Where do we go from the SM?

One good answer: The Next-to-Minimal Supersymmetric Model with a SM-like Higgs of mass $\sim \mathbf{100}\ \mathrm{GeV}$.

Jack Gunion
U.C. Davis

GGI, Florence, June 7, 2006

Based largely on:
R. Dermisek and J. Gunion, hep-ph/0510322
R. Dermisek and J. Gunion, hep-ph/0502105
J. Gunion, D. Hooper and B. McElrath, hep-ph/0509024

See also: J. Gunion, D. Miller, A. Pilaftsis, forthcoming CPNSH (CP-violating and Non-Standard Higgses) CERN Yellowbook Report.

Outline

1. **Reality is coming.**

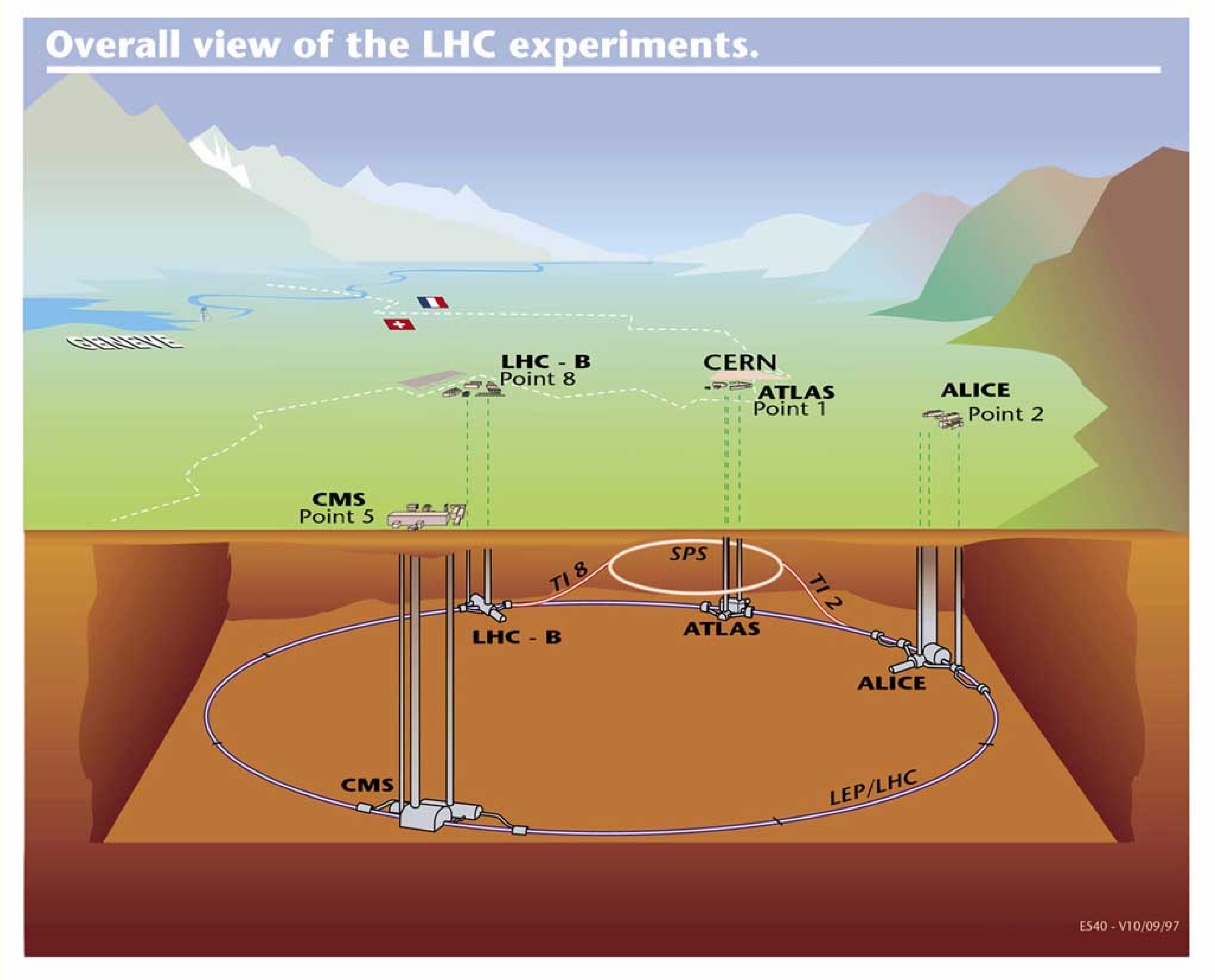
2. **SUSY solves the hierarchy problem.**

3. **The Minimal SUSY Model (MSSM) is very attractive, but LEP limits on the lightest Higgs and the gluino imply that it is in a fine-tuned part of parameter space.**

4. **The Next to Minimal Supersymmetric Model (NMSSM) maintains all the attractive features of the MSSM while avoiding fine tuning, especially if $m_{h_1} \sim 100~\mathrm{GeV}$, as preferred by LEP data (precision and direct search).**

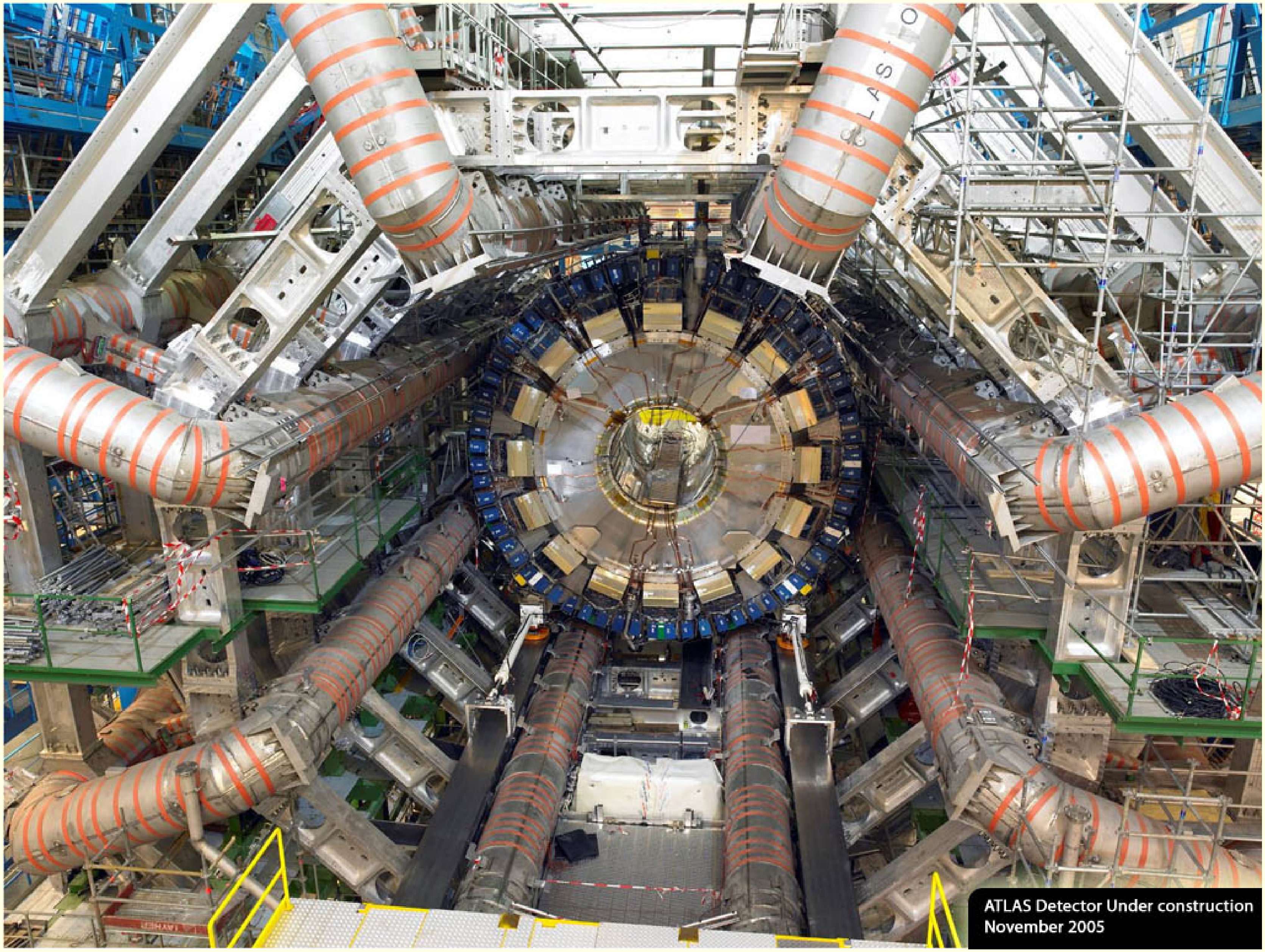
5. **Low-fine-tuning NMSSM models change how to search for the Higgs at the LHC and imply that one should look again at the LEP data for $h \to aa$ Higgs signals.**

Reality is at hand

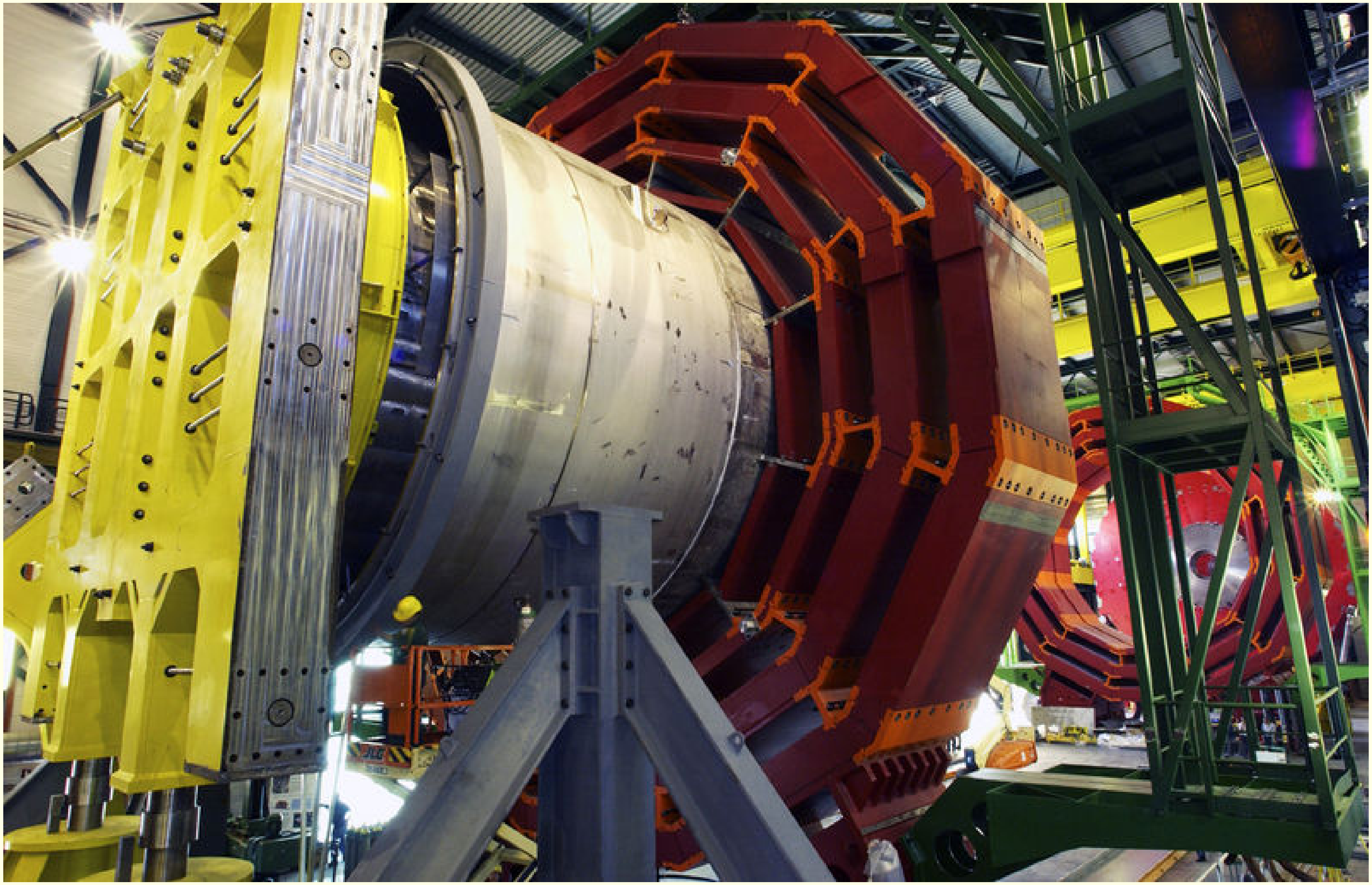


The Tunnel

The Magnets



The ATLAS Detector



The CMS Detector

So shouldn't we get real!

The Beauty of Supersymmetry

- **SUSY is mathematically intriguing.**
- **SUSY is naturally incorporated in string theory.**
- **Scalar fields have a natural place in SUSY, and so there are candidates for the spin-0 fields needed for electroweak symmetry breaking and Higgs bosons.**
- **SUSY cures the naturalness / hierarchy problem provided the SUSY breaking scale is of order $\sim 1~\mathrm{TeV}$.**
- **The MSSM comes close to being very nice.**

  If we assume that all sparticles reside at the $\mathcal{O}(1~\mathrm{TeV})$ scale and that μ is also $\mathcal{O}(1~\mathrm{TeV})$, then, the MSSM has two particularly wonderful properties.

1. Gauge Coupling Unification

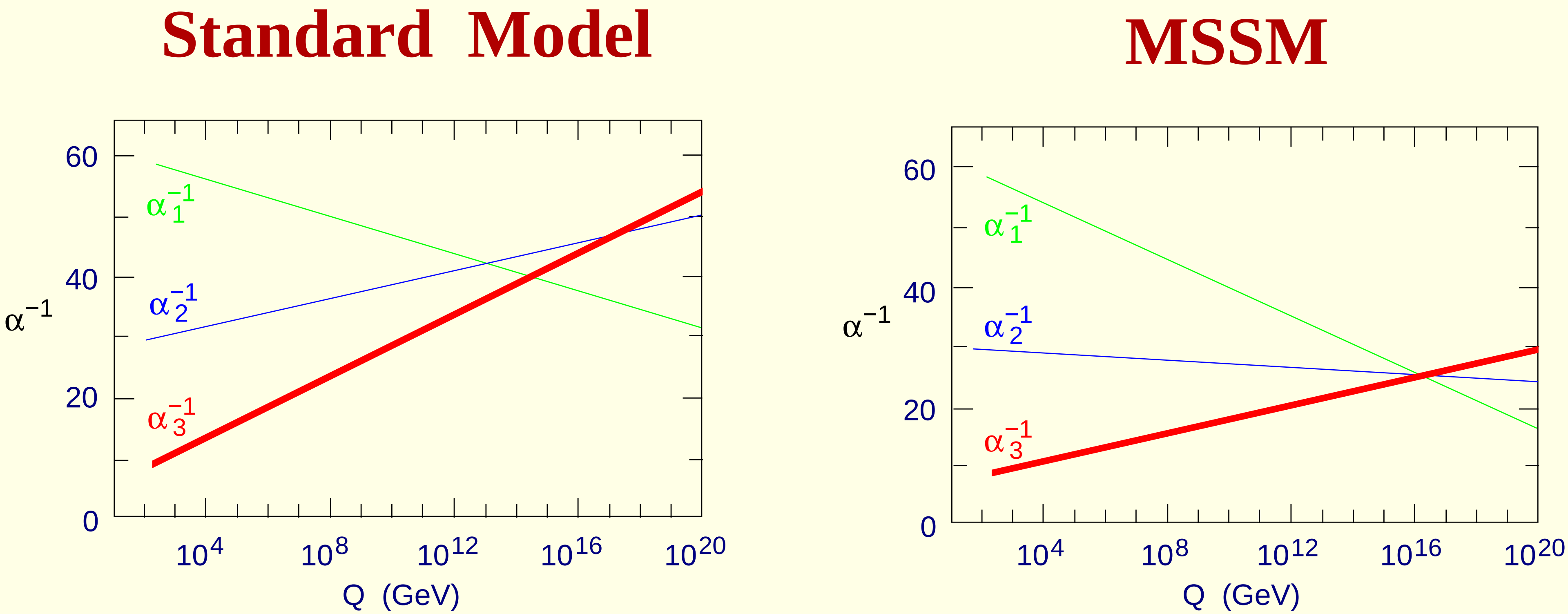


Figure 1: Unification of couplings constants ($\alpha_i = g_i^2/(4\pi)$) in the minimal supersymmetric model (MSSM) as compared to failure without supersymmetry.

The MSSM sparticle content + two-doublet Higgs sector $\Rightarrow$ gauge coupling unification at $M_U \sim few \times 10^{16}\,\mathrm{GeV}$, close to M_P. High-scale unification correlates well with the attractive idea of gravity-mediated SUSY breaking.

2. RGE EWSB

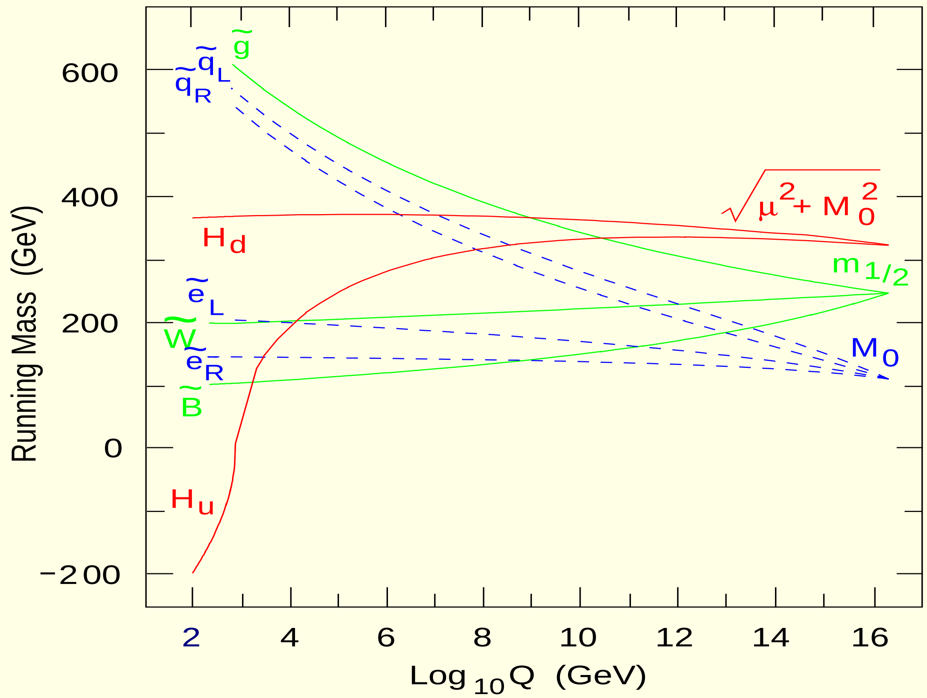


Figure 2: Evolution of SUSY-breaking masses or masses-squared, showing how $m^2_{H_u}$ is driven < 0 at low $Q \sim \mathcal{O}(m_Z)$.

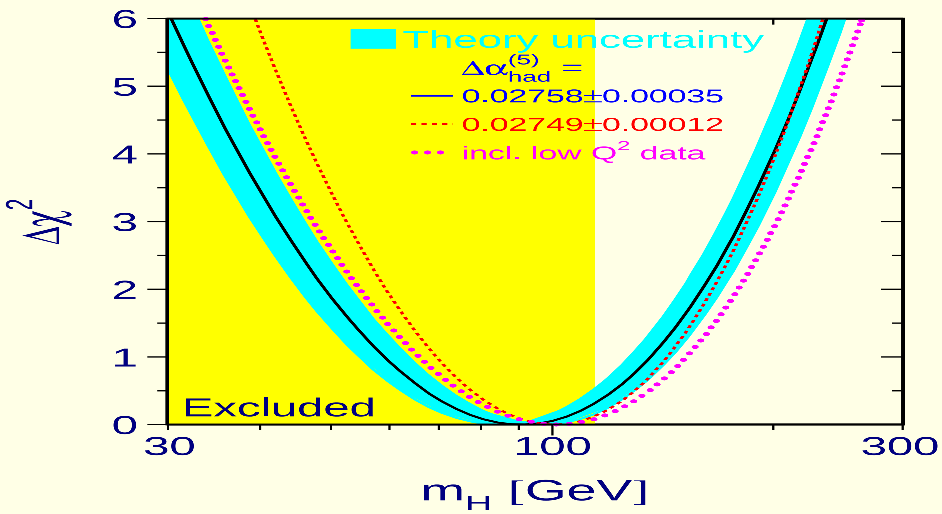
Starting with universal soft-SUSY-breaking masses-squared at M_U, the RGE's predict that the top quark Yukawa coupling will drive one of the soft-SUSY-breaking Higgs masses squared ($m^2_{H_u}$) negative at a scale of order $Q \sim m_Z$, thereby automatically generating electroweak symmetry breaking ($\langle H_u \rangle = h_u, \langle H_d \rangle = h_d$), BUT MAYBE m_Z IS FINE-TUNED.

- **The Higgs Mass**

In the presence of soft-SUSY-breaking, the light Higgs has ($\tan\beta = h_u/h_d$)

$$\begin{aligned} m_h^2 &\sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right) + \ldots \\ &\overset{\text{large}\tan\beta}{\sim} (91~\mathrm{GeV})^2 + (38~\mathrm{GeV})^2 \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right). \end{aligned} \tag{1}$$

A Higgs mass of order $100~\mathrm{GeV}$, as predicted for stop masses $\sim 2m_t$, is in wonderful accord with precision electroweak data.



So, why haven't we seen the Higgs? Is SUSY wrong, are stops heavy, or is the MSSM too simple?

MSSM Problems

- **The μ parameter in $W \ni \mu \widehat{H}_u \widehat{H}_d$,[1] is dimensionful, unlike all other superpotential parameters. A big question is why is it $\mathcal{O}(1~\mathrm{TeV})$ (as required for EWSB and $m_{\widetilde{\chi}_1^\pm}$ lower bound), rather than $\mathcal{O}(M_U, M_\mathrm{P})$ or 0.**

- **LEP limits:**

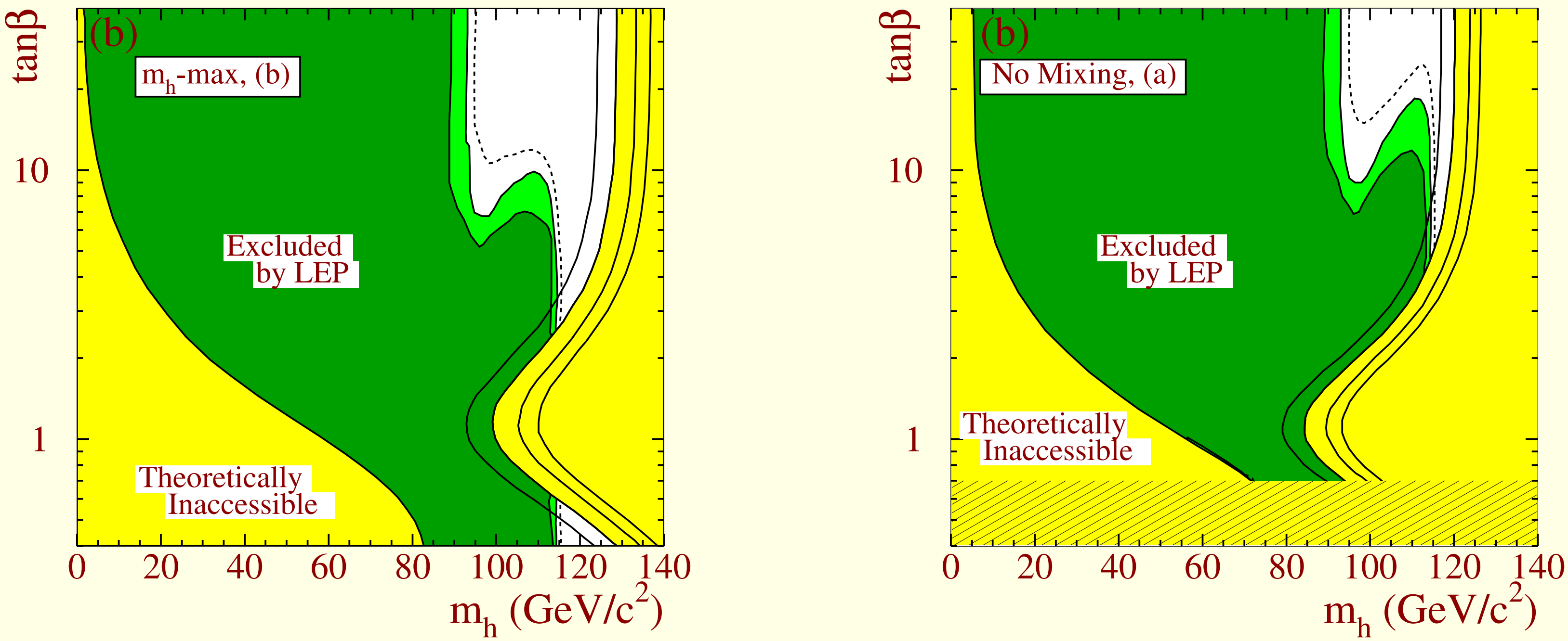


Figure 3: **Maximal-mixing ($X_t = A_t - \mu \cot\beta = -2 m_{\mathrm{SUSY}} = -2~\mathrm{TeV}$, $\mu > 0$) and no-mixing (with $\mu > 0$) LEP exclusions at 90% CL. From CERN-PH-EP/2006-001.**

[1] Hatted (unhatted) capital letters denote superfields (scalar superfield components).

The LEP limits on Higgs bosons have pushed the CP-conserving MSSM into an awkward corner of parameter space characterized by large $\tan\beta$ and large $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$. For $m_{\widetilde{t}_L} = m_{\widetilde{t}_R} = 1~\mathrm{TeV} \equiv m_{\mathrm{SU\not{S}Y}}$, we have the MSSM exclusion plots shown.

There is still room, but we need $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \gtrsim 900~\mathrm{GeV}$.

- **Fine-tuning**

Minimization of the Higgs potential gives (at scale m_Z)

$$\frac{1}{2} m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - \tan^2\beta m_{H_u}^2}{\tan^2\beta - 1} \tag{2}$$

and the m_Z-scale $\mu, m_{H_u}^2, m_{H_d}^2$ parameters are sensitive to their GUT scale values yielding at $\tan\beta = 10$ (similar to $\tan\beta = 2.5$ results in Kane and King hep-ph/9810374 and Bastero-Gil, Kane, and King hep-ph/9910506)

$$\begin{aligned} m_Z^2 &= -2.0\mu^2(M_U) + 5.9 M_3^2(M_U) + 0.8 m_Q^2(M_U) + 0.6 m_U^2(M_U) \\ &\quad -1.2 m_{H_u}^2(M_U) - 0.7 M_3(M_U) A_t(M_U) + 0.2 A_t^2(M_U) + \ldots \end{aligned}$$

One would expect that $m_Z \sim 2M_3(M_U), m_Q(M_U), m_u(M_U) \sim m_{\tilde{g}}, m_{\tilde{t}}$,
$\Rightarrow$ we need a very light gluino and a rather light stop to avoid fine-tuning OR we need highly correlated cancellations and large A_t (Nomura's talk).
A rigorous measure is F plotted below.

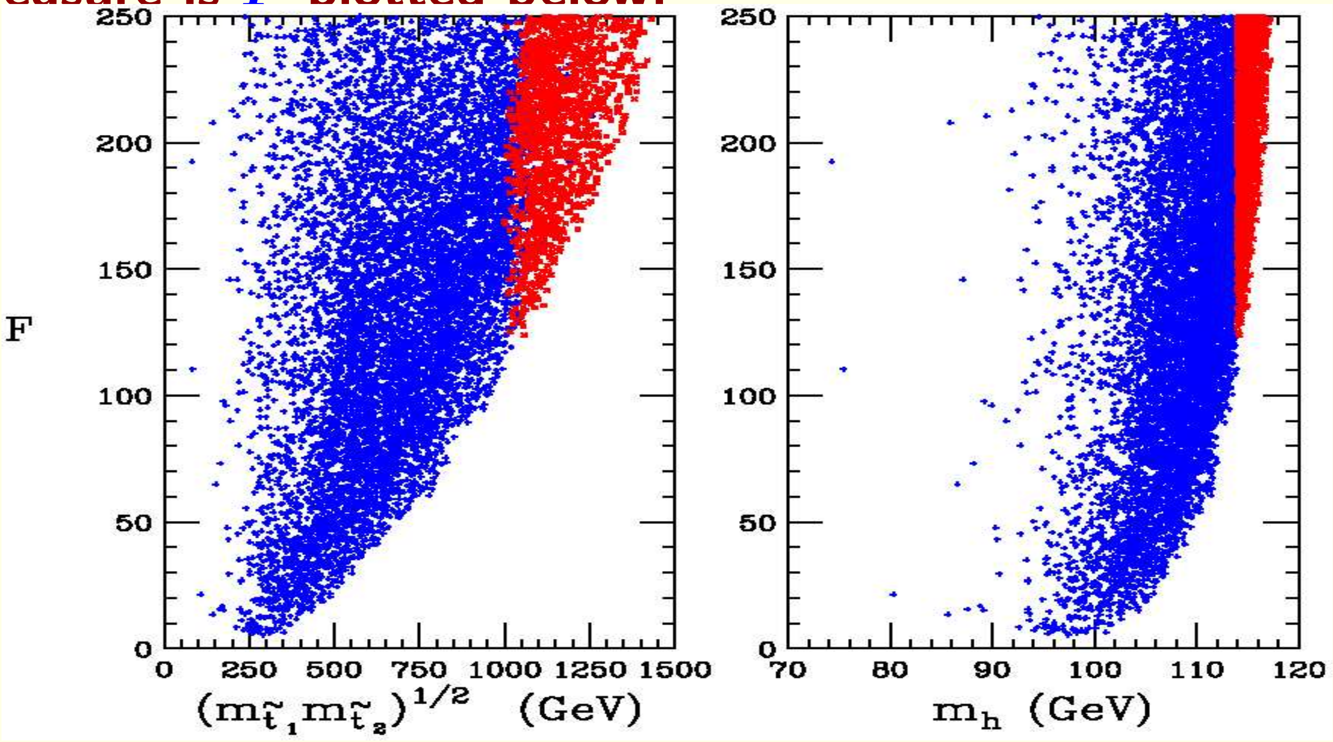


Figure 4: MSSM results for $F = \mathrm{Max}_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right|$, where $p \in \left\{ M_{1,2,3}, m^2_Q, m^2_U, m^2_D, m^2_{H_u}, m^2_{H_d}, \mu, A_t, B\mu, \ldots \right\}$ (all at M_U). The $+$ points have $m_h < 114$ GeV, and are experimentally excluded. The $\times$ points have $m_h \geq 114$ GeV. Plot is for $\tan\beta = 10$, $M_{1,2,3} = 100, 200, 300$ GeV (at scale m_Z). All parameters scanned, but $|A_t| < 500$ GeV is imposed.

Essentially all the blue points fail LEP limits due to $m_h < 114$ GeV.

Note that if $m_h \sim 100$ GeV were ok, then smallest F occurs there.

One can do better by taking very large A_t values.

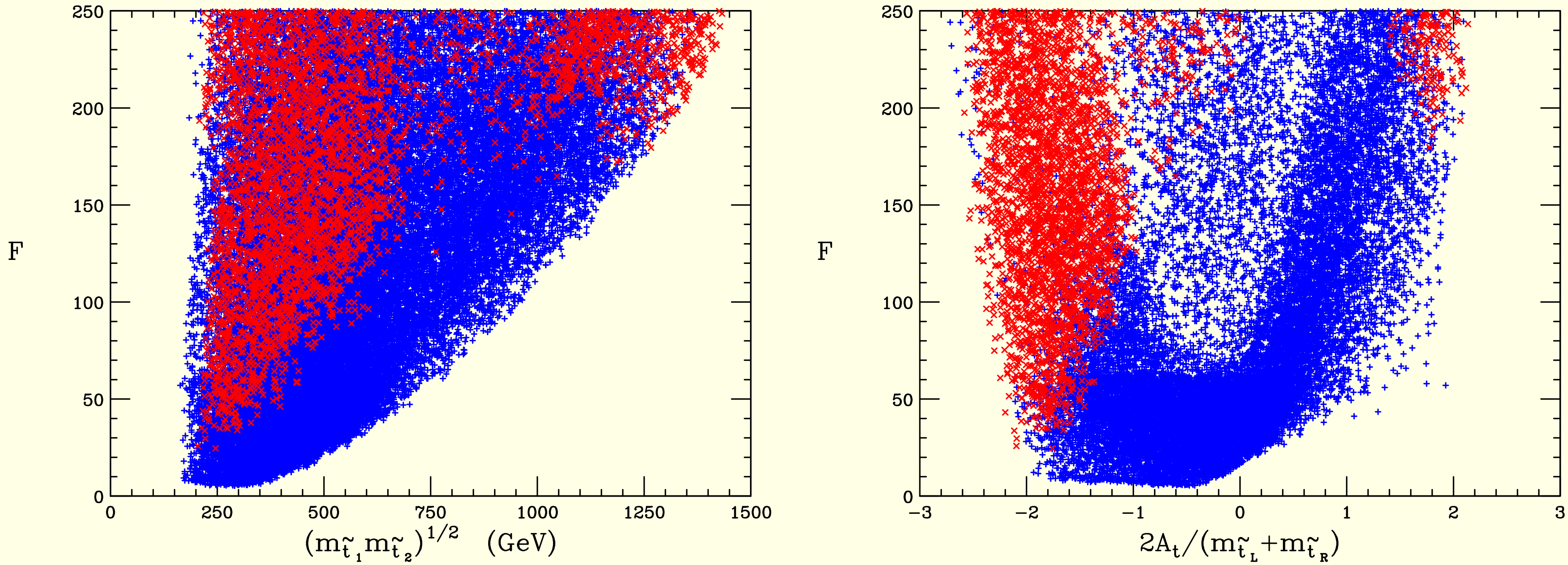


Figure 5: F in the MSSM. The $+$ points have $m_h < 114$ GeV. The $\times$ points have $m_h \geq 114$ GeV. Plot is for $\tan\beta = 10$, $M_{1,2,3} = 100, 200, 300$ GeV (at scale m_Z). All other parameters were scanned over, with $|A_t| < 4$ TeV imposed.

The figure shows clearly that large negative $A_t(m_Z)$ is required to get anything like reasonable F for allowed $m_h \geq 114\ \mathrm{GeV}$ points, and even then $F \gtrsim 30$.

- $\boxed{\text{So, what direction should one head in?}}$
  - **CP-violating MSSM, e.g. CPX-like scenarios?**
    These don't solve the μ issue, and nature has shown very little inclination for CP-violation as large as that needed to significantly alter the CP-conserving situation.
  - **Large extra dimensions, little Higgs, Higgsless, ...**
    All worth exploring, but these models are complicated and typically have problems of one kind or another, especially precision EW data.
  - **Hints from string theory.**
    In particular, it is very clear that extra singlet superfields are common in string models.
    Let's make use of singlets and let's do it in the simplest possible way (i.e. no associated gauge group and no dimensionful superpotential parameters) $\Rightarrow$ the NMSSM.

The NMSSM

- **The NMSSM introduces just one extra singlet superfield, with superpotential $\lambda\widehat{S}\widehat{H}_u\widehat{H}_d$. The μ parameter is then automatically generated by $\langle S\rangle$ leading to $\mu_{eff}\widehat{H}_u\widehat{H}_d$ with $\mu_{eff} = \lambda\langle S\rangle$. The only requirement is that $\langle S\rangle$ be of order the SUSY-breaking scale at $\sim 1~\mathrm{TeV}$.**

- **However, $\lambda\widehat{S}\widehat{H}_u\widehat{H}_d$ cannot be the end.**

  Including Yukawa W terms, there is a PQ symmetry that will spontaneously break when the Higgs scalars gain vevs, and a pseudo[2]-Nambu-Goldstone boson, known as the PQ axion (it is actually one of the pseudoscalar Higgs bosons), will be generated.

  For values of $\lambda \sim \mathcal{O}(1)$, this axion would have been detected in experiment and this model ruled out.

- **Gauging the $U(1)_{\mathrm{PQ}}$ (so that axion is absorbed in Z' mass) typically leads to FCNC problems.**

[2] The axion is only a "pseudo"-Nambu-Goldstone boson since the PQ symmetry is explicitly broken by the QCD triangle anomaly. The axion then acquires a small mass from its mixing with the pion.

- In the NMSSM, the PQ symmetry is explicitly broken by $W \ni \frac{1}{3}\kappa\widehat{S}^3$.

  Other possible superpotential terms with dimensionful parameters are absent if one demands that the superpotential be invariant under a Z_3 symmetry.

  If the Z_3 is applied also to soft SUSY breaking terms, only $\frac{1}{3}\kappa A_\kappa S^3$ is allowed in addition to $\lambda A_\lambda S H_u H_d$.

- However, this $\mathbb{Z}_3$ symmetry cannot be completely unbroken. If it were, a cosmological "domain wall problem" would arise.

- To avoid this problem (Panagiotakopoulos and Tamvakis), one introduces a Z_2^R symmetry that is broken by the soft-SUSY breaking terms, giving rise to harmless tadpoles of order $\frac{1}{(16\pi^2)^n} M_{\rm SUSY}^3$, with $2 \leq n \leq 4$. For example, a superpotential term of form $\widehat{S}^7/M_{\rm P}^4$ (which is ok under Z_2^R) generates at 4-loops (Abel) the tadpole form $\delta V \sim \left(\frac{1}{16\pi^2}\right)^4 m_{\rm SUSY}^3 (S+S^*)$.

  Although these terms are phenomenologically irrelevant, they are entirely sufficient to break the global Z_3 symmetry and make the domain walls collapse.

- Net Result

Since the only *relevant* superpotential terms that are introduced have dimensionless couplings, the scale of the vevs (i.e. the scale of EWSB) is determined by the scale of SUSY-breaking.

- **Further**, all the good properties of the MSSM (coupling unification and RGE EWSB, in particular) are preserved under singlet addition.

- **New Particles**

  The single extra singlet superfield of the NMSSM contains an extra neutral gaugino (the singlino) ($\Rightarrow \widetilde{\chi}^0_{1,2,3,4,5}$), an extra CP-even Higgs boson ($\Rightarrow h_{1,2,3}$) and an extra CP-odd Higgs boson ($\Rightarrow a_{1,2}$).

- **The parameters of the NMSSM**

  Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\ \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3}\ \widehat{S}^3 \tag{3}$$

  depending on two dimensionless couplings λ, κ beyond the MSSM. The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3\,. \tag{4}$$

The final two input parameters are

$$\tan\beta = h_u/h_d\,, \quad \mu_{\rm eff} = \lambda s\,, \tag{5}$$

where $h_u \equiv \langle H_u \rangle$, $h_d \equiv \langle H_d \rangle$ and $s \equiv \langle S \rangle$. These, along with m_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S (denoted $m_{H_u}^2$, $m_{H_d}^2$ and m_S^2) through the three minimization equations of the scalar potential. (From the model building point of view, we emphasize the reverse — i.e. the SUSY-breaking scales $m_{H_u}^2$, $m_{H_d}^2$ and m_S^2, along with A_λ and A_κ determine the EWSB vevs, λ and κ being dimensionless.)

Thus, as compared to the three independent parameters needed in the MSSM context (often chosen as μ, $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda\ ,\ \kappa\ ,\ A_\lambda\ ,\ A_\kappa,\ \tan\beta\ ,\ \mu_{\rm eff}\ . \tag{6}$$

In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

The NMSSM is much less constrained than the MSSM, and is not necessarily forced into awkward/fine-tuned corners of parameter space either by LEP limits or by theoretical reasoning.

$\Rightarrow$ the NMSSM should be adopted as the more likely benchmark minimal SUSY model and it should be explored in detail.

- **To further this study, Ellwanger, Hugonie and I constructed NMHDECAY**

  `http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

  `http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

  It computes all aspects of the Higgs sector and checks against many (but, as we shall see, not all) LEP limits and various other constraints.

- **We also developed a program to examine the LHC observability of Higgs signals in the NMSSM.**

  A significant hole in the LHC no-lose theorem for Higgs discovery emerges: only if we avoid that part of parameter space for which $h \to aa$ and similar decays are present is there a guarantee for finding a Higgs boson at the LHC in one of the nine "standard" channels (e.g. $h \to \gamma\gamma$, $t\bar{t}h, a \to t\bar{t}b\bar{b}$, $t\bar{t}h, a \to t\bar{t}\gamma\gamma$, $b\bar{b}h, a \to b\bar{b}\tau^+\tau^-$, $WW \to h \to \tau^+\tau^-$, ...

A series of papers (beginning with JFG+Haber+Moroi at Snowmass 1996 and continued by JFG, Ellwanger, Hugonie, Moretti, Miller, .. .) has demonstrated the general nature of this LHC no-lose theorem "hole".

- The portion of parameter space with $h \to aa, \ldots$ is small $\Rightarrow$ one is tempted to ignore it were it not for the fact that it is where fine-tuning can be absent.

  As before, the canonical measure of fine-tuning employed is

$$F = \mathrm{Max}_p F_p \equiv \mathrm{Max}_p \left| \frac{d \log m_Z}{d \log p} \right| , \tag{7}$$

  where the parameters p comprise the GUT-scale values of λ, κ, A_λ, A_κ, and the usual soft-SUSY-breaking gaugino, squark, slepton, . . . masses.

- How do we get small fine-tuning?

  1. F is minimum for $m_{h_1} \sim 100 \div 104~\mathrm{GeV}$ (in a totally unconstrained scan of parameter space this is just what one finds). Neither lower nor higher!

For $m_{h_1} \sim 100$ GeV, $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \sim 350$ GeV.

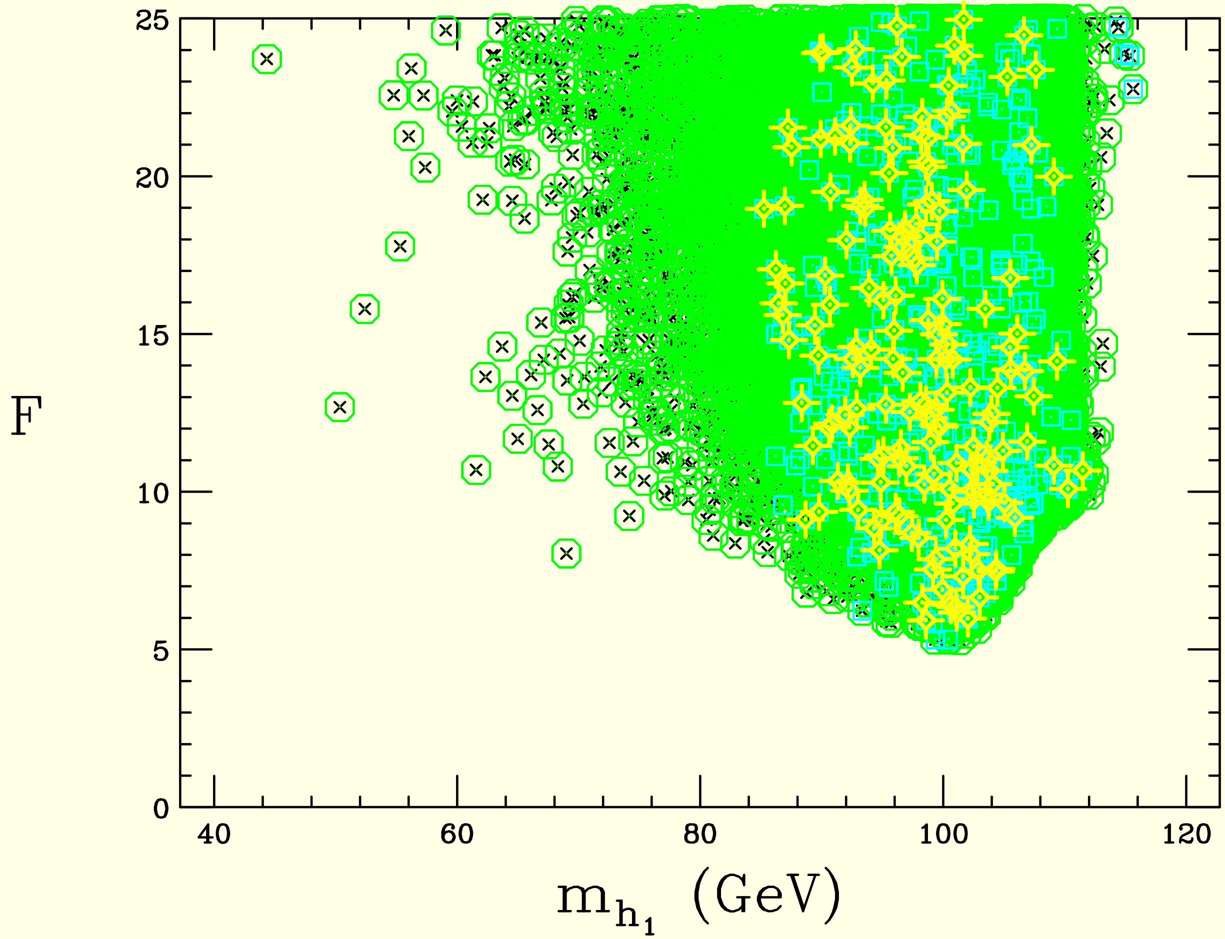


Figure 6: F vs. m_{h_1} for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan\beta = 10$. Small $\times$ = no constraints other than global and local minimum, no Landau pole before M_U and neutralino LSP. The O's = stop and chargino limits imposed, but NO Higgs limits. The □'s = all single channel Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{a_1} < 2m_b$.

2. $m_{h_1} \sim 100$ GeV is only LEP-allowed if $h_1 \to a_1 a_1$ and $a_1 \to \tau^+\tau^-$ ($2m_\tau < m_{a_1} < 2m_b$) or $gg, q\bar{q}$ ($m_{a_1} < 2m_\tau$) so as to hide the h_1 in

this mass range (more later).

3. **A light a_1 is natural.** In fact, a_1 is a pseudo-Nambu-Goldstone boson associated with a $U(1)_R$ symmetry of the superpotential, whose spontaneous breaking by the vevs of H_u, H_d and S would yield $m_{a_1} = 0$ were it not that the $U(1)_R$ is explicitly broken by the A_κ and A_λ soft-SUSY-breaking terms, implying $m_{a_1} \to 0$ for $A_\kappa, A_\lambda \to 0$ (ignoring the small one-loop contributions to $U(1)_R$ breaking from gaugino masses). (Dobrescu, Matchev)

$$m_{a_1}^2 \simeq 3s\left(\kappa A_\kappa \sin^2\theta_A + \frac{3\lambda A_\lambda \cos^2\theta_A}{2\sin 2\beta}\right) \quad \text{where}$$

$$a_1 \equiv \cos\theta_A a_{MSSM} + \sin\theta_A a_S\,, \quad \text{with} \quad \cos\theta_A \simeq \frac{2v}{s\tan\beta}\,. \tag{8}$$

Further, the RGE's

$$\begin{aligned} 16\pi^2\frac{dA_\lambda}{dt} &= -6A_t\lambda_t^2 + 8\lambda^2 A_\lambda - 4\kappa^2 A_\kappa - 6g_2^2 M_2 - \frac{6}{5}g_1^2 M_1 \\ 16\pi^2\frac{dA_\kappa}{dt} &= 12(-\lambda^2 A_\lambda + \kappa^2 A_\kappa) \end{aligned} \tag{9}$$

imply that if $A_\kappa(M_U), A_\lambda(M_U) \sim 0$ then $A_\kappa(m_Z) \ll A_\lambda(m_Z) \sim M_2$. However, since $\cos\theta_A$ is small, the contributions of the A_κ and A_λ terms to $m_{a_1}^2$ are comparable for $A_\kappa(m_Z) \ll A_\lambda(m_Z)$.

Net Result: A light a_1 that is mainly singlet is natural.

4. However, correlated $A_\kappa, A_\lambda \neq 0$ needed for really small m_{a_1} and large enough $BR(h_1 \to a_1 a_1)$ to escape LEP limits on $Zh_1 \to Zb\bar{b}$.

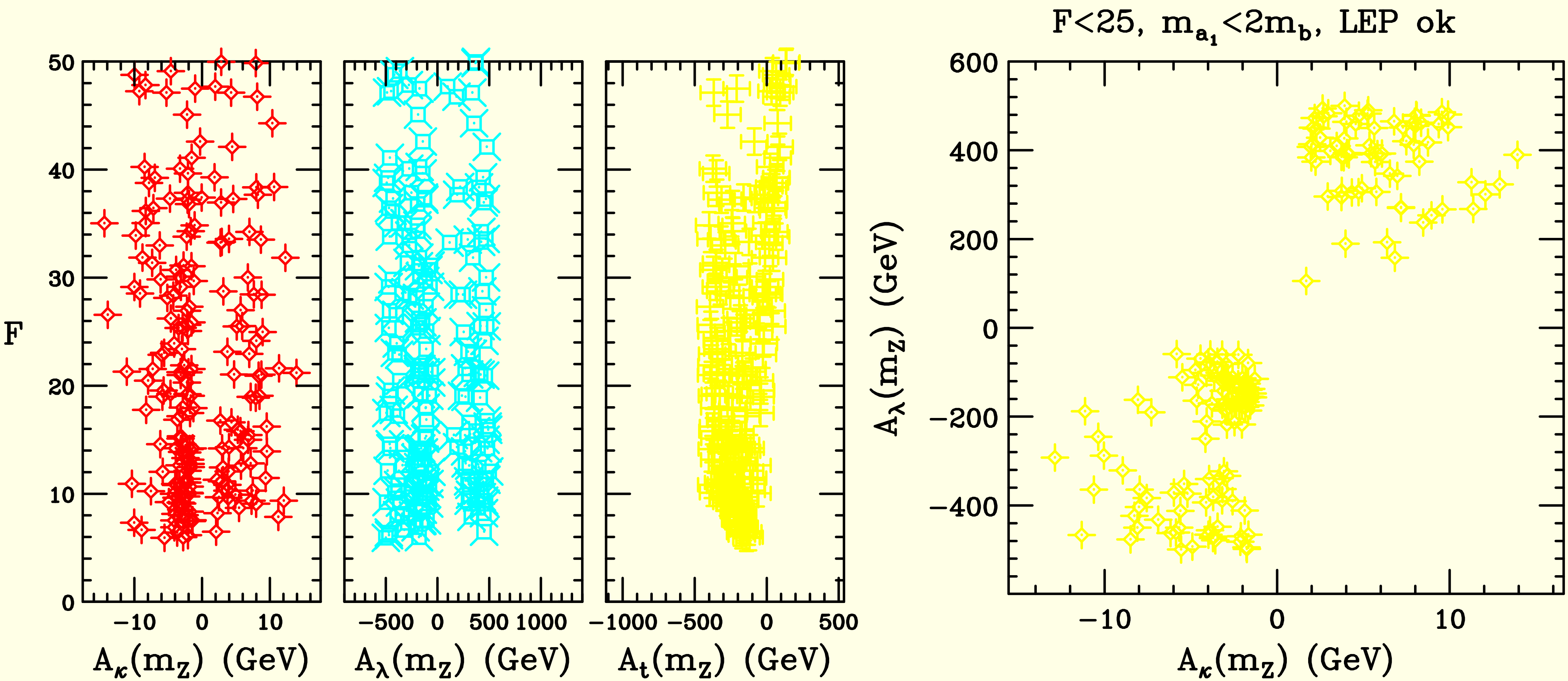


Figure 7: F vs. $A_{\kappa,\lambda,t}(m_Z)$ for $M_{1,2,3} = 100, 200, 300\ \mathrm{GeV}$ and $\tan\beta = 10$ for fully ok $m_{a_1} < 2m_b$ solutions. Note: A_κ, A_λ exactly 0 is not ok.

How does $B(h_1 \to a_1 a_1)$ depend on A_κ and A_λ?

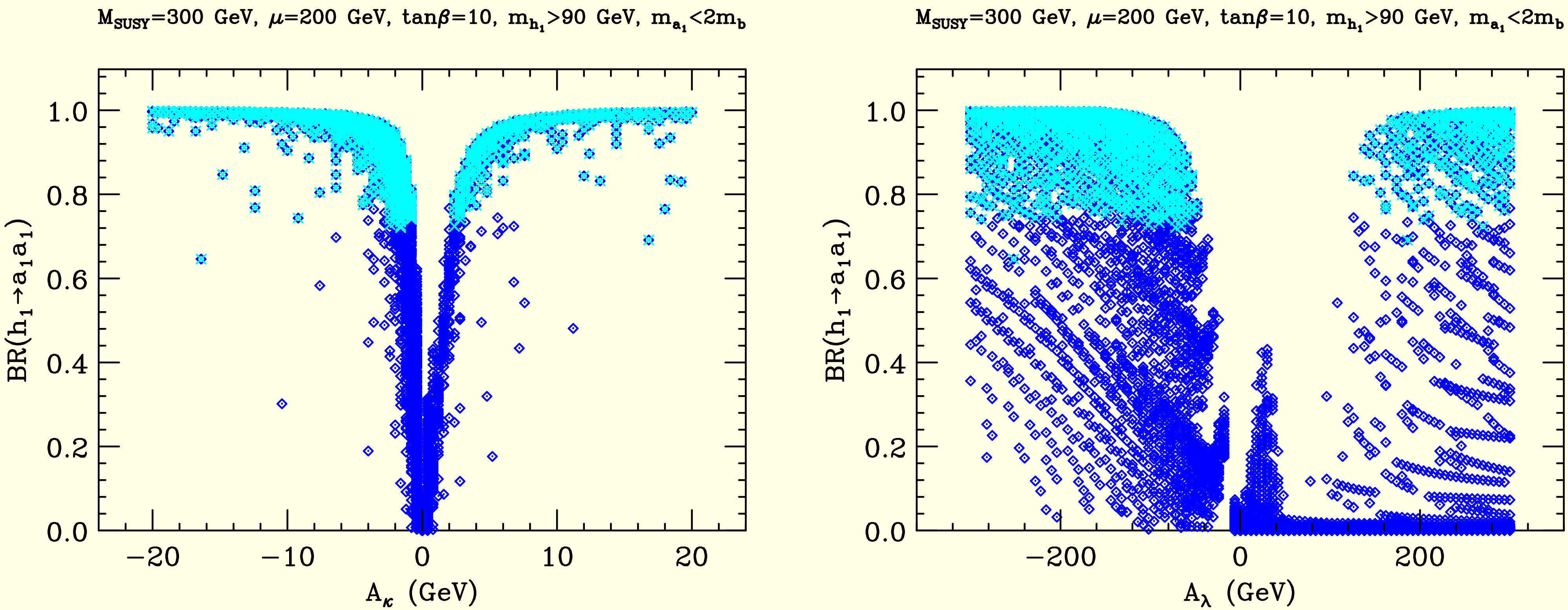


Figure 8: $B(h_1 \to a_1 a_1)$ vs. A_κ and A_λ for $M_{\rm SUSY} = 300~{\rm GeV}$, $\tan\beta = 10$, $\mu = 150~{\rm GeV}$ after scanning over λ and κ values. All points have $m_{a_1} < 2m_b$. Light cyan $\times$ points have large enough $B(h_1 \to a_1 a_1)$ to escape LEP limits on $Zh_1 \to Zb\bar{b}$.

We see again that $|A_\kappa|$ of order a few GeV is needed for large enough $B(h_1 \to a_1 a_1)$ to escape LEP limits.

Is there a new fine-tuning associated with getting both a very light m_{a_1} and large enough $B(h_1 \to a_1 a_1)$? Define

$$F_{\rm MAX} = {\rm Max}\left\{ \left| \frac{A_\lambda}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\lambda} \right|, \left| \frac{A_\kappa}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\kappa} \right| \right\} . \quad (10)$$

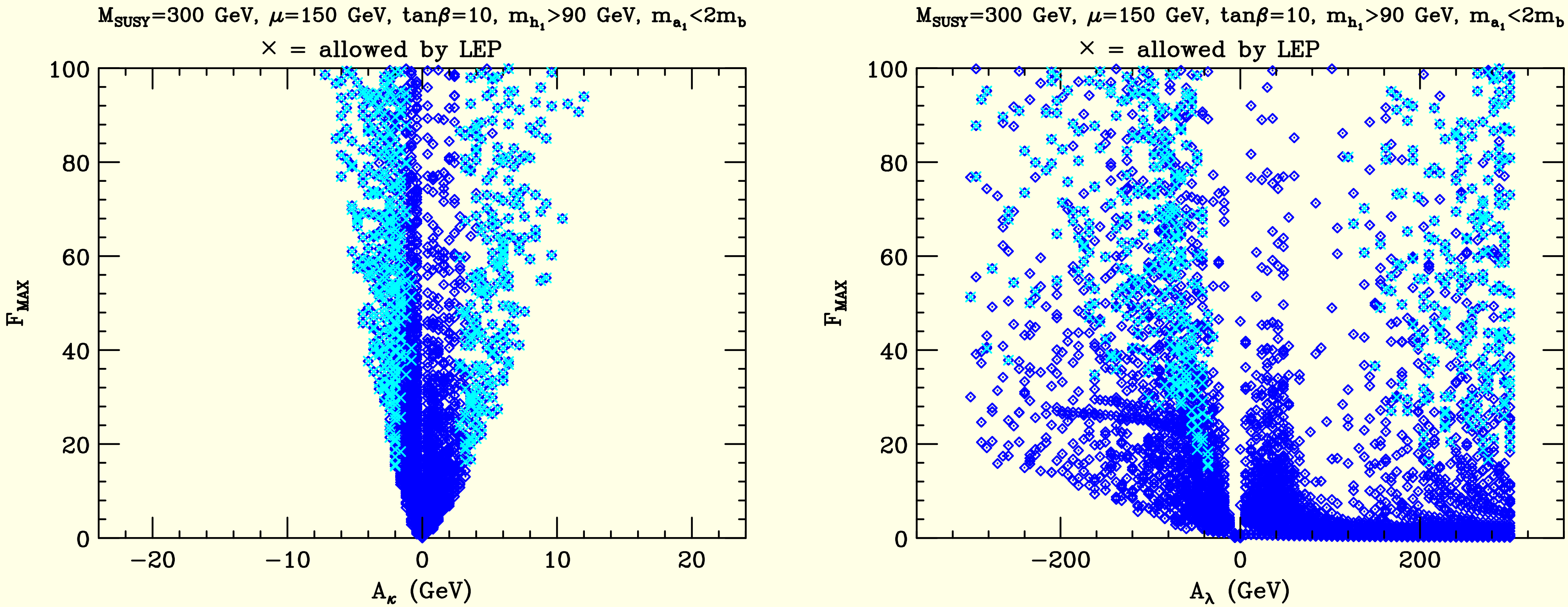


Figure 9: $F_{\rm MAX}$ **vs.** A_κ **and** A_λ **for** $M_{\rm SUSY} = 300$ GeV, $\tan\beta = 10$, $\mu = 150$ GeV **after scanning over** λ **and** κ **values. All points have** $m_{a_1} < 2m_b$**. Light cyan** $\times$ **points have large enough** $B(h_1 \to a_1 a_1)$ **to escape LEP limits on** $Zh_1 \to Zb\bar{b}$**.**

$\Rightarrow$, 5% **to** 10% **fine tuning to get** $m_{a_1} < 2m_b$ **and ok** $B(h_1 \to a_1 a_1)$.

5. **Small F is associated with small values for $m^2_{H_u}(M_U)$, $m^2_{H_d}(M_U)$ and $m^2_S(M_U)$.**

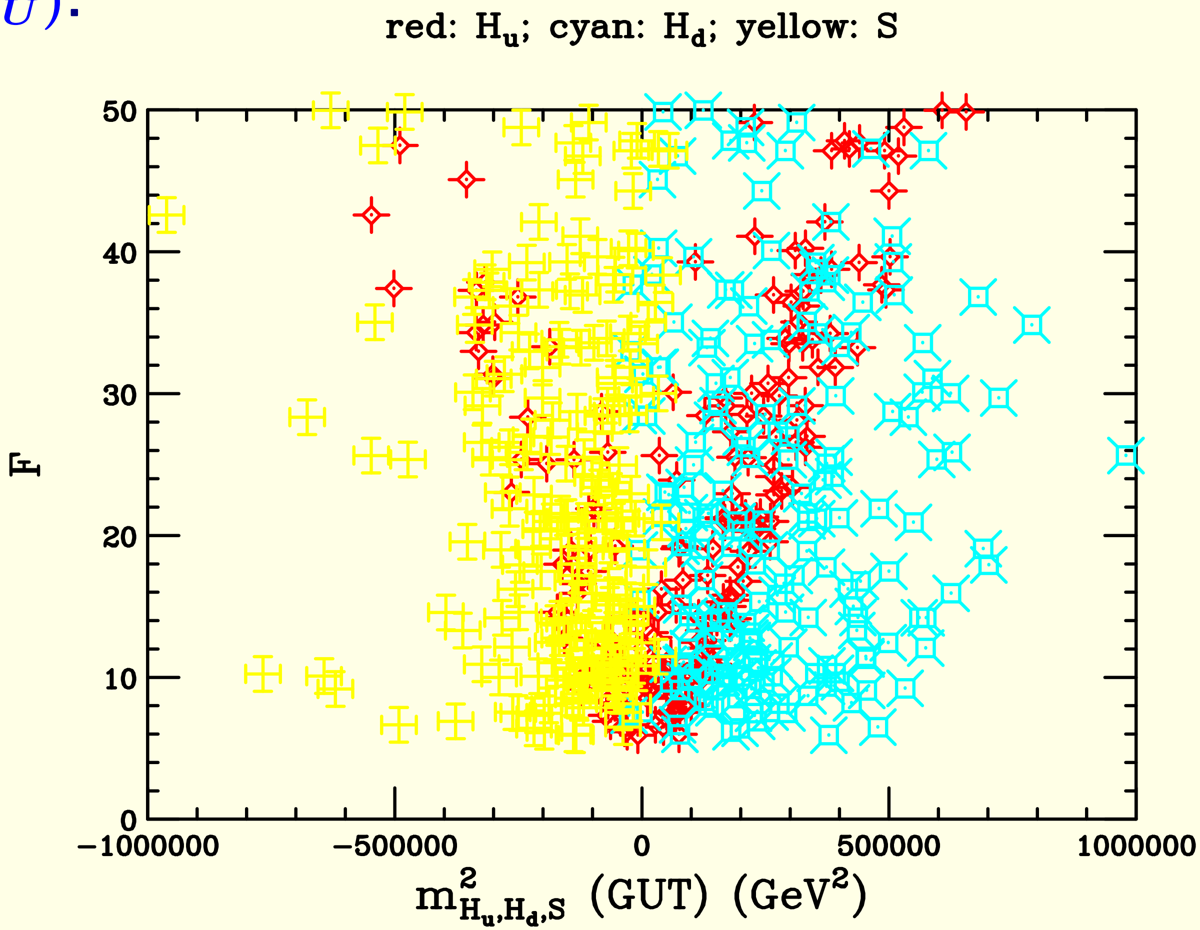


Figure 10: F vs. $m^2_{H_u,H_d,S}(M_U)$ for $M_{1,2,3} = 100, 200, 300\ \mathrm{GeV}$ and $\tan\beta = 10$ for fully ok $m_{a_1} < 2m_b$ solutions.

Fine-Tuning and new LEP limits

- Thus, Dermisek and I find that fine-tuning is absent in the NMSSM for precisely those parameter choices for which $m_{h_1} \sim 100~\mathrm{GeV}$ (and is SM-like) and yet the h_1 escapes LEP limits due to the presence of $h_1 \to a_1 a_1$ decays. (There is little improvement in F per se associated with $m_{a_1} < 2m_b$, but as we now review LEP limits require this.)

  We illustrate LEP constrained results for $\tan\beta = 10$, and $M_{1,2,3} = 100, 200, 300~\mathrm{GeV}$.

  After incorporating the latest LEP single-channel limits (to be discussed), we find the results shown in the following figure after doing a large scan. The $+$ points have $m_{h_1} < 114~\mathrm{GeV}$ and the $\times$ points have $m_{h_1} \geq 114~\mathrm{GeV}$.

  As already noted, $m_{h_1} < 114~\mathrm{GeV}$, and in particular $m_{h_1} \sim 100~\mathrm{GeV}$, one can achieve very low F values.

  An h_1 with $m_{h_1} \sim 100~\mathrm{GeV}$ and SM-like couplings to gauge bosons and fermions is, of course, exactly the value preferred by precision electroweak constraints.

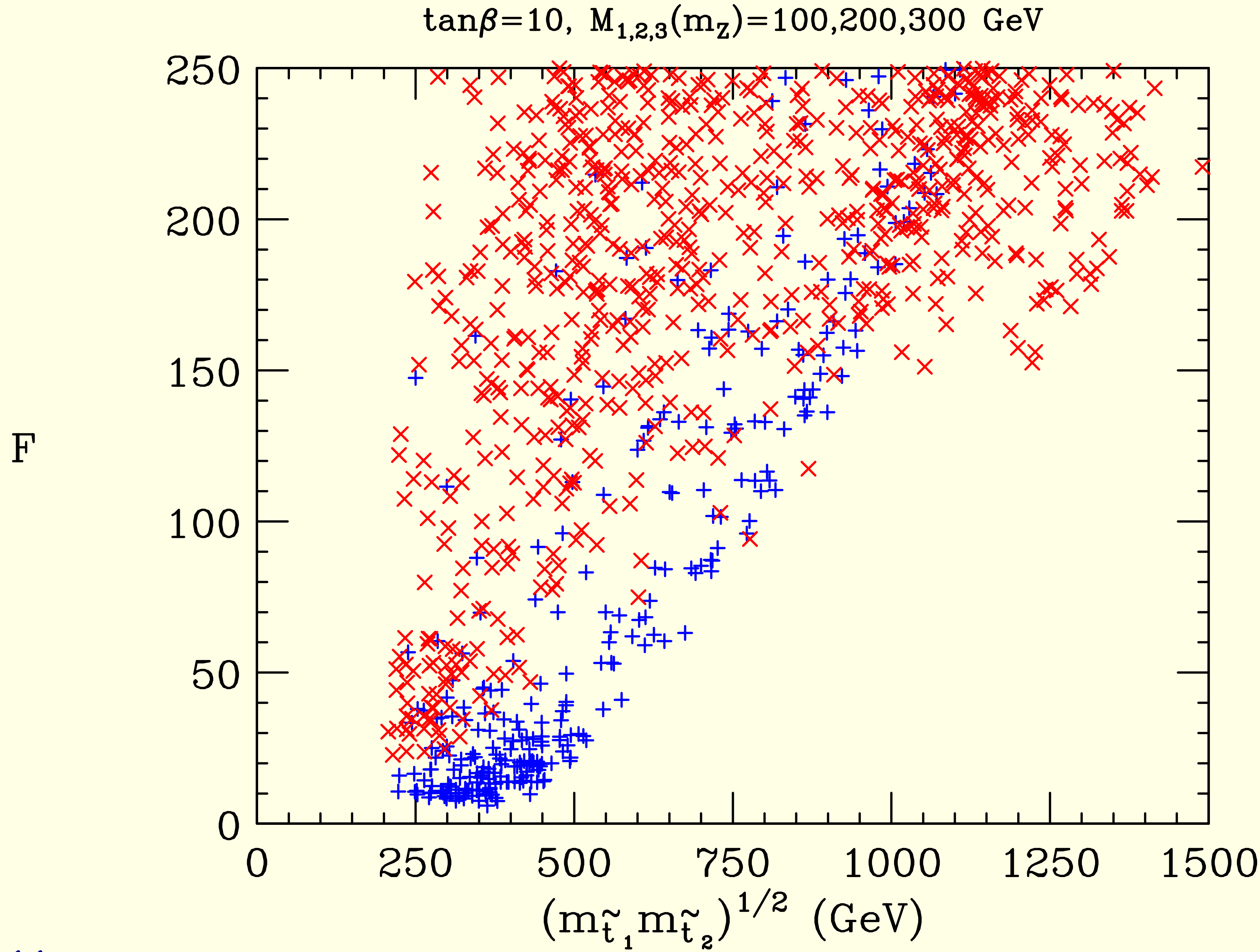


Figure 11: F as a function of root mean stop mass after latest *single-channel* LEP limits. Both $m_{h_1} < 114$ GeV ($+$) and $m_{h_1} \geq 114$ GeV ($\times$) points are allowed.

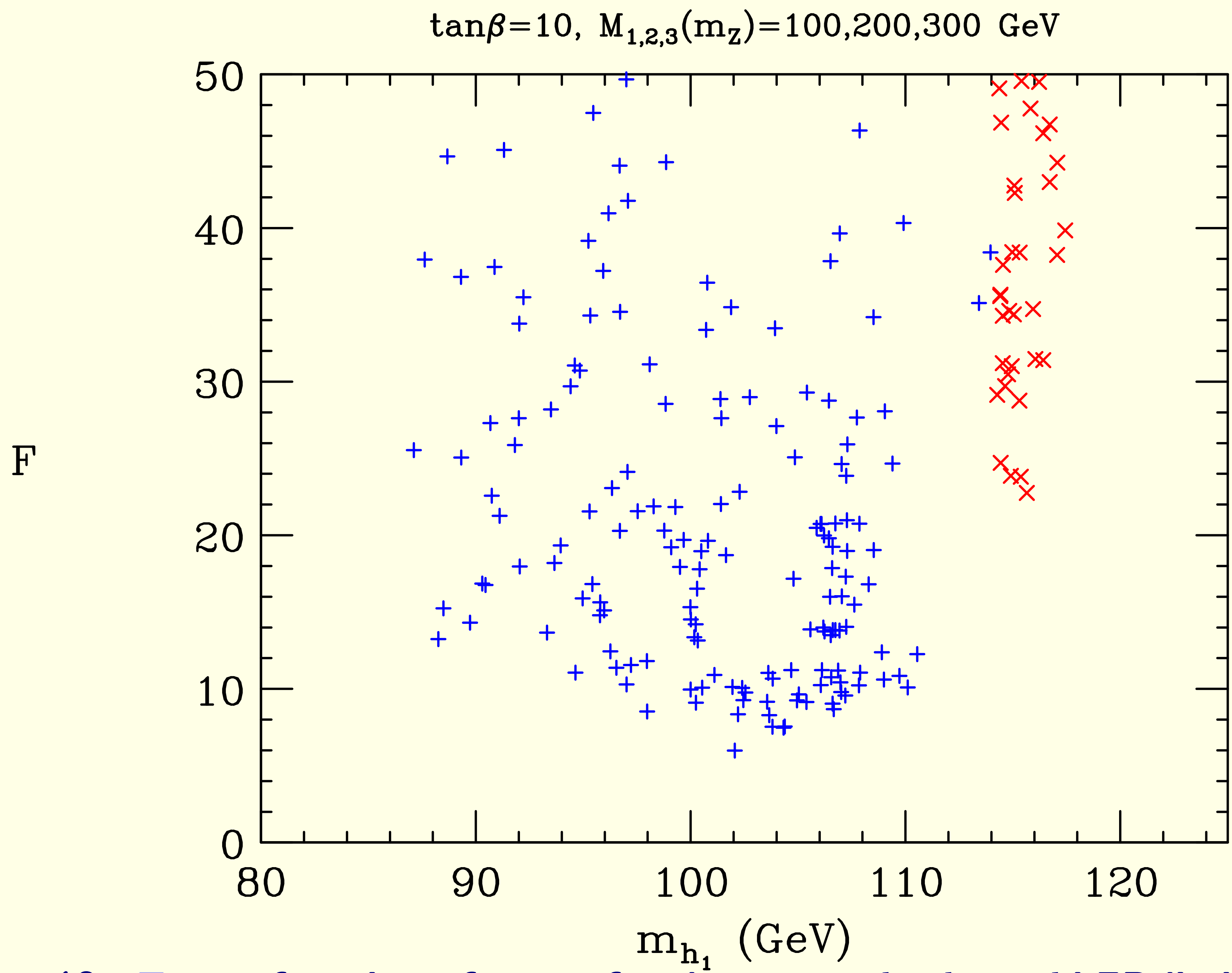


Figure 12: F **as a function of** m_{h_1} **after latest** *single-channel* **LEP limits.**

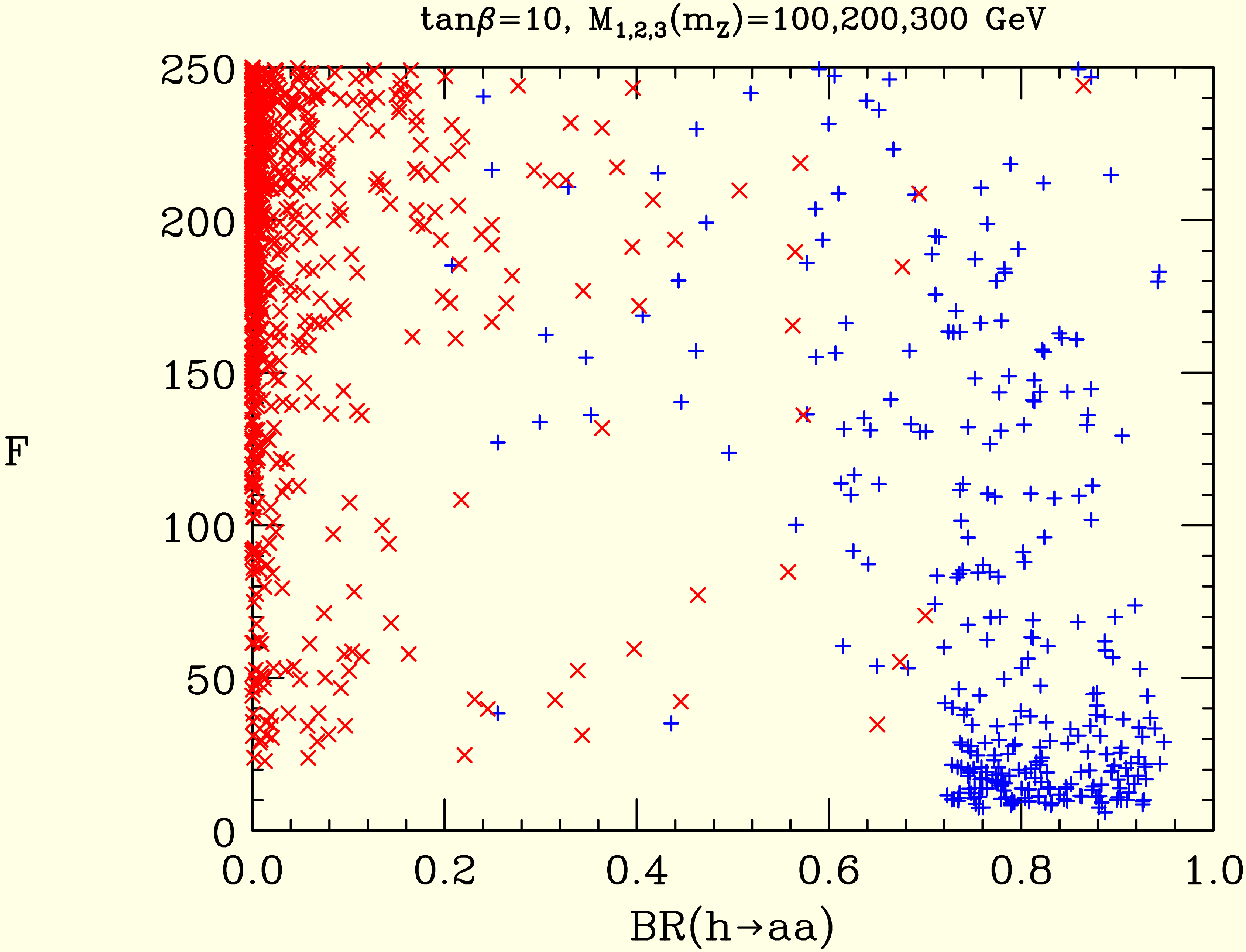


Figure 13: F **as a function of** $B(h_1 \to a_1 a_1)$ **after latest** *single-channel* **LEP limits. Note that** $h_1 \to a_1 a_1$ **can be dominant even when** m_{h_1} **is large enough that the decay is not needed to escape LEP limits.**

Among the points shown in the preceding few plots with low F, there are ones with $m_{a_1} > 2m_b$ and ones with $m_{a_1} < 2m_b$. The former have problems unless $m_{h_1} \gtrsim 110~\mathrm{GeV}$.

In particular, the $Z2b$ and $Z4b$ channels are not actually independent.

- Putting the $F < 10$ scenarios with $m_{a_1} > 2m_b$ through the full LHWG analysis, one finds that all are excluded at somewhat more than the 99% CL.

  In fact, all the $m_{a_1} > 2m_b$ scenarios with $m_{h_1} \lesssim 108 \div 110~\mathrm{GeV}$ are ruled out at a similar level. What is happening is that you can change the $h_1 \to b\overline{b}$ direct decay branching ratio and you can change the $h_1 \to a_1 a_1 \to 4b$ branching ratio, but roughly speaking $B(h_1 \to b's) \gtrsim 0.85$ (a kind of sum rule). So, if the ZZh_1 coupling is full strength (as is the case in all the scenarios with any kind of reasonable F) there is no escape except high enough m_{h_1}.

- The only way to achieve really low F, which comes with low $m_{h_1} \sim 100~\mathrm{GeV}$, and remain consistent with LEP is to have $m_{a_1} < 2m_b$.

  The relevant limit from LEP is then only that from the $Z2b$ channel. (It turns out that LEP has never placed limits on the $Z4\tau$ channel for h masses larger than about $87~\mathrm{GeV}$.)

- **Note:** Such a light to very light a_1 is not excluded by $\Upsilon, \ldots$ precision decay measurements since, as remarked earlier, the a_1 is very singlet-like for all the low-F scenarios.

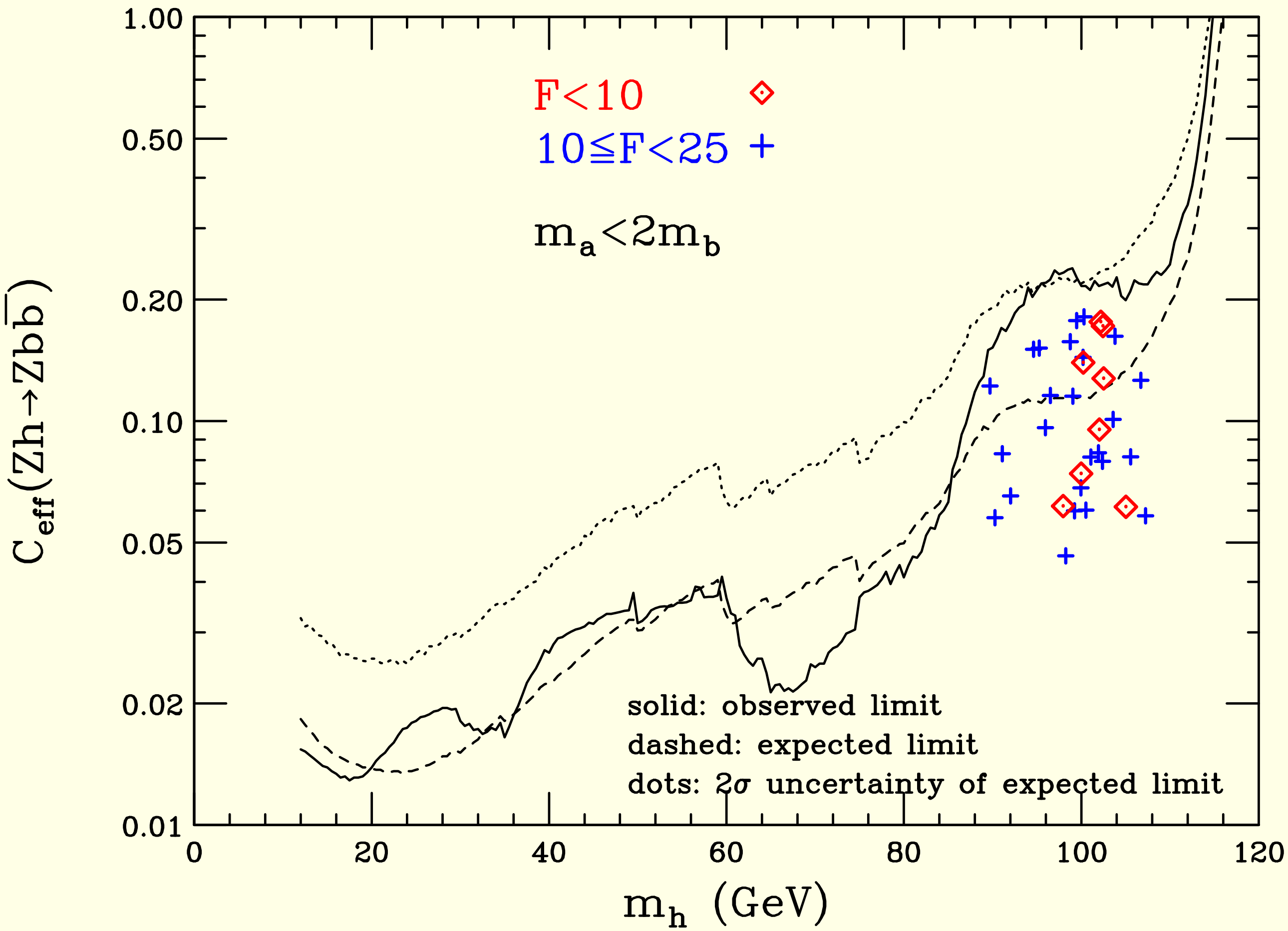


Figure 14: Observed LEP limits on C^{2b}_{eff} for the low-F points with $m_{a_1} < 2m_b$.

So just how consistent are the $F < 10$ points with the observed event excess. Although it is slightly misleading, a good place to begin is to recall the famous $1-CL_b$ plot for the $Z2b$ channel. (Recall: the smaller $1-CL_b$ the less consistent is the data with expected background only.)

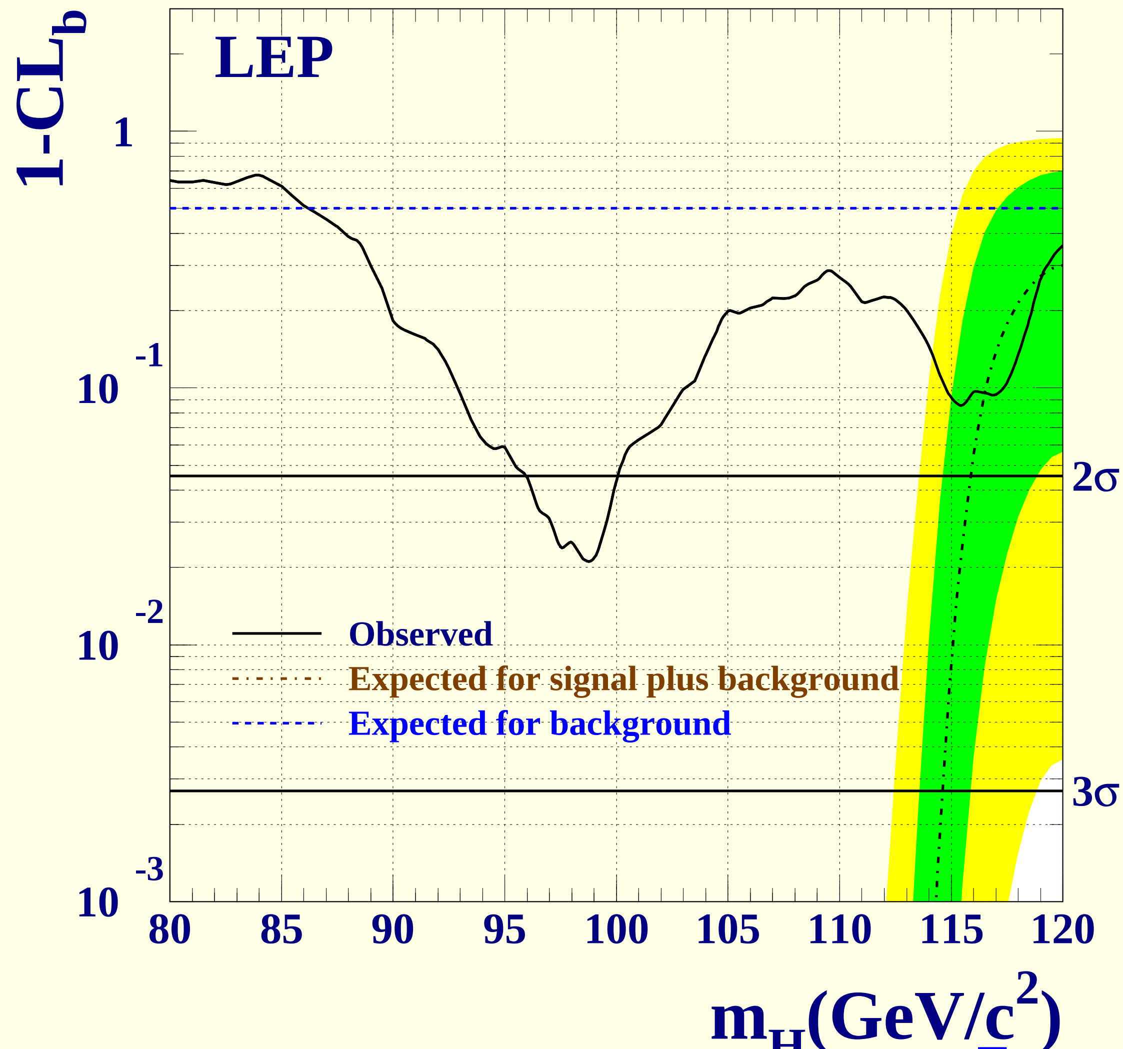


Figure 15: Plot of $1 - CL_b$ for the $Zb\bar{b}$ final state.

- **There is an observed vs. expected discrepancy exactly where we want it! And because $B(h_1 \to b\bar{b})$ is $1/10$ the SM value, the discrepancy is of about the right size.**

- **Are there other relevant limits on the kind of scenario we envision?**

  If the $a_1 a_1 \to 4\tau$ decay is the relevant scenario, the LEP limits run out for $m_h > 87$ GeV.

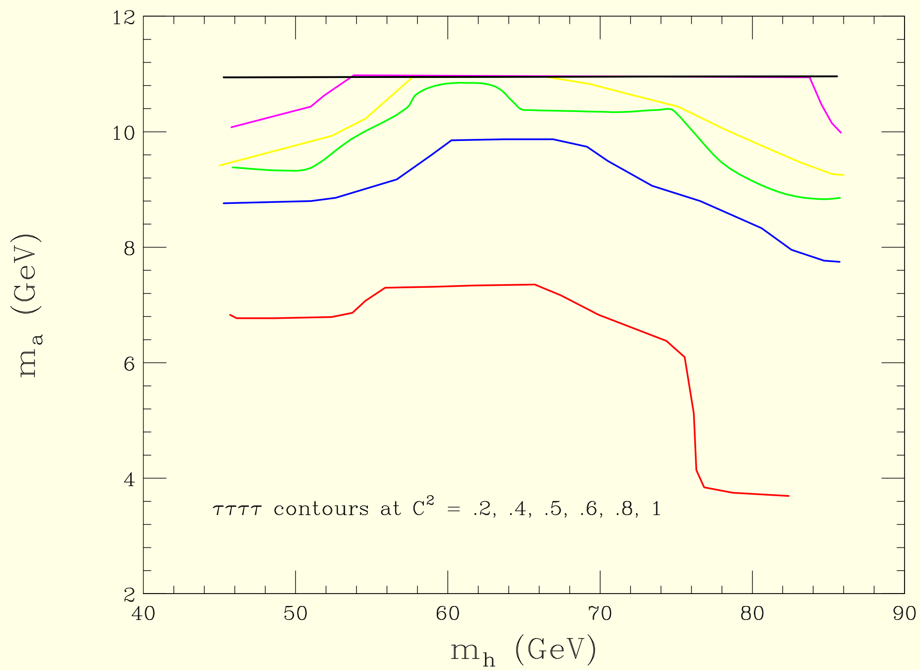


Figure 16: Contours of limits on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to aa) \times [BR(a \to \tau^+\tau^-)]^2$ at $C^2 = 0.2$, 0.4, 0.5, 0.6, 0.8 and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if $C^2 > 0.2$, then the region below the $C^2 = 0.2$ contour is excluded at 95% CL.

If the $a_1 a_1 \to (gg, q\overline{q}) + (gg, q\overline{q})$ decay is relevant, then we have the hadronic decay limits. They run out for $m_h > 80$ GeV.

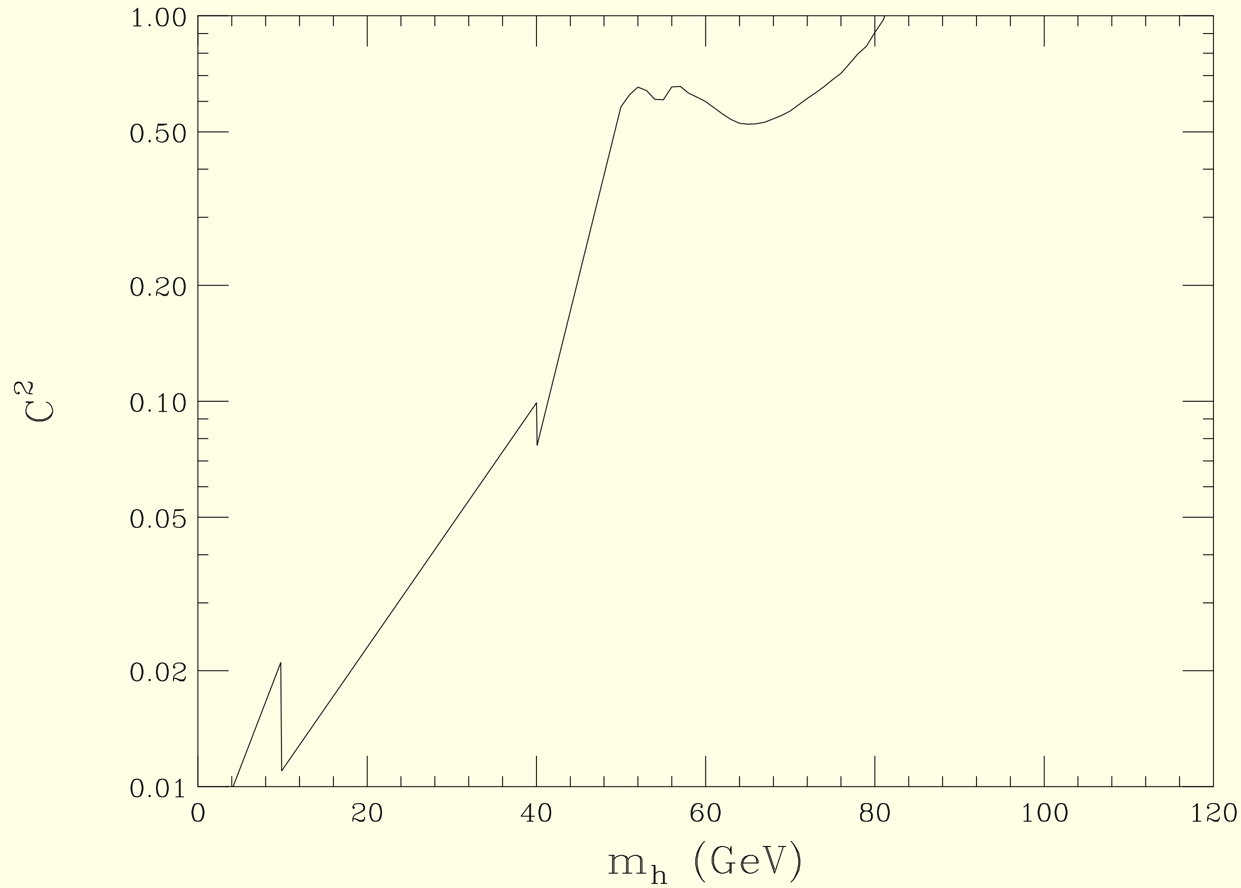


Figure 17: **Plot of the 95% CL limit on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to$ hadrons$)$, where h is only assumed to decay to hadrons, not any specific number of jets.**

Or, if we say that the gg or $q\overline{q}$ from each a_1 overlap to form a single 'jet', then we have the limits in the 'jet-jet' channel. They give $C^2 \lesssim 0.4$ for $m_h \sim 100~\mathrm{GeV}$ and might be relevant. A detailed analysis is needed.

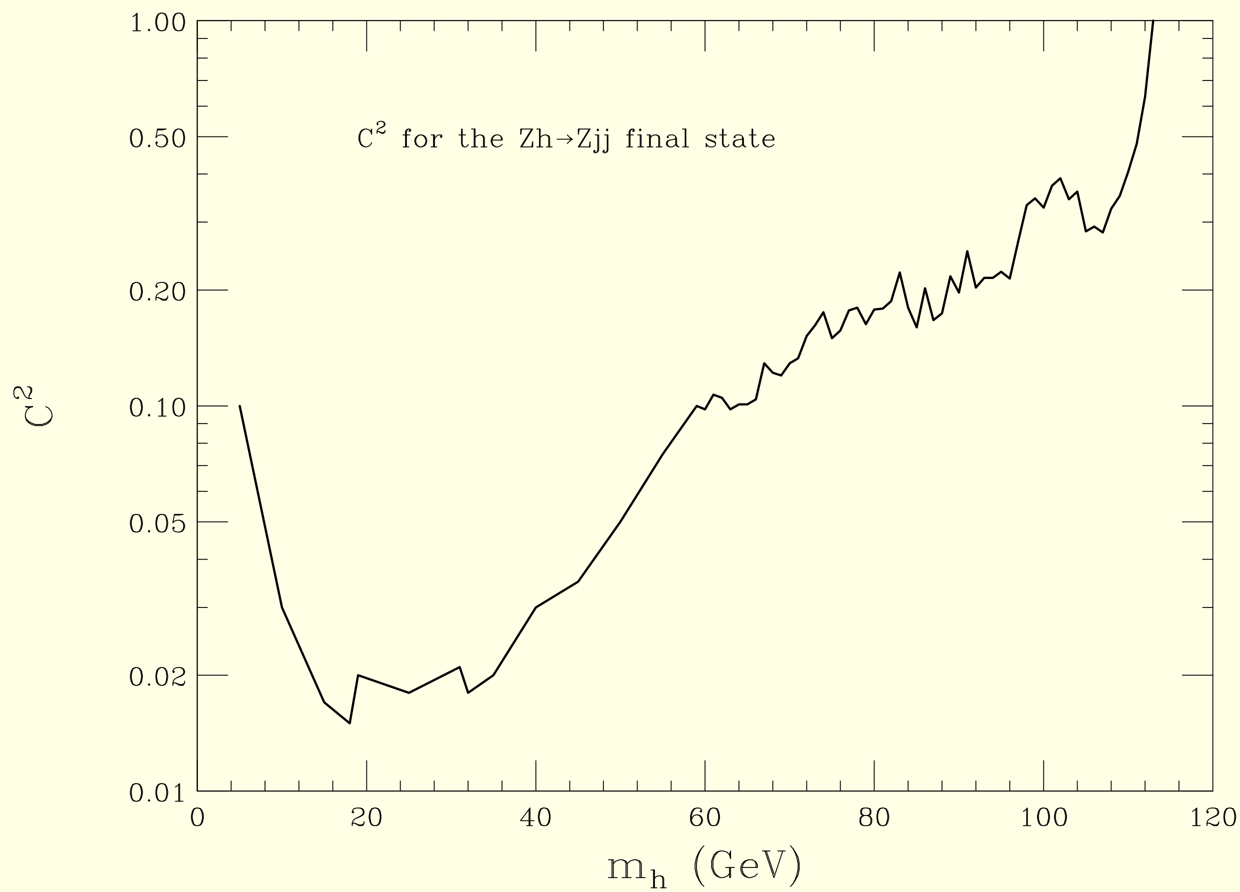


Figure 18: 95% CL upper limit on $C^2 = [g^2_{Zh}/[g^2_{Zh}]_{SM}] \times BR(h \to jj)$ from LEP analyzes.

- To see how well the $F < 10$, $m_{a_1} < 2m_b$ points describe the LEP excesses we have to run them through the full LHWG code. Well, we didn't do it, but Philip Bechtle did it for us.

  In Table 1, we give the precise masses and branching ratios of the h_1 and a_1 for all the $F < 10$ points.

  We also give the number of standard deviations, $n_{\rm obs}$ ($n_{\rm exp}$) by which the observed rate (expected rate obtained for the predicted signal+background) exceeds the predicted background. The numbers are obtained after full processing of all Zh final states using the preliminary LHWG analysis code (thanks to P. Bechtle). They are derived from $(1 - CL_b)_{\rm observed}$ and $(1 - CL_b)_{\rm expected}$ using the usual tables: e.g. $(1 - CL_b) = 0.32$, 0.045, 0.0027 correspond to 1σ, 2σ, 3σ excesses, respectively.

  The quantity $s95$ is the factor by which the signal predicted in a given case would have to be multiplied in order to exceed the 95% CL. All these quantities are obtained by processing each scenario through the full preliminary LHWG confidence level/likelihood analysis.

m_{h_1}/m_{a_1} (GeV)	Branching Ratios $h_1 \to b\bar{b}$	$h_1 \to a_1 a_1$	$a_1 \to \tau\bar{\tau}$	$n_{\rm obs}/n_{\rm exp}$ units of 1σ	$s95$	N_{SD}^{LHC}
98.0/2.6	0.062	0.926	0.000	2.25/1.72	2.79	1.2
100.0/9.3	0.075	0.910	0.852	1.98/1.88	2.40	1.5
100.2/3.1	0.141	0.832	0.000	2.26/2.78	1.31	2.5
102.0/7.3	0.095	0.887	0.923	1.44/2.08	1.58	1.6
102.2/3.6	0.177	0.789	0.814	1.80/3.12	1.03	3.3
102.4/9.0	0.173	0.793	0.875	1.79/3.03	1.07	3.6
102.5/5.4	0.128	0.848	0.938	1.64/2.46	1.24	2.4
105.0/5.3	0.062	0.926	0.938	1.11/1.52	2.74	1.2

Table 1: Some properties of the h_1 and a_1 for the eight allowed points with $F < 10$ and $m_{a_1} < 2m_b$ from our $\tan\beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300\ {\rm GeV}$ NMSSM scan. N_{SD}^{LHC} is the statistical significance of the best "standard" LHC Higgs detection channel for integrated luminosity of $L = 300\ {\rm fb}^{-1}$.

Comments

- **If $n_{\rm exp}$ is larger than $n_{\rm obs}$ then the excess predicted by the signal plus background Monte Carlo is larger than the excess actually observed and vice versa.**
- **The points with $m_{h_1} \lesssim 100\ {\rm GeV}$ have the largest $n_{\rm obs}$.**

- Point 2 gives the best consistency between $n_{\rm obs}$ and $n_{\rm exp}$, with a predicted excess only slightly smaller than that observed.
- Points 1 and 3 also show substantial consistency.
- For the 4th and 7th points, the predicted excess is only modestly larger (roughly within 1σ) compared to that observed.
- The 5th and 6th points are very close to the 95% CL borderline and have a predicted signal that is significantly larger than the excess observed.
- LEP is not very sensitive to point 8.

Thus, a significant fraction of the $F < 10$ points are very consistent with the observed event excess.

- In our scan there are many, many points that satisfy all constraints and have $m_{a_1} < 2m_b$. The remarkable result is that those with $F < 10$ have a substantial probability that they predict the Higgs boson properties that would imply a LEP $Zh \to Z + b$'s excess of the sort seen.

Collider Implications

- An important question is the extent to which the type of $h \to aa$ Higgs scenario (whether NMSSM or other) described here can be explored at the Tevatron, the LHC and a future e^+e^- linear collider.

  At the first level of thought, the $h_1 \to a_1 a_1$ decay mode renders inadequate the usual Higgs search modes that might allow h_1 discovery at the LHC.

  Since the other NMSSM Higgs bosons are rather heavy and have couplings to b quarks that are not greatly enhanced, they too cannot be detected at the LHC. The last column of Table 1 shows the statistical significance of the most significant signal for *any* of the NMSSM Higgs bosons in the "standard" SM/MSSM search channels for the eight $F < 10$ NMSSM parameter choices.

  For the h_1 and a_1, the most important detection channels are $h_1 \to \gamma\gamma$, $Wh_1 + t\bar{t}h_1 \to \gamma\gamma\ell^\pm X$, $t\bar{t}h_1/a_1 \to t\bar{t}b\bar{b}$, $b\bar{b}h_1/a_1 \to b\bar{b}\tau^+\tau^-$ and $WW \to h_1 \to \tau^+\tau^-$.

  Even after $L = 300~\mathrm{fb}^{-1}$ of accumulated luminosity, the typical maximal signal strength is at best 3.5σ. For the eight points of Table 1, this largest signal derives from the $Wh_1 + t\bar{t}h_1 \to \gamma\gamma\ell^{\pm}X$ channel.

There is a clear need to develop detection modes sensitive to the $h_1 \to a_1a_1 \to \tau^+\tau^-\tau^+\tau^-$ and (unfortunately) $4j$ decay channels.

I will focus on 4τ in my discussion of possibilities below, but keep in mind the $4j$ case.

Hadron Colliders

The LHC

1. An obvious possibility is $WW \to h_1 \to a_1a_1 \to 4\tau$.

   Study under way with Schumacher. Looks moderately promising but far from definitive results at this time.

2. Another mode is $t\bar{t}h_1 \to t\bar{t}a_1a_1 \to t\bar{t}\tau^+\tau^-\tau^+\tau^-$.

   Study begun.

3. A third possibility: $\widetilde{\chi}_2^0 \to h_1\widetilde{\chi}_1^0$ with $h_1 \to a_1a_1 \to 4\tau$.

(Recall that the $\widetilde{\chi}_2^0 \longrightarrow h_1\widetilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \longrightarrow b\overline{b}$ decays are dominant.)

4. **Last, but definitely not least: diffractive production** $pp \longrightarrow pph_1 \longrightarrow ppX$.

   The mass M_X can be reconstructed with roughly a $1-2$ GeV resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs.

   Preliminary results are that one expects about 3 clean, i.e. reconstructed and tagged, events per $30~\text{fb}^{-1}$ of luminosity. $\Rightarrow$ clearly a high luminosity game.

Tevatron

1. It is possible that Zh_1 and Wh_1 production, with $h_1 \longrightarrow a_1a_1 \longrightarrow 4\tau$, will provide a small signal. (Wacker et.al; JG, McElrath, Conway).

   Backgrounds can be made small, but efficiencies are low and one must simply accumulate enough events.

2. Wacker et. al. and JG+McElrath have considered $gg \longrightarrow h_1 \longrightarrow a_1a_1 \longrightarrow 4\tau$ which would have substantially larger rate. But cuts etc. imply low efficiencies.

Wacker et. al. suggest hints are possible in the all lepton channel with $6~\mathrm{fb}^{-1}$. We estimated $15~\mathrm{fb}^{-1}$ would be needed for believable signal.

Further points

- **If supersymmetry is detected at the Tevatron, but no Higgs is seen, and if LHC discovery of the h_1 remains uncertain, the question will arise of whether Tevatron running should be extended so as to allow eventual discovery of $h_1 \to 4\tau$.**

  However, rates imply that the h_1 signal could only be seen if Tevatron running is extended until $L > 10 - 15~\mathrm{fb}^{-1}$ (our estimates) has been accumulated.

  And, there is the risk that $m_{a_1} < 2m_\tau$, in which case Tevatron backgrounds to $a_1 a_1 \to 4 - jet$ would be impossibly large regardless of how the h_1 is produced.

- **Of course, even if the LHC is unable to see any of the NMSSM Higgs bosons, it *would* observe numerous supersymmetry signals and *would* confirm that $WW \to WW$ scattering is perturbative, implying that something like a light Higgs boson must be present.**

Lepton Colliders

- Of course, discovery of the h_1 will be straightforward at an e^+e^- linear collider via the inclusive $Zh \to \ell^+\ell^- X$ reconstructed M_X approach (which allows Higgs discovery independent of the Higgs decay mode).

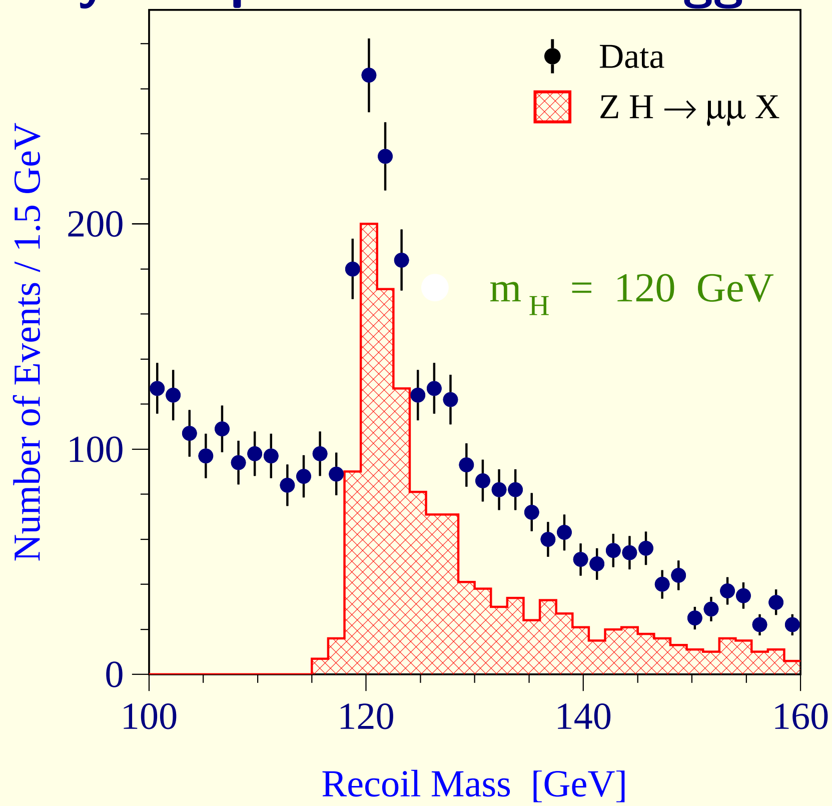


Figure 19: Decay-mode-independent Higgs M_X peak in the $Zh \to \mu^+\mu^- X$ mode for $L = 500~\mathrm{fb}^{-1}$ at $\sqrt{s} = 350~\mathrm{GeV}$, taking $m_h = 120~\mathrm{GeV}$..

There are lots of events in just the $\mu^+\mu^-$ channel (which you may want to restrict to since it has the best mass resolution).

- Although the $h \to b\bar{b}$ and $h \to \tau^+\tau^-$ rates are $1/10$ of the normal, the number of Higgs produced will be such that you can certainly see $Zh \to Zb\bar{b}$ and $Zh \to Z\tau^+\tau^-$ in a variety of Z decay modes.

  This is quite important, as it will allow you to subtract these modes off and get a determination of $B(h_1 \to a_1 a_1)$, which will provide unique information about $\lambda, \kappa, A_\lambda, A_\kappa$.

- Presumably direct detection in the $Zh \to Z a_1 a_1 \to Z 4\tau$ mode will also be possible although I am unaware of any actual studies.

  This would give a direct measurement of $B(h_1 \to a_1 a_1 \to \tau^+\tau^-\tau^+\tau^-)$. Error?

- Coupled with the indirect measurement of $B(h_1 \to a_1 a_1)$ from subtracting the direct $b\bar{b}$ and $\tau^+\tau^-$ modes would give a measurement of $B(a_1 \to \tau^+\tau^-)$.

  This would allow a first unfolding of information about the a_1 itself.

Of course, the above assumes we have accounted for all modes.

- Maybe, given the large event rate, one could even get a handle on modes such as $h_1 \to a_1 a_1 \to \tau^+\tau^- jj$ ($j = c, g$), thereby getting still more cross checks.

- At a $\gamma\gamma$ collider, the $\gamma\gamma \to h_1 \to 4\tau$ signal will be easily seen (Gunion, Szleper).

  This could help provide still more information about the h.

- In contrast, since (as already noted) the a_1 in these low-F NMSSM scenarios is fairly singlet in nature, its *direct* (i.e. not in h_1 decays) detection will be very challenging even at the ILC.

- Further, the low-F points are all such that the other Higgs bosons are fairly heavy, typically above $400~\mathrm{GeV}$ in mass, and essentially inaccessible at both the LHC and all but a $\gtrsim 1~\mathrm{TeV}$ ILC.

A few notes on $m_{a_1} > 2m_b$.

- We should perhaps also not take describing the LEP excess and achieving extremely low fine tuning overly seriously.

  Indeed, scenarios with $m_{h_1} > 114~\mathrm{GeV}$ (automatically out of the reach of LEP) begin at a still modest (relative to the MSSM) $F \gtrsim 25$.

  In fact, one can probably push down to as low as $m_{h_1} \gtrsim 108 \div 110~\mathrm{GeV}$ when $m_{a_1} > 2m_b$.

  $\Rightarrow$ must be on the lookout for the $4b$ and $2b2\tau$ final states from h_1 decay, with $h_1 \to 4b$ being the largest when $m_{a_1} > 2m_b$.

- At the LHC, the modes that seem to hold some promise are:
  1. $WW \to h_1 \to a_1 a_1 \to b\bar{b}\tau^+\tau^-$.
     Our (JFG, Ellwanger, Hugonie, Moretti) work suggested some hope. Experimentalists (esp. D. Zerwas) are working on a fully realistic evaluation but are not that optimistic.
  2. $t\bar{t}h_1 \to t\bar{t}a_1a_1 \to t\bar{t}4b$.
     This I imagine will be viable.

3. Gluino cascades containing $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$.
4. Doubly diffractive $pp \to pph_1$ followed by $h_1 \to a_1 a_1 \to 4b$ or $2b2\tau$. These modes are also under consideration by JFG, Khoze, ...

- At the Tevatron, perhaps the lack of overlapping events and lower background rates might allow some sign of a signal in modes such as Wh_1 and Zh_1 production with $h_1 \to a_1 a_1 \to 4b$ or $2b2\tau$. There is a study underway by G. Huang, Tao Han and collaborators.

General Considerations

- We should note that much of the discussion above regarding Higgs discovery is quite generic. Whether the a is truly the NMSSM CP-odd a_1 or just a lighter Higgs boson into which the SM-like h pair-decays, hadron collider detection of the h in its $h \to aa$ decay mode will be very challenging — only an e^+e^- linear collider can currently guarantee its discovery.

Conclusions

- If low fine-tuning is imposed for an acceptable model, we should expect:
  - a $m_{h_1} \sim 100\ \mathrm{GeV}$ Higgs decaying via $h_1 \to a_1 a_1$.
    Higgs detection will be quite challenging at a hadron collider.
    Higgs detection at the ILC is easy using the missing mass $e^+e^- \to ZX$ method of looking for a peak in M_X.
    Higgs detection in $\gamma\gamma \to h_1 \to a_1 a_1$ will be easy.
  - The very smallest F values are attained when:
    * h_2 and h_3 have "moderate" mass, i.e. in the $300\ \mathrm{GeV}$ to $700\ \mathrm{GeV}$ mass range;
    * the a_1 mass is $< 2m_b$ and the a_1 has a substantial singlet component.
    * the stops and other squarks are light;
    * the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;
- Detailed studies of the $WW \to h_1 \to a_1 a_1$, $t\bar{t}h_1 \to t\bar{t}a_1 a_1$, diffractive $pp \to pph_1$ and $\widetilde{g}$ cascades with $\widetilde{\chi}^0_2 \to h_1 \widetilde{\chi}^0_1$ channels (with $h_1 \to 4b$ or 4τ) by the experimental groups at both the Tevatron and the LHC should receive significant priority.

- It is likely that other models in which the MSSM μ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

- In general, very natural solutions to the fine-tuning and little hierarchy problems are possible in relatively simple extensions of the MSSM.

  One does not have to employ more radical approaches or give up on small fine-tuning!

  Further, small fine-tuning probably requires a light SUSY spectrum in all such models and SUSY should be easily explored at both the LHC (and very possibly the Tevatron) and the ILC and $\gamma\gamma$ colliders.

  Only Higgs detection at the LHC will be a real challenge.

  Ability to check perturbativity of $WW \to WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY.