Lorentz-gauge vector, $D_\mu A^\mu = 0$, with 3 components, and ϕ and $h = h^\mu{}_\mu$ two scalars.

- Not all need be propagating modes!
- To linear order, vectors decouple by gauge symmetry, and the only modes responding to brane matter are TT-tensors and scalars.
- Write down the TT-tensor and scalar Lagrangian.

Gauge symmetry III

- Note: there still remain residual gauge transformations

$$\zeta = \frac{f(x)}{a}, \qquad \chi^\mu = \mathcal{E}^\mu(x) + \frac{1}{2} D^\mu \omega(x) + \frac{1}{\epsilon H a} D^\mu f(x)$$

under which

$$\begin{aligned} h'^{\rm TT}_{\mu\nu} &= h^{\rm TT}_{\mu\nu}\,, \quad A'_\mu = A_\mu - a^{3/2}\mathcal{E}_\mu\,, \\ h' &= h - a^{3/2} D^2\omega - \frac{2a^{1/2}}{\epsilon H} D^2 f - 8\epsilon H a^{1/2} f\,, \\ \phi' &= \phi - a^{3/2}\omega - \frac{2a^{1/2}}{\epsilon H} f\,, \quad F' = F + f\,. \end{aligned}$$

so we can go to a brane-fixed gauge *F′=0* and

$$h_{\mu\nu} = h^{\rm TT}_{\mu\nu} + D_\mu A_\nu + D_\nu A_\mu + \left(\mathcal{O}_{\mu\nu} - \tfrac{1}{4}\mathcal{O}^\lambda{}_\lambda \bar\gamma_{\mu\nu}\right)\phi + \frac{2a^{1/2}}{\epsilon H}\mathcal{O}_{\mu\nu} F + \tfrac{1}{4} h \bar\gamma_{\mu\nu}$$

Forking

- Direct substitution into field equations yields the spectrum; use mode decomposition

$$h_{\mu\nu}(x,y) \sim u_m(y)\chi^{(m)}_{\mu\nu}(x)\,, \qquad (D^2 - 2H^2)\chi^{(m)}_{\mu\nu} = m^2\chi^{(m)}_{\mu\nu}$$

- Get the bulk eigenvalue problem

$$u''_m + [m^2 - \frac{9H^2}{4} + (\frac{M_4^2}{M_5^3}m^2 - 3\epsilon H)\delta(y)]u_m = 0$$

- A constant potential with an attractive δ-function well.
- This is self-adjoint with respect to the norm

$$\langle u|v\rangle = \int_{-\infty}^{\infty} dy\,(M_5^3 + M_4^2\delta(y))u(y)v(y)$$

Brane-localized modes: Tensors

- Gapped continuum:

$$u_m(y) = \alpha_m \sin(\omega_m y + \delta_m)\,, \quad \alpha_m = \sqrt{\frac{m}{\pi M_5^3 \omega_m}}\,, \quad m^2 \geq \frac{9H^2}{4}$$

$$\omega_m = \sqrt{m^2 - \frac{9H^2}{4}}\,, \quad \tan\delta_m = \frac{2M_5^3\omega_m}{3M_5^3\epsilon H - m^2 M_4^2}\,, \langle u_m | u_{\bar m}\rangle = \delta(m - \bar m)$$

- Bound state:

$$u_m(y) = \alpha_m \exp(-\lambda_m y)\,, \qquad \alpha_m = \frac{1}{M_4}\left[\frac{3M_4^2 H - 2M_5^3(1+\epsilon)}{3M_4^2 H - 2M_5^3\epsilon}\right]^{\frac{1}{2}}$$

$$\lambda_m = \sqrt{\frac{9H^2}{4} - m^2}\,, \quad m_d^2 = \frac{M_5^3}{M_4^2}\left[3H - \frac{2M_5^3}{M_4^2}\right](1+\epsilon)\,, \quad \langle u|u\rangle = 1$$

Bound state specifics

- On the normal branch, $\varepsilon=-1$, the bound state is massless! This is the normalizable graviton zero mode, arising because the bulk volume ends on a horizon, a finite distance away. It has additional residual gauge invariances, and so only 2 propagating modes, with matter couplings $g \sim H$. It decouples on a flat brane.
- On the self-accelerating branch, $\varepsilon=1$, the bound state mass is not zero! Instead, it has Pauli-Fierz mass term and 5 components,

$$m_d^2 = \frac{2M_5^3}{M_4^2}\left[3H - \frac{2M_5^3}{M_4^2}\right] \to \begin{cases} 0 < m_d^2 < 2H^2\,, & \text{for} \quad \sigma > 0; \\ m_d^2 > 2H^2\,, & \text{for} \quad \sigma < 0. \end{cases}$$

- Perturbative *ghost*: $m^2<2H^2$, helicity-0 component has negative kinetic term (Deser, Nepomechie, 1983; Higuchi, 1987; I. Bengtsson, 1994; Deser, Waldron 2001).

Brane-localized modes: Scalars

- Single mode, with $m^2 = 2H^2$, obeying

$$h^{(\phi)}_{\mu\nu}(x,y) = W(y)\mathcal{O}_{\mu\nu}\hat{\phi}(x)\,, \quad \left(D^2 - 2H^2\right)h^{(\phi)}_{\mu\nu} = 2H^2 h^{(\phi)}_{\mu\nu}\,, \; W''(y) - \frac{H^2}{4}W(y) = 0$$

with the brane boundary condition

$$\left(W'(0) - (\frac{3}{2}\epsilon H - \frac{M_4^2 H^2}{M_5^3})W(0)\right)\hat{\phi} = 2(1 - \epsilon\frac{HM_4^2}{M_5^3})F$$

- Subtlety: interplay between normalizability, brane dynamics and gauge invariance. On the normal branch, the normalizable scalar can always be gauged away by residual gauge transformations; not so on the self-accelerating branch. There one combination survives:

$$h^{(\phi)}_{\mu\nu} = -\frac{2}{H}\left[\frac{M_5^3 - M_4^2 H}{2M_5^3 - M_4^2 H}\right] e^{-Hy/2}\mathcal{O}_{\mu\nu}F$$

Full perturbative solution

- Full perturbative solution of the problem is

$$h_{\mu\nu}(x,y) = \alpha_{m_d} e^{-\lambda_{m_d} y} \chi^{(m_d)}_{\mu\nu}(x) + \int_{\frac{3H}{2}}^{\infty} dm \; u_m(y) \chi^{(m)}_{\mu\nu}(x)$$

$$+\frac{(1+\epsilon)}{H}\left\{ a^{1/2} \mathcal{O}_{\mu\nu} F - \left[\frac{M_5^3 - M_4^2 H}{2M_5^3 - M_4^2 H}\right] a^{-1/2} \mathcal{O}_{\mu\nu} F \right\}$$

- On the normal branch, this solution has no scalar contribution, and the bound state tensor is a zero mode. Hence there are no ghosts.
- On the self-accelerating branch, the bound state is massive, and when $\sigma>0$ its helicity-0 mode is a ghost; for $\sigma<0$, the surviving scalar is a ghost (its kinetic term is proportional to σ).
- Zero tension is tricky.

Zeroing in

- Zero tension corresponds to $m^2 = 2H^2$ on SA branch. The lightest tensor and the scalar become completely degenerate. In Pauli-Fierz theory, there is an accidental symmetry (Deser, Nepomechie, 1983)

$$\chi_{\mu\nu}^{(\sqrt{2}H)} \to \chi_{\mu\nu}^{(\sqrt{2}H)} + \mathcal{O}_{\mu\nu}\vartheta\,, \qquad (D^2 + 4H^2)\vartheta = 0$$

so that helicity-0 is pure gauge, and so it decouples – ghost gone!

- With brane present, this symmetry is spontaneously broken! The brane Goldstone mode becomes the Stuckelberg-like field, and as long as we demand normalizability the symmetry lifts to

$$h'^{\rm TT}_{\mu\nu} = h^{\rm TT}_{\mu\nu} + a^{-1/2}\mathcal{O}_{\mu\nu}\vartheta\,, \qquad \phi' = \phi - a^{-1/2}\vartheta\,.$$

- We can't gauge away both helicity-0 and the scalar; the one which remains is a ghost (see also Dubovsky, Koyama, Sibiryakov, 2005).

(d)Effective action

- This analysis is borne out by the direct calculation of the quadratic effective action for the localized modes:

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}h^{(m_d)\mu\nu}X^{(m_d)}_{\mu\nu} + \frac{1}{2}u_{m_d}(0)h^{(m_d)\mu\nu}\tau_{\mu\nu} + \int_{\frac{3H}{2}}^{\infty} dm \left[-\frac{1}{2}\, h^{(m)\mu\nu}X^{(m)}_{\mu\nu} + \frac{1}{2}u_m(0)h^{(m)\mu\nu}\tau_{\mu\nu}\right] - \frac{3(1+\epsilon)}{2}M_5^3 H^2\,\frac{\hat{\phi}}{\alpha}(D^2+4H^2)\hat{\phi}\,,$$

- where $\alpha = -4\dfrac{1 - M_4^2\epsilon H/M_5^3}{(1+3\epsilon)H - 2M_4^2H^2/M_5^3}$ and

$$X^{(m)}_{\mu\nu} = -\frac{1}{2}\left(D^2 - 2H^2\right)h_{\mu\nu} + D_{(\mu}D^{\alpha}h_{\nu)\alpha} - \frac{1}{2}D_\mu D_\nu h - \frac{1}{2}\bar{\gamma}_{\mu\nu}\left[D^\alpha D^\beta h_{\alpha\beta} - \left(D^2+H^2\right)h\right] + \frac{1}{2}m^2\left(h^{(m)}_{\mu\nu} - h^{(m)}\bar{\gamma}_{\mu\nu}\right)$$

(d)Effective action II

- By focusing on the helicity zero mode, we can check that in the unitary gauge (see Deser, Waldron, 2001; CGKP, 2006) its Hamiltonian is

$$\mathcal{H} = \frac{m^2\nu^2}{6H^2}\pi^2 + \frac{3H^2}{2m^2\nu^2}\varphi\left(m^2 - \frac{9H^2}{4} - e^{-2Ht}\Delta\right)\varphi$$

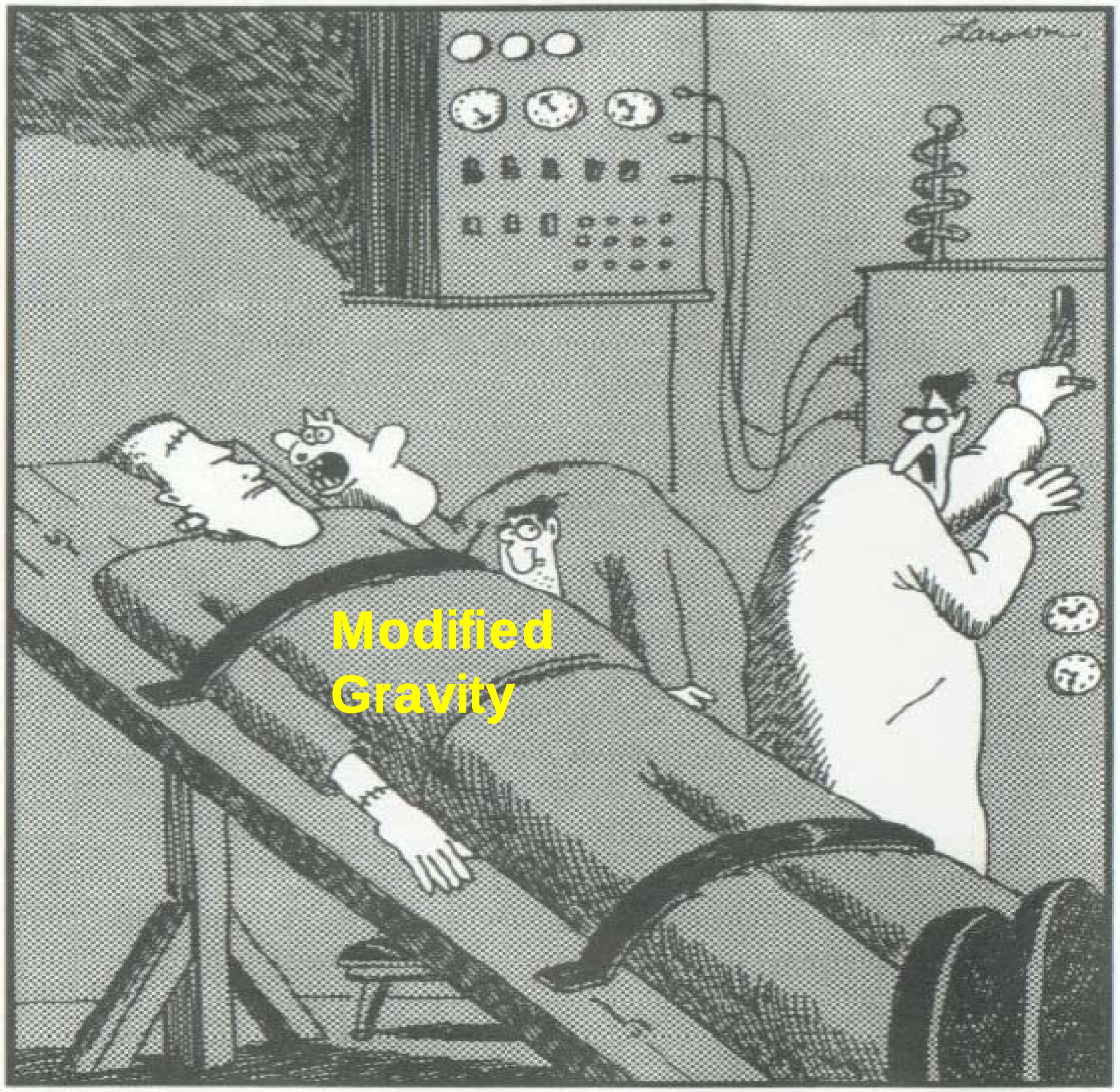
where $\nu^2 = m^2 - 2H^2$, and therefore this mode is a ghost when $m^2 < 2H^2$; by mixing with the brane bending it does not decouple even when $m^2 = 2H^2$.

- In the action, the surviving combination is

$$h_{\mu\nu}(x,y) \propto e^{-\frac{H}{2}y}(\mathcal{H}_{\mu\nu}(x) - y\,\mathcal{O}_{\mu\nu}F) \qquad (D^2 - 4H^2)\mathcal{H}_{\mu\nu} = -H\,\mathcal{O}_{\mu\nu}F$$

Shocking nonlocalities

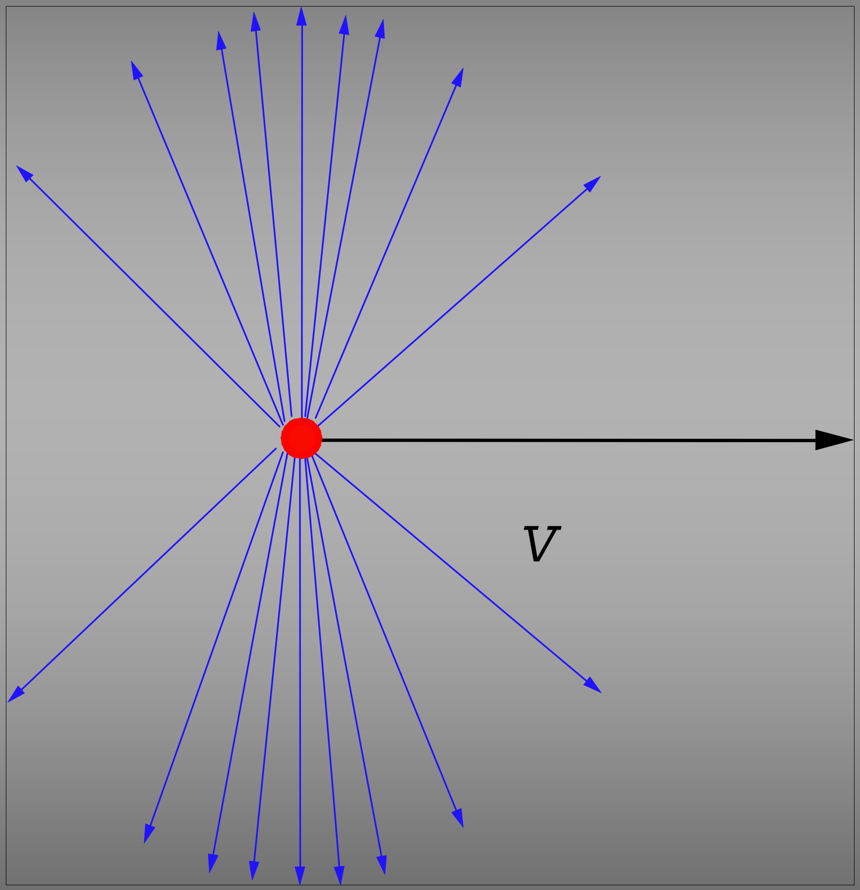
- What does this ghost imply? In the Lagrangian in the bulk, there is no explicit negative norm states; the ghost comes about from brane boundary conditions - brane does not want to stay put.
- Can it move and/or interact with the bulk and eliminate the ghost?
- In shock wave analysis (NK, 2005) one finds a singularity in the gravitational wave field of a massless brane particle in the localized solution. It can be smoothed out with a non-integrable mode.
- But: this mode *GROWS* far from the brane – it lives at asymptotic infinity, and is sensitive to the boundary conditions there.
- Can we say anything about what goes on there? (Gabadadze,...)



“Igor! Get that Wolfman doll out of his face! …
Boy, sometimes you really are bizarre.”

Trick: shock waves

- Physically: because of the Lorentz contraction in the direction of motion, the field lines get pushed towards the instantaneous plane which is orthogonal to *V*.
- The field lines of a massless charge are confined to this plane! (P.G Bergmann, 1940's)
- The same intuition works for the gravitational field. (Pirani; Penrose; Dray, 't Hooft; Ferrari, Pendenza, Veneziano; Sfetos;…)



DGP in a state of shock

- The starting point for 'shocked' DGP is (NK, 2005)

$$ds_5^2 = e^{-2\epsilon H|z|}\{\frac{4dudv}{(1+H^2uv)^2} - \frac{4\delta(u) f du^2}{(1+H^2uv)^2} + + (\frac{1-H^2uv}{1+H^2uv})^2 \frac{d\Omega_2}{H^2} + dz^2\}$$

- Term $\sim f$ is the discontinuity in dv . Substitute this metric in the DGP field equations, where the new brane stress energy tensor includes photon momentum

$$T^\mu{}_\nu = -\lambda\delta^\mu{}_\nu + 2\frac{p}{\sqrt{g_5}} g_{4\,uv}\delta(\theta)\delta(\phi)\delta(u)\delta^\mu_v\delta^u_\nu$$

- Turn the crank!

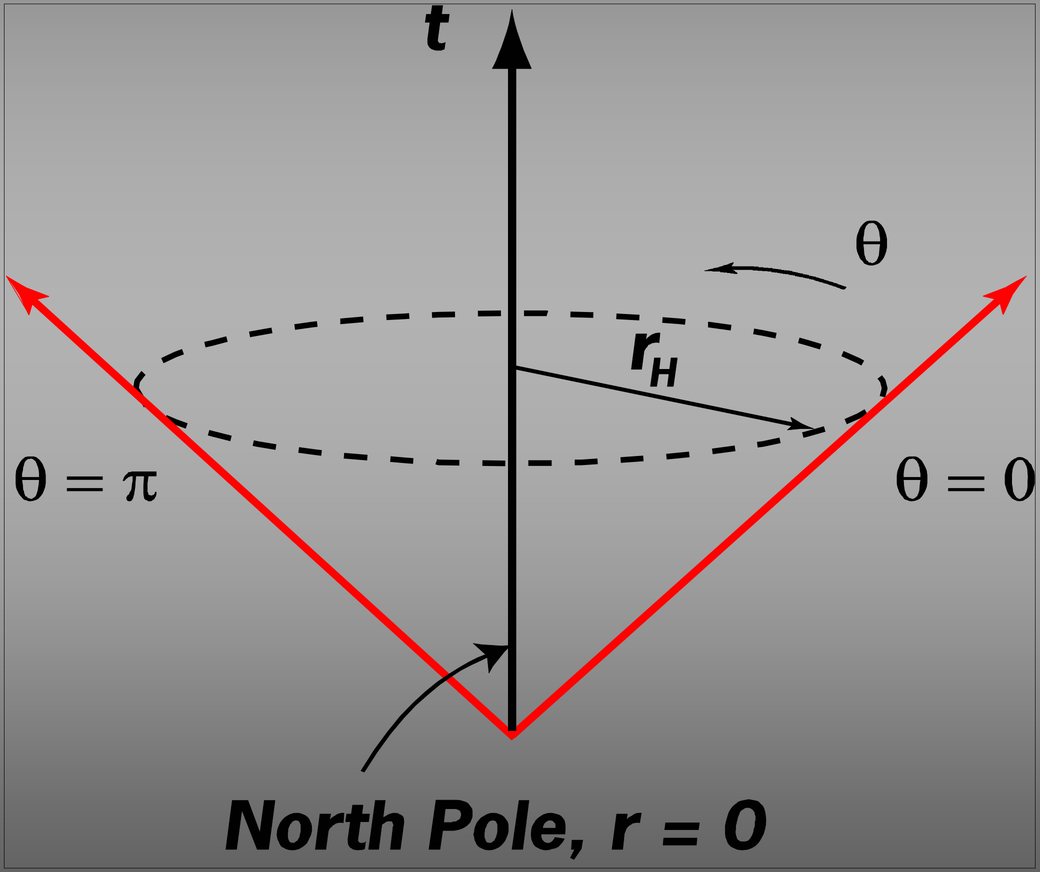
Chasing shocks

- Best to work with two 'antipodal' photons, that zip along the past horizon (ie boundary of future light cone) in opposite directions. This avoids problems with spurious singularities on compact spaces. It is also the correct infinite boost limit of Schwarzschild-dS solution in 4D (Hotta, Tanaka, 1993) . The field equation is (NK, 2005)

$$\frac{M_5^3}{M_4^2 H^2}(\partial_z^2 f - 3\epsilon H \partial_{|z|} f + H^2(\Delta_2 f + 2f)) +$$

$$+(\Delta_2 f + 2f)\delta(z) = \frac{2p}{M_4^2}(\delta(\Omega) + \delta(\Omega'))\delta(z)$$

“Antipodal” photons in the static patch on de Sitter brane



Shocking solutions I

- Thanks to the symmetries of the problem, we can solve the equations by mode expansion:

$$\Phi = 2\sum_{l=0}^{\infty} \Phi_{2l}(y) P_{2l}(\cos\theta)$$

where the radial wavefunctions are

$$\Phi_{2l} = A_{2l} e^{-(2l+\frac{3\epsilon+1}{2})H|y|} + B_{2l} e^{(2l-\frac{3\epsilon-1}{2})\,H|y|}$$

- Here A_{2l} is normalizable: it describes gravitons localized on the brane. The mode B_{2l} is not normalizable. Its amplitude diverges at infinity. This mode lives far from the brane, and is sensitive to boundary conditions *there*.

Shocking solutions II

- Defining $g = 2M_5^3/(M_4^2 H) = 1/(H r_c)$, using the spherical harmonic addition theorem,

$$\frac{2n+1}{4\pi} P_n(\cos\theta) = \sum_{m=-n}^{n} Y^*_{nm}(0,0)\, Y_{nm}(\theta,\phi)$$

and changing normalization to $B_{2l} = -\frac{p}{4\pi M_4^2}\frac{4l+1}{(2l-1-\frac{1-\epsilon}{2}g)(l+1-\frac{1+\epsilon}{4}g)}\alpha_{2l}$ we can finally write the solution down as:

$$\Phi = -\frac{2p}{M_4^2}\sum_{l=0}^{\infty}\sum_{m=-2l}^{2l}\left(\frac{1-\alpha_{2l}}{(2l-1+g)(l+1)} + \frac{\alpha_{2l}}{(2l-1)(l+1-\frac{g}{2})}\right) Y^*_{2l\,m}(0,0)\, Y_{2l\,m}(\theta,\phi)$$

- The parameter α_{2l} controls the contribution from the nonintegrable modes. This is like choosing the vacuum of a QFT in curved space.
- At short distances: the solution is well approximated by the Aichelburg-Sexl 4D shockwave - so the theory *does* look 4D!
- But at large distances, one finds that low-l (large wavelength) are *repulsive* - they resemble ghosts, from 4D point of view.

More on shocks…

- For integer *g* there are poles similar to the pole encountered on the SA branch in the tensionless limit $g=1$ for the lightest brane mode.
- This suggests that the general problem has more resonances, once the door is opened to non-integrable modes.
- Once a single non-integrable mode is allowed, how is one to stop all of them from coming in, without breaking bulk general covariance?

In contrast, normal branch solutions are completely well behaved. One can use them as a benchmark for looking for cosmological signatures of modified gravity. Once a small cosmological term is put in by hand,

  - it simulates $w<-1$ (Sahni, Shtanov, 2002; Lue, Starkman, 2004)
  - it changes cosmological structure formation

Codimension-2 DGP

- Higher codimension models are different. A lump of energy of codimension greater than unity gravitates. This lends to gravitational short distance singularities which must be regulated.
- The DGP gravitational filter may still work, confining gravity to the defect. However the crossover from 4D to higher-D depends on the short distance cutoff. (Dvali, Gabadadze, Hou, Sefusatti, 2001)
- There were concerns about ghosts, and/or nonlocal effects.
- We find a very precise and simple description of the cod-2 case. The shocks show both the short distance singularities and see-saw of the cross-over scale by the UV cutoff. (NK, D. Kiley, in preparation)
- We suspect: no ghosts (very preliminary - no proof yet, but...)! There are light gravitationally coupled modes so that the theory is Brans-Dicke. Can the BD field be stabilized?

Shocking codimension-2

- Background equations:

$$M_6^4 G^A_{6B} + M_4^2 G^\mu_{4\nu} \delta^A_\mu \delta^\nu_B \frac{\delta^2(\vec{y})}{\sqrt{\det h}} = T^\mu_\nu \delta^A_\mu \delta^\nu_B \frac{\delta^2(\vec{y})}{\sqrt{\det h}}$$

- Select 4D Minkowski vacuum x 2D cone:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + (1-b)^2 \rho^2 \, d\phi^2 \qquad b = \frac{\lambda}{2\pi M_6^4}$$

- *b* measures deficit angle: far from the core, $g_{\phi\phi} \sim B^2 \rho^2 d\phi^2$, where

$$B = 1 - \frac{\lambda}{2\pi M_6^4}$$

- Thus: the tension (*a.k.a. brane-localized vacuum energy*) dumped into the bulk (e.g. just like in Sundrum, 1998, or in self-tuning)
- But to have static solution, one *MUST* have $B>0$! Thus, arguably, one needs $M_6 \geq TeV$, and $M_4 \sim 10^{19}\ GeV$; how is $r_c \sim H_0^{-1}$ generated?
- M_4/M_6^2 only a millimeter… $4D \rightarrow 6D$ at a millimeter?… No! One has gravitational see-saw! (DGHS, 2001)

Unresolved cone

- Put a photon on the brane:

$$ds_6^2 = 4dudv - 4\delta(u) f du^2 + d\vec{x}_\perp^2 + d\rho^2 + d\rho^2 + (1-b)^2 \rho^2 \, d\phi^2$$

- Field equation, using $l = M_4/M_6^2$:

$$\nabla_4^2 f + l^2 \nabla_2^2 f \frac{1}{2\pi(1-b)\rho} \delta(\rho) = \frac{2p}{2\pi(1-b) M_6^4 \rho} \delta^2(\vec{x}_\perp)\delta(\rho)$$

- "Solution":

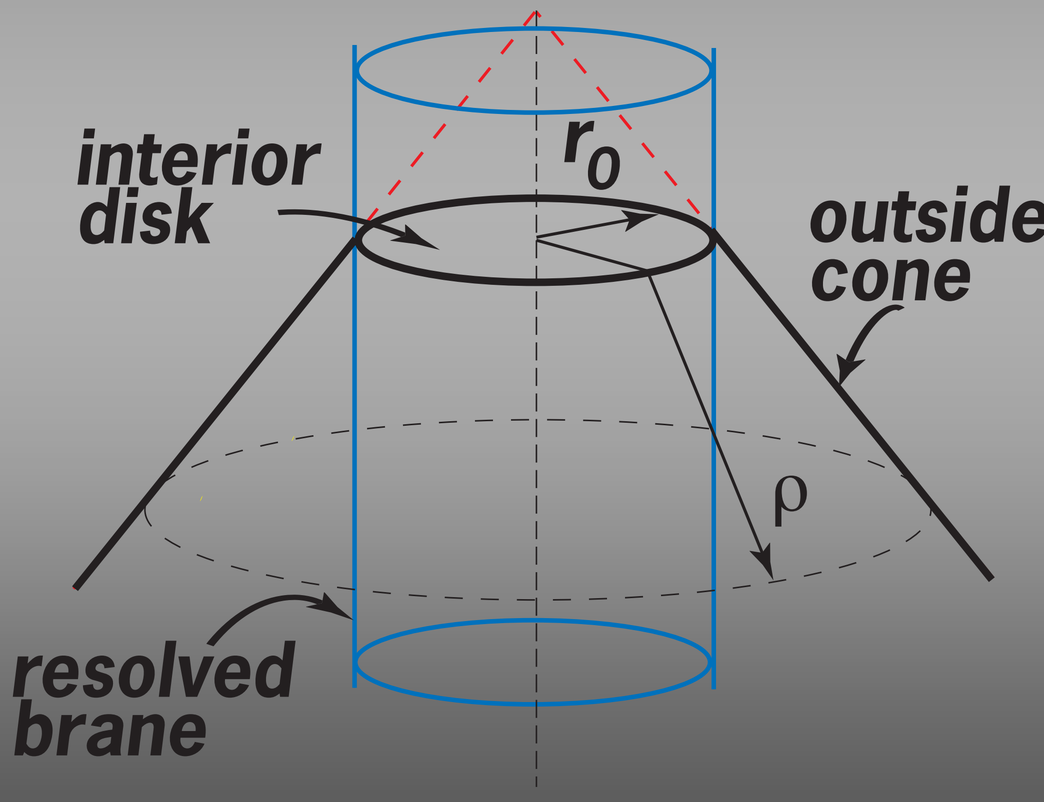
$$f(r,\rho) = -\frac{p}{2\pi^2(1-b)M_6^4} \int_0^\infty dk \, k \frac{I_0(0)K_0(k\rho)J_0(k|\vec{x}_\perp|)}{1 + \frac{l^2k^2}{2\pi(1-b)} I_0(0)K_0(0)}$$

where r is the longitudinal and ρ transverse distance. Now both I and K are divergent at small argument; but on the brane $(\rho=0)$ divergences cancel, and for $r < l/(1-b)$ (can be large!) one finds the leading behavior of 4D Aichelburg-Sexl shockwave!

- But for any $\rho \neq 0$ the divergence in the denominator fixes $f=0$ - very singular!
- Begs to be regulated!

Resolving the cone

An example of an ill-defined exterior boundary value problem in electrostatics! Resolution: replace the point charge with a ring source and solve by imposing regular boundary conditions in and out! This can be done by taking a 4-brane with a massless scalar and wrapping it on a circle of a fixed radius r_0.



Shocking resolved cone

- Put a photon (a massless loop) on the brane:

$$ds_6^2 = 4dudv - 4\delta(u) f du^2 + d\vec{x}_\perp^2 + d\rho^2 + d\rho^2 + \Big[(1 - b\,\theta(\rho - r_0))\rho + b\,r_0\theta(\rho - r_0)\Big]^2 d\phi^2$$

- Field equation, using $\ell = M_4/M_6^2$ and $R=\rho+br_0/(1-b)$, with r_0 brane radius:

$$\nabla_4^2 f + l^2 \nabla_2^2 f \frac{1}{2\pi(1-b)R}\delta\Big(R - \frac{r_0}{1-b}\Big) = \frac{2p}{2\pi(1-b)M_6^4 R}\delta^2(\vec{x}_\perp)\delta\Big(R - \frac{r_0}{1-b}\Big)$$

- Solution!

$$f(r,R) = -\frac{p}{2\pi^2(1-b)M_6^4}\int_0^\infty dk\, k \frac{I_0\Big(k\frac{r_0}{1-b}\Big)K_0(kR)J_0(k|\vec{x}_\perp|)}{1 + \frac{l^2k^2}{2\pi(1-b)} I_0\Big(k\frac{r_0}{1-b}\Big)K_0\Big(k\frac{r_0}{1-b}\Big)}$$

everywhere regular! At distances $r < r_c$ one finds the 4D Aichelburg-Sexl shock wave! At $r > r_c$ changes to 6D (of Ferrari, Pendenza,Veneziano, 1988).

- The crossing scale r_c is exactly the see-saw scale of DGHS:

$$r_c = \frac{M_4^2}{4\pi r_0 M_6^4}$$

Summary

- The keystone of DGP : **gravitational filter** - hides the extra dimension. But: longitudinal scalar is **tricky**!
- On SA brane, the localized mode is a perturbative ghost. Cosmology with it running loose is unreliable.
- What does the ghost do?
  - Can it catalyze transition from SA to normal branch?
  - Can it `condense'?
  - What do strong couplings do? At short scales? At long scales?
  - ...
- Cod-2: is the simple wrapped 4-brane resolution ghost-free? Can it resurrect self-tuning?
- More work: we may reveal interesting new realms of gravity!

Time to call in heavy hitters?...