Towards NLO Parton Shower MC

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What is NLO parton shower?

A litle bit of warm-up: What is the LO parton shower?

- The LO parton shower MC is built using LO class evolution kernels and/or LO PDFs for each incomming/ougoing shower/ladder.
- LO PS MC implements LO DGLAP evolution of the total cross section and of semi-inclusive distributions (structure functions).
- If hard process is corrected to the NLO level (N+LO), the all collinear/soft singularities of the LO PS MC are subtracted from the hard proces ME in the exclusive form.
- In N+LO schemes certain partons originally generated by the LO PS MC get promoted to the hard process, where their distributions get corrected to NLO level.

What is NLO parton shower?

Now everything one order higher:

- The NLO parton shower MC is built using NLO class evolution kernels and/or NLO PDFs for each shower/ladder.
- NLO PS MC implements NLO DGLAP evolution of the total cross section and of semi-inclusive distributions (structure functions).
- If hard process is corrected to the N^2LO level (N+NLO), collinear/soft singularities of the NLO PS MC are subtracted from the hard proces ME in the exclusive form.
- In N+NLO scheme certain partons originally generated by the NLO PS MC get incorporated into the hard process, where their distributions get corrected to N^2LO level.

Problems and solutions

- NLO kernels have to be recalculated in the exclusive form.
  - We have recalculatet all NLO kernels using Curci-Furmanski-Petronzio (CFP) scheme – explicit diagramatic calculation in axial gauge (also Ellis+Voghesang, Kunst+Heinrich).
  - Technical improvements were proposed (Skrzypek+Gituliar)
- LO parton shower may miss some phase space regions which are present in NLO kernels/evolution, like $q \to qG^*$, $G^* \to GG$ spliting
  - One could add $G^* \to GG$ after LO PS generation is finished,
  - Luckily, some modern LO PS MCs already populated this ph.sp.
- Introducing complete NLO real and virtual corrections into PS MC in the exclusive form, in accordance with the collinear factorization theorems (CFP), a formidable problem, theoreticaly and practicaly.
  - Theoretical framework CFP-compatible formulated and tested,
  - 3 methods of practical implementation of NLO corrections in the PS MC formulated and tested. One of them quite promissing.

Remarks on NLO kernel re-calculation

- Why CFP? Because there is nothing else in the literature.
- All inclusive $\overline{MS}$ kernels were reproduced, but we have listed/exploited all exclusive 2-real and 1real+1virtual distributions, before the phase space integration.
- CFP was modified in order to eliminate spurious $1/\varepsilon^3$ poles obscuring relation to MC at $d = 4$ dimensions. The so-called NPV prescription by Skrzypek and Gituliar, pulished recently.
- For subsets of diagrams in 2-real parton contributions, soft gluon limit was analyzed carefully. Expected gauge cancellations found.
- In CFP NLO kernel is extracted as coefficient of $1/\varepsilon$. An alternative method of taking derivative $\partial/\partial(\ln \mu^2)$ was tested.
- $\overline{MS}$ scheme produces technical artefact $\sim \varepsilon/\varepsilon^2$, which are source of the problems in the MC implementation of NLO corrections. These terms were clasified and their role was analyzed.

Theoretical framewework of PS MC: Collinear Factorization

- What is collinear factorization?

$$F_{bare}(q_h/\mu,\varepsilon) = \frac{\sigma_{Bare}}{\sigma_{Born}} = \prod_{Ladders} C^{(\infty)}\left(\alpha, \frac{q_h}{\mu}\right) \otimes \Gamma^{(\infty)}_{ladder}(\alpha,\varepsilon)$$

$\otimes$ in lightcone x and parton type, Γ inclusive, C can be kept unintegrated/exclusive.

$$\text{Case LO}: \quad F^{(1)}_{bare}(q_h/\mu,\varepsilon) = [\mathbb{1} + C^{[1]}(\alpha, q_h/\mu)] \otimes [\mathbb{1} + \Gamma^{[1]}(\alpha,\varepsilon)]$$

- Physical distributions: $\Gamma \to \mathrm{PDF}$. LO example:

$$F_{Phys.} = [\mathbb{1} + C^{[1]}(\alpha, q_h/\mu)] \otimes \mathrm{PDF}(\mu), \quad C^{[1]}(q_h/\mu) \equiv F^{[1]}_{bare}(q_h/\mu,\varepsilon) - \Gamma^{[1]}(\varepsilon)$$

$F_{Phys.}$ factor. scheme independent; both C and PDFs are dependent:

$$\Gamma^{[1]}(\varepsilon) \to \Gamma^{[1]} + \Delta\Gamma^{[1]}, \quad C^{[1]} \to C^{[1]} - \Delta\Gamma^{[1]}, \quad \Delta C^{[1]} = -\Delta\Gamma^{[1]}.$$

- Evolution of F and/or PDFs and evolution kernels:

$$\frac{\partial}{\partial \ln \mu} F(\mu) = P \otimes F(\mu), \quad P = \alpha P^{[0]} + \alpha^2 P^{[1]} + \dots = \mathrm{Res}_1 \Gamma(\varepsilon) = \frac{\partial \ln_\otimes C(q/\mu)}{\partial \ln \mu}$$

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$$\text{Case NLO}: \quad F^{(2)}_{bare}(q_h/\mu, \varepsilon) = [\mathbb{1} + C^{[1]} + C^{[2]}] \otimes [\mathbb{1} + \Gamma^{[1]} + \Gamma^{[2]}]$$

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Collinear Factorization – Fixed order calculations

- Fixed order calculation (like MCFM):

$$F(q_h) = [\mathbb{1} + C^{[1]}]_J \otimes \mathrm{PDF}(\mu), \quad C^{[1]} \equiv [F^{[1]}_{bare}(q_h/\mu, \varepsilon) - \Gamma^{[1]}(\varepsilon)]_{q_h=\mu}$$

  $[...]_J$ means experimental acceptance $J(x, y)$ kept in integrand.

- Typical example: ISR gluonstrahlung part of DIS, def. $y = q/q_h \in (1, 0)$:

$$C^{[1]}(z, y) = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+ + \beta(z, y) + \delta_{y=0}\Sigma(z)$$

$$\beta(z, y) = |\mathrm{ME}_{exact}|^2 - \frac{C_F\alpha}{\pi}\frac{1}{y}\frac{\bar{P}(z)}{1-z}, \qquad \Sigma(z) = \frac{C_F\alpha}{\pi}\left(\frac{\bar{P}(z)}{1-z}\frac{(1-z)^2}{z}\right)_+$$

  where $\bar{P}(z) = (1-z)P_{qq}(z) = (1+z^2)/2$.

- Soft-collinear counterterm technique (eg. Catani-Seymour) often used to facilitate computing codes (MCFM):

$$C^{[1]} = [F^{[1]}_{bare} - C_{SC}]_{d=4} + [C_{SC} - \Gamma^{[1]}]_{d\neq 4}, \quad C_{SC}(z, y) = \frac{C_F\alpha}{\pi}\frac{1}{y^{1-2\varepsilon}}\left(\frac{\bar{P}(z)}{1-z}\right)_+$$

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LO and N+LO parton shower MC

- Pure LO parton shower MC, again *ep* with single ISR ladder:

$$F(q_h) = G_{q_0 \to q_h} \otimes \mathrm{PDF}_{\mu = q_0 \simeq 1 GeV}$$

$$G_{q_0 \to q_h} = \exp_{y-ordering} \left\{ \int_0^1 dy \left(\frac{1}{y}\right)_+ \int_0^{2\pi} d\phi \int_0^1 dz \, \frac{C_F \alpha}{\pi} (P^{[0]}(z))_+ \right\}$$

  where $y = q/q_h$ and $(1/y)_+$ regulated using $y > \Delta = q_0/q_h$.

- N+LO parton shower (POWHEG or MCatNLO) is schematicaly:

$$F(q_h) = [\mathbb{1} + \tilde{C}^{[1]}] \otimes G_{q_0 \to q_h} \otimes \mathrm{PDF}_{\mu = q_0 \simeq 1 GeV}$$

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- The above is forward evol. Backward evolution PS MC starts from q_h:

$$F(q_h) = \mathrm{PDF}_{\mu=q_h} \otimes (G_{q_0 \to q_h})^{-1}$$

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$$\tilde{C}^{[1]} = \delta_{z=1} \delta_{y=0} (1 + \Delta_{SV}) + \beta(z, y) + \delta_{y=0} \Sigma(z)$$

but the <u>peculiar</u> $\Sigma(z)$, $\overline{\mathrm{artefact}}$ of $\overline{MS}$, due to ε/ε terms remains!

KRLnlo variant of N+LO parton shower MC
A simpler alternative to POWHEG or MC@NLO

- Backward evolution version with NLO corrected hard process

$$F(q_h) = [\mathbb{1} + \tilde{C}^{[1]}] \otimes \mathrm{PDF}^{\overline{MS}}_{\mu=q_h} \otimes (G_{q_0\to q_h})^{-1}$$

$$\tilde{C}^{[1]} = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \beta(z,y) + \delta_{y=0}\Sigma(z)$$

- is reorganized as follows:

$$F(q_h) = [\mathbb{1} + \bar{C}^{[1]}] \otimes \mathrm{PDF}^{\mathrm{MC}}_{\mu=q_h} \otimes (G_{q_0\to q_h})^{-1},$$

$$\bar{C}^{[1]}(y,z) = \delta_{z=1}\delta_{y=0}(1+\Delta_{SV}) + \beta(z,y),$$

- where $\mathrm{PDF}^{\overline{MS}}$ is translated to MC factorization scheme <u>outside PS MC</u>:

$$\mathrm{PDF}^{\mathrm{MC}}(\mu) \equiv (\mathbb{1} - \Sigma) \otimes \mathrm{MC}^{\overline{MS}}(\mu)$$

- In reality Σ is matrix in fravour space and mixes $q \leftrightarrow G \leftrightarrow \bar{q}$.
  Its element are fixed from inspecting at least two processes.
- **It was tested for DY process, see later on...**

NLO Fixed order variant of KRLnlo

- On may notice that collecting all step, we have

$$\bar{C}^{[1]} = F^{[1]}_{bare}(\varepsilon)|_{q_h=\mu} - C^{\rm MC}_{\rm SC}(\varepsilon) = (1+\Delta_{SV})\mathbb{1} + \beta(z,y).$$

$$C^{\rm MC}_{\rm SC}(y,z,\varepsilon) = \delta_{y=0}\Gamma^{[1]}(z,\varepsilon) + \delta_{y=0}\Sigma(z) + \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+,$$

- where $C^{\rm MC}_{\rm SC}(\varepsilon)$ is the 1-st order part of the evolution operator of the LO PS MC in $d = 4+2\varepsilon$:

$$G^{d=4+2\varepsilon}_{q_0\to q_h} = \mathbb{1} + G^{[1]}(\varepsilon) + ..., \qquad C^{\rm MC}_{\rm SC}(\varepsilon) = G^{[1]}(\varepsilon) \text{ !!!}$$

- $C^{\rm MC}_{\rm SC}$ may be also employed/tested as a soft-collinear counterterm in the NLO fixed order calculation (MCFM-style), with PDFs in the MC scheme:

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NLO ladder in N+NLO parton shower MC

- Use collinear factoriz. of Curci-Furmanski-Petronzio (CFP) as a basis. The 2-nd order version reads:

$$F^{(2)}_{bare}(q_h,\varepsilon) = C^{(2)}(q_h/\mu) \otimes \prod_{Ladders} \Gamma^{(2)}_L(\varepsilon), \quad C^{(2)} = F^{(2)}_{bare} \otimes \prod_L (\Gamma^{(2)}_L)^{-1}$$

and exploit the experience gained in the previous N+LO case.

- Fixed order N^{2}LO with collinear $\overline{MS}$ PDFs (one ladder) is now:

$$F^{(2)}_{phys.}(q_h) = C^{(2)}|_{q_h=\mu} \otimes \mathrm{PDF}^{\overline{MS}}(\mu)$$

- Generalizing N+LO case, we define/use MC distribution truncated to 2-nd order $G^{(2)}_{MC}$ as a soft-collinear counreterm:

$$F^{(2)}(q_h) = \{F^{(2)}_{bare} \otimes (G^{(2)}_{MC})^{-1}\}_{d=4} \otimes \{G^{(2)}_{MC}(\varepsilon) \otimes (\Gamma^{(2)}(\varepsilon))^{-1}\} \otimes \mathrm{PDF}^{\overline{MS}}(\mu)$$

- The key point is to construct the NLO evolution operator G_{MC} such that
  - $G^{(\infty)}_{MC,D=4}$ represents NLO parton shower MC (single ladder) and
  - $G^{(2)}_{MC,d=4+2\varepsilon}$ encalsulates ALL of collinear and soft singularies in the CFP construction of the NLO $\overline{MS}$ evolution kernel $P^{(2)}(z)$.

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- Explicit example of NLO evolution operator G_{MC} in $d=4$, again for the gluonstrahlung branch (extension to $d=4+2\varepsilon$ is trivial).

$$dG^{(2)}_{MC,d=4} = \mathbb{1} + dy_1 dz_1\; g^{[1]}_{MC}(y_1,z_1)\big(1+V^{[1]}(z_1)\big)$$
$$+ dy_1 dz_1 dy_2 dz_2 \theta_{y_2>y_1} \big[g^{[1]}_{MC}(y_1,z_1) g^{[1]}_{MC}(y_2,z_2) + \beta^{[1]}(y_2/y_1, z_2/z_1)\big]\big\}$$
$$g^{[1]}_{MC}(y,z) = \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+,$$

  where LO component $g^{[1]}_{MC}$ is already known from N+LO exercise.
- NLO corrections $V^{[1]}(z)$ and $\beta^{[1]}(y,z)$ from comparing/matching/analyzing $G^{(2)}_{MC,d\neq 4}$ and elements of CFP scheme.
- The above matching procedure is formulated, but still getting consolidated.
- Basic elements of CFP, see next slide...

NLO ladder in N+NLO parton shower MC

- Explicit example of NLO evolution operator G_{MC} in $d = 4$, again for the gluonstrahlung branch (extension to $d = 4 + 2\varepsilon$ is trivial).

$$dG^{(2)}_{MC,d=4} = \mathbb{1} + dy_1 dz_1\; g^{[1]}_{MC}(y_1, z_1)\big(1 + V^{[1]}(z_1)\big)$$
$$+ dy_1 dz_1 dy_2 dz_2 \theta_{y_2>y_1}\big[g^{[1]}_{MC}(y_1, z_1) g^{[1]}_{MC}(y_2, z_2) + \beta^{[1]}(y_2/y_1, z_2/z_1)\big]\}$$
$$g^{[1]}_{MC}(y, z) = \frac{C_F\alpha}{\pi}\left(\frac{1}{y}\right)_+ \left(\frac{\bar{P}(z)}{1-z}\right)_+,$$

where LO component $g^{[1]}_{MC}$ is already known from N+LO exercise.

- NLO corrections $V^{[1]}(z)$ and $\beta^{[1]}(y, z)$ from comparing/matching/analyzing $G^{(2)}_{MC,d\neq 4}$ and elements of CFP scheme.
- The above matching procedure is formulated, but still getting consolidated.
- Basic elements of CFP, see next slide...

Elements of the CFP (EGMPR) scheme

- CFP actorization formula for sigle ladder with two-particle-iredudible (2PI) kernels K_0 in the axial gauge:

$$F = C_0 \cdot \frac{1}{1-K_0} = C_0 \cdot \sum_{n=0} K_0^n.$$

- It is reorganized using projection operator $\mathbb{P} = P_{spin}\, P_{kin}\, \mathrm{PP}$, with kinematic P_{kin}, P_{spin} spin parts and PP extracting pole part $\sim 1/\epsilon^k$.

$$F = C\left(\alpha, \frac{Q^2}{\mu^2}\right) \otimes \Gamma\left(\alpha, \frac{1}{\epsilon}\right) = C_0 \cdot \frac{1}{1-[(1-\mathbb{P})K_0]} \otimes \frac{1}{1-\left\{\mathbb{P} K_0 \cdot \frac{1}{1-[(1-\mathbb{P})K_0]}\right\}_{\otimes}}.$$

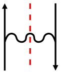
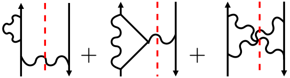
- Second order truncation exploiting 2-nd order $K_0^{(2)} = K_0^{[1]} + K_0^{[2]}$:

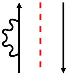
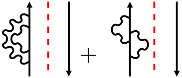
$$\Gamma^{(2)} = \mathbb{1} + \mathbb{P} K_0^{(2)} + \mathbb{P}(K_0^{[1]} \cdot (1-\mathbb{P})K_0^{[1]}) + (\mathbb{P} K_0^{(1)}) \otimes (\mathbb{P} K_0^{(1)})$$

- An example of the diagramatic content of $K_0^{(2)} = K_0^{[1]} + K_0^{[2]}$ for gluonstrahlung is shown on the next slide...

Contributions to example 2PI $\sim C_F^2$ kernel $K_0(q \to q)$:

$$K_0 = K_0^{[1]} + K_0^{[2]},$$

$K_0^{[1]} =$  , $\quad K_0^{[2]} =$ 

$Z_F = 1 + Z_F^{[1]} + Z_F^{[2]}, \quad Z_F^{[1]} =$  , $\quad Z_F^{[2]} =$ 

Determining NLO corrections $V^{[1]}(z)$ and $\beta^{[1]}(y,z)$ for the MC ladder

- As a calibration exercise we apply CFP machinery of extracting $\Gamma(\varepsilon)$ and NLO kernel, to MC distributions in $d = 4+2\varepsilon$ for $V^{[1]} = 0$ and $\beta^{[1]} = 0$
- Surprisingly (?) a non-zero NLO corrections to kernel is found:

$$\Delta P(z) = -\Big(\frac{C_F\alpha}{\pi}\Big)^2 \Delta_{CFP}(z)$$

$$\Delta_{CFP}(z) = \int_0^1 dz_1 dz_2\ (P(z_1))_+\ \ln(z_2)P(z_2)\delta(z-z_1z_2)$$

$$= \frac{1+z^2}{2(1-z)}\ln z\left[\ln\frac{1-z}{z^{1/2}}+\frac{3}{4}\right]+\frac{1}{8}\ln z\,[(1+z)\ln z - 2(1-z)],$$

which (up to normalization) is the $\Delta(z)$ function in CFP paper, eq. (6.44), responsible for violation of the Gribov rule relating NLO kernels of the intitial and final state ladders.

- Its origin is traced back to kinematics: for instance in DY, 1st real emission (going backwards toward hadron beam), changes assignment $\mu^2 = \hat{s} = q_h^2$ into $\mu^2 = \hat{s}/z$. This induces $\sim \Delta_{CFP}(z)$ to NLO kernel.
- In the standard CFP kernel calculation. this contribution is cancelled in the end by another similar term, but in the MC it may be kept or not in $V^{[1]}$, depending how NLO PDFs are defined and used.

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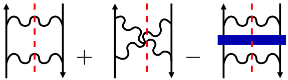
Determining NLO 2-real corrections $\beta^{[1]}(y,z)$ for the MC ladder

- Within the same gluonstrahlung example, determination of $\beta^{[1](y,z)}$ is rather simple:

$$\beta^{[1]}(y_2/y_1, z_2/z_1) = |\mathrm{ME}_{2r}|^2 - g^{[1]}_{MC}(y_2,z_2) g^{[1]}_{MC}(y_1,z_1).$$

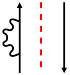
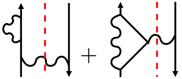
- The same RHS diagramatically:



- NB. The internal subtraction of the $(\mathrm{LO_{MC}})^2$ contribution is necessary only for a small subset of NLO diagrams.

Determining NLO corrections $V^{[1]}(z)$ for the MC ladder

- 1real + 1virtual contribution to $V^{[1]}(z)$ comes from diagrams:

$Z_F^{[1]} =$  , $K^{[2]}_{0,1r1v} =$ 

all the time $\sim C_F^2$ glunstrahlung example...

- Determination of $V^{[1]}(z)$ is not easy – it involves several issues:
  - Extracting $\Gamma(\varepsilon)$ from 1r1v part of MC in $d = 4 + 2\varepsilon$ requires
    (i) either extension of CFP subtraction recipe or
    (ii) adjusting IR cut-off $(1-z) < \delta$ in such that some terms disappear.
  - CFP subtraction have to be done separately for the virtual Sudakov formfactor.
  - In principle $V^{[1]}$ could also depend on y variable.
    This dependence in fact materializes from the UV subtraction.
    However, such terms contribute only pure $1/\varepsilon^2$ to Γ (pure $(\mathrm{LO_{MC}})^2$ in finite part) and have to be removed, to avoid double counting with the exponetiated LO MC.
  - The presence/absence of $\sim \Delta_{\mathrm{CFP}}$ has to be decided.
- Finally we find: $\bar{P}(z) \equiv (1+z^2)/2$

$$V^{[1]}(z) = -\frac{1}{2}\frac{\bar{P}(z)}{1-z}\ln(z)\ln(1-z) + \frac{1}{2}\frac{\bar{P}(z)}{1-z}\mathrm{Li}_2(1-z) + \frac{z}{8}.$$

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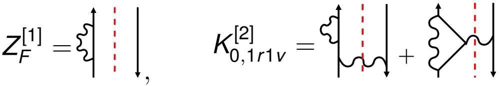

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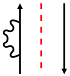
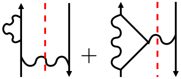
$Z_F^{[1]} =$  , $K^{[2]}_{0,1r1v} =$ 

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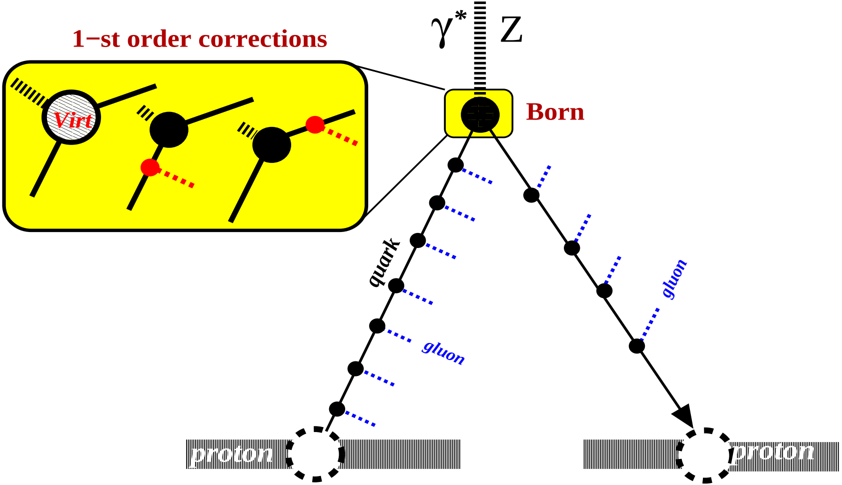
Examples of numerical implementations

A litle bit of numerical implementation results for:

- NLO corrections to hard process
  (an alternative to MCatNLO and/or POWHEG)
- NLO corrections in the ladder
  (for NLO parton shower MC + NNLO hard process)

N+LO correcting HARD process, KRKnlo method



MC weight with NLO corrs. to DY hard proc.

NLO correction introduced using simple positive MC weight (only one term in the sums may be kept in case of kT-ordering):

$$W_{MC}^{NLO} = 1 + \Delta_{S+V} + \sum_{j\in F} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Fj})}{\bar{P}(z_{Fj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega} + \sum_{j\in B} \frac{\tilde{\beta}_1(\hat{s}, \hat{p}_F, \hat{p}_B; a_j, z_{Bj})}{\bar{P}(z_{Bj})\, d\sigma_B(\hat{s},\hat{\theta})/d\Omega},$$

$\bar{P}(z) \equiv \frac{1+z^2}{2}$. **The IR/Col.-finite real emission part is**

$$\tilde{\beta}_1(\hat{p}_F, \hat{p}_B; q_1, q_2, k) = \left[\frac{(1-\alpha)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{F1}) + \frac{(1-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \theta_{B2})\right]$$
$$- \theta_{\alpha>\beta} \frac{1+(1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}) - \theta_{\alpha<\beta} \frac{1+(1-\alpha-\beta)^2}{2} \frac{d\sigma_B}{d\Omega_q}(\hat{s}, \hat{\theta}),$$

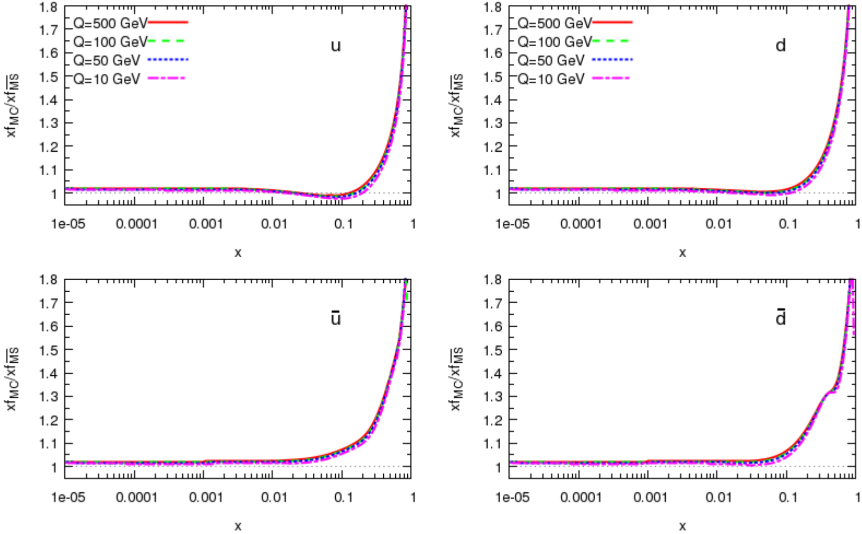
the kinematics independent virtual+soft correction is

$$\Delta_{V+S} = \frac{C_F \alpha_s}{\pi} \left(\frac{1}{3}\pi^2 - 4\right) + \frac{C_F \alpha_s}{\pi} \frac{1}{2}$$

Terms like $\left(\frac{f(z)}{1-z}\right)_+$ in virt. corrs completely absent!

4. Redefine PDFs: $\overline{MS} \to$ MC scheme

Ratios with respect to standard $\overline{MS}$ PDFs for light quarks.



MCFM $\overline{MS}$ vs. MCFM in MC scheme at NLO

Technical cross-check (using modified MCFM)

$$\sigma_{\rm tot}^{\overline{\rm MS}} = f_q \otimes (1 + \alpha_s\, C_q^{\overline{\rm MS}}) \otimes f_{\bar q}$$

$$\begin{aligned}\sigma_{\rm tot}^{\rm MC} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\rm MC}) \otimes (f_{\bar q} + \alpha_s \Delta f_{\bar q})\\ &= f_q \otimes f_{\bar q} + \alpha_s \left(\Delta f_q \otimes f_{\bar q} + \Delta f_{\bar q} \otimes f_q + C_q^{\rm MC} \otimes f_q \otimes f_{\bar q}\right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)\end{aligned}$$

Drell-Yan, $q\bar q$ channel, $\alpha_s = \alpha_s(m_Z)$:

$$C_q^{\overline{\rm MS}} \otimes f_q \otimes f_{\bar q} = \Delta f_q \otimes f_{\bar q} + \Delta f_{\bar q} \otimes f_q + C_q^{\rm MC} \otimes f_q \otimes f_{\bar q}$$

$$(336.36 \pm 0.09)\,{\rm pb} = \underbrace{25.79\,{\rm pb} + 25.79\,{\rm pb} + 284.77\,{\rm pb}}_{(336.35 \pm 0.09)\,{\rm pb}}$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s^2)$.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,$pb, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,$pb.

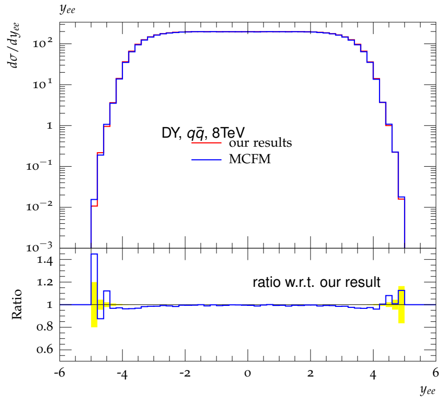
MCFM $\overline{MS}$ vs. MCFM in MC scheme at NLO

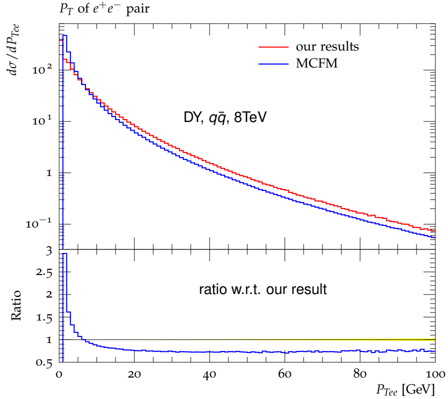
Total cross section for DY, $q\bar{q}$ channel, 8 TeV

	σ_{tot} [pb]
MCFM ($\overline{MS}$ PDFs)	1344.1 ± 0.1
MCFM (MC PDFs)	1361.6 ± 0.3
PS+full NLO (MC PDFs)	1355.9 ± 0.8

- The difference between fully corrected PS+NLO is at the level of 0.8% w.r.t. MCFM in $\overline{MS}$ scheme and 0.4% w.r.t. to MCFM in MC scheme.

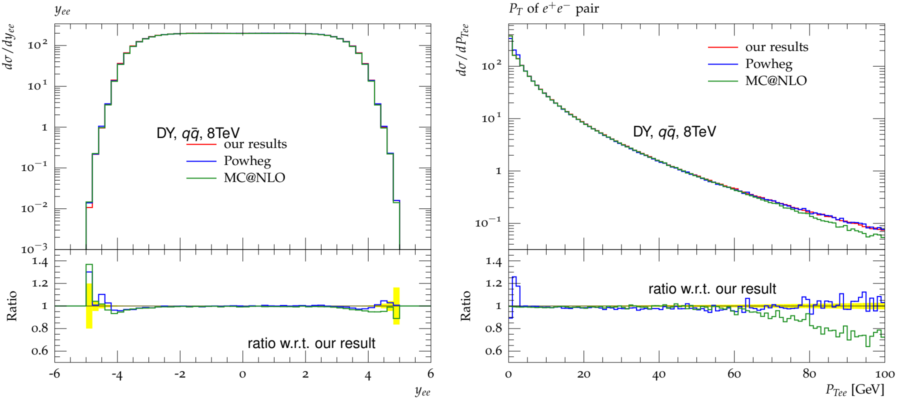
pT and rapidity distributions, KRKnlo vs MCFM





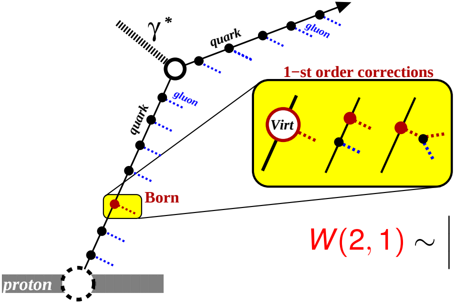
- Our KRKnlo on top of Sherpa LO MC, $q\bar{q}$ chanel only.
- y_Z distribution from KRKnlo agrees with MCFM at NLO.
- p_T distribution suppressed at low p_T due to Sudakov.
- Virtual correction spread over a range of p_T.

KRKnlo vs. POWHEG and MC@NLO

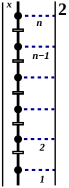
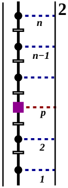
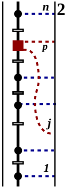


- y_Z and p_T distributions very close to POWHEG (difference at low p_T due to slightly different evolution variable)
- y_Z very close to MC@NLO, same for low and intermediate p_T (differences for the tail of p_T distributions due to higher orders as expected)
- The above is for $q\bar{q}$ chanel. Results for qG chanel still validated.

NLO-corrected middle-of-the-ladder kernel, C_F^2

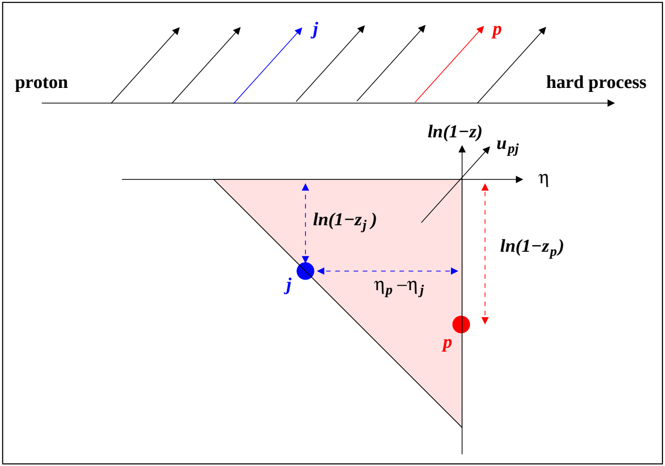


$$W(2,1) \sim \left| \;\cdot\; \right|^2 = \left| \;\cdot + \cdot\; \right|^2 - \left| \;\cdot\; \right|^2$$

$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$  $$+\sum_{p=1}^{n}$$  $$+\sum_{p=1}^{n}\sum_{j=1}^{p-1}$$  $$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+\sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[1 + \sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}.$$

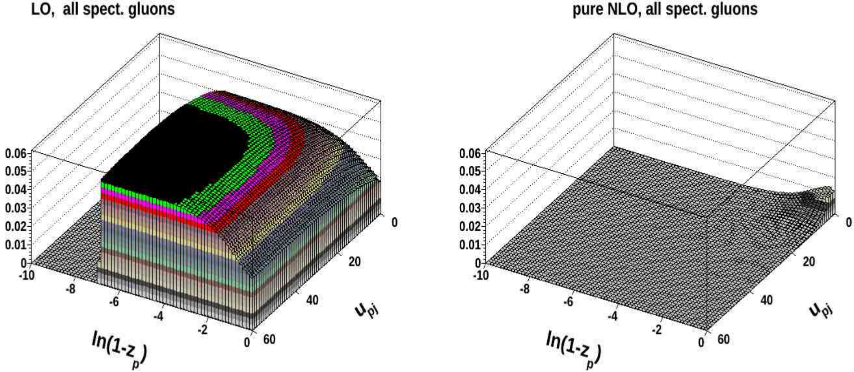
Define variable u_{pj} for "u-ordering" in the middle of the ladder



$u_{pj} = |\eta_p - \eta_j| + \lambda \ln(1 - z_j), \quad \lambda \sim 1 - 2.$

Variable η is rapitity, z is conventional lightcone variable.

Location and size of the (real) NLO correction in the ladder on the Sudakov log space

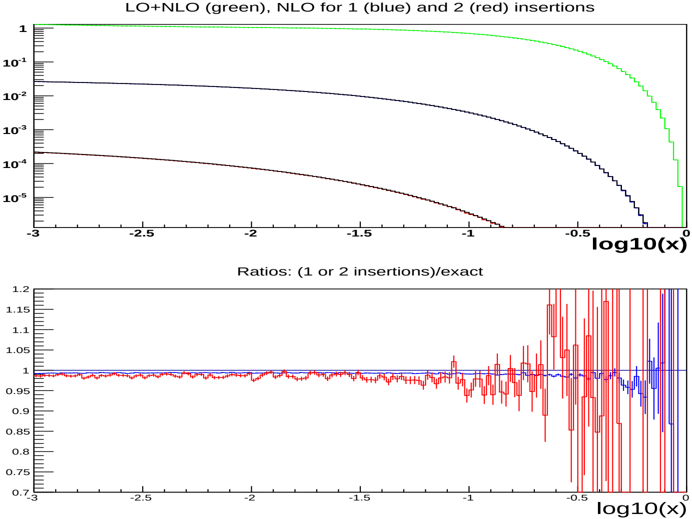


LO inclusive distribution features triple-log IR/coll. singularity, seen as a plateau in 2-dim. projection.

NLO correction IR/coll. finite, nonzero in the corner of the size ~ 1.

Repetition of test for NLO-corrected ladder

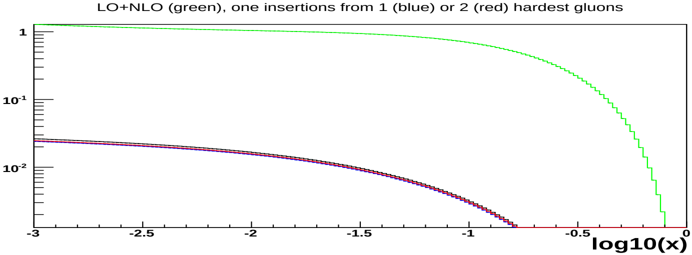
OLD: NLO MC vs. analyt. NLO kernels. Perfect agreement

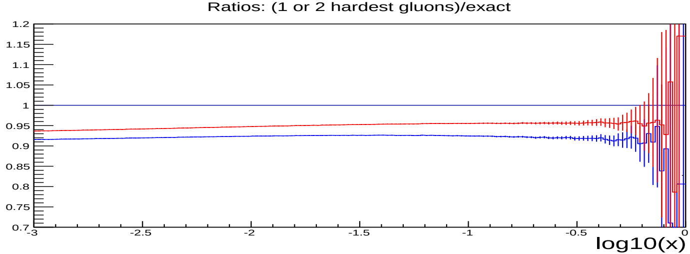


Single ladder, 1GeV-1TeV, 1 or 2 kernels NLO-corrected. (Slow in CPU time).

Repetition of test for NLO-corrected ladder

NEW: NLO contrib. to 1 kernel, 1 and 2 gluons with max. kT





This difference ~ 15% is formally the NNLO/NLO class. (Faster in CPU time).

Summary

- **An alternative (to MC@NLO or POWHEG) scenario for NLO-corrected hard proc. and LO PSMC is worked out.**
- **Parton shower MC implementing complete NLO DGLAP in the ladders in exclusive way is progressing well.**
- Long term N+NLO: NLO ladder + NNLO hard process, (but LO ladder + NLO hard proc. to be optimized first!)
- Most likely application: high quality QCD+EW+QED MC with hard process like $W/Z/H$ boson production.
- Potential gains from new QCD methods are:
  – reducing h.o. QCD uncertainties
  – easier implementation of NLO and NNLO corrections to hard process.
  – better environment for low x resumm. (BFKL, CCFM),
  – and more...