Towards N^{3}LO QCD Higgs production cross section

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Based on work in collaboration with:
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[Some results are from a sub-project paper, arXiv:1407.4049 Maik Höschele, Jens Hoff, TU]

Karlsruhe Institute of Technology

HP2.5
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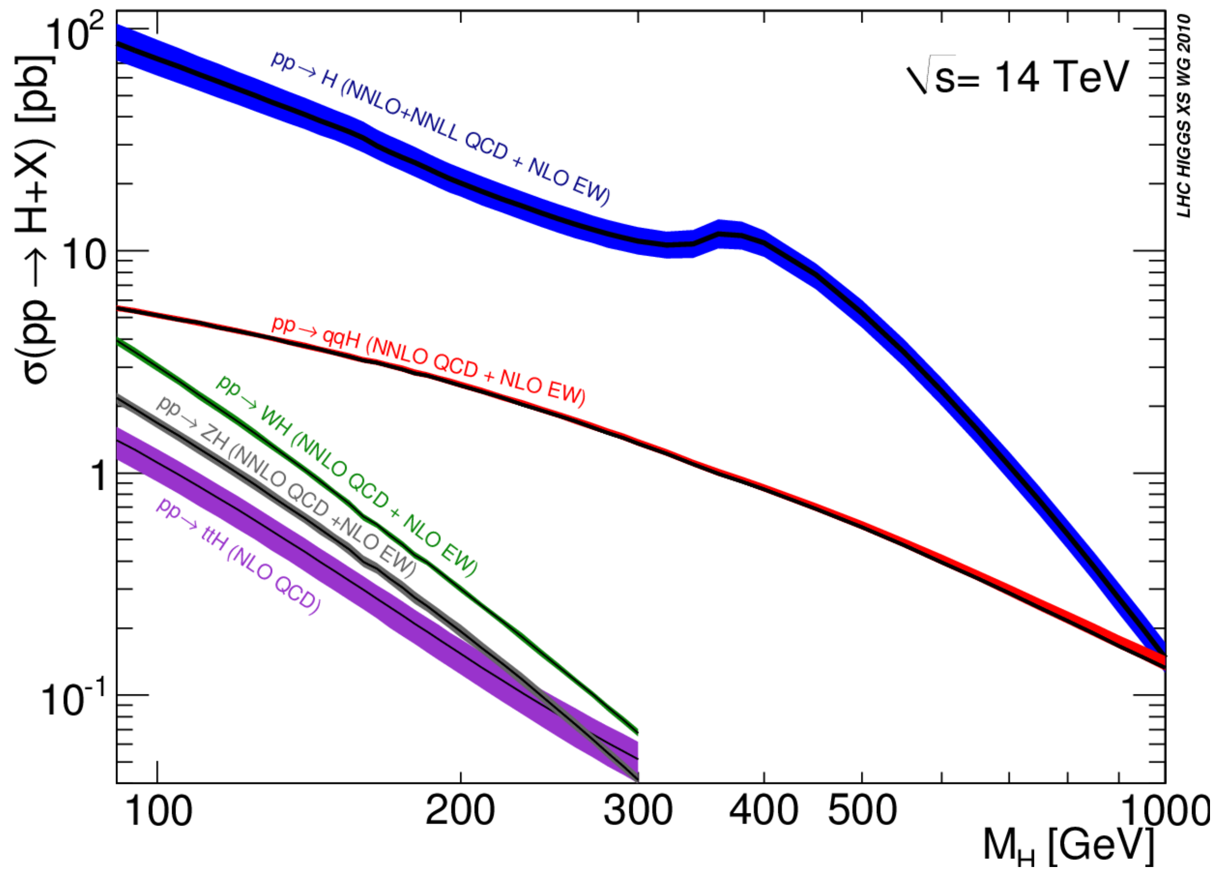
Introduction

- The discovery of a Higgs boson SM? BSM?

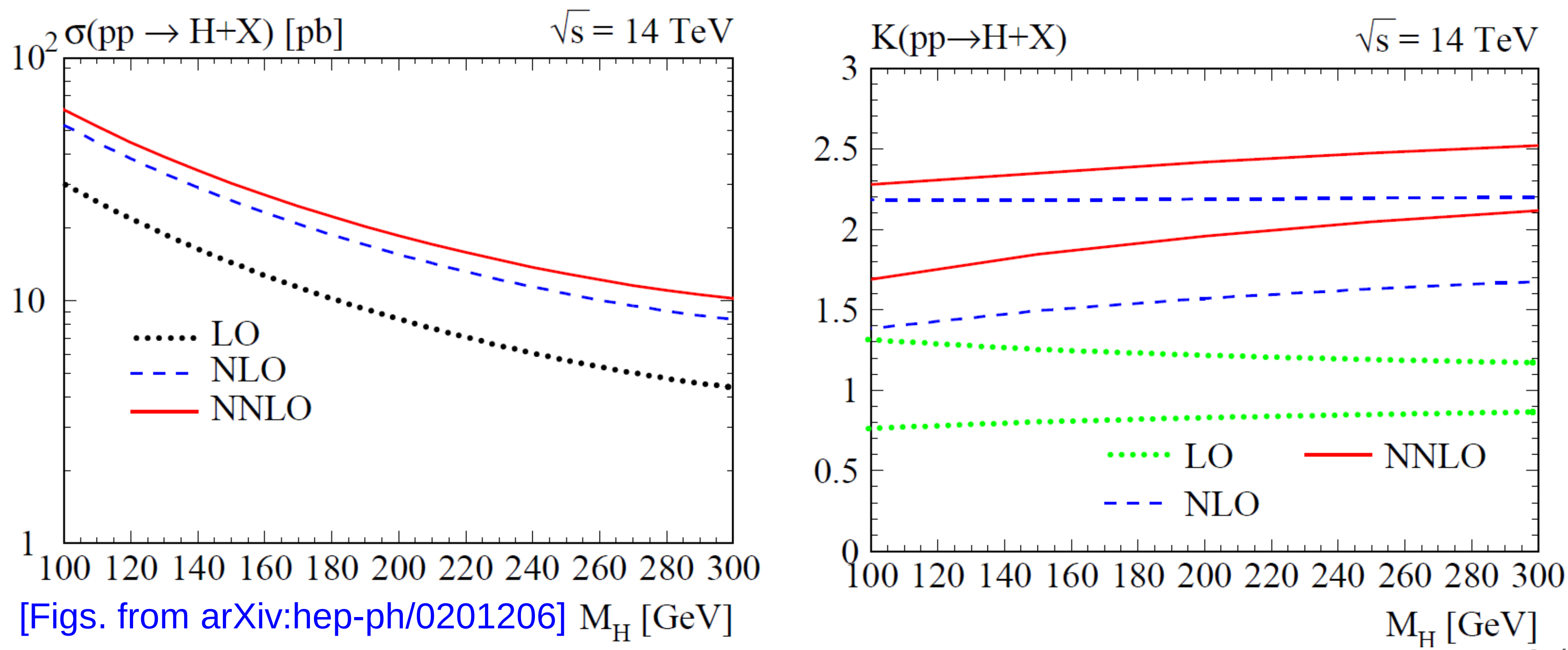
  ⇨ Needs for studying its production and decay processes

- Higgs production @ LHC
- Dominated by the gluon fusion channel
- Mainly QCD corrections



Introduction

- NNLO QCD corrections to the partonic cross sections in the gluon fusion channel are known
  [Harlander, Kilgore ’02; Anastasiou, Melnikov ’02; Ravindran, Smith, van Neerven ’03]
- Converge slowly ⇨ The scale uncertainty is the main source of theoretical uncertainty
- In total: large theoretical uncertainty $\sim$ 10-15 %



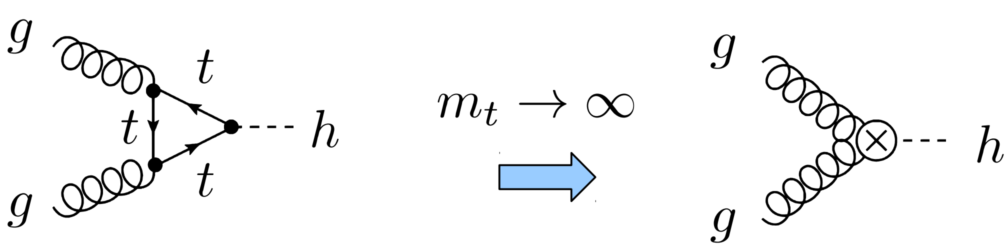
[Figs. from arXiv:hep-ph/0201206]

Improve TH prediction

- To improve the theoretical prediction:
  - Resummations
  - Approximated N^{3}LO
  - PDFs or α_s
  - Other contributions (finite top mass, bottom, EW etc.)
- Our ultimate goal:
  Improve the the theoretical prediction by computing N^{3}LO QCD corrections in the gluon fusion channel

Heavy-Top Effective Theory

- Top-loop induced ggh vertex



$$\mathcal{L}_{\text{eff}} = -\frac{C_1}{4v} G^a_{\mu\nu} G^{a,\mu\nu} h$$

v : VEV

- Wilson coefficient C_1 is known up to 3-loop

[Chetyrkin, Kniehl, Steinhauser ’98; Krämer, Laenen, Spira ’98]

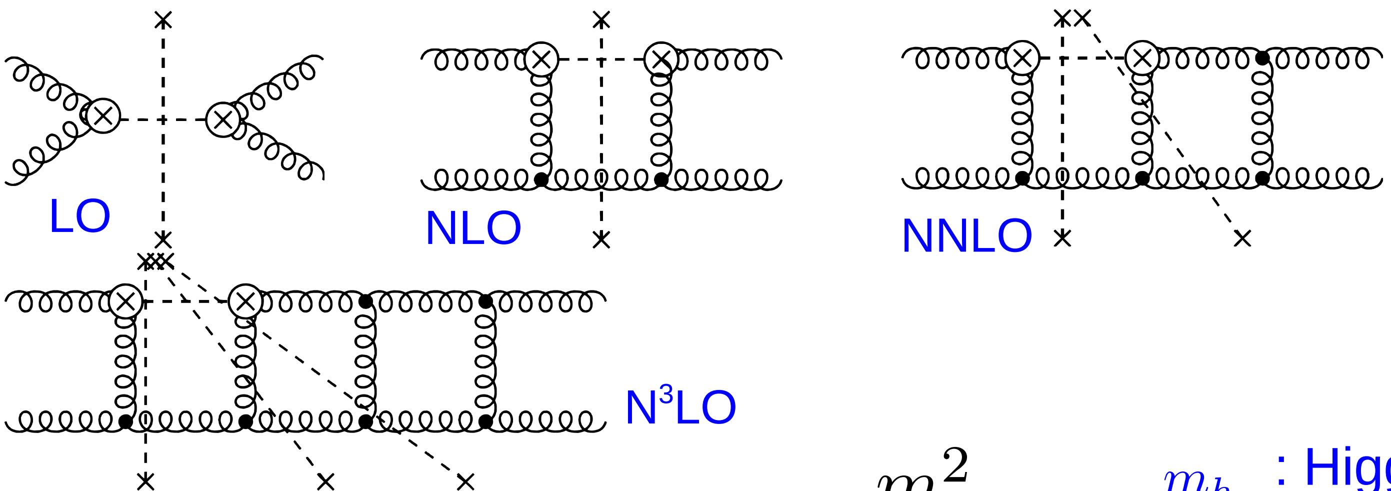
- Small power corrections in $\dfrac{m_h}{m_t}$ at NNLO

[Harlander, Ozeren ’09; Harlander, Mantler, Marzani, Ozeren ’10; Pak, Rogal, Steinhauser ’09-’11]

- Good approximation to the result in the full theory

Overview

- Dimensional regularization $D = 4 - 2\epsilon$
- Consider all possible cuts in forward scattering diagrams



- One kinematic variable $x = \dfrac{m_h^2}{s}$

m_h : Higgs mass

s : partonic center-of-mass energy

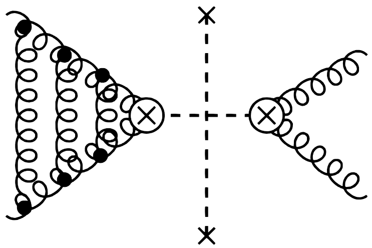
$x \to 1$: Higgs threshold / soft partons

- (Unrenormalized) partonic cross sections have expansions like

$$\sigma^{\text{partonic}}_{i+j\to h+X}(\alpha_s, x, \epsilon) = \frac{1}{v^2} \sum_{k,l} \alpha_s^k \epsilon^l \sigma^{\text{partonic}(k,l)}_{i+j\to h+X}(x)$$

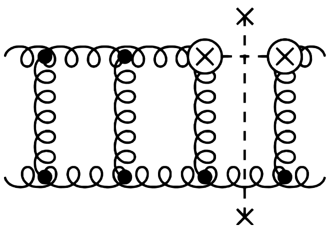
Status of N^{3}LO Calculations

- Triple-virtual (form factor) known



[Baikov, Chetyrkin, Smirnov[2], Steinhauser '09; Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]

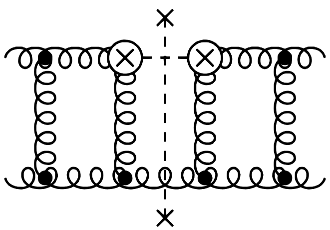
- Double-virtual real Soft limit $x \to 1$ known



[Duhr, Gehrmann '13; Li, Zhu '13]

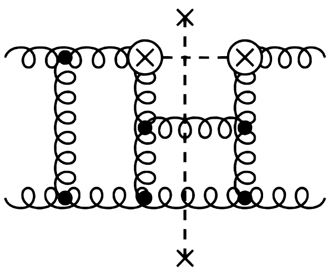
Dulat's talk

- Real virtual-squared known



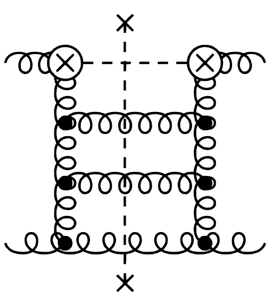
[Anastasiou, Duhr, Dulat, Herzog, Mistlberger '13; Kilgore '13]

- Double-real virtual Soft limit known



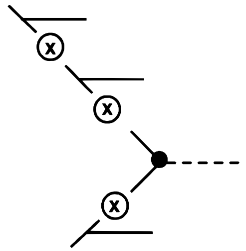
[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger '14; Li, Manteuffel, Schabinger, Zhu '14]

- Triple-virtual First two terms in soft expansion known



[Anastasiou, Duhr, Dulat, Mistlberger '13]

- IR subtraction terms known



$P_{im}^{(1)} \otimes (\tilde{\sigma}_{ij}^{(0)}/x) \otimes P_{jk}^{(1)} \otimes P_{kl}^{(1)}$

[Höschele, Hoff, Pak, Steinhauser, TU '13; Buehler, Lazopoulos '13]

- Hadronic Higgs production cross section at threshold up to N^{3}LO

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger '14]

Mistlberger's talk

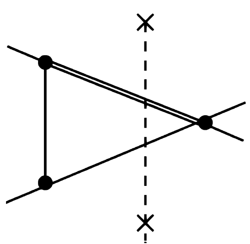
Reverse Unitarity Technique

[Anastasiou, Melnikov '02]

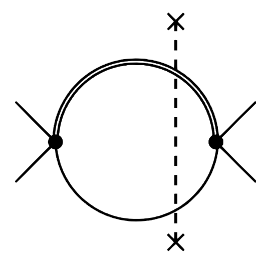
- In the same way as loop integrals, cut integrals can be reduced using integration-by-parts (IBP) relations

  ⇒ Reduction to master integrals (MIs)

  [1-loop example]



$$= -\frac{1-2\epsilon}{\epsilon(1-x)}$$



- We used
  - an in-house implementation of Laporta algorithm with TopoID for automatization and symmetries [Hoff, Pak]
  - FIRE [Smirnov]

Differential Equation Method

[Kotikov ’91; Bern, Dixon, Kosower ’94; Remiddi ’97; Gehrmann, Remiddi ’00]

- Consider the derivatives of MIs with respect to x
  - Dependence on x comes only from the Higgs propagator

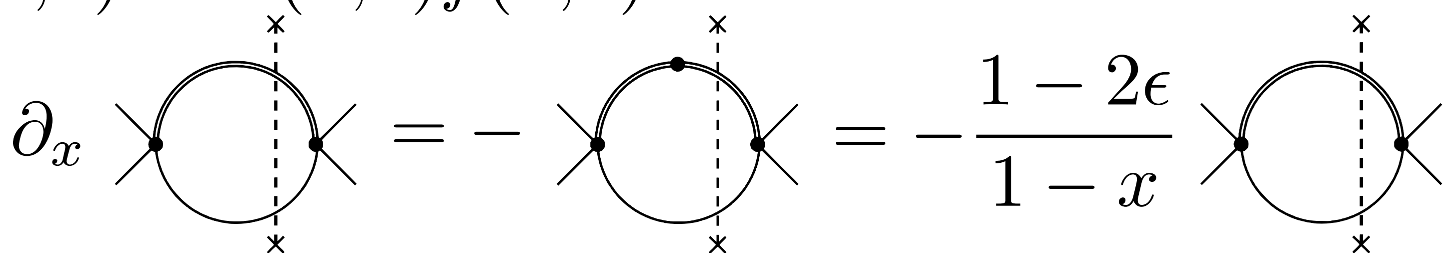
$$\partial_x \frac{1}{q^2 + x} = -\frac{1}{(q^2 + x)^2}$$

  - Equivalent to increasing the index of the Higgs propagator (put a dot) up to a constant factor
  - Resultant integrals can be reduced to the MIs

System of linear differential equations (DEs) for MIs

$$\partial_x \vec{f}(x, \epsilon) = A(x, \epsilon) \vec{f}(x, \epsilon)$$

[1-loop example]



- Can be quite complicated for higher loop orders

Change of Basis

- The choice of MIs are not unique. We can choose another basis of MIs:

$$\vec{f}_{\rm new}(x,\epsilon) = B(x,\epsilon)\vec{f}_{\rm old}(x,\epsilon)$$

- The system of DEs for the new MIs are:

$$\partial_x \vec{f}_{\rm new}(x,\epsilon) = A_{\rm new}(x,\epsilon)\vec{f}_{\rm new}(x,\epsilon)$$

$$A_{\rm new} = \Big[(\partial_x B) + B A_{\rm old}\Big] B^{-1}$$

i.e., $A_{\rm new}$ is determined by

  - $A_{\rm old}$: the system of DEs in the old basis
  - B : how the new basis integrals are reduced into the old basis integrals

Canonical Basis

- Conjecture: MIs for QCD integrals can be chosen as pure functions of uniform weight, which makes it strikingly simple to solve the system of DEs [Henn '13]
- Choose a basis satisfying the following canonical form

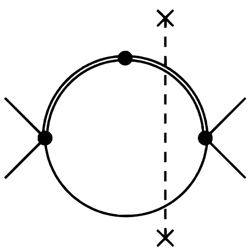
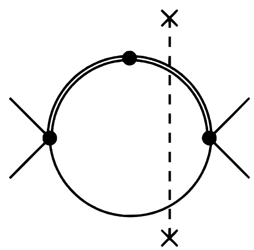
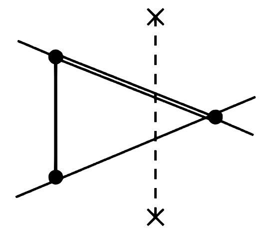
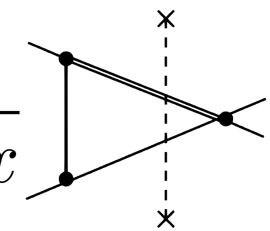
$$\partial_x \vec{f}(x,\epsilon) = \epsilon A(x) \vec{f}(x,\epsilon) \qquad A(x) = \frac{a}{x} + \frac{b}{1-x} + \frac{c}{1+x}$$

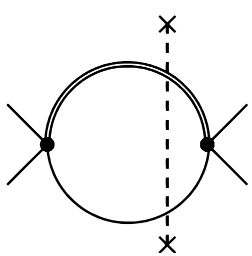
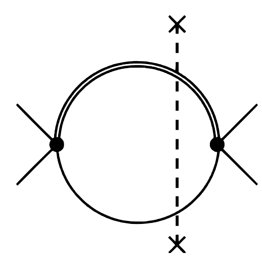
  - The expansion of the system of DEs in ϵ is triangular; easy to solve if the boundary condition is fixed at some $x = x_0$
We use the values at the soft limit $x = 1$
  - The solutions are expressed in iterated integrals of uniform weight; in this case, harmonic polylogarithms
[Remiddi, Vermaseren '00]

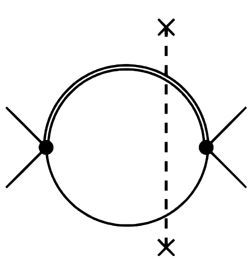
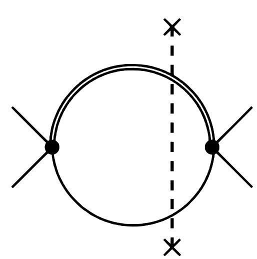
Canonical Basis

$$\partial_x \vec{f}(x,\epsilon) = \epsilon A(x) \vec{f}(x,\epsilon) \qquad A(x) = \frac{a}{x} + \frac{b}{1-x} + \frac{c}{1+x}$$

[1-loop example]

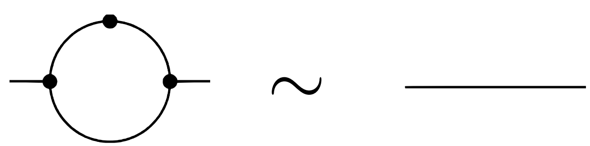
∂_x  $= \dfrac{2\epsilon}{1-x}$  or ∂_x  $= \dfrac{2\epsilon}{1-x}$ 

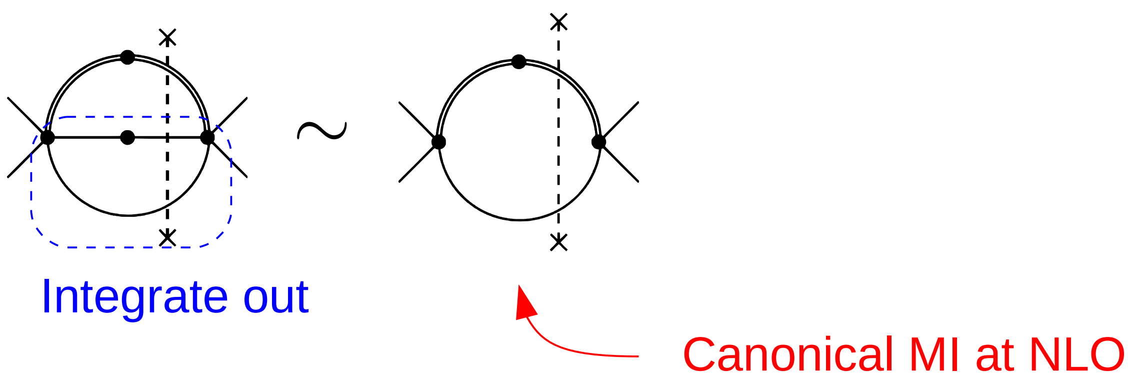
or $\partial_x \left[\dfrac{1}{1-x} \right.$  $\left. \right] = \dfrac{2\epsilon}{1-x} \left[\dfrac{1}{1-x} \right.$  $\left. \right]$

cf. ∂_x  $= -\dfrac{1-2\epsilon}{1-x}$ 

Find Canonical MIs

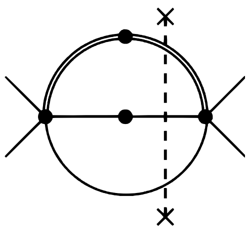
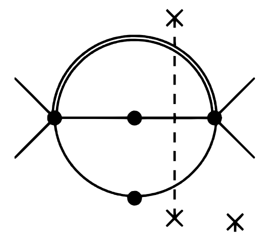
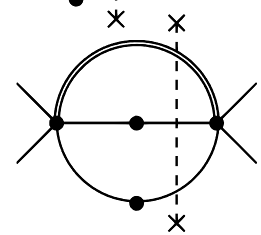
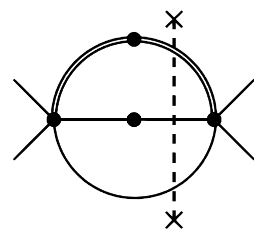
- No general recipe, but some tips exist to find candidates that may give the canonical form
- Hints from the lower loop orders results
  - Massless bubble with a dot $$\int \frac{d^d k}{(k^2)^2 (k+q)^2} \sim \frac{1}{(q^2)^{1+\epsilon}}$$

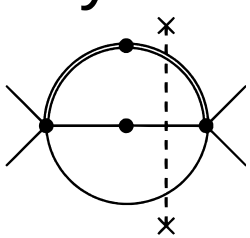
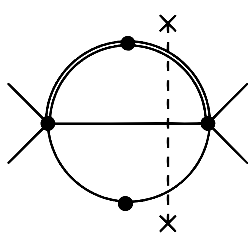
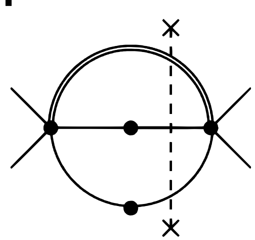


  - 

  is a good candidate at NNLO

Find Canonical MIs

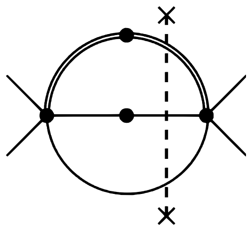
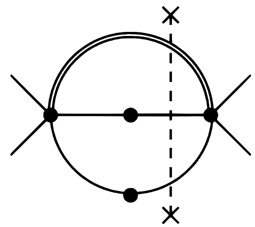
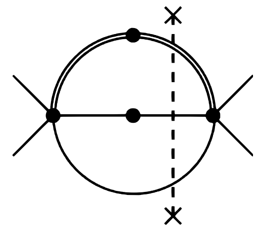
-  is one of 2-coupled MIs. Need another MI
- One may try a similar integral  but not canonical
- It turns out a linear combination  $+2$  is better
  - Feynman parameter representation tells us

 $+$  $+$ 

$$\sim \int \prod_j d\alpha_j \frac{\delta(1-\sum_j \alpha_j)}{U(\{\alpha_j\})W(x,\{\alpha_j\})}(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)$$

$$U(\{\alpha_j\}) = \alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3$$

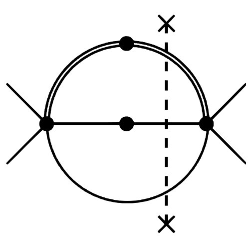
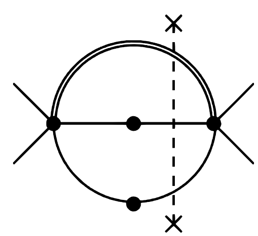
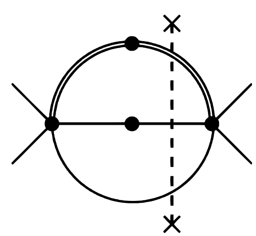
Find Canonical MIs

- $\vec{f} = \Big\{$  ,  $+2$  $\Big\}$

  gives

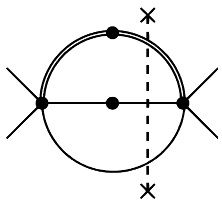
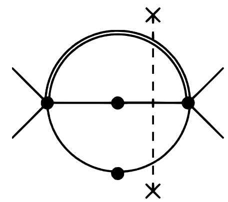
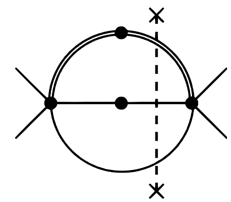
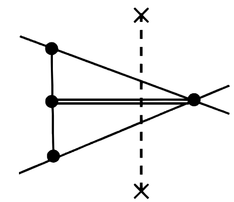
$$A = \begin{pmatrix} -\frac{3\epsilon}{x} & \frac{\epsilon}{x} \\ -\frac{6\epsilon}{x} - \frac{6\epsilon}{1-x} & \frac{2\epsilon}{x} + \frac{1+4\epsilon}{1-x} \end{pmatrix} = \begin{pmatrix} -\frac{3\epsilon}{x} & (1-x)\left(\frac{\epsilon}{x} + \frac{\epsilon}{1-x}\right) \\ -\frac{6\epsilon}{(1-x)x} & \frac{2\epsilon}{x} + \frac{1+4\epsilon}{1-x} \end{pmatrix}$$

- Need a normalization factor

$\vec{f} = \Big\{$  $, (1-x) \Big[$  $+2$  $\Big] \Big\}$

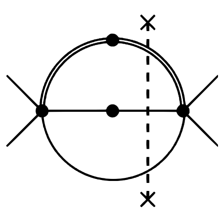
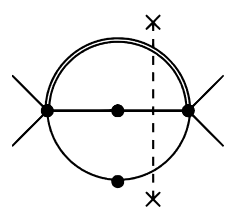
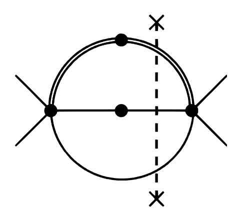
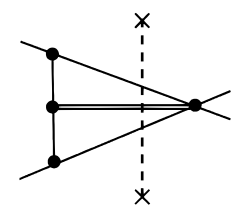
gives a canonical basis $\quad A = \begin{pmatrix} -\frac{3\epsilon}{x} & \frac{\epsilon}{x} + \frac{\epsilon}{1-x} \\ -\frac{6\epsilon}{x} & \frac{2\epsilon}{x} + \frac{4\epsilon}{1-x} \end{pmatrix}$

Add Another MI

$\vec{f} = \Big\{$  $, (1-x)\Big[$  $+2$  $\Big],$  $\Big\}$

gives $A = \begin{pmatrix} -\frac{3\epsilon}{x} & \frac{\epsilon}{x} + \frac{\epsilon}{1-x} & 0 \\ -\frac{6\epsilon}{x} & \frac{2\epsilon}{x} + \frac{4\epsilon}{1-x} & 0 \\ -\frac{1}{\epsilon x} & \frac{2}{\epsilon x} & -\frac{2\epsilon}{x} \end{pmatrix}$

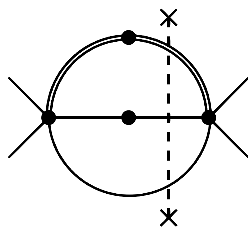
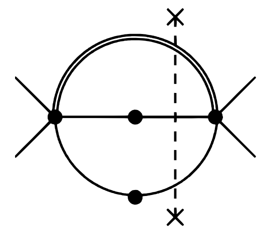
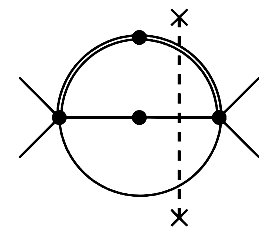
- Need a normalization factor

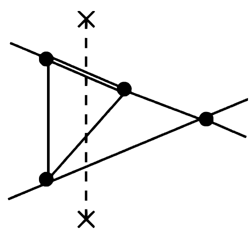
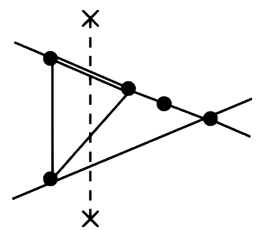
$\vec{f} = \Big\{$  $, (1-x)\Big[$  $+2$  $\Big], \epsilon^2$  $\Big\}$

gives a canonical basis $A = \begin{pmatrix} -\frac{3\epsilon}{x} & \frac{\epsilon}{x} + \frac{\epsilon}{1-x} & 0 \\ -\frac{6\epsilon}{x} & \frac{2\epsilon}{x} + \frac{4\epsilon}{1-x} & 0 \\ -\frac{\epsilon}{x} & \frac{2\epsilon}{x} & -\frac{2\epsilon}{x} \end{pmatrix}$

- Adding a MI to a system is relatively easy:
  Trial & error for each candidate, adjusting normalization

Add Coupled MIs

$\vec{f} = \Big\{$  $, (1-x) \Big[$  $+2$  $\Big],$

ϵ^2  $, \epsilon$  $\Big\}$

$$A = \begin{pmatrix} -\frac{3\epsilon}{x} & \frac{\epsilon}{x} + \frac{\epsilon}{1-x} & 0 & 0 \\ -\frac{6\epsilon}{x} & \frac{2\epsilon}{x} + \frac{4\epsilon}{1-x} & 0 & 0 \\ -\frac{\epsilon(2-5\epsilon)}{2(1-\epsilon)x} - \frac{\epsilon(1-4\epsilon)}{(1-\epsilon)(1-x)} & \frac{\epsilon(1-3\epsilon)}{4(1-\epsilon)x} + \frac{\epsilon(1-4\epsilon)}{4(1-\epsilon)(1-x)} & -\frac{\epsilon}{x} + \frac{2\epsilon}{1-x} & \frac{1}{2(1-x)} \\ \frac{2\epsilon(1-4\epsilon)}{(1-\epsilon)x} - \frac{3\epsilon^2}{(1-\epsilon)x^2} + \frac{2\epsilon(1-4\epsilon)}{(1-\epsilon)(1-x)} & -\frac{\epsilon(1-4\epsilon)}{2(1-\epsilon)x} + \frac{\epsilon^2}{(1-\epsilon)x^2} - \frac{\epsilon(1-4\epsilon)}{2(1-\epsilon)x} & -\frac{4\epsilon}{x} - \frac{4\epsilon}{1-x} & -\frac{1+2\epsilon}{x} - \frac{1}{1-x} \end{pmatrix}$$

- Can one of the added coupled MIs be a canonical MI if the other is adequately chosen?

Characteristic Form of Higher Order DE

- Point: from a set of DEs of f_1 and f_2 , we can eliminate f_2 and get a higher order DE of f_1

$$f_1' = a_{11} f_1 + a_{12} f_2 + \sum_i r_{1i} g_i \qquad g_i' = \sum_j \alpha_{ij} g_j$$

$$f_2' = a_{21} f_1 + a_{22} f_2 + \sum_i r_{2i} g_i$$

$$\Longrightarrow \quad f_1'' = -\left(-a_{11} - \frac{a_{12}'}{a_{12}} - a_{22}\right) f_1' + \left(a_{11}' - \frac{a_{11} a_{12}'}{a_{12}} + a_{12} a_{21} - a_{11} a_{22}\right) f_1$$
$$+ \sum_i \left(-\frac{a_{12}' r_{1i}}{a_{12}} - a_{22} r_{1i} + a_{12} r_{2i} + r_{1i}' + \sum_j r_{1j} \alpha_{ji}\right) g_i$$
$$=: -C_1 f_1' + C_0 f_1 + \sum_i C_{0i} g_i$$

- The coefficients C_1, C_0 and C_{0i} are independent on the choice of f_2

Characteristic Form of Higher Order DE

- For f_1 and f_2, we have A
- If f_1 is a canonical MI (there exists a proper choice of f_2^{p}, and f_1 and f_2^{p} gives A^{p} in the canonical form), the coefficients have the following specific form because A^{p} is linear in ϵ

$$C_1 = C_1^{(0)} + \epsilon C_1^{(1)} \qquad C_0 = \epsilon C_0^{(1)} + \epsilon^2 C_0^{(2)} \qquad C_{0i} = \epsilon C_{0i}^{(1)} + \epsilon^2 C_{0i}^{(2)}$$

- Equating the C's both from A and A^{p}, we get a set of (differential) equations for A^{p} even though f_2^{p} is unknown (at this point)

$$a_{12}^{\mathrm{p}\,\prime} + C_1^{(0)} a_{12}^{\mathrm{p}} = 0 \qquad a_{11}^{\mathrm{p}\,\prime} - \frac{a_{12}^{\mathrm{p}\,\prime}}{a_{12}^{\mathrm{p}}} a_{11}^{\mathrm{p}} = \epsilon C_0^{(1)} \qquad a_{22}^{\mathrm{p}} = -a_{11}^{\mathrm{p}} - \epsilon C_1^{(1)}$$

$$a_{21}^{\mathrm{p}} = \frac{a_{11}^{\mathrm{p}} a_{22}^{\mathrm{p}}}{a_{12}^{\mathrm{p}}} + \epsilon^2 \frac{C_0^{(2)}}{a_{12}^{\mathrm{p}}} \qquad r_{1i}^{\mathrm{p}\,\prime} - \frac{a_{12}^{\mathrm{p}\,\prime}}{a_{12}^{\mathrm{p}}} r_{1i}^{\mathrm{p}} = \epsilon C_{0i}^{(1)} \quad r_{2i}^{\mathrm{p}} = \frac{a_{22}^{\mathrm{p}} r_{1i}^{\mathrm{p}}}{a_{12}^{\mathrm{p}}} - \sum_j \frac{r_{1j}^{\mathrm{p}} \alpha_{ji}}{a_{12}^{\mathrm{p}}} + \epsilon^2 \frac{C_{0i}^{(2)}}{a_{12}^{\mathrm{p}}}$$

Characteristic Form of Higher Order DE

- Once all the elements in A^{p} are obtained, one can get the transformation B from $\{g_i, f_1, f_2\}$ to $\{g_i, f_1, f_2^{\mathrm{p}}\}$:

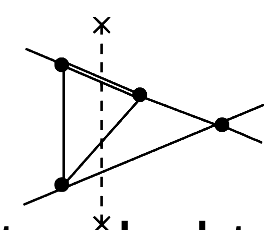
$$B = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & 1 & 0 \\ \beta_{2i} & b_{21} & b_{22} \end{pmatrix} \qquad B' = A^{\mathrm{p}} B - B A$$

Give linear equations
(not DEs)

cf. $A_{\mathrm{new}} = \Big[(\partial_x B) + B A_{\mathrm{old}} \Big] B^{-1}$

- f_2^{p} is expressed as a linear combination of $\{g_i, f_1, f_2\}$

Algorithm for Coupled MIs

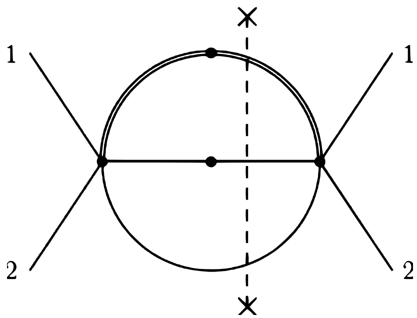
- Input: a basis $\{g_i, f_1, f_2\}$ (and A)
- Assume f_1 is a canonical MI. This leads $\{g_i, f_1, f_2^{\mathrm{p}}\}$, where f_2^{p} is expressed as a linear combination of $\{g_i, f_1, f_2\}$
- Any contradiction means f_1 cannot be a canonical MI
  - C's do not have the specific form
  - Obtained A^{p} is not of the canonical form
  - No consistent solution for B
- Answer to the previous question: ϵ^2  is canonical
- In principle, this algorithm can be extended to 3 or more coupled MIs

Canonical MIs at NNLO

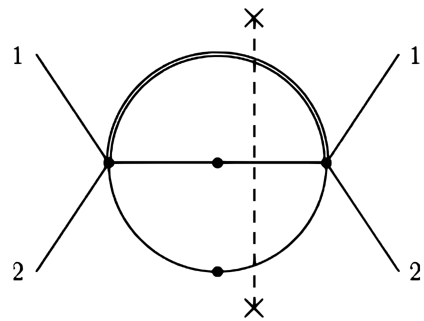
3-particle cut diagrams (17 MIs)

$$\partial_x \vec{V}(x,\epsilon) = \epsilon A_V(x) \vec{V}(x,\epsilon)$$

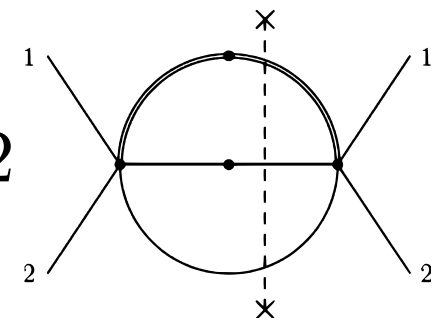
$$V_1 = \epsilon$$



$$V_2 = \epsilon(1-x) \Big[$$



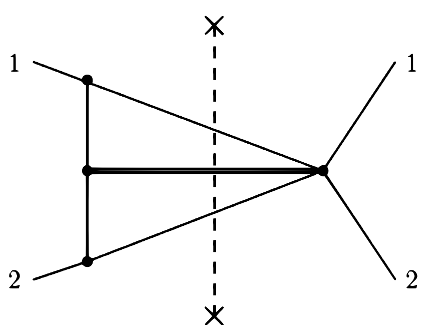
$$+\ 2$$



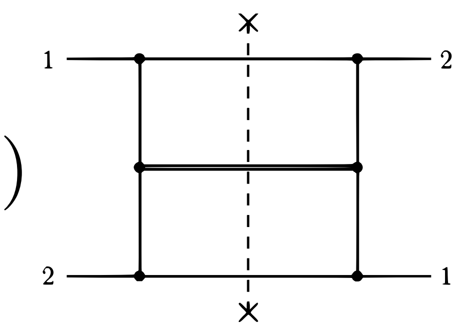
$$\Big]$$

coupled

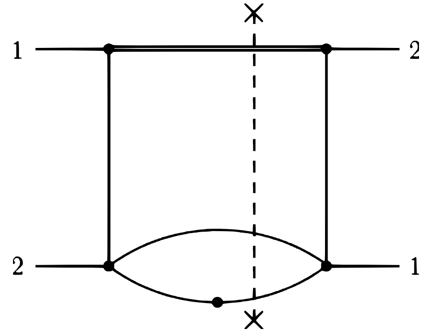
$$V_3 = \epsilon^3$$



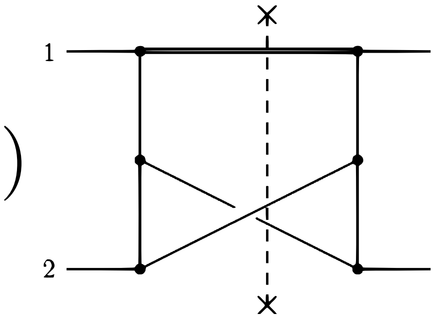
$$V_4 = \epsilon^3(1-x)$$



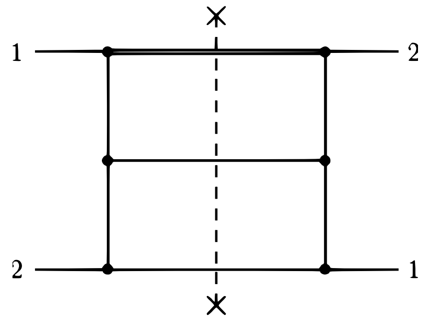
$$V_5 = \epsilon^2(1-x)$$



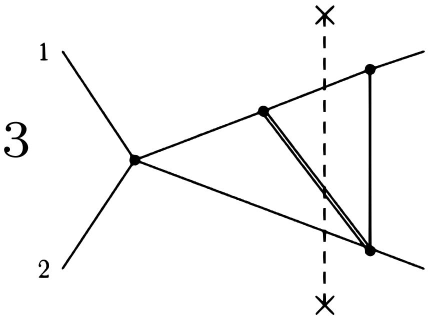
$$V_6 = \epsilon^3 x(1-x)$$



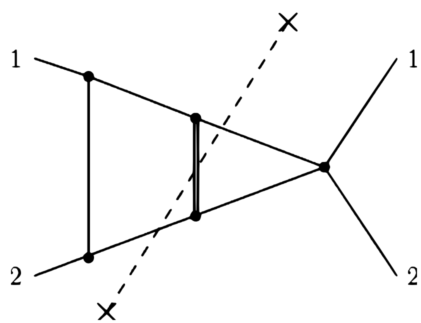
$$V_7 = \epsilon^3(1-x)$$



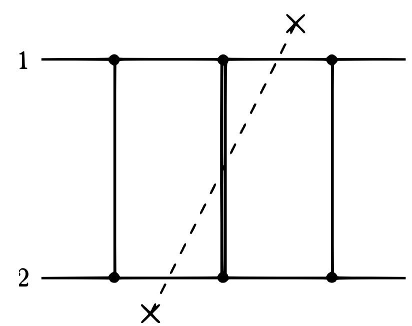
$$V_8 = \epsilon^3$$



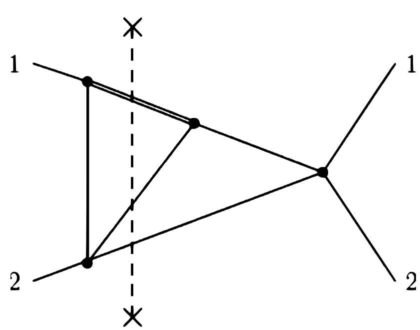
$$V_9 = \epsilon^3$$



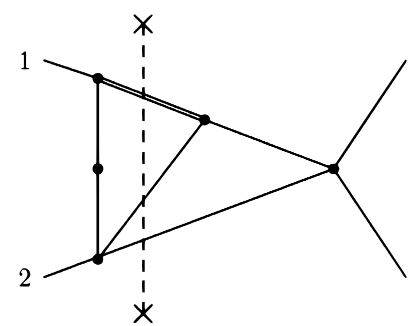
$$V_{10} = \epsilon^3 x$$



$$V_{11} = \epsilon^3$$

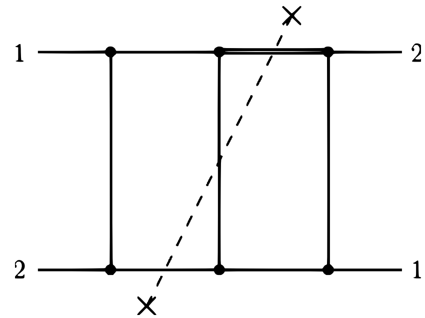


$$V_{12} = \epsilon^2 x$$

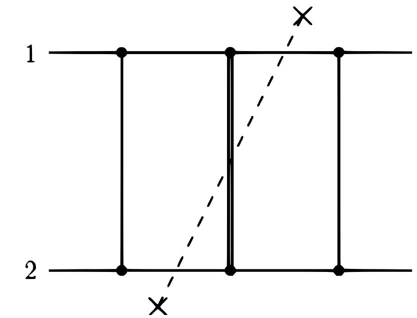


coupled

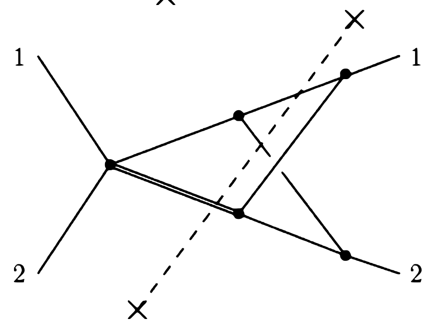
$$V_{13} = \epsilon^3(1-x)$$



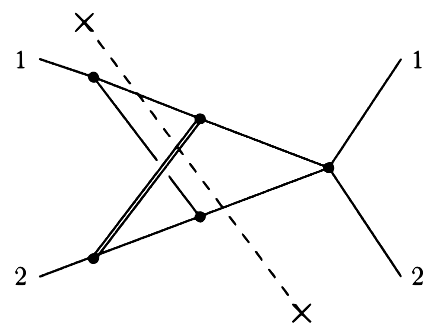
$$V_{14} = \epsilon^3(1+x)$$



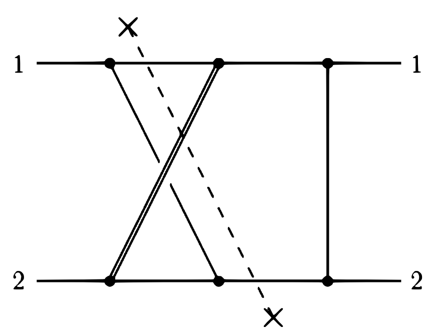
$$V_{15} = \epsilon^3(1-x)$$



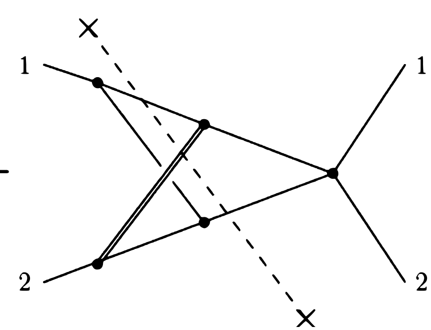
$$V_{16} = \epsilon^3(1+x)$$



$$V_{17} = \epsilon^3 x \Big[$$



$$-$$



$$\Big]$$

Canonical MIs at NNLO

3-particle cut diagrams (17 MIs)

$$\partial_x \vec{V}(x,\epsilon) = \epsilon A_V(x) \vec{V}(x,\epsilon)$$

$$A_V = \frac{a_V}{x} + \frac{b_V}{1-x} + \frac{c_V}{1+x}$$

$$a_V = \begin{pmatrix}
-3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-6 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & \frac{1}{2} & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & \frac{1}{2} & -2 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-5 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & -2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\
-4 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{2} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{2} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2
\end{pmatrix}$$

$$b_V = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{2} & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
-4 & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

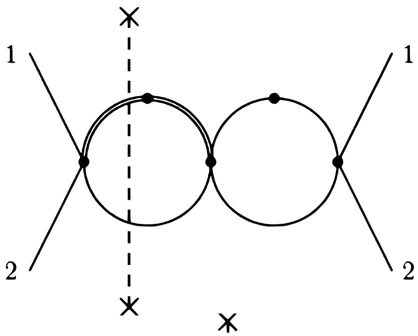
$$c_V = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-4 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & -2 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0
\end{pmatrix}$$

Canonical MIs at NNLO

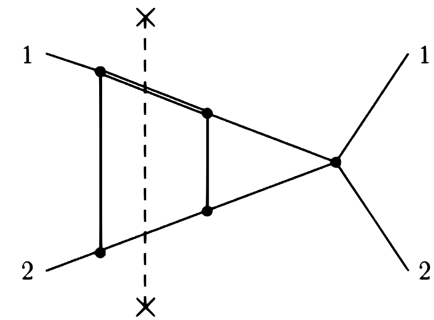
2-particle cut diagrams (6 MIs)

$$\partial_x \vec{W}(x,\epsilon) = \epsilon A_W(x)\vec{W}(x,\epsilon)$$

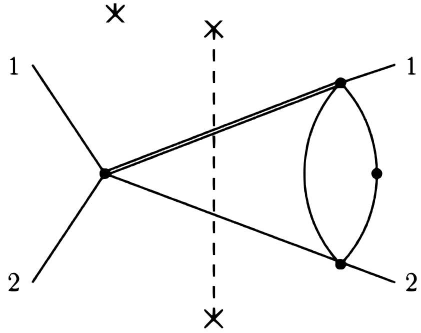
$$W_1 = \epsilon$$



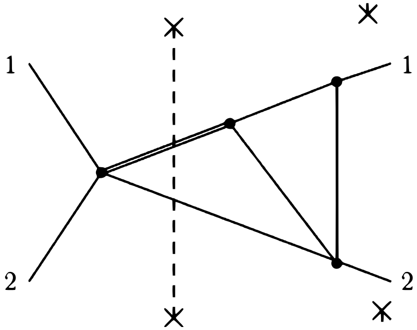
$$W_2 = \epsilon^3(1-x)$$



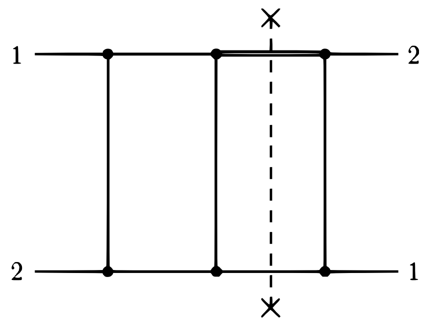
$$W_3 = \epsilon^2$$



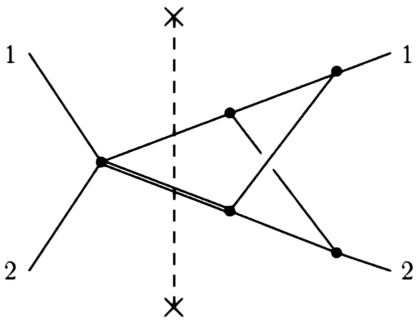
$$W_4 = \epsilon^3$$



$$W_5 = \epsilon^3(1-x)$$



$$W_6 = \epsilon^3(1-x)$$



Canonical MIs at NNLO

2-particle cut diagrams (6 MIs)

$$\partial_x \vec{W}(x,\epsilon) = \epsilon A_W(x) \vec{W}(x,\epsilon)$$

$$A_W = \frac{a_W}{x} + \frac{b_W}{1-x}$$

$$a_W = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -2 & -2 & 0 & 0 \\ 2 & -2 & 2 & 2 & 0 & 0 \\ 0 & 0 & -4 & -4 & 0 & 0 \end{pmatrix} \qquad b_W = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

Canonical MIs at NNLO

- The canonical basis and the reduction basis:

$$\vec{f}_{\text{canonical}} = B\vec{f}_{\text{reduction}}$$

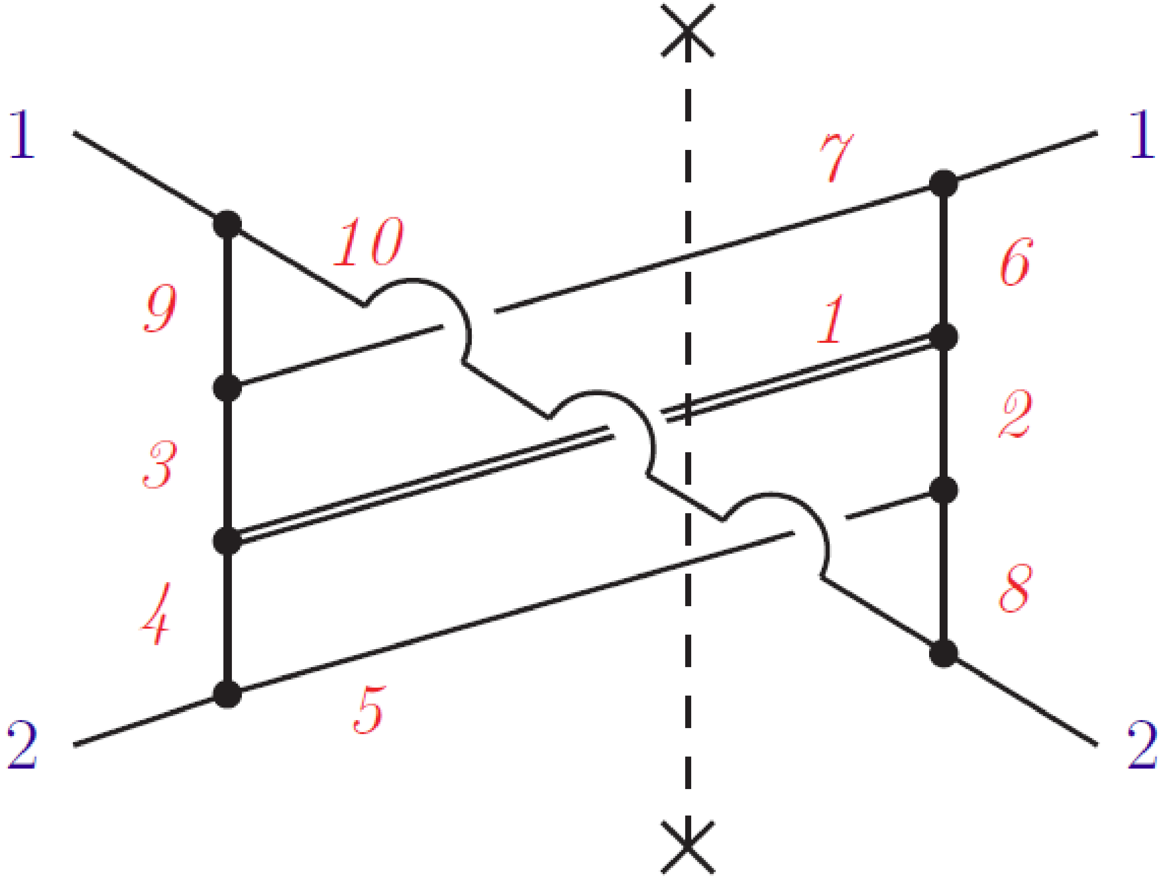
- In the reduction basis, the values of the soft limit are obtained from the corresponding phase space integrals
- Translate the soft limits in reduction basis into those in the canonical basis (Avoid direct evaluation of soft limits of diagrams with dotted cut propagators)

$$\vec{f}^{\text{soft}}_{\text{canonical}} = B\vec{f}^{\text{soft}}_{\text{reduction}}$$

- Solve MIs in the canonical basis with the soft limits as the boundary condition
- Checked $\vec{f}_{\text{reduction}} = B^{-1}\vec{f}_{\text{canonical}}$ with known results

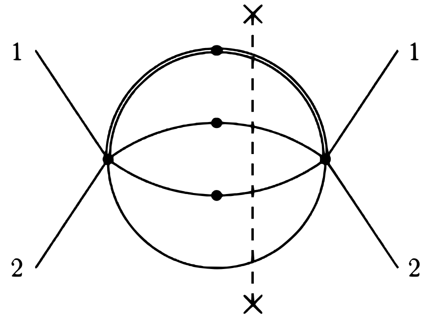
Example at N^{3}LO

- Sea-snake topology (11 MIs)

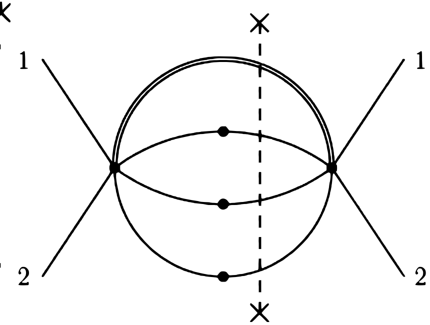


Example at N^{3}LO

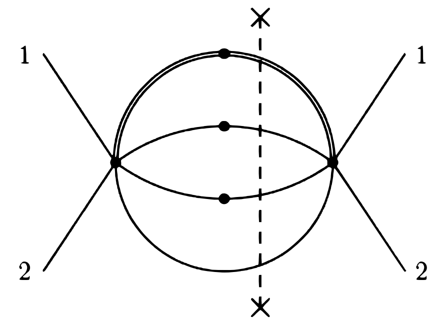
$S_1 = \epsilon^2$



$S_2 = \epsilon^2(1-x)\,\Big[$



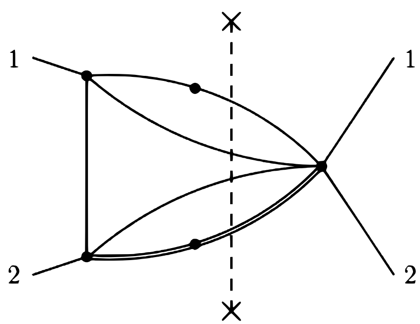
$+\;3$



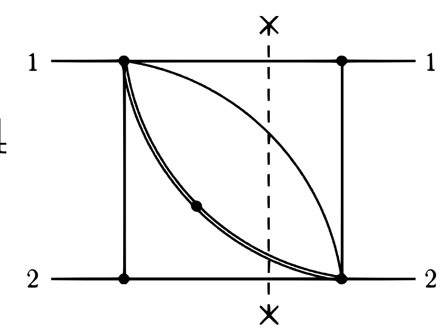
$\Big]$

coupled

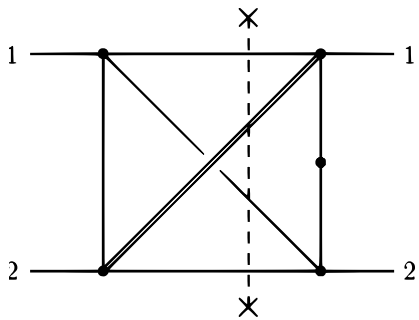
$S_3 = \epsilon^3$



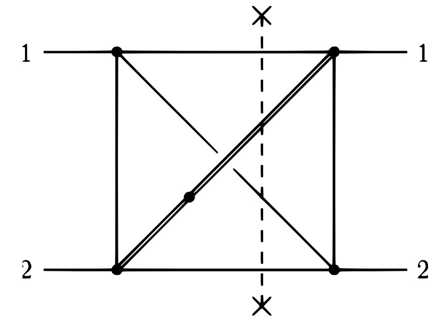
$S_4 = \epsilon^4$



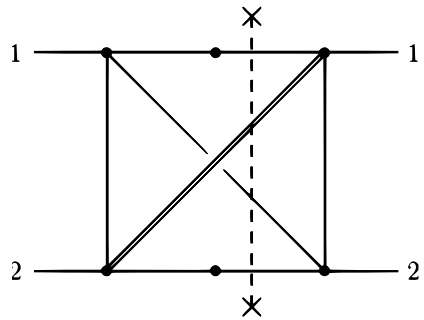
$$S_5 = \epsilon^4$$



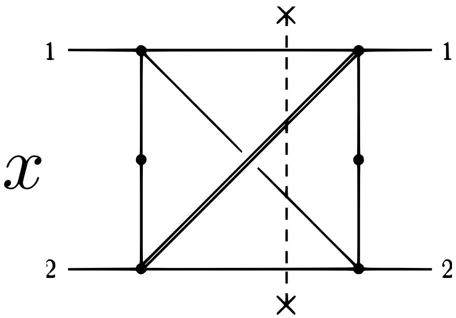
$$S_6 = \epsilon^4$$



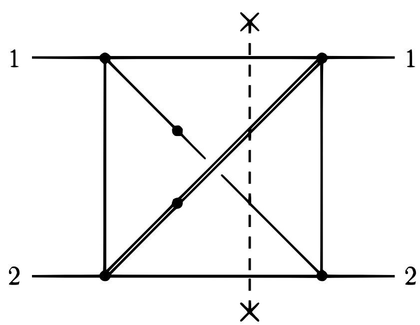
$$S_7 = \epsilon^3(1-x)$$



$$S_8 = \epsilon^3 x$$



$$S_9 = \epsilon^3 x$$

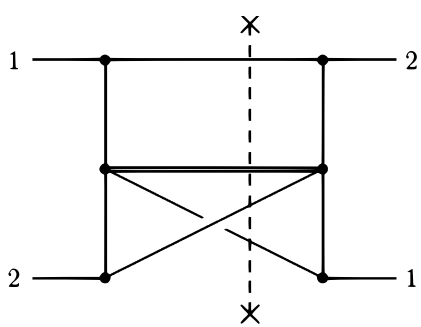


diagrammatically similar to MIs of off-shell K_4

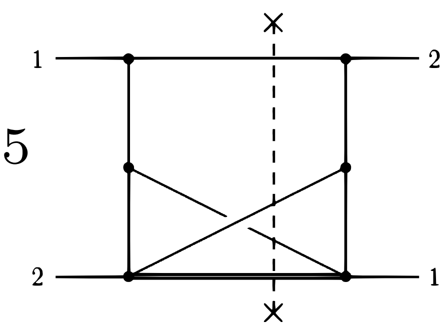
[Henn, Smirnov^2 '14]

coupled

$$S_{10} = \epsilon^5$$



$$S_{11} = \epsilon^5$$

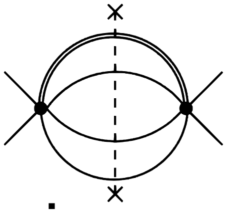


Example at $\mathrm{N}^3\mathrm{LO}$

$$A_S = \frac{a_S}{x} + \frac{b_S}{1-x}$$

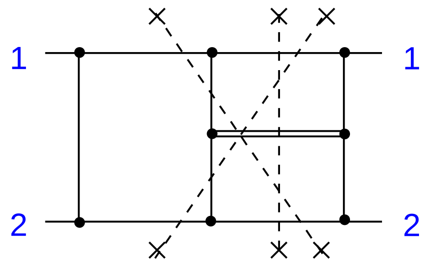
$$a_S = \begin{pmatrix}
-4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -\frac{1}{6} & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{6} & 0 & 0 & 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 & 0 \\
\frac{5}{6} & 0 & 1 & 0 & \frac{7}{3} & -\frac{7}{3} & \frac{7}{12} & \frac{1}{4} & -\frac{1}{6} & 0 & 0 \\
\frac{11}{3} & -\frac{1}{2} & 2 & 0 & \frac{14}{3} & -\frac{14}{3} & \frac{7}{6} & \frac{5}{2} & \frac{8}{3} & 0 & 0 \\
\frac{7}{3} & -\frac{1}{2} & 2 & 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{6} & -\frac{7}{2} & -\frac{2}{3} & 0 & 0 \\
\frac{7}{3} & -\frac{1}{2} & 2 & 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{6} & -\frac{1}{2} & -\frac{11}{3} & 0 & 0 \\
0 & -\frac{1}{6} & 0 & 2 & \cancel{0}\ 24 & \cancel{0} & 0 & 0 & 0 & -3 & 0 \\
-3 & 0 & -7 & 0 & -13 & -2 & -\frac{3}{2} & -\frac{5}{2} & -2 & 0 & -3
\end{pmatrix}
\qquad
b_S = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{11}{3} & -\frac{1}{2} & 6 & 0 & \frac{50}{3} & \frac{10}{3} & \frac{37}{6} & \frac{7}{2} & \frac{5}{3} & 0 & 0 \\
\frac{11}{3} & -\frac{1}{2} & 6 & 0 & \frac{50}{3} & \frac{10}{3} & \frac{1}{6} & \frac{7}{2} & \frac{5}{3} & 0 & 0 \\
-\frac{11}{3} & -\frac{1}{2} & -6 & 0 & -\frac{50}{3} & -\frac{10}{3} & -\frac{19}{6} & -\frac{7}{2} & -\frac{5}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Example at N^3LO

- As the boundary condition, compute values of the soft limit in the reduction basis, translated into those in the canonical basis
  - For this purpose, the method of soft MIs are used, where the coefficients in soft expansions are expressed in a small number of soft MIs [Anastasiou, Duhr, Dulat, Mistlberger '13]
  - The soft MI for this topology is only the phase-space 
  - Without the reduction to the soft MI, the leading terms in the soft expansions of the MIs can be computed with the standard MB techniques
- Solved up to weight 6, enough for N^3LO

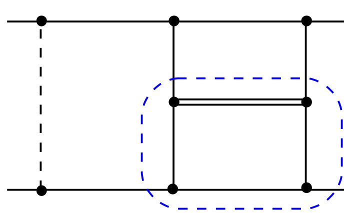
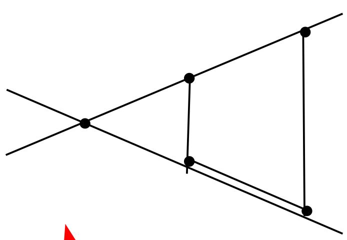
Work in Progress

- Other topologies: example: Tennis coat topology



  - 3- and 4-particle cuts (36 MIs)

- As a preliminary result, we have obtained a canonical basis
  - The last 2 MIs are coupled. Applying aforementioned algorithm to a candidate for the 35$^{\text{th}}$ MI succeeded

$\epsilon^5(1-x)$  $\sim \epsilon^3(1-x)$ 

Canonical MI at NNLO

- For the boundary condition, the standard MB technique works?? MB Tools (MB.m and Mbasymptotics.m by Czakon; barnesroutines.m by Kosower)

Summary

- We studied MIs needed for the Higgs production via the DE method with canonical bases
  - Recomputed MIs up to NNLO
  - Application to MIs at N^3LO
- We developed an algorithm to see whether or not an integral in a coupled system of DEs can be a canonical MI
- Work in progress for other topologies at N^3LO