TOWARDS DIFFERENTIAL TOP PAIR PRODUCTION AT NNLO

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Based on work done in collaboration with:
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Top Pair Production At The LHC

- Interesting signal. Rich phenomenology. Important in new physics searches, PDF fits, …

- Top quark pairs are copiously produced at the LHC

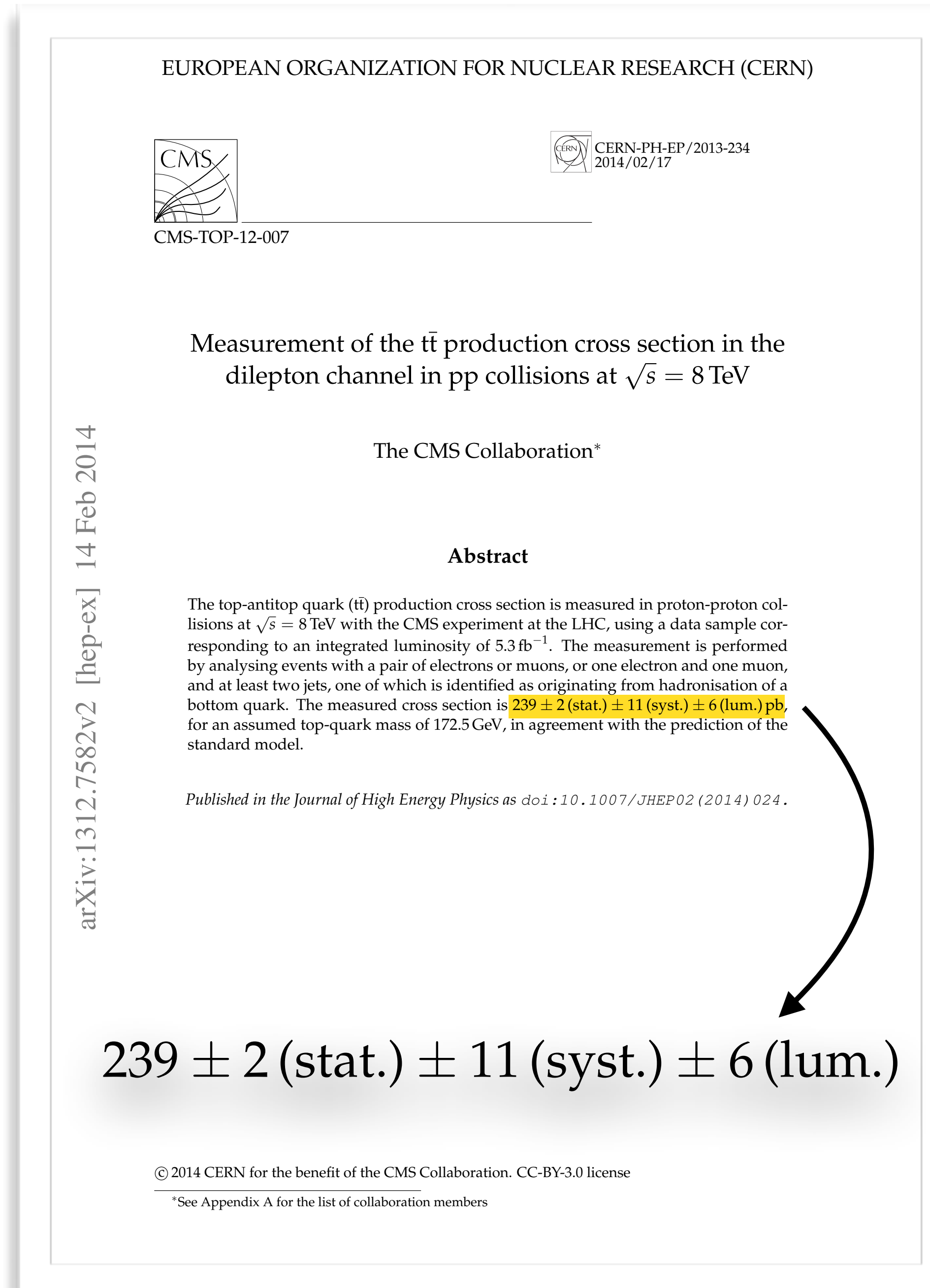
$$\sigma_{t\bar{t}+X}(\sqrt{s} = 7\,\text{TeV}) \sim 170\,\text{pb}$$

$$\sigma_{t\bar{t}+X}(\sqrt{s} = 8\,\text{TeV}) \sim 250\,\text{pb}$$

$$\sigma_{t\bar{t}+X}(\sqrt{s} = 14\,\text{TeV}) \sim 950\,\text{pb}$$

- Abundant statistics. Expected experimental error ~5%

- Need theoretical predictions with similar accuracy
  - Requires computations through higher orders in perturbation theory


EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-PH-EP/2013-234
2014/02/17

CMS-TOP-12-007

Measurement of the $t\bar{t}$ production cross section in the dilepton channel in pp collisions at $\sqrt{s} = 8$ TeV

The CMS Collaboration*

Abstract

The top-antitop quark ($t\bar{t}$) production cross section is measured in proton-proton collisions at $\sqrt{s} = 8$ TeV with the CMS experiment at the LHC, using a data sample corresponding to an integrated luminosity of 5.3 fb^{-1}. The measurement is performed by analysing events with a pair of electrons or muons, or one electron and one muon, and at least two jets, one of which is identified as originating from hadronisation of a bottom quark. The measured cross section is $239 \pm 2\,(\text{stat.}) \pm 11\,(\text{syst.}) \pm 6\,(\text{lum.})$ pb, for an assumed top-quark mass of 172.5 GeV, in agreement with the prediction of the standard model.

Published in the Journal of High Energy Physics as doi:10.1007/JHEP02(2014)024.

arXiv:1312.7582v2 [hep-ex] 14 Feb 2014

$$239 \pm 2\,(\text{stat.}) \pm 11\,(\text{syst.}) \pm 6\,(\text{lum.})$$

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*See Appendix A for the list of collaboration members

Top Pair Production At The LHC: State Of The Art

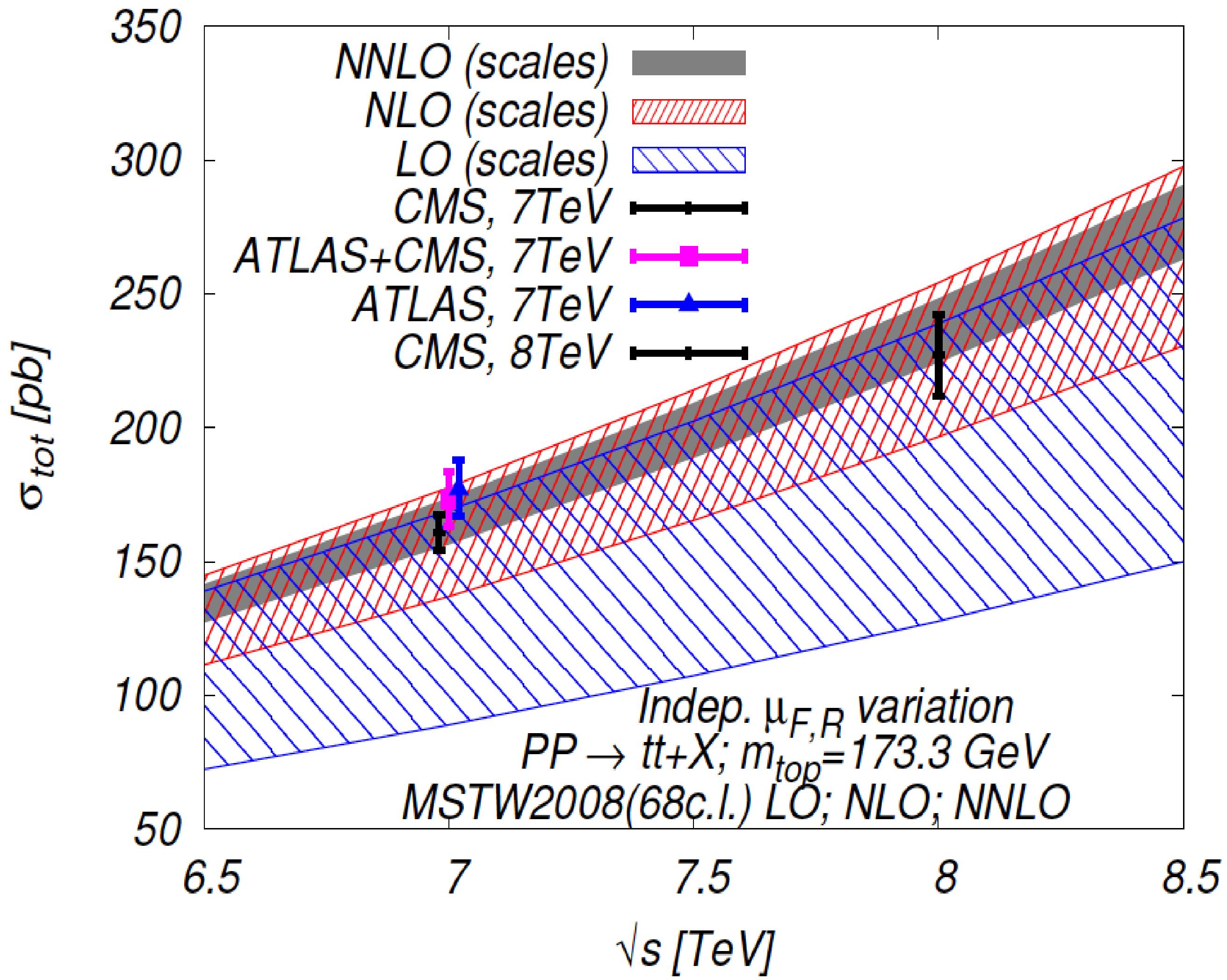
- **NLO QCD corrections:** Ellis, Dawson, Nason; Beenakker, Kuijf, van Neerven, Smith '89
- **NLO EW corrections:** Beenakker, Bernreuther, Denner, Fuecker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackenroth, Yuan
- **Threshold resummation and Coulomb corrections:** Ahrens, Banfi, Berger, Bonciani, Catani, Contopanagos, Czakon, Ferroglia, Frixione, Kidonakis, Kiyo, Kühn, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak Ridolfi, Steinhauser, Sterman, Uwer, Vogt, Yang

Yield a theoretical uncertainty of ~10%

To match theory and experimental accuracies at the LHC, cross sections for top pair production must be calculated through NNLO in pQCD

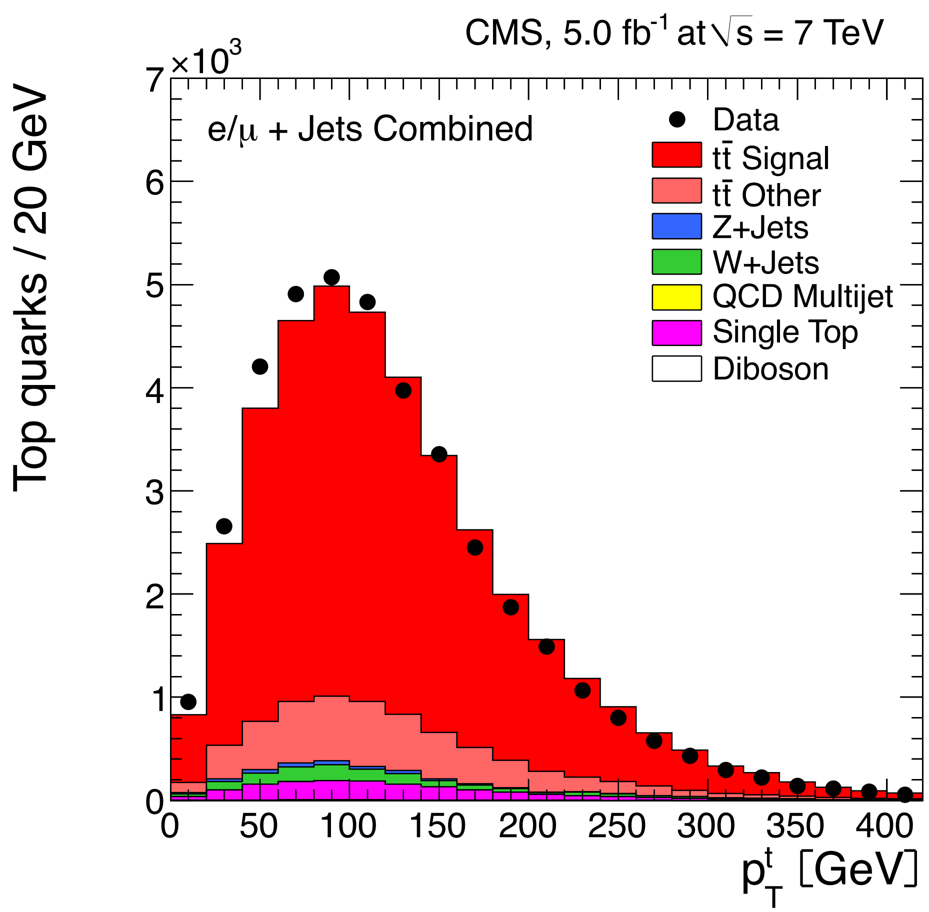
Top Pair Production At The LHC: State Of The Art

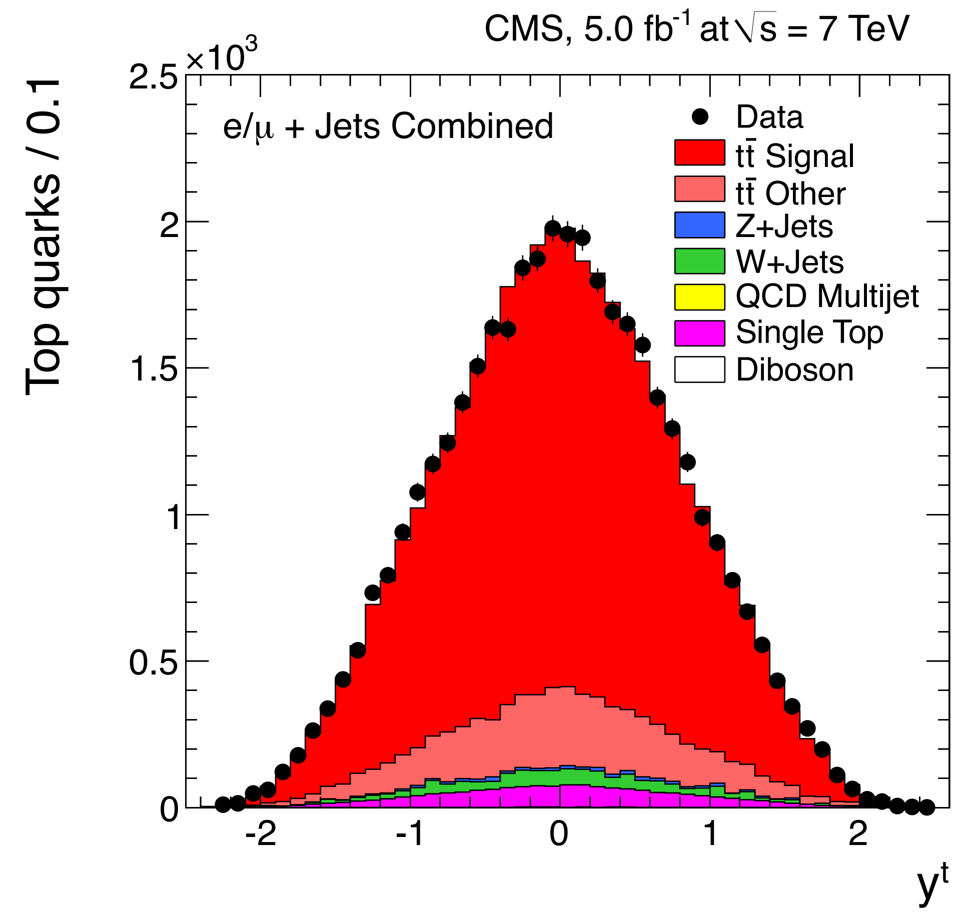
- Calculation of the total NNLO cross section completed [Czakon, Fiedler, Mitov '13]
  - Combined with NNLL resummation
  - Theoretical and experimental uncertainties of similar sizes (percent level)



Differential Top Pair Production

- Differential distributions probe the dynamics of top quark production
  - Important in order to search for new physics as deviations from SM predictions





Need NNLO predictions for $\frac{d\sigma}{dX}$ with $X = p_T^t,\, p_T^{t\bar{t}},\, y^t,\, y^{t\bar{t}},\, m_{t\bar{t}}$

Differential Top Pair Production

- Goal: fully differential event generator for $t\bar{t}$ production at NNLO

- This talk:
  - Status of our NNLO calculation for the $q\bar{q}$ channel

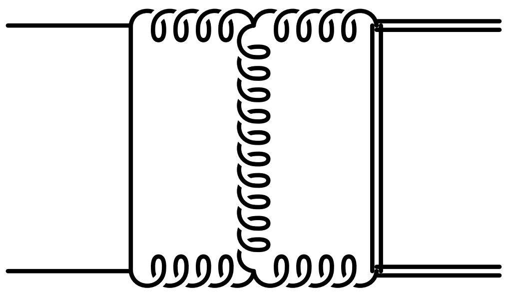
$$\mathrm{d}\hat{\sigma}_{q\bar{q},NNLO} = C_A\, C_F \Big[N_c^2 A + N_c B + C + \frac{D}{N_c} + \frac{E}{N_c^2} + N_l \Big(N_c F_l + \frac{G_l}{N_c} \Big) + N_h \Big(N_c F_h + \frac{G_h}{N_c} \Big) + N_l^2 H_l + N_l N_h H_{lh} + N_h^2 H_h \Big]$$

  - Preliminary differential distributions. N_l contributions only

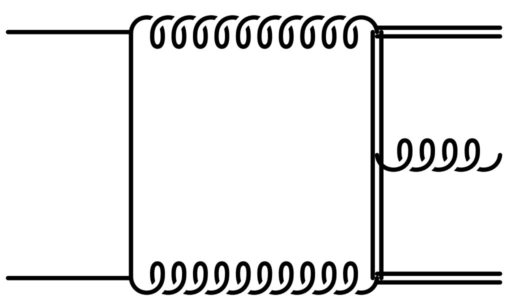
Ingredients For Top Pair Production At NNLO

- LO and NLO fully differential cross sections
  - Known [Ellis, Dawson, Nason ‘89; Beenakker, Kuijf, van Neerven, Smith ’89]
  - Re-derived using NLO antenna subtraction [GA, Gehrmann-De Ridder ’11]

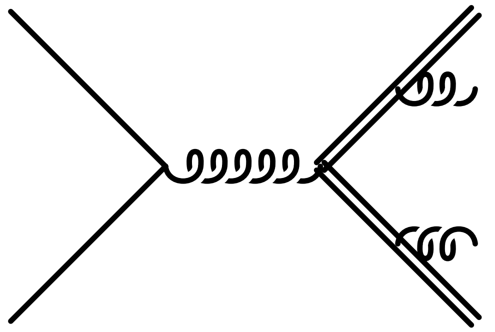
- Two-loop 2→2 matrix elements for $q\bar{q} \to t\bar{t}$
  - Use analytic results
    [Bonciani, Ferroglia, Gehrmann, Maître, v. Manteuffel, Studerus]



- One-loop 2→3 matrix elements for $q\bar{q} \to t\bar{t}g$
  - Obtained numerically with OpenLoops
    [Cascioli, Meierhöfer, Pozzorini]
    - ✓Color structure handled algebraically
    - ✓Quadruple precision evaluation in soft limit
      (P. Meierhöfer’s Talk)



- Tree-level 2→4 matrix elements for
  $q\bar{q} \to t\bar{t}gg \quad q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q}$



Ingredients For Top Pair Production At NNLO

$$\mathrm{d}\hat{\sigma}_{NNLO} = \int_{\mathrm{d}\Phi_4} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_3} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} \right) + \int_{\mathrm{d}\Phi_2} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO} \right)$$

- $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$, $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$ ⟶ explicit IR poles from loop integration
- $\int_{\mathrm{d}\Phi_4} \mathrm{d}\hat{\sigma}^{RR}_{NNLO}$, $\int_{\mathrm{d}\Phi_3} \mathrm{d}\hat{\sigma}^{RV}_{NNLO}$ ⟶ implicit IR poles from PS integration over single and double unresolved regions

Need a procedure to isolate and cancel all IR singularities, and assemble all parts in a parton-level event generator

Antenna Subtraction At NNLO

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} = &\int_{\mathrm{d}\Phi_4} \left(\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO} \right) \\
&+ \int_{\mathrm{d}\Phi_3} \left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} + \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{NNLO} \right) \\
&+ \int_{\mathrm{d}\Phi_2} \left(\mathrm{d}\hat{\sigma}^{VV}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO} + \int_1 \mathrm{d}\hat{\sigma}^{VS}_{NNLO} + \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO} \right)
\end{aligned}
$$

- Introduce double real and real-virtual subtraction terms $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$, $\mathrm{d}\hat{\sigma}^{VS}_{NNLO}$ and add them back in integrated form

- The integrated double real subtraction term is split as

$$
\int_{\mathrm{d}\Phi_4} \mathrm{d}\hat{\sigma}^{S}_{NNLO} = \int_{\mathrm{d}\Phi_3} \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{NNLO} + \int_{\mathrm{d}\Phi_2} \int_2 \mathrm{d}\hat{\sigma}^{S,2}_{NNLO}
$$

- Each PS integrand is free of explicit poles, well behaved in singular regions, and can be integrated numerically in D=4

Antenna Subtraction At NNLO

Antenna subtraction terms constructed with

- Antenna functions
  - Smoothly interpolate many unresolved limits
  - Constructed from physical matrix elements

$$X_3^0(i,j,k) = S_{ijk,IK}\frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2}$$

$$X_3^1(i,j,k) = S_{ijk,IK}\frac{|\mathcal{M}_3^1(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2} - X_3^0(i,j,k)\frac{|\mathcal{M}_2^1(I,K)|^2}{|\mathcal{M}_2^0(I,K)|^2}$$

$$X_4^0(i,j,k,l) = S_{ijkl,IL}\frac{|\mathcal{M}_4^0(i,j,k,l)|^2}{|\mathcal{M}_2^0(I,L)|^2}$$

- 3➞2 and 4➞2 on-shell momentum mappings, (different for FF, IF, II configurations)

$$\{p_i,p_j,p_k\} \to \{p_I,p_K\} \qquad \{p_i,p_j,p_k,p_l\} \to \{p_I,p_L\}$$

  - Conserve momentum in reduce matrix elements
  - Collapse to appropriate kinematics in each unresolved limit

Antenna Subtraction At NNLO

Integrated form of subtraction terms obtained with

- Phase space factorisations (different for FF, IF, II configurations). E.g. (FF)

$$\mathrm{d}\Phi_m(\ldots,p_i,p_j,p_k,\ldots) = \mathrm{d}\Phi_{m-1}(\ldots,p_I,p_K,\ldots) \times \mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k)$$

$$\mathrm{d}\Phi_m(\ldots,p_i,p_j,p_k,p_l,\ldots) = \mathrm{d}\Phi_{m-2}(\ldots,p_I,p_L,\ldots) \times \mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l)$$

- Integrated antennae. Inclusive phase space integrals

$$\mathcal{X}^{0,1}_{ijk}(\epsilon,s_{IK}) = \left(8\pi^2\,(4\pi)^{\epsilon}\,e^{\epsilon\gamma_E}\right)\int \mathrm{d}\Phi_{X_{ijk}} X^{0,1}_3(i,j,k)$$

$$\mathcal{X}^{0}_{ijkl}(\epsilon,s_{IL}) = \left(8\pi^2\,(4\pi)^{\epsilon}\,e^{\epsilon\gamma_E}\right)^2\int \mathrm{d}\Phi_{X_{ijkl}} X^{0}_4(i,j,k,l)$$

  - Phase space integrals reduced to masters
  - Master integrals evaluated directly when possible. With differential equations in external kinematic invariants otherwise

NNLO Antenna Subtraction With Massive Quarks

All tools available for processes with massless partons

- Phase space mappings [Kosower '03; Daleo, Gehrmann, Maître '07]
- Antenna functions X_3^0, X_4^0, X_3^1 [Gehrmann-De Ridder, Gehrmann, Glover '04, '05]
- Integrated antennae:
  - FF X_3^0, X_4^0, X_3^1 [Gehrmann-De Ridder, Gehrmann, Glover '05]
  - IF, II X_3^0 [Daleo, Gehrmann, Maître '07]
  - IF X_4^0, X_3^1 [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni '10]
  - II X_3^1 [Gehrmann, Monni '11]
  - II X_4^0 [Boughezal, Gehrmann-De Ridder, Ritzmann '11; Gehrmann-De Ridder, Gehrmann, Ritzmann '12]

Challenge: extend NNLO antenna subtraction method to treat massive quarks.

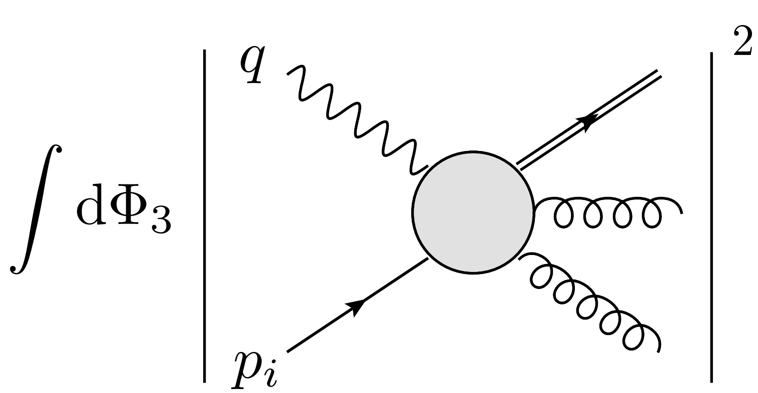
- Re-derive phase space mappings and factorizations [G.A., Gehrmann-De Ridder '11]
- Compute and integrate NLO and NNLO massive antennae. So far incomplete

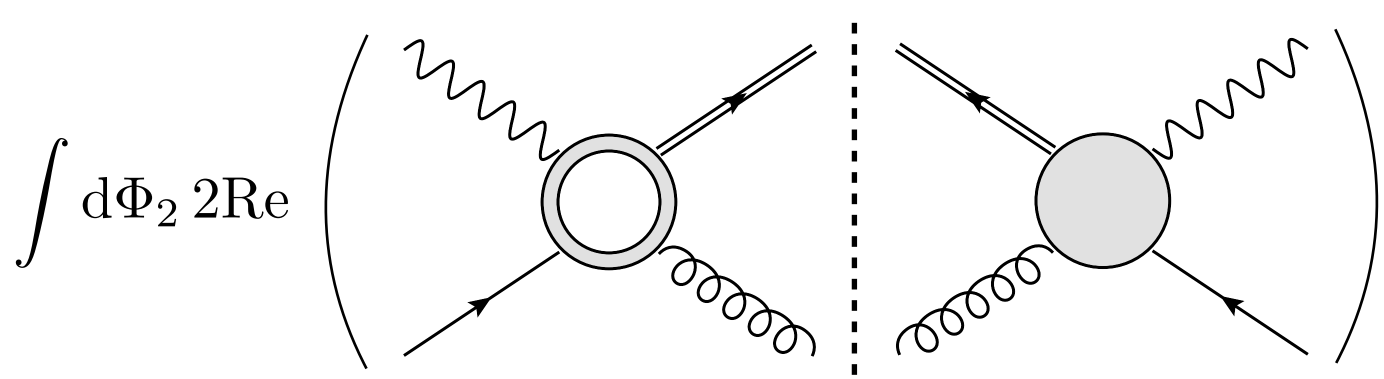
Integrated Massive Antennae

- All massive three-parton tree-level antennae (X_3^0) known
  - FF [Gehrmann-De Ridder, Ritzmann '09]
  - IF [GA, Gehrmann-De Ridder '11]

- Some four-parton antennae (X_4^0) known
  - FF: $\mathcal{A}^0_{Qgg\bar{Q}}$ $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}$ [Bernreuther, Bogner, Dekkers '11, '13]
  - IF: $\mathcal{B}^0_{q,Qq'\bar{q}'}$ $\mathcal{E}^0_{g,Qq\bar{q}}$ $\widetilde{\mathcal{E}}^0_{g,Qq\bar{q}}$ [GA, Dekkers, Gehrmann-De Ridder '12]

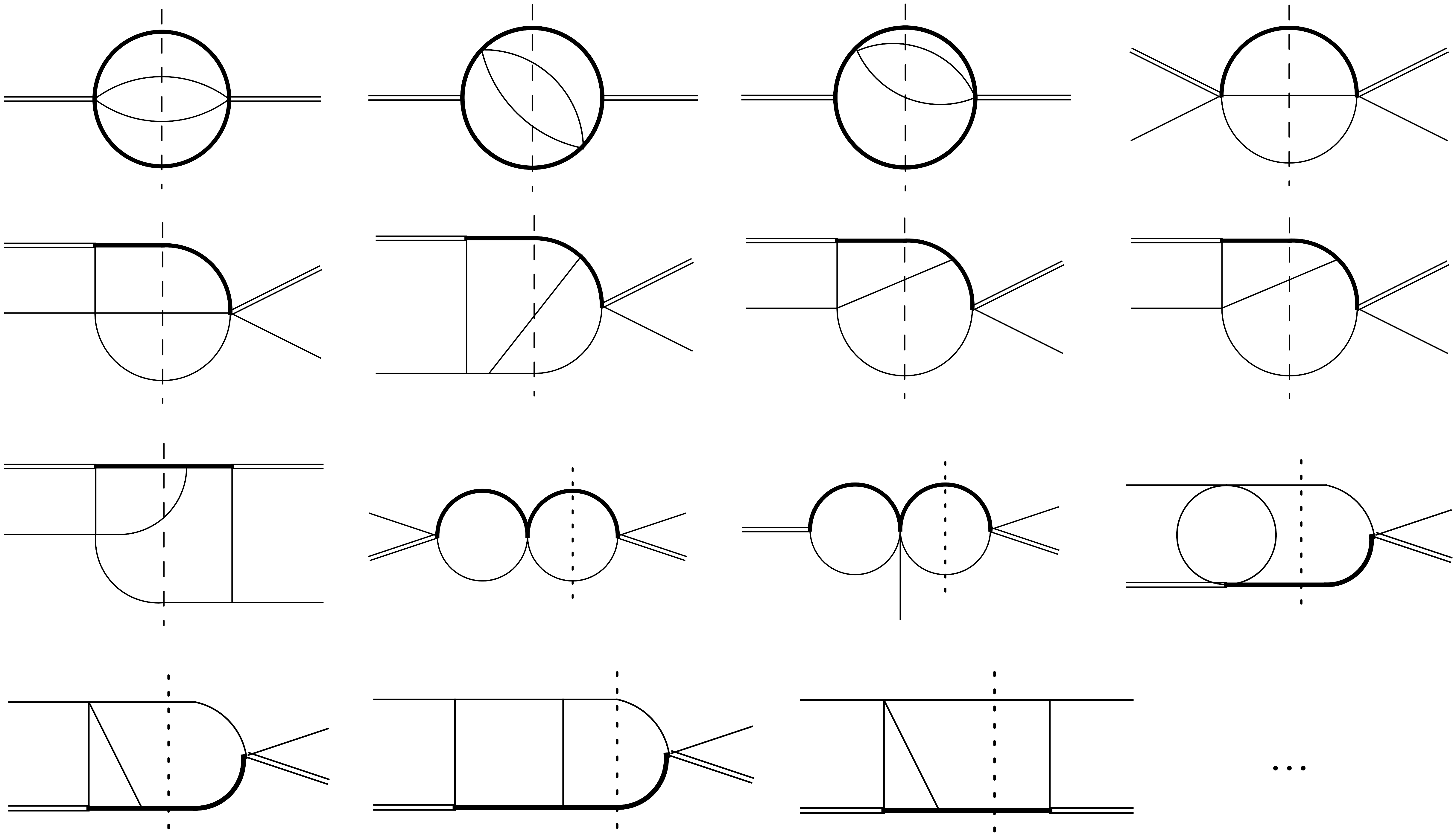
- In progress (almost there) $\mathcal{A}^0_{q,Qgg}$ $\mathcal{A}^1_{q,Qg}$





Integrated Massive Antennae

- Many new master integrals involved



Double Real Contributions

- Subtraction terms for partonic processes
  - $q\bar{q} \to t\bar{t}q'\bar{q}'$ $\quad q\bar{q} \to t\bar{t}q\bar{q}$ [GA, Gehrmann-De Ridder '11]
  - $q\bar{q} \to t\bar{t}gg$ (leading-color only) [GA, Gehrmann-De Ridder, Meierhöfer, Pozzorini '14]

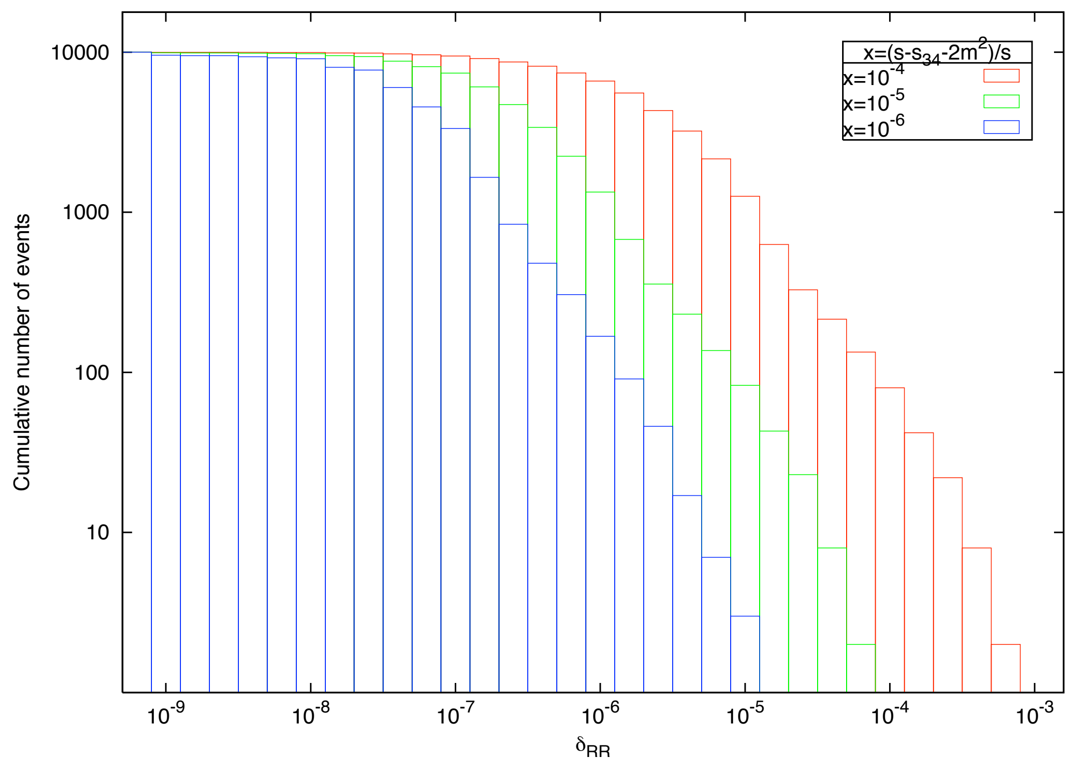
- Check of convergence
  - Generate events near every singular region
  - Control proximity to singularities with a control variable x (specific to each limit)
  - For each event, compute

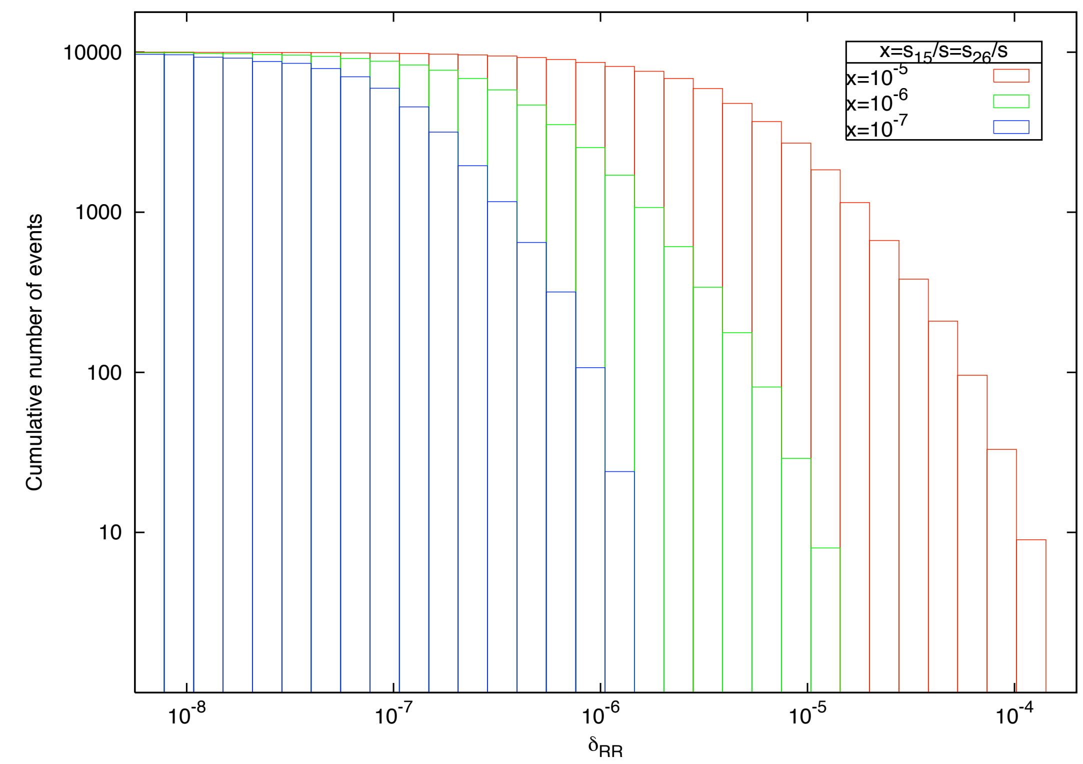
$$\delta_{RR} = \left| \frac{\mathrm{d}\hat{\sigma}^{RR}_{NNLO}}{\mathrm{d}\hat{\sigma}^{S}_{NNLO}} - 1 \right|$$

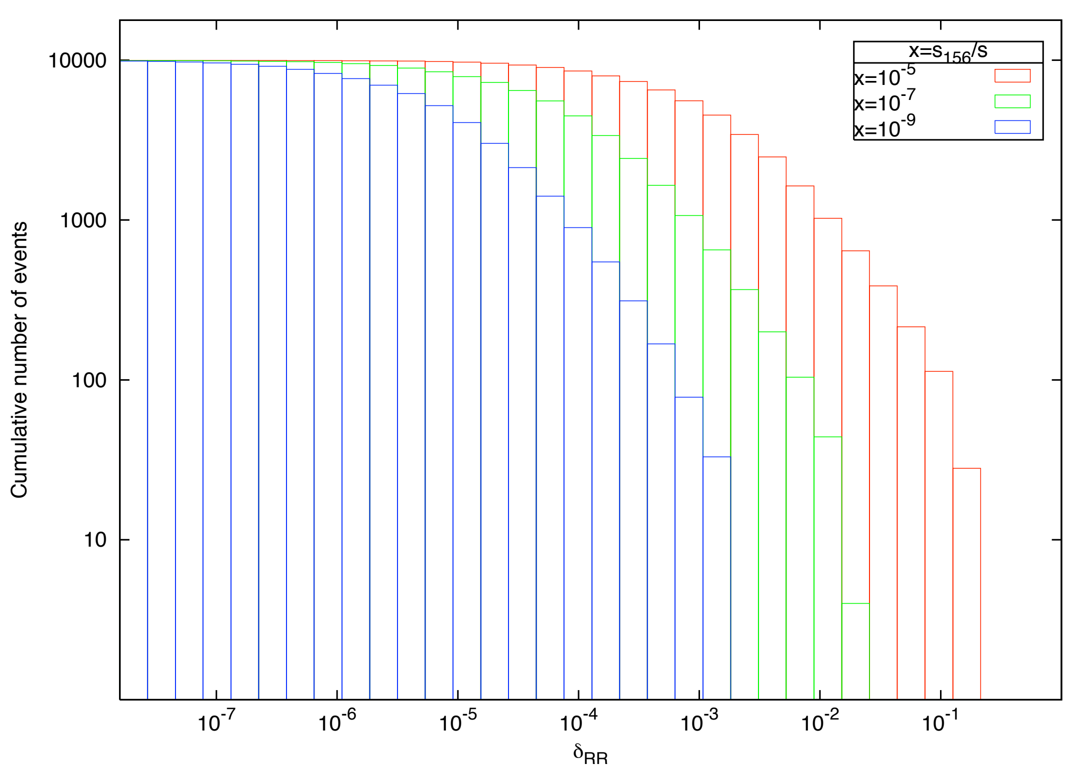
  - Convergence of $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ to $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$ observed in cumulative histograms in δ_{RR}

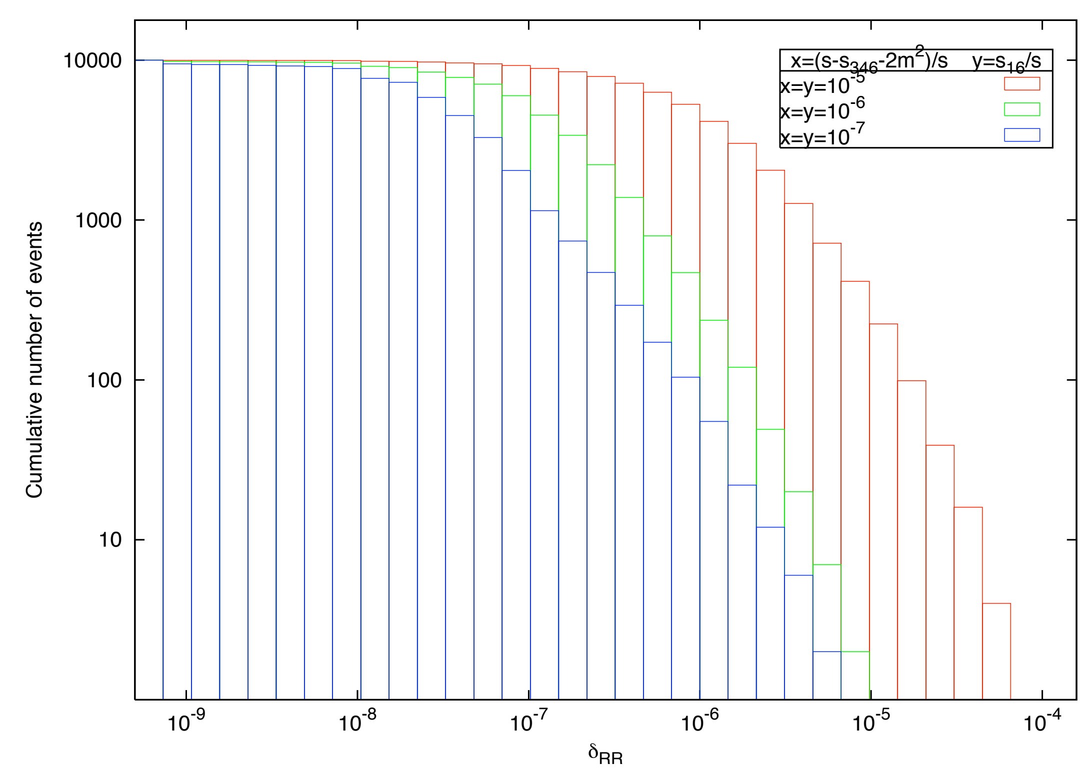
- Good convergence observed in all single and double unresolved limits

Double Real Contributions









Real Virtual Contributions

- Partonic process $q\bar{q} \to t\bar{t}g$ at one-loop
- One-loop amplitudes computed
  - Numerically with OpenLoops for leading-color contributions
  - Analytically for N_l, N_h pieces
- Subtraction terms constructed and implemented
  - Leading-color: [GA, Gehrmann-De Ridder, Meierhöfer, Pozzorini '14]
  - Fermionic contributions: [GA, Gehrmann-De Ridder (in preparation)]
- Pointwise cancellation of explicit IR poles checked analytically

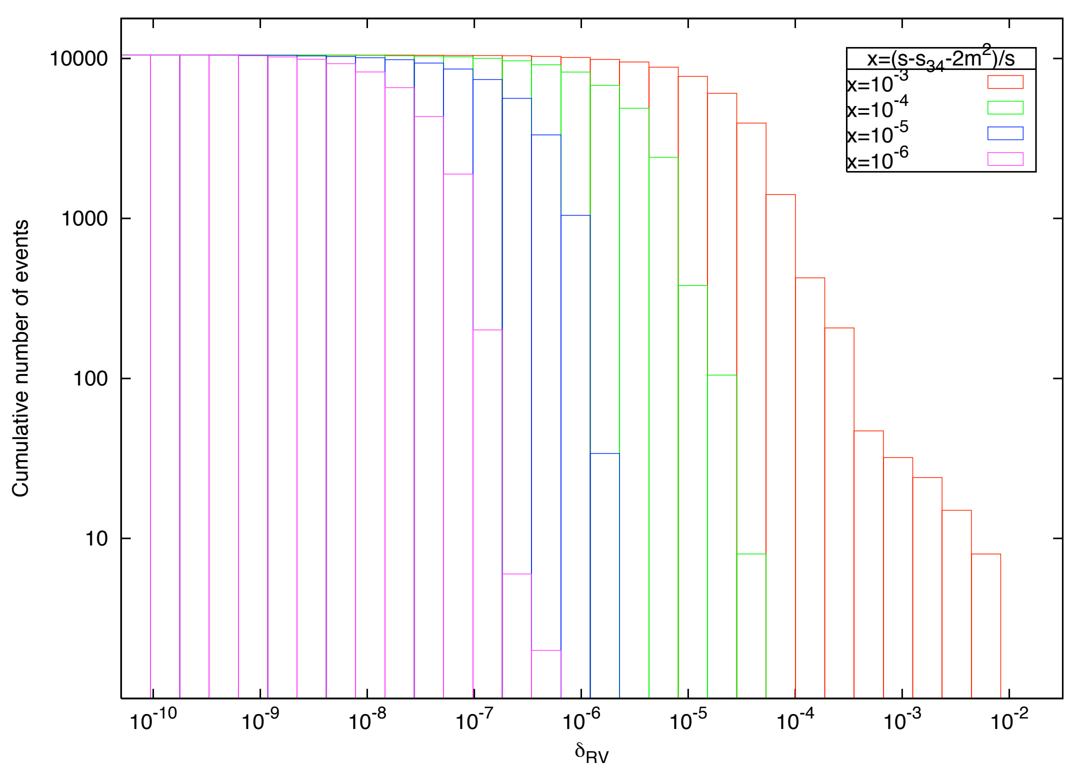
$$\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} + \int_1 \mathrm{d}\hat{\sigma}^{S,1}_{NNLO}\right) = 0$$

Real Virtual Contributions

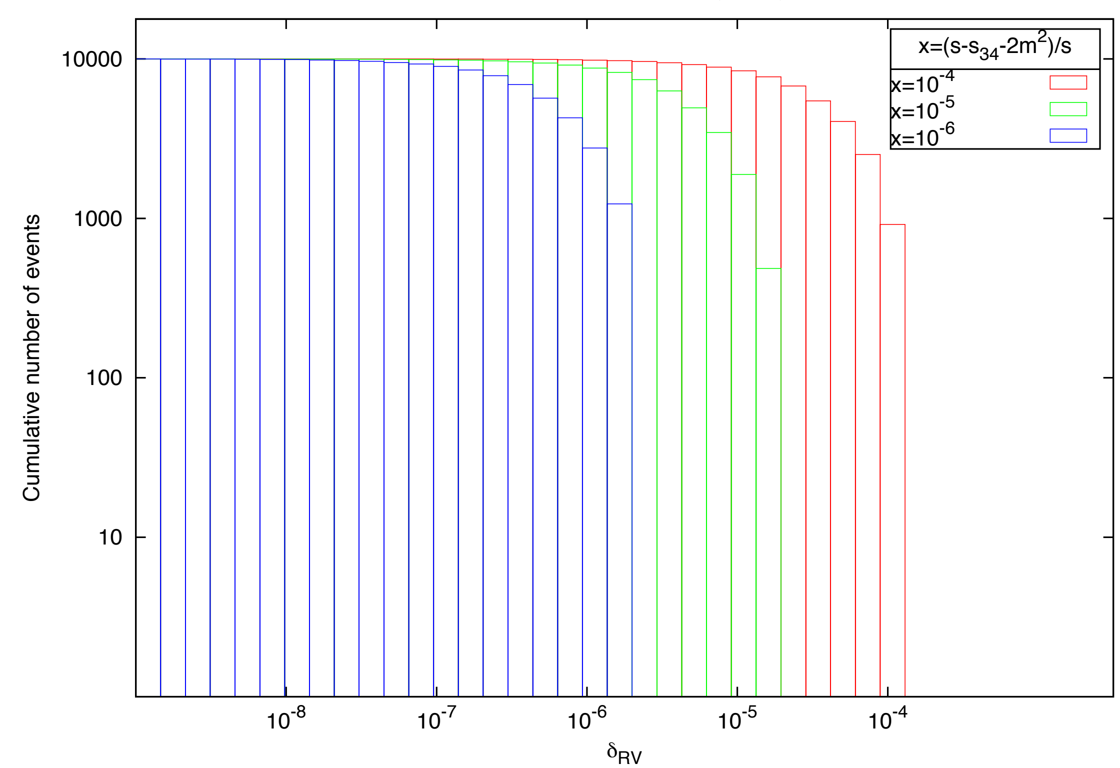
- Check of convergence. Analogous to double real check
  - Good convergence observed in fermionic pieces in soft and collinear limits
  - Good convergence observed in collinear limit of leading color piece
  - Convergence in soft limit of leading-color piece only achieved evaluating $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$ in quadruple precision

$$\delta_{RV} = \left| \frac{\mathcal{F}inite(\mathrm{d}\hat{\sigma}^{RV}_{NNLO})}{\mathcal{F}inite(\mathrm{d}\hat{\sigma}^{T}_{NNLO})} - 1 \right|$$

Soft limit (leading-color)



Soft limit (Nl)



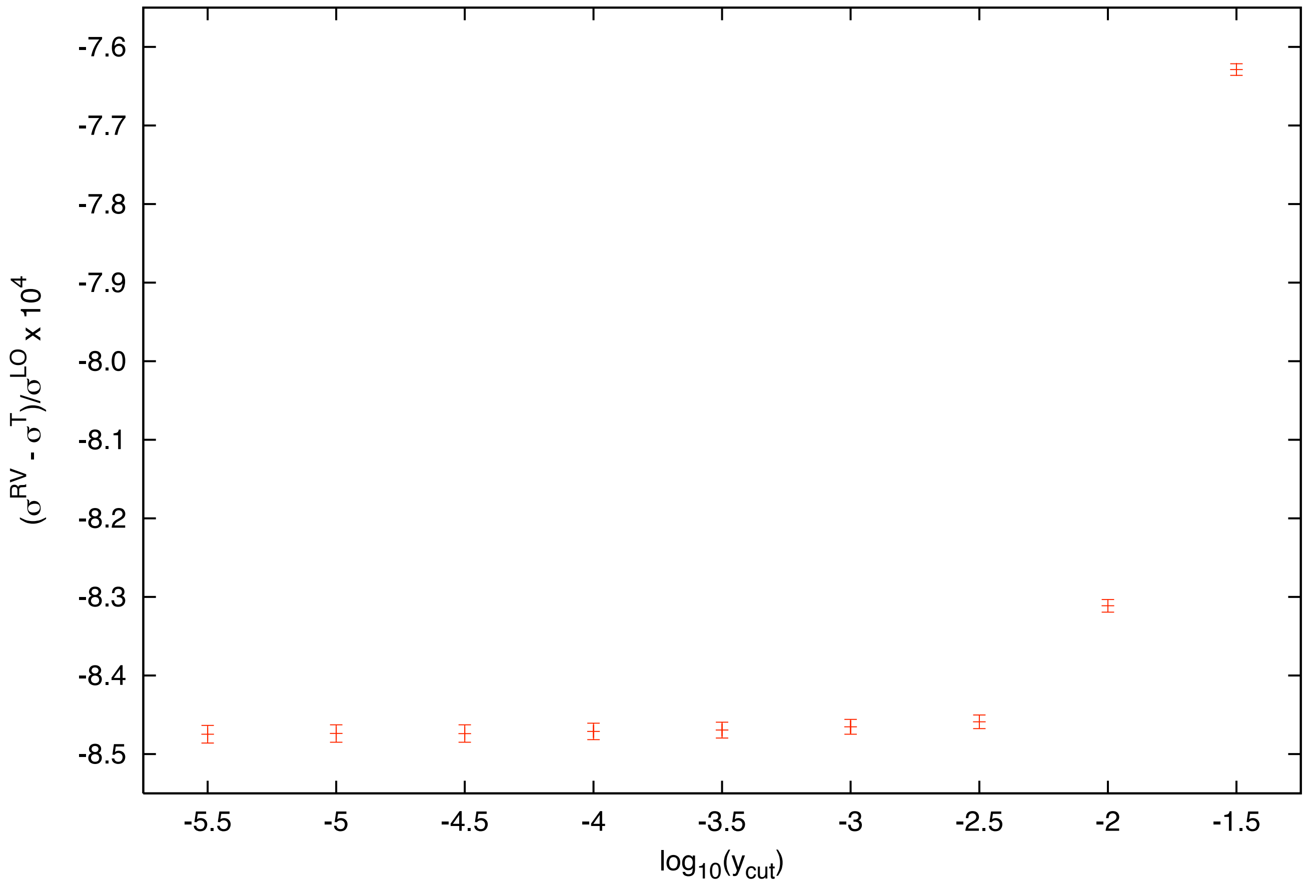
Real Virtual Contributions

Only "bad points" are (re)evaluated by OpenLoops in quadruple precision

- Fraction of quadruple precision evaluation in $\int_{d\Phi_3} \left(d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO} \right)$?
- Is the integration stable?

Stability check: Evaluate $R = \left(\sigma^{RV}_{NNLO} - \sigma^{T}_{NNLO} \right) / \sigma_{LO}$ as a function of $y_{cut} = p_T^g / \sqrt{\hat{s}}$

- Integration is stable
- R has a plateau for $y_{cut} < y_{cut}^{max} \sim 10^{-3}$
- Strong check of our subtraction terms
- We can run with $y_{cut} \sim 10^{-4}$. Only ~0.01% points require quadruple precision.
- Efficient evaluation in double precision for the vast majority of points



Double Virtual Contributions

The ultimate check:

$$\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}_{NNLO}^{VV} + \mathrm{d}\hat{\sigma}_{NNLO}^{MF,2} + \int_1 \mathrm{d}\hat{\sigma}_{NNLO}^{VS} + \int_2 \mathrm{d}\hat{\sigma}_{NNLO}^{S,2}\right) = 0$$

- Pole cancellation verified analytically in N_l piece

```
Pole1LC =
 Simplify[
  Simplify[Coefficient[TwoLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] + Coefficient[OneTimesOneLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] -
     Coefficient[SubTermLC, ep, -1]] //. RepsLogsLC]
0
```

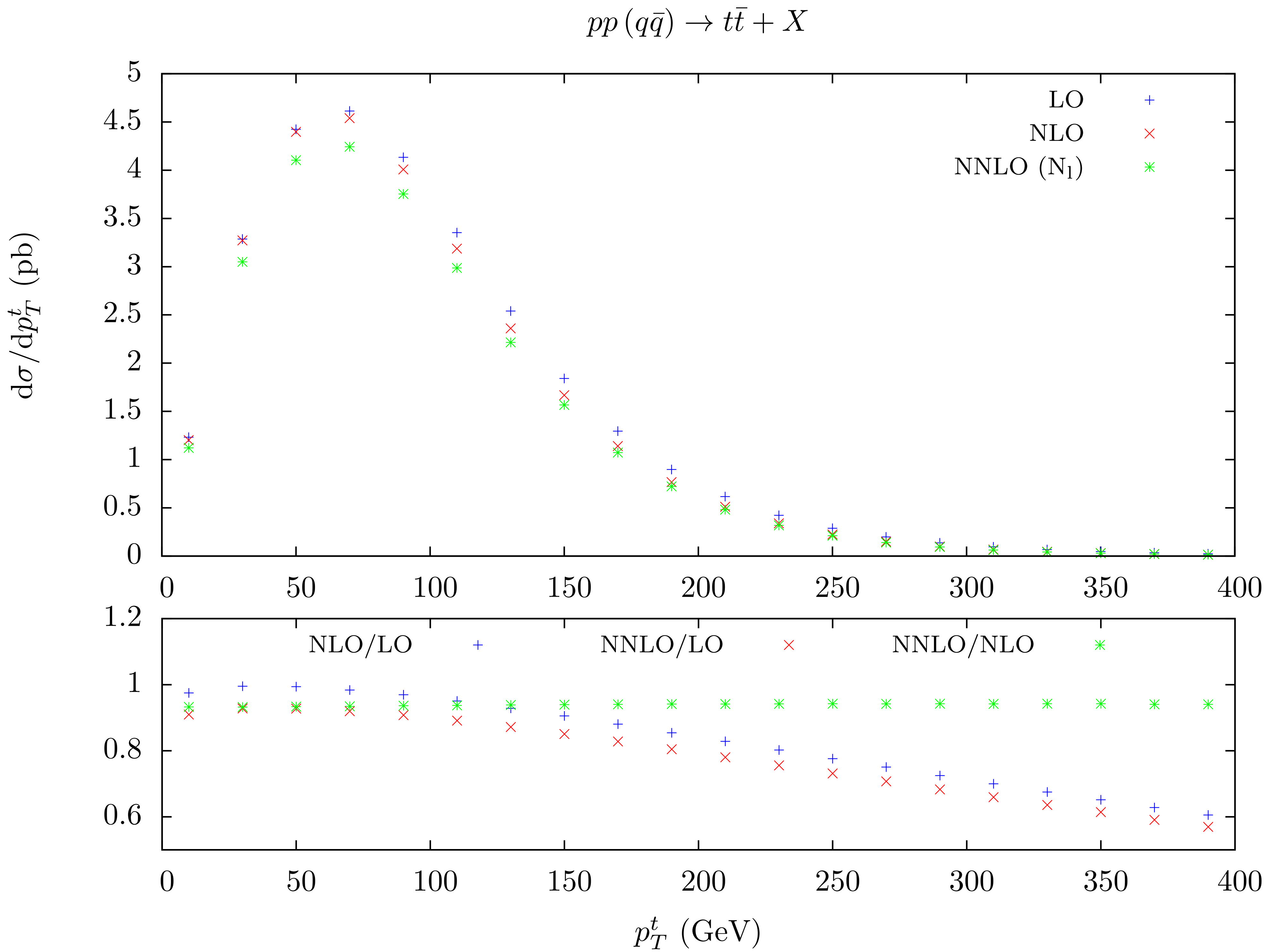
  - Non-trivial check on new integrated massive antennae
  - Proves applicability of NNLO antenna subtraction to reactions with massive fermions

- Pole cancellation in remaining color factors in progress

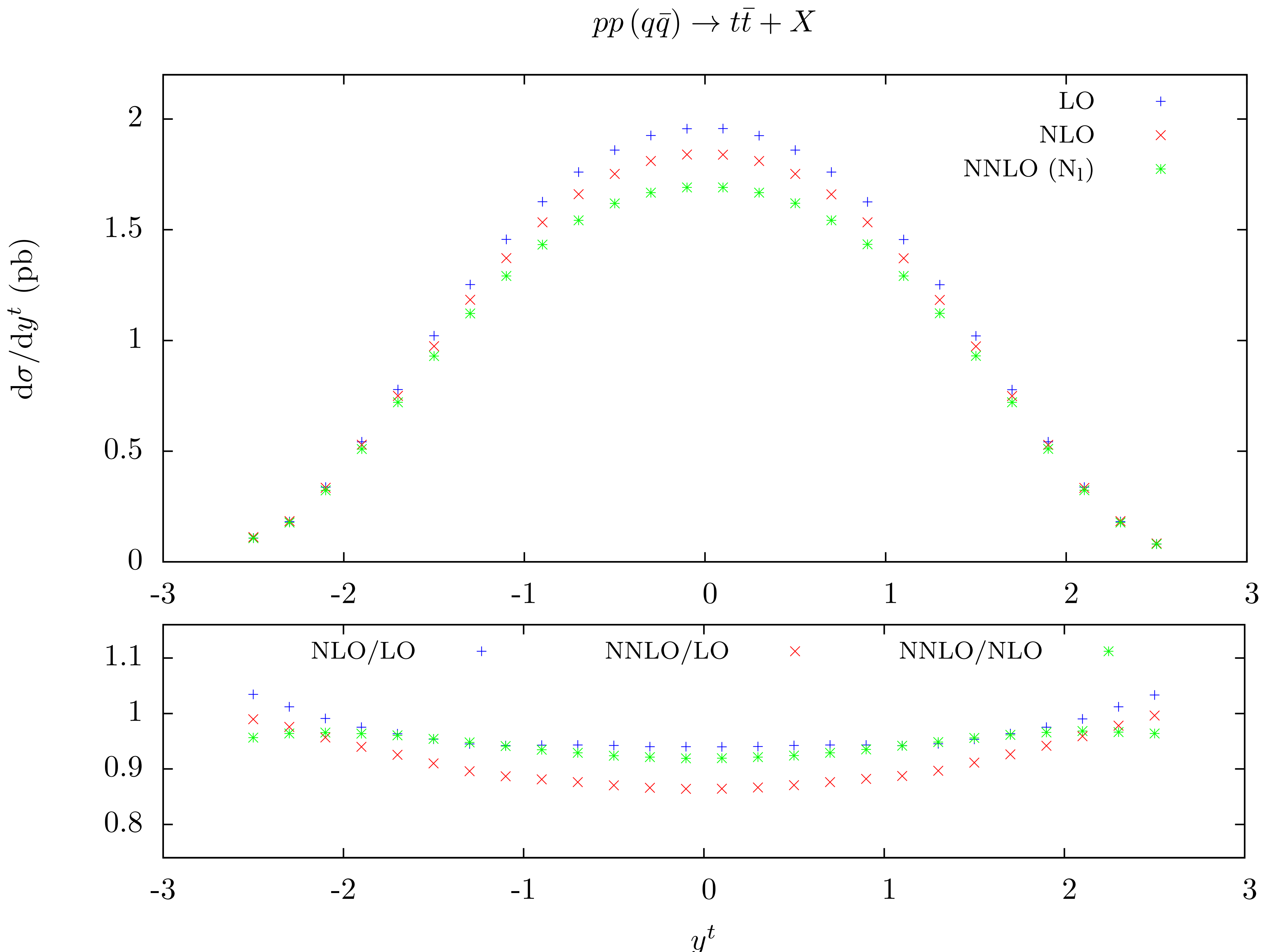
Preliminary Results

- Preliminary results for $pp(q\bar{q}) \to t\bar{t} + X$ (N_l only)
  - $\sqrt{s} = 7\,\mathrm{TeV}$
  - $m_{top} = 173.5\,\mathrm{GeV}$
  - $\mu = m_{top}$
  - PDF sets: MSTW2008LO, MSTW2008NLO, MSTW2008NNLO

Preliminary Results

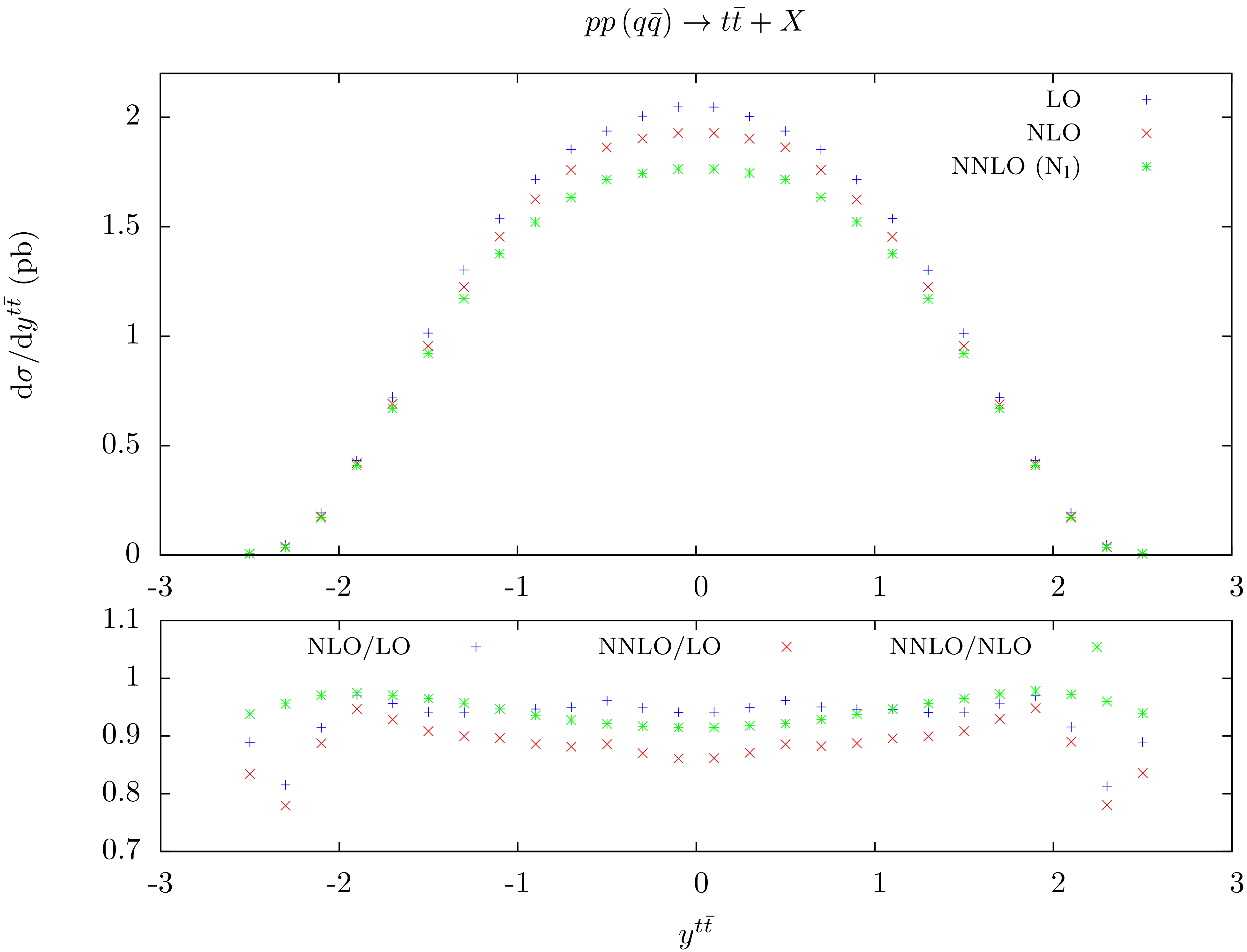


Preliminary Results

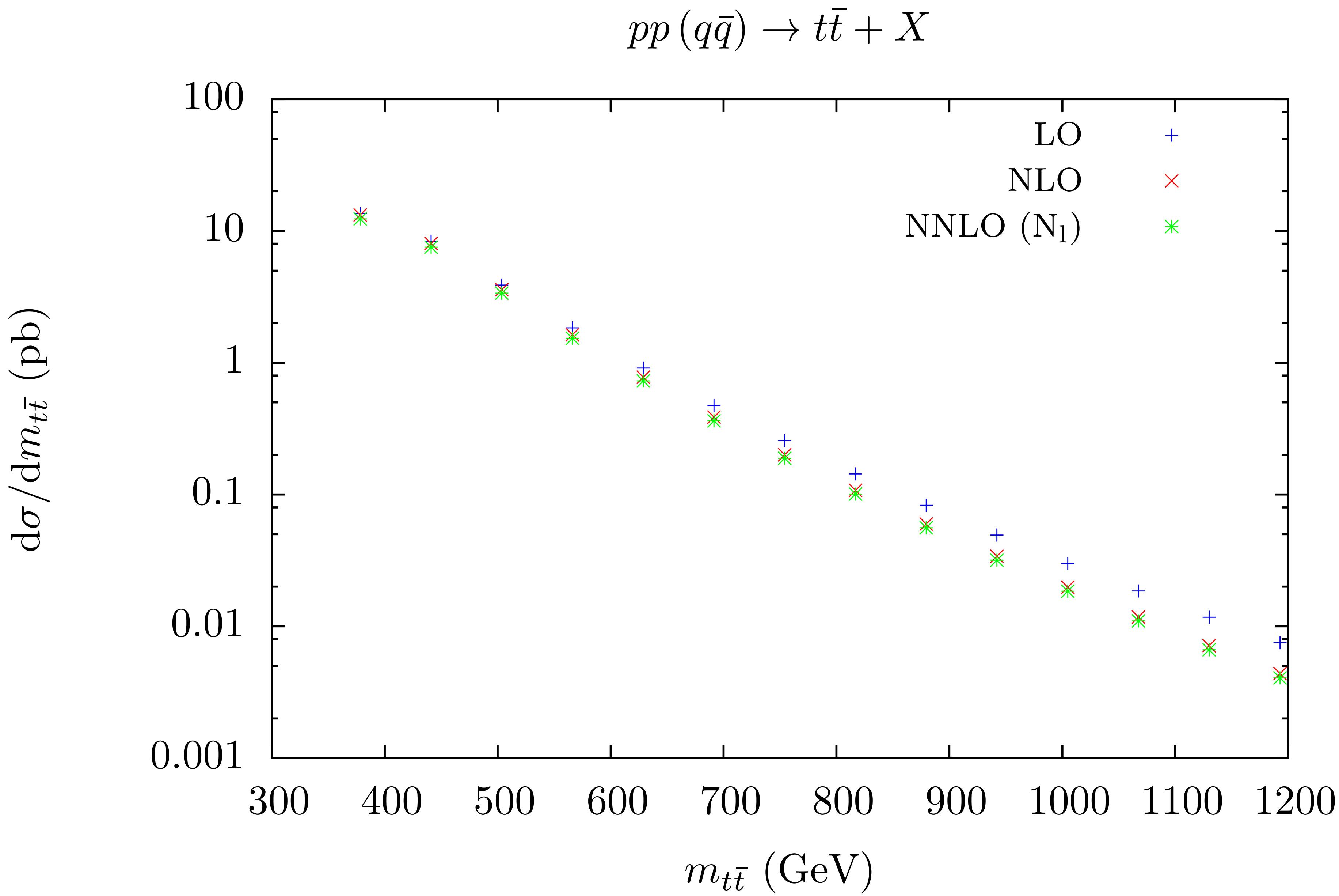


NNLO effects on A_{FB}?

Preliminary Results



Preliminary Results



Summary And Outlook

- Fully differential NNLO calculation for $t\bar{t}$ production in the $q\bar{q}$ channel within reach (leading-color + fermionic contributions)
- Double real contributions: subtraction terms implemented and tested
- Real-virtual contributions:
  - Subtraction terms implemented and tested
  - Precise and stable one-loop amplitudes from OpenLoops in leading-color part
- Double virtual contributions:
  - Two-loop amplitudes available (for leading color and fermionic pieces)
  - Analytic pole cancelation in N_l part
- Event generator implemented and working (with N_l part for the moment)

Outlook

- Complete leading-color double virtual contributions in $q\bar{q}$ channel
- Phenomenology in $q\bar{q}$ channel. A_{FB}, main goal
- Include gg and qg channels