$pp \to \gamma^*\gamma^*$ in the large N_F limit

Romain Mueller, ETH Zurich

Based on arXiv:1408.4546, in collaboration with
Ch. Anastasiou, J. Cancino, F. Chavez, C. Duhr, A. Lazopoulos, and B. Mistlberger

HP^2 2014, GGI Firenze

Colourless final states @ LHC

The production processes of colourless particles at the LHC are of prime importance as many of them are probe the electroweak sector of the Standard Model.

For example:

- Higgs production
- Drell-Yan
- Vector bosons pair production
- …

In particular: The production of two off-shell vector bosons is important to assess background contributions in Higgs searches.

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In particular: The production of two off-shell vector bosons is important to assess background contributions in Higgs searches.

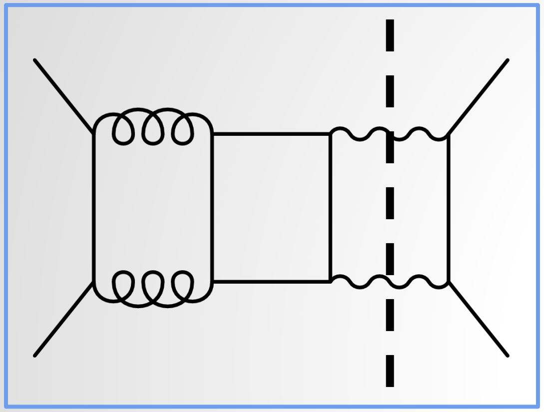
Note: Inclusive production of on-shell W^+W^- and ZZ recently computed at N^2LO.

[Gehrmann, Grazzini, Kallweit, Maierhofer, von Manteuffel, Pozzorini, Rathlev, Tancredi]

[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer , von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs]

Contributions @ N^2LO

Double virtual



Emission and reabsorption of two virtual particles:

- Usually the bottleneck of N^2LO computations.
- Recent progress in analytic tools for master integrals.
- All integrals necessary for diboson production @ N^2LO are known.

[Caola, Melnikov, Henn, Smirnov]
[Gehrmann, von Manteuffel, Tancredi, Weihs]
[Duhr, Chavez]

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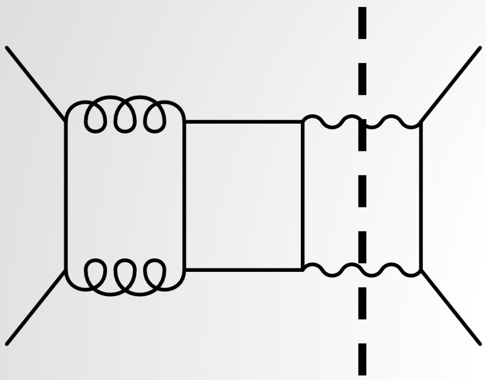
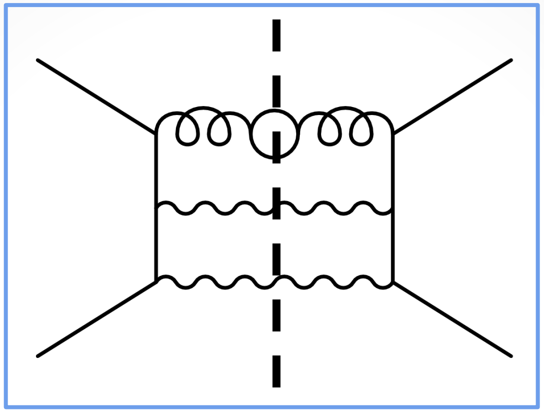


Double real



Emission of two real particles:

- Subtraction of infrared divergences is a difficult problem.
- General methods are becoming available:
  - ➢ q_T subtraction, sector decomposition based methods (STRIPPER), antenna subtraction, non-linear mappings, etc.

Contributions @ N^2LO

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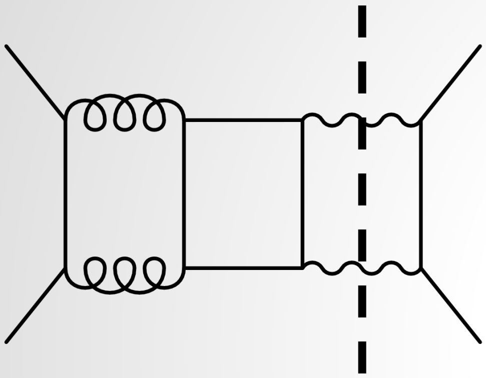

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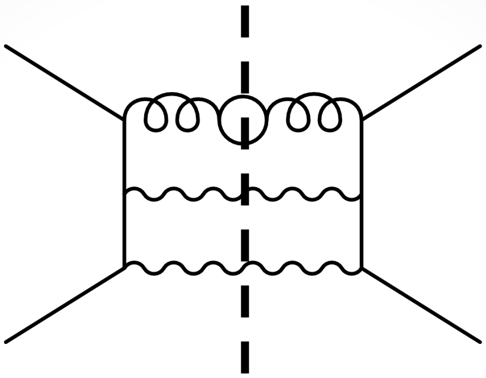
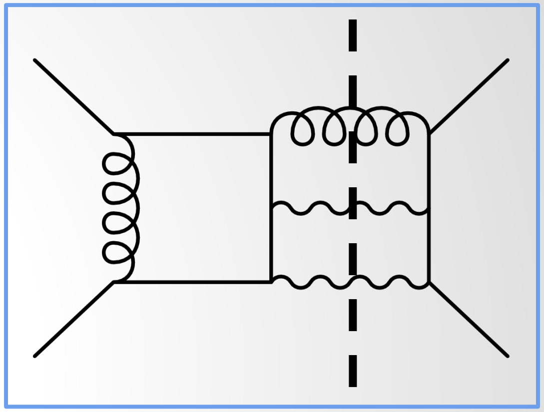


Real-virtual



Emission of a real particle and emission + reabsorption of a virtual particle:

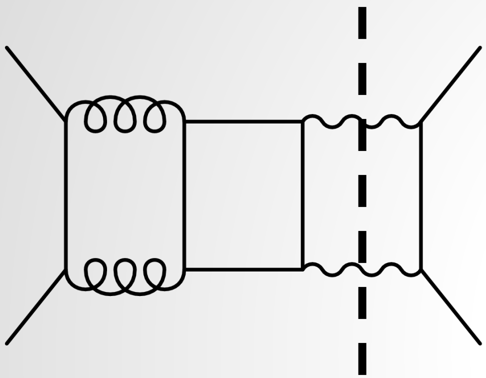
- Soft and collinear limits necessary for subtraction are known in principle.

  [Bern, Chalmers; Kosower ; Kosower, Uwer]

- Implementation may still be challenging.

Contributions @ N^{2}LO

Double virtual



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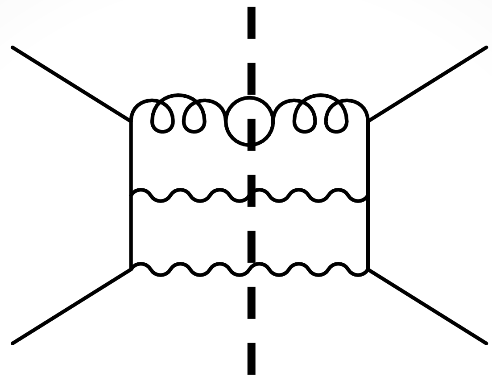

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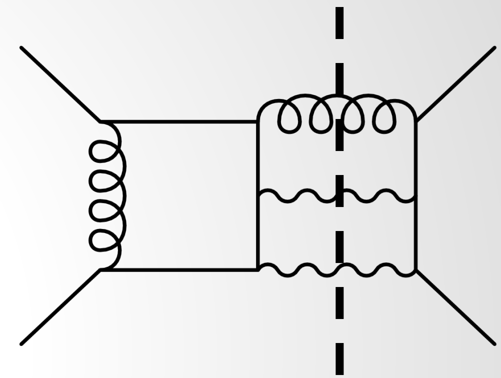

Here want to look at a simple physical process with two different masses in the final states:

$pp \to \gamma^*\gamma^*$ in the large NF limit

Which already possesses some of the complications of the full calculation.

The large N_F limit @ N^2LO

The large N_F (= number of light-quark flavors) limit is not necessarily dominant but can serve as an excellent means to develop analytic and numeric methods.

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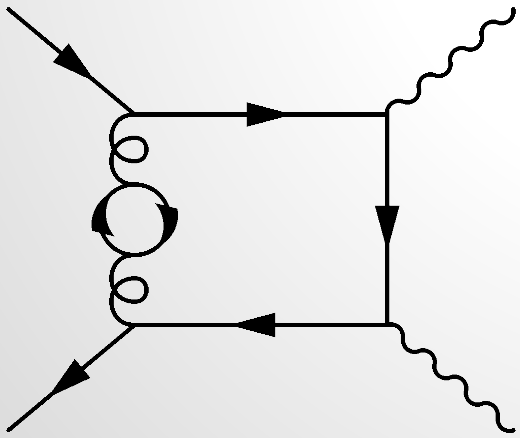

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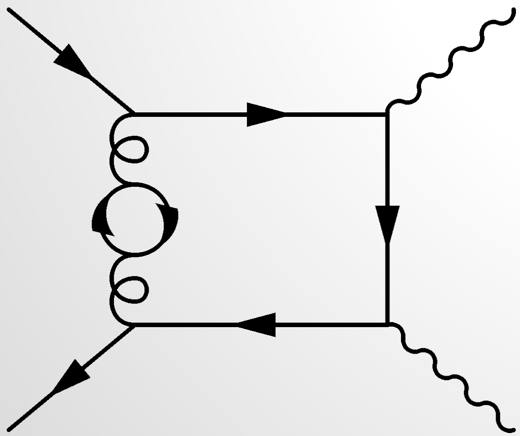
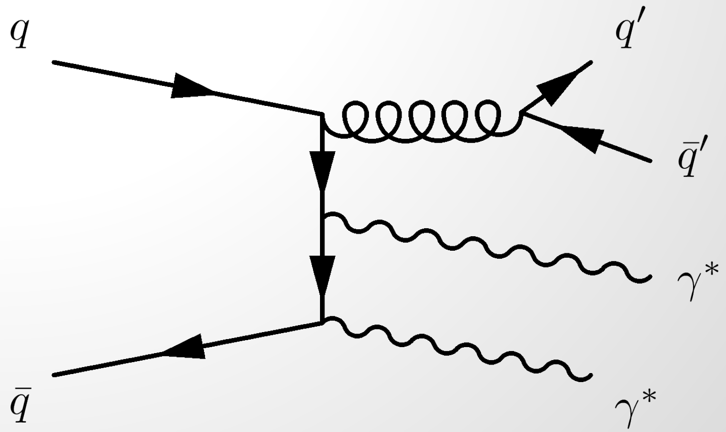
- Double virtual is challenging but not too difficult (bubble insertions).
- Double real consists only of the $q\bar{q} \to \gamma^*\gamma^*q'\bar{q}'$ channel.





Virtual: Reduction

Well-established method to deal with the virtual contributions:

- The different integrals appearing are not independent but related by Integration-by-parts identities (IBPs).

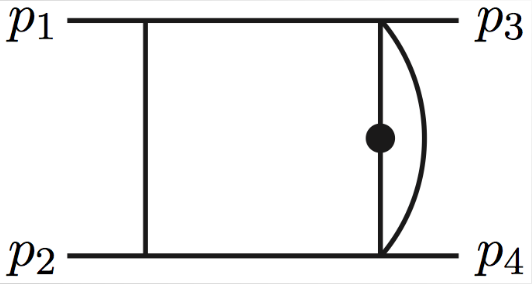
[Chetyrkin, Tkachov]

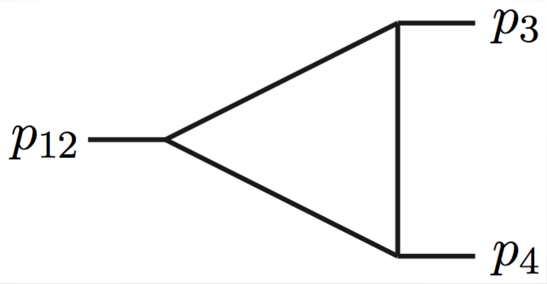
- These identities can be used to reduce algorithmically any integral to a linear combination of ‘master integrals’.

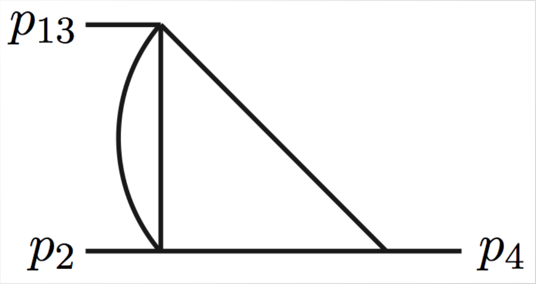
[Laporta]

- ‘The only thing left to do’: compute the master integrals analytically.

Some master integrals:







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- ‘The only thing left to do’: compute the master integrals analytically.

- We computed the master integrals in the spirit of Chavez & Duhr (direct integration), arXiv:1209.2722, and Brown arXiv:0804.1660.
- Independent computation by Caola, Melnikov, Henn & Smirnov (differential equations) arXiv:1404.5590, arXiv:1402.7078.

Virtual: Master integrals

Master integrals are generally complicated functions, especially when many scales are involved.

- Expansion in ε usually involves logarithms, (classical-)polylogarithms, HPLs, etc. → Whole zoo of functions!
- These functions are not independent (but relations are very complicated).
- The symbol/coproduct approach allowed to clean up this mess a bit, by making hidden identities among these functions explicit.
- However: there is still some arbitrariness in the choice of basis functions.
- Can we find a basis which is ‘<u>as simple as possible</u>’?

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Idea:

Identify *a priori* a basis of functions with the correct analytic structure.

Construction of the basis

Algorithm:

- Obtain the alphabet of the symbol/coproduct for the master integrals.
  - Either by direct integration, or by inspection of the differential equations.
- A basis of function with the right analytic properties can then be constructed recursively, weight by weight.

[Brown]

- Moreover, this basis is ‘as simple as possible’ in the sense that no linear combination of the new functions appearing at each weight can be written as a linear combination of product of functions of lower weight.

This restricted set of basis functions can then be studied, in order to:

- Perform the analytic continuation,
- Achieve efficient numerical evaluation.

Example: Triangles

arXiv:1209.2722

It can be shown that triangles can be expressed through single-valued functions

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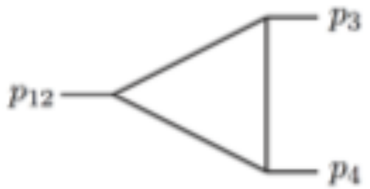
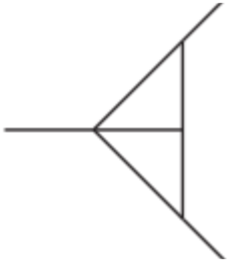


$$= -2c_\Gamma \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)^2} (-p_3^2)^{-1-\epsilon} \frac{u^{-\epsilon} v^{-\epsilon}}{z-\bar{z}} \Big\{ \mathcal{P}_2(z) + 2\epsilon \mathcal{Q}_3(z) + \epsilon^2 \Big[\Big(\frac{1}{6} \ln u \ln v - \zeta_2 \Big) \mathcal{P}_2(z) + 2\, \mathcal{Q}_4^-(z) \Big] + \mathcal{O}(\epsilon^3) \Big\}.$$



$$= c_\Gamma^2 (-p_3^2)^{-1-2\epsilon} \frac{6}{z-\bar{z}} \mathcal{P}_4(1-\frac{1}{z}) + \mathcal{O}(\epsilon)$$

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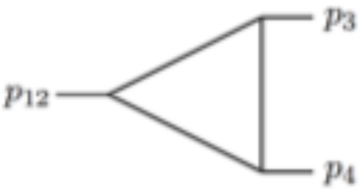
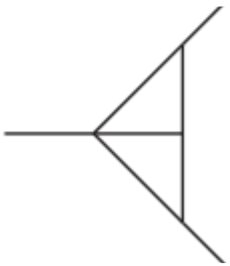


$$= -2c_\Gamma \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)^2}(-p_3^2)^{-1-\epsilon}\frac{u^{-\epsilon}v^{-\epsilon}}{z-\bar{z}}\left\{\boxed{\mathcal{P}_2(z)}+2\epsilon\boxed{\mathcal{Q}_3(z)}\right.$$

$$\left.+\epsilon^2\left[\left(\frac{1}{6}\ln u\ln v-\zeta_2\right)\boxed{\mathcal{P}_2(z)}+2\boxed{\mathcal{Q}_4^-(z)}\right]+\mathcal{O}(\epsilon^3)\right\}.$$



$$= c_\Gamma^2(-p_3^2)^{-1-2\epsilon}\frac{6}{z-\bar{z}}\boxed{\mathcal{P}_4}(1-\frac{1}{z})+\mathcal{O}(\epsilon)$$

In red: the single-valued basis functions.

Only 12 indecomposable basis functions.
(up to 2 loops, weight 4)

Example: Triangles

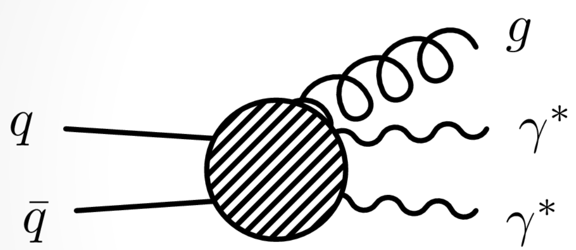
Example of a basis function for weight 3:

$$\begin{aligned}
\mathcal{Q}_3(r) = &\frac{1}{2}\left[G\left(0,\frac{1}{\bar{r}},\frac{1}{r},1\right) - G\left(0,\frac{1}{r},\frac{1}{\bar{r}},1\right)\right] \\
&+ \frac{1}{2}\Big[\mathrm{Li}_3(1-r) - \mathrm{Li}_3(1-\bar{r})\Big] \\
&+ \frac{1}{4}\log|r|^2\left[G\left(\frac{1}{r},\frac{1}{\bar{r}},1\right) - G\left(\frac{1}{\bar{r}},\frac{1}{r},1\right)\right] + \mathrm{Li}_3(r) - \mathrm{Li}_3(\bar{r}) \\
&+ \frac{1}{4}\Big[\mathrm{Li}_2(r) + \mathrm{Li}_2(\bar{r})\Big]\log\frac{1-r}{1-\bar{r}} + \frac{1}{4}\Big[\mathrm{Li}_2(r) - \mathrm{Li}_2(\bar{r})\Big]\log|1-r|^2 \\
&+ \frac{1}{16}\log\frac{r}{\bar{r}}\log^2\frac{1-r}{1-\bar{r}} \\
&+ \frac{1}{8}\log^2|r|^2\log\frac{1-r}{1-\bar{r}} + \frac{1}{4}\log|r|^2\,\log|1-r|^2\,\log\frac{1-r}{1-\bar{r}} \\
&+ \frac{1}{16}\log^2|1-r|^2\log\frac{r}{\bar{r}} - \frac{\pi^2}{12}\log\frac{1-r}{1-\bar{r}}\,.
\end{aligned}$$

Real contributions

Production of $\gamma^*\gamma^*$ in association with additional massless coloured particles in the final state:

NLO



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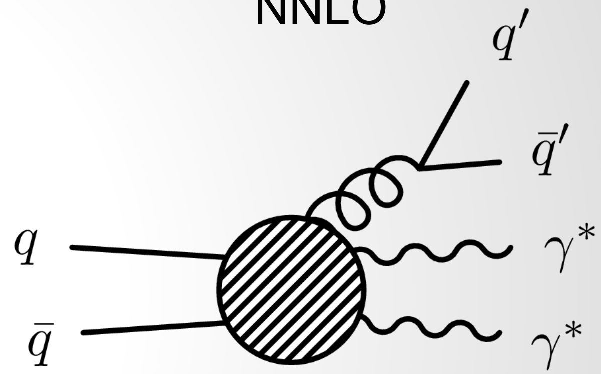

The (squared) amplitudes become singular when external particles become soft or collinear to each other

- Integration over the phase space introduces divergences.
- These divergences need to be extracted to obtain a finite cross-section.

Kinematics I

Spin structure of the g* → q'q' vertex puts strong constraints on the singularity structure:

- The off-shell parent gluon controls completely the singular behaviour of the amplitude.
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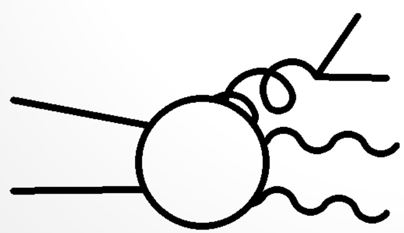
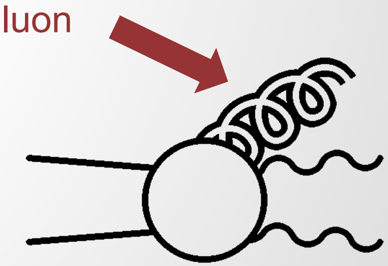
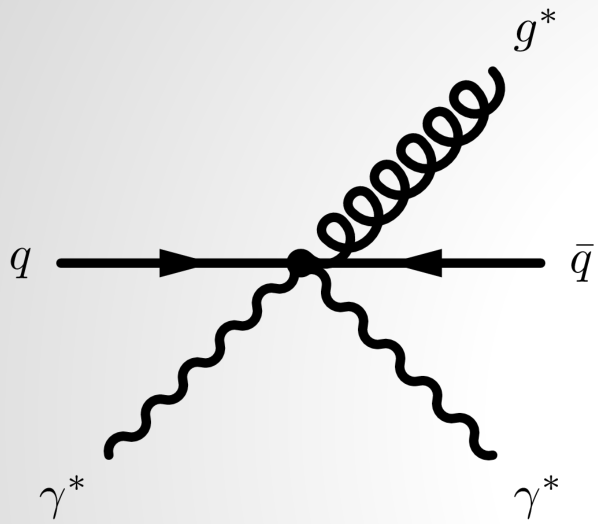
⇒ As far as the singularity structure is concerned, we can integrate over the phase-space of the final-state quarks:

$$\int d\Phi_{g^* \to q' \bar{q}'} \left| \vphantom{\int} \right.$$

$$\left. \vphantom{\int} \right|^2 = \frac{A(\epsilon)}{(p_{g^*}^2)^{1+\epsilon}} \left| \vphantom{\int} \right.$$


$$\left. \vphantom{\int} \right|^2$$

- Full kinematics will be restored in a second time.

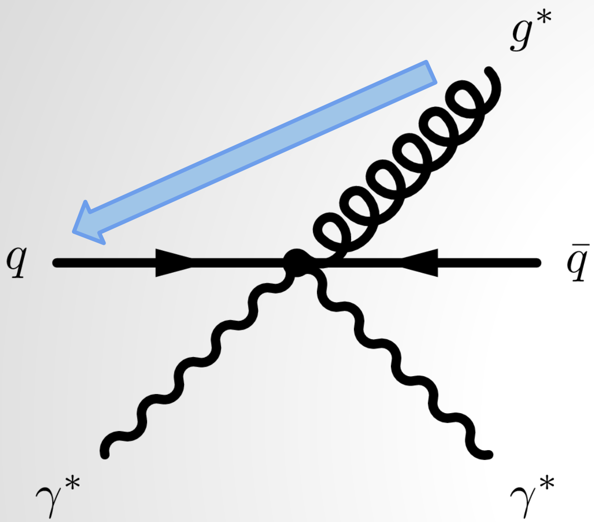
Kinematics II



$$p_{g^*} = (1-z)\Big[(1-\lambda)p_1 + \lambda \frac{1-\rho(1-z)(1-\lambda)}{1-(1-z)(1-\lambda)} p_2 + \sqrt{s\rho\lambda(1-\lambda)}\; e_T\Big]$$

$(z, \lambda, \rho \in [0,1])$

Kinematics II



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Singular limits: $\lambda \to 0$: $p_{g^*} \parallel p_1$ (coll.)

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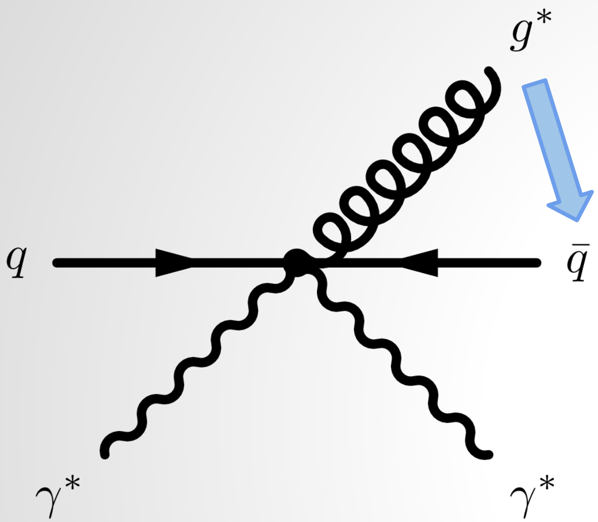

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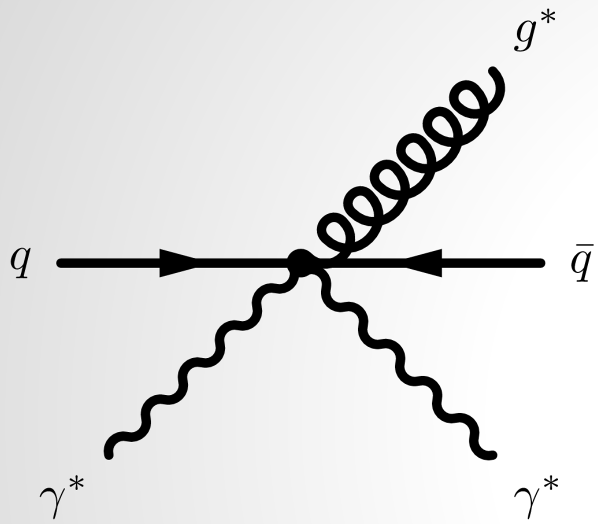

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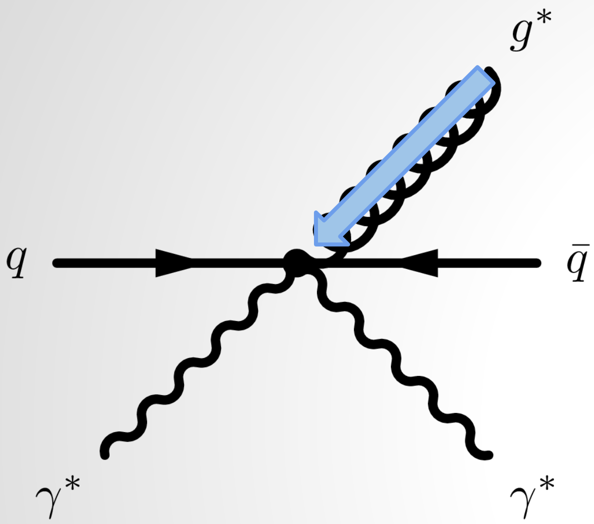

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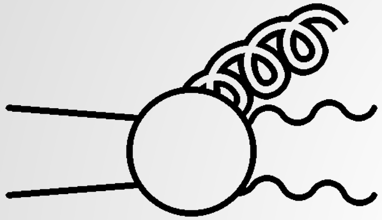
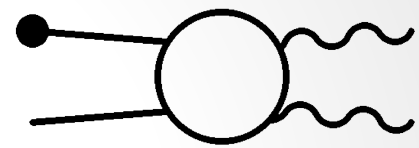
$\rho \to 1$	:	$p_{g^*}^2 = 0$	(massless)
$z \to 1$	:	$p_{g^*} = 0$	(soft)

Asymptotic behaviour

It is a well-known fact that amplitudes factorize in singular limits:

$\Big|$  $\Big|^2 \sim \frac{S_{g^* q}}{s(1-z)^2\lambda} \times \Big|$  $\Big|^2$

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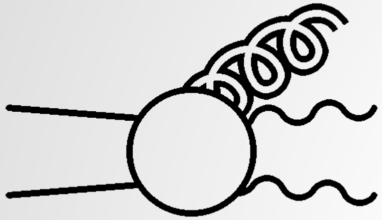
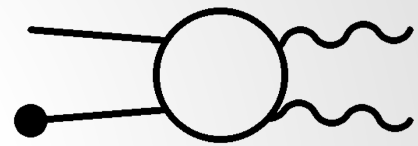
$\Bigg|$  $\Bigg|^2 \sim \dfrac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)} \times \Bigg|$  $\Bigg|^2$

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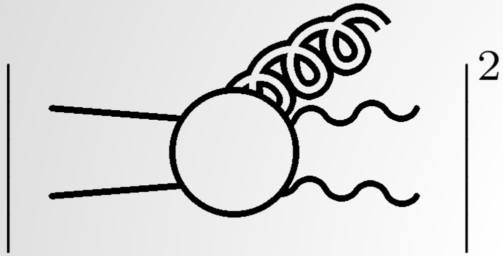
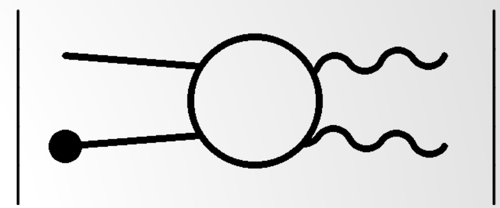


$$\sim \frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)} \times$$



The $S_{ijk\ldots}$ are universal functions, in the sense that they are identical among all colourless final-states.

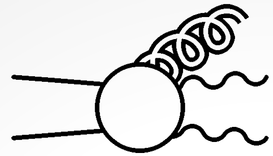
[Catani, Grazzini]

Here, we use a pragmatic approach to extract the singularities:

- Parameterize the phase-space.
- Subtract the residue at every singular limit.
- Integrate the counterterms analytically.

Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{\;\Bigg|$$



$$\Bigg|^2 \qquad \Bigg\}$$

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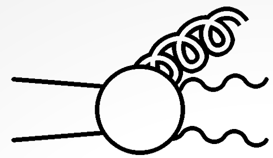
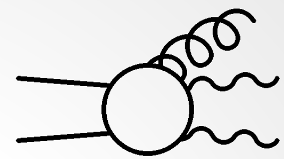
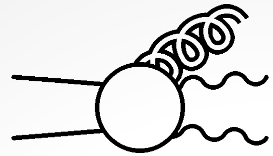


$$\Bigg|^2 - \Bigg|$$



$$\Bigg|^2 \Bigg\}$$

Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{\ \Bigg|$$

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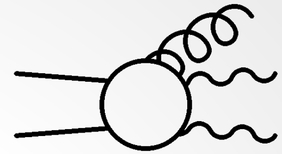
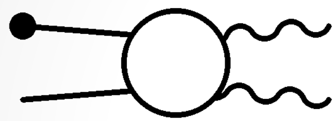
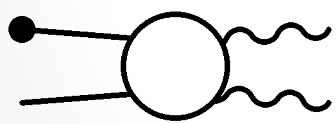
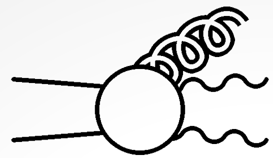

$$\Bigg|^2$$

$$-\ \frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2$$

$$+\ \frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2 \qquad \Bigg\}$$

Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{\ \Bigg|$$

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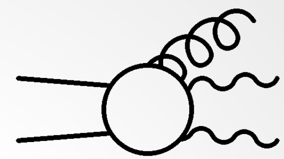
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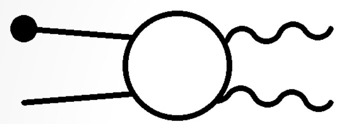
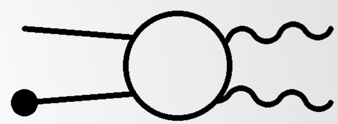
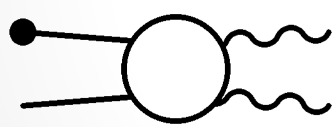
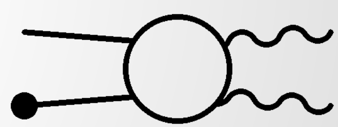
$$-\ \frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2 -\ \frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$

$$\Bigg|^2$$

$$+\ \frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2 +\ \frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$

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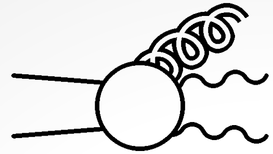
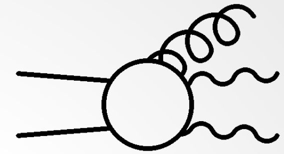
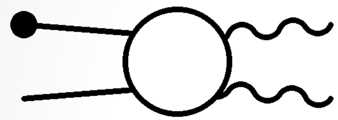
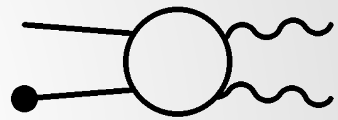
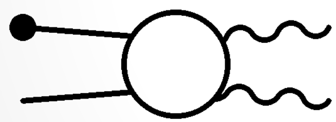
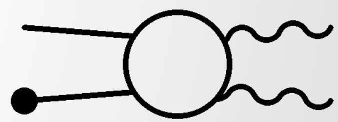
Subtraction

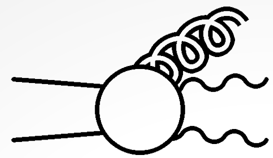
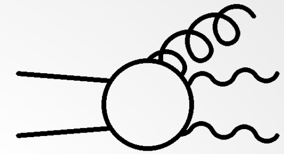
$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{\ \Bigg|$$  $$\Bigg|^2 - \Bigg|$$  $$\Bigg|^2$$

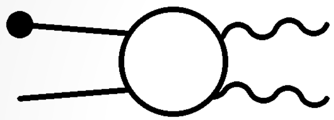
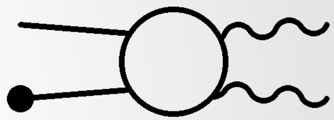
$$-\frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$  $$\Bigg|^2 - \frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$  $$\Bigg|^2$$

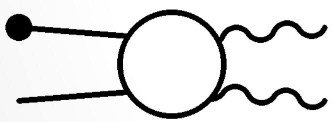
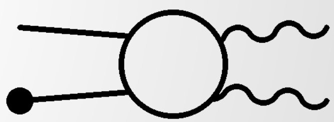
$$+\frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$  $$\Bigg|^2 + \frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$  $$\Bigg|^2\ \Bigg\}$$

- Singular limits commute → counter-terms combine in a non trivial way.

Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{\Bigg|$$

$$\Bigg|^2 - \Bigg|$$

$$\Bigg|^2$$

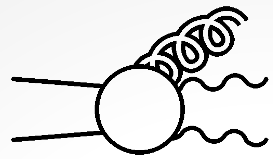
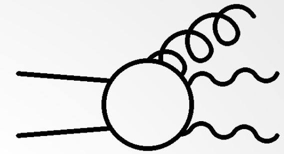
$$-\frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2 - \frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$

$$\Bigg|^2$$

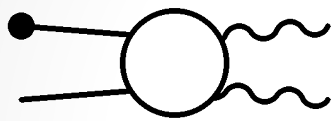
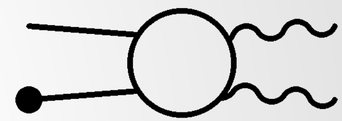
$$+\frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times\Bigg|$$

$$\Bigg|^2 + \frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Bigg|$$

$$\Bigg|^2\Bigg\}$$

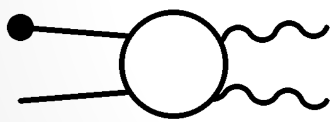
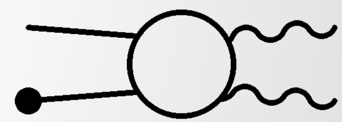
- Singular limits commute → counter-terms combine in a non trivial way.

Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{$$

$\Big|$  $\Big|^2$ $-\Big|$  $\Big|^2$

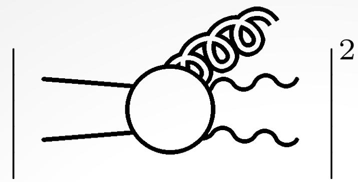
$-\frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times\Big|$  $\Big|^2$ $-\frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Big|$  $\Big|^2$

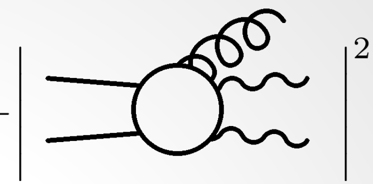
$+\frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times\Big|$  $\Big|^2$ $+\frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)}\times\Big|$  $\Big|^2\Bigg\}$

- Singular limits commute → counter-terms combine in a non trivial way.

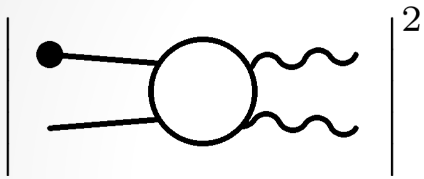
Subtraction

$$\int dz\,d\lambda\,d\rho\,\frac{(\ldots)}{1-\rho}\Bigg\{$$

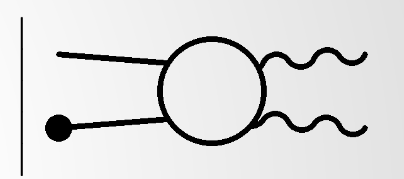


$-$ 

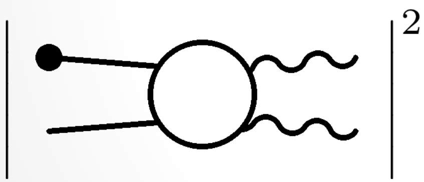
$$-\frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda}\times$$



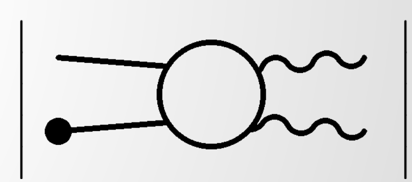
$$-\frac{S_{g^*\bar{q}}}{s(1-z)^2(1-\lambda)}\times$$



$$+\frac{S_{g\bar{q}}}{s(1-z)^2\lambda}\times$$



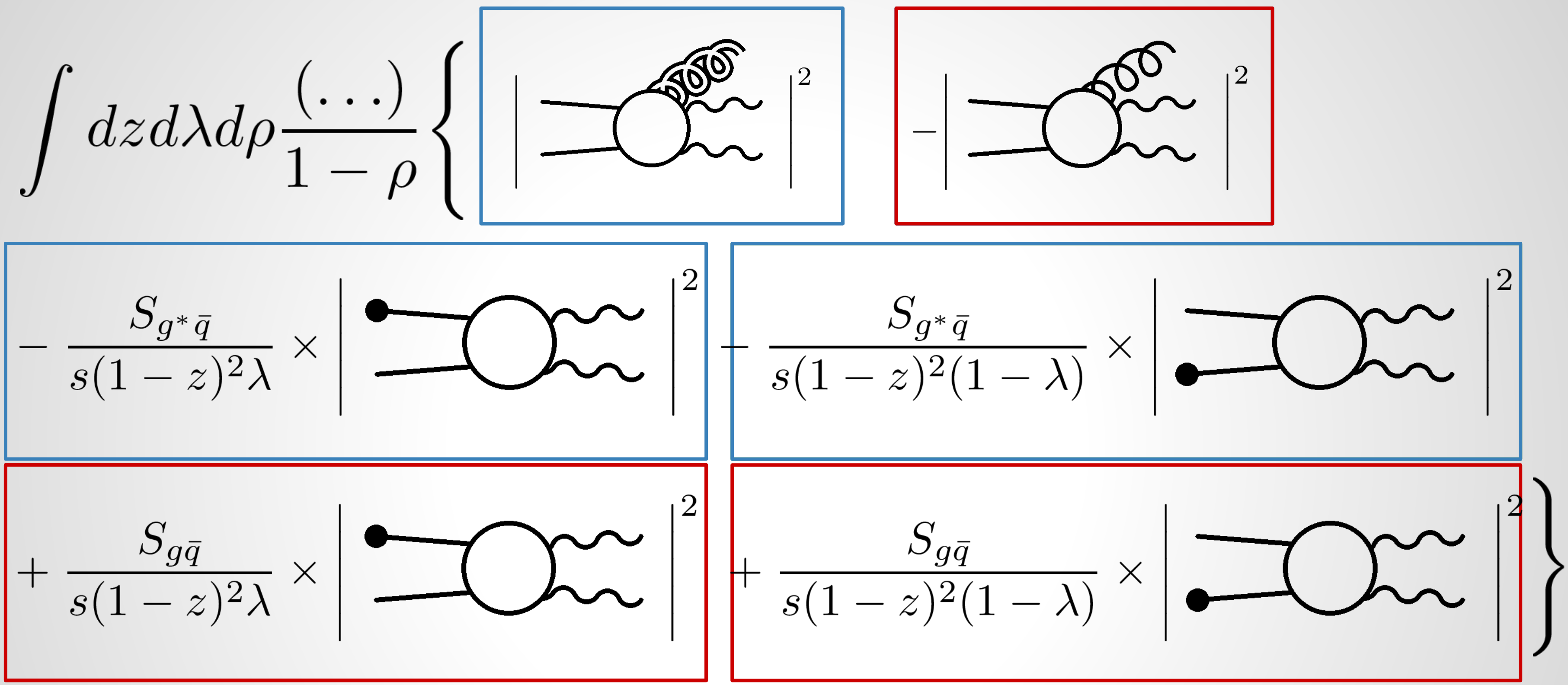
$$+\frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)}\times$$



$$\Bigg\}$$

- Singular limits commute → counter-terms combine in a non trivial way.

Subtraction

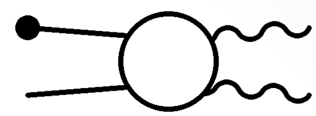


- Singular limits commute → counter-terms combine in a non trivial way.
- No explicit subtraction of the soft limit is needed.

Integrated counterterms I

The triple-collinear counterterms can be integrated analytically:

$$\int dz d\lambda d\rho \frac{(\ldots)}{1-\rho}\left\{ - \frac{S_{g^*\bar{q}}}{s(1-z)^2\lambda} \times \right|$$



$$\left|^2 \right\}$$

$$= \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\mu^2}{s}\right)^{2\epsilon} \int dz\, G^{(1)}_{qq;1}(z)\, \sigma_{LO}(zp_1, p_2)$$

For the other leg:

$$\left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\mu^2}{s}\right)^{2\epsilon} \int dz\, G^{(1)}_{qq;2}(z)\, \sigma_{LO}(p_1, zp_2)$$

- The functions G are identical for every colourless final-state.
- However: they are parameterization dependent.
- Same form as the PDF convolutions → analytic cancellation of ε-poles.

Integrated counterterms II

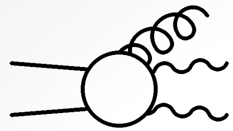
$$
\begin{aligned}
G^{(1)}_{qq;1}(z) \;&=\; \frac{C_F}{48}\Bigg\{-\frac{\delta(\bar z)}{\epsilon^3}+\frac{1}{\epsilon^2}\left[4\mathcal{D}_0(\bar z)-\frac{5}{3}\delta(\bar z)-2(1+z)\right] \\
&+\frac{1}{\epsilon}\Bigg[-16\mathcal{D}_1(\bar z)+\frac{20}{3}\mathcal{D}_0(\bar z)-\frac{1}{18}(56-21\pi^2)\delta(\bar z) \\
&-\frac{10}{3}(1+z)+8(1+z)\ln\bar z+2(1+z^2)\frac{\ln z}{\bar z}\Bigg] \\
&+32\mathcal{D}_2(\bar z)-\frac{80}{3}\mathcal{D}_1(\bar z)+\frac{2}{9}(56-21\pi^2)\mathcal{D}_0(\bar z) \\
&-\frac{1}{54}(328-105\pi^2-1116\zeta_3)\delta(\bar z) \\
&-4(1+z^2)\frac{\mathrm{Li}_2(\bar z)}{\bar z}-16(1+z)\ln^2\bar z-(1+z^2)\frac{\ln^2 z}{\bar z}-8(1+z^2)\frac{\ln z\ln\bar z}{\bar z} \\
&+\frac{40}{3}(1+z)\ln\bar z+\frac{10}{3}(1+z^2)\frac{\ln z}{\bar z} \\
&-\frac{1}{9}(38+74z-21\pi^2(1+z))\Bigg\}+\mathcal{O}(\epsilon), \\
G^{(1)}_{qq;2}(z) \;&=\; G^{(1)}_{qq;1}(z)-\frac{C_F}{48}\left(4(1+z^2)\frac{\mathrm{Li}_2(\bar z)}{\bar z}-4\ln z-4\bar z\right)+\mathcal{O}(\epsilon).
\end{aligned}
$$

$$
\boxed{\begin{gathered}\bar z \equiv 1-z \\ \mathcal{D}_n(\bar z)=\left[\frac{\log^n(1-z)}{1-z}\right]_+\end{gathered}}
$$

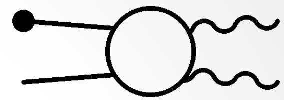
Integrated counterterms III

The final-state collinear counterterms can be integrated together

$$\int dz\, d\lambda\, d\rho \frac{(\dots)}{1-\rho}\Bigg\{ -\Bigg|$$



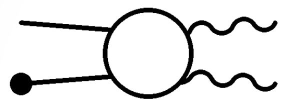
$$\Bigg|^2 + \frac{S_{g\bar{q}}}{s(1-z)^2\lambda} \times \Bigg|$$



$$\Bigg|^2$$

NLO real subtracted

$$+ \frac{S_{g\bar{q}}}{s(1-z)^2(1-\lambda)} \times \Bigg|$$



$$\Bigg|^2 \Bigg\} = -\frac{1}{6\epsilon}\left(\frac{\alpha_s}{\pi}\right)^2 \sigma_{NLO} + \tilde{\sigma}_{NLO} + \mathcal{O}(\epsilon)$$

NLO real subtracted, with modified measure:

$$\int d\Phi_{NLO} \rightarrow -\frac{1}{6}\frac{\alpha_s}{\pi} \int d\Phi_{NLO} \left(\frac{5}{3} - \log\left(\frac{s}{\mu^2} f\right)\right)$$

In summary:

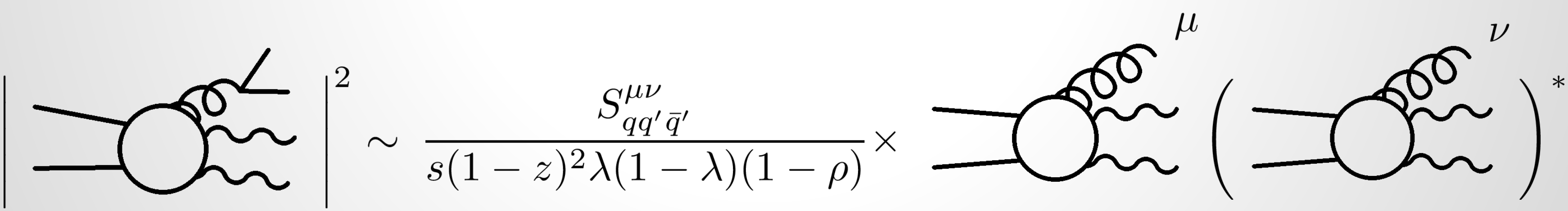
- Very small number of counterterms.
- Poles can be cancelled analytically → 4-dimensional scheme!
- Universality of singular limits → valid for all colourless final-states.

Fully differential subtraction

Restore the full kinematics by extending parametrization to the final-state quarks, while keeping

$$p_{q'} + p_{\bar{q}'} = p_{g^*}$$

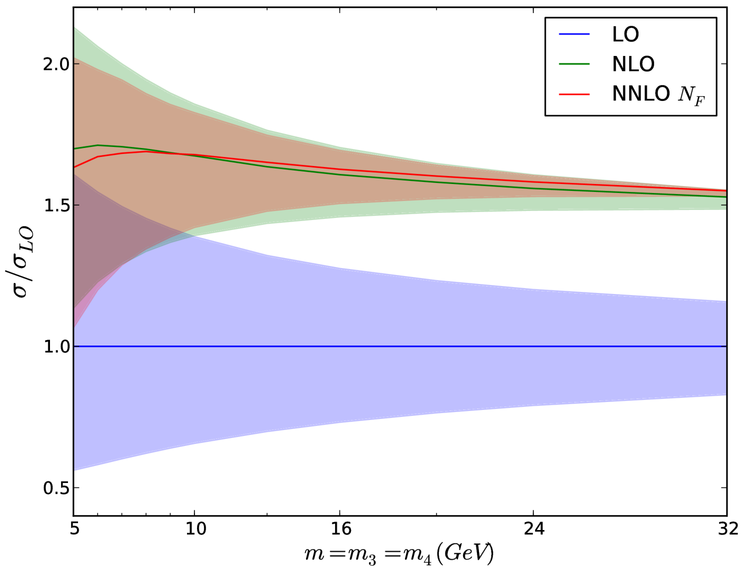
- Singularity structure remains the same
- Triple-collinear counterterms: Singular limits are slightly more complicated but factorization is identical.
- Factorization in the final-state collinear limits gets modified because of spin correlations:

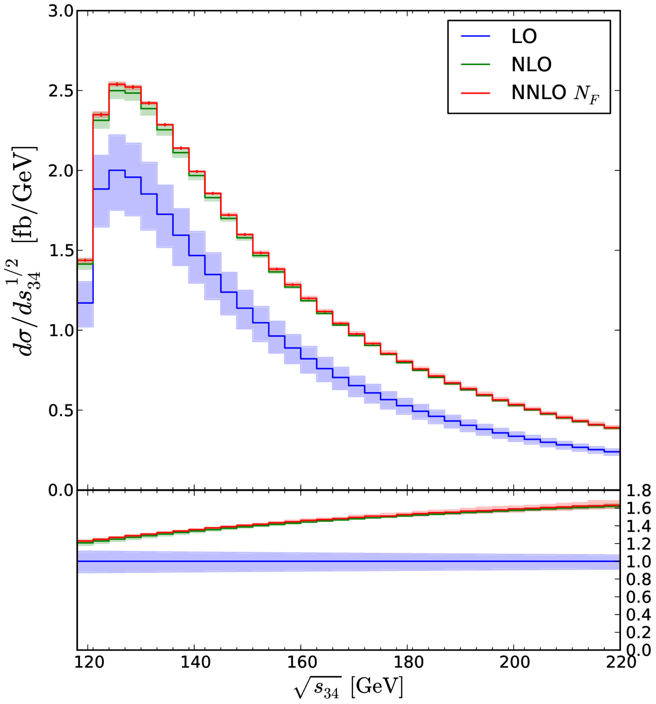


Consistent: All integrated counterterms are identical.

Results

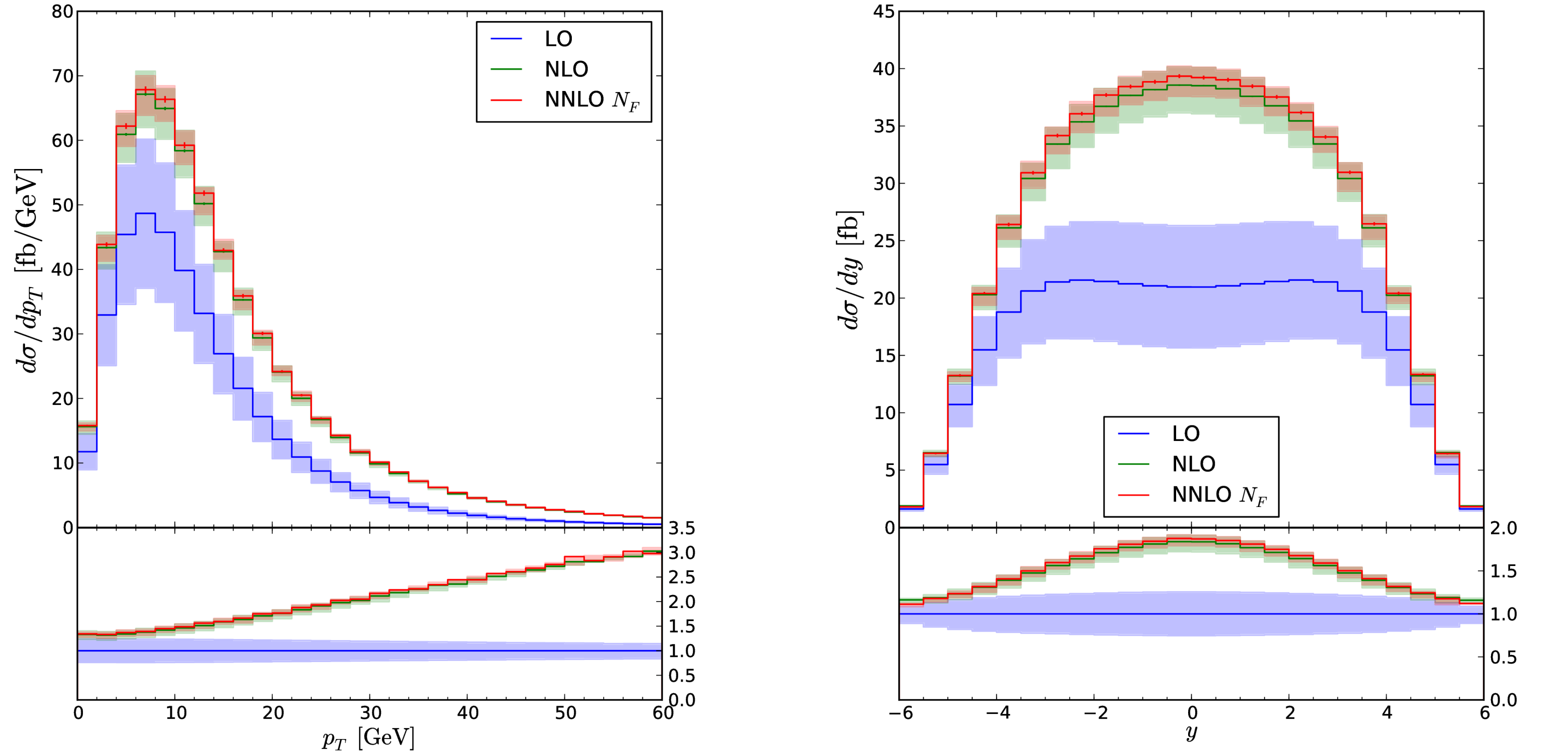
Implemented in a new Monte Carlo program (→ framework !)





- The corrections turn out to be very small (1-2%) in the large N_F limit.
- Scale variation decreases, but not drastically.

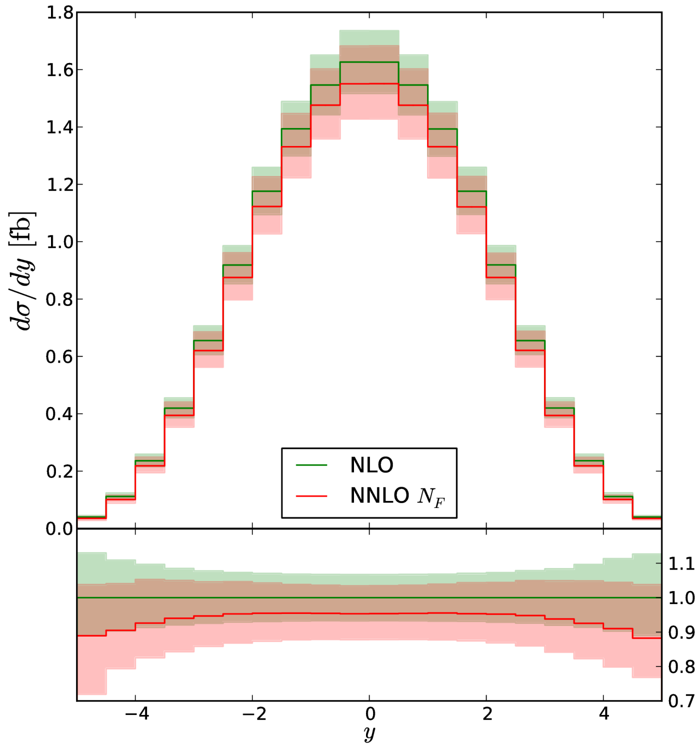
Results: Differential distributions

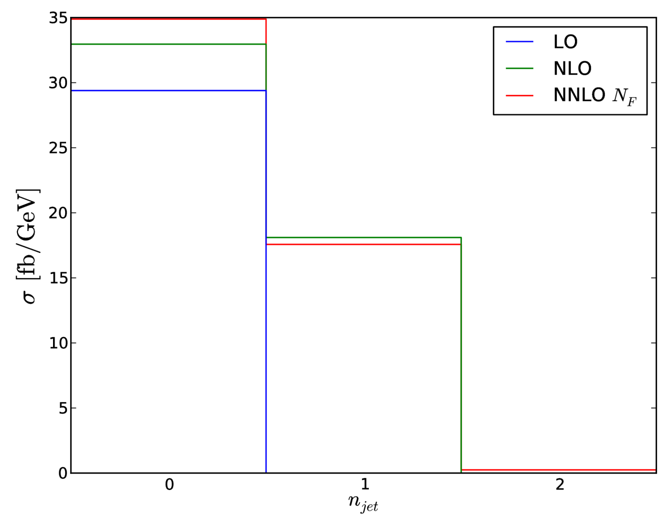


$(m_3 = 30GeV, \quad m_4 = 15GeV)$

- Negligible effect on differential distributions.
- Good convergence of the integrals, even at the differential level:
  - ➢ You get disgusting plots in ~5 min, and nice plots in ~20 min on a desktop computer.

Results: Jets





- Interestingly the N^2LO N_F piece decreases the 1-jet cross section.

Summary

We looked at a simple N^2LO computation with two massive particles in the final state (with different masses), as a means to develop analytic and numeric methods.

Double virtual:

- Understanding of the analytic structure a priori, allows to identify the natural space of functions in which our master integrals are expressible.
- Can be extended to basically any class of master integral, but construction of the basis becomes increasingly complicated.

Double real:

- Fully differential subtraction with low number of counterterms.
- Analytic integration of counterterms $\mapsto$ 4-dimensional scheme.
- However, does not face the most challenging issues of double real computations…

Thank you for your attention