CALCULATING THE DOUBLE-VIRTUAL REAL CORRECTIONS TO HIGGS AT N3LO

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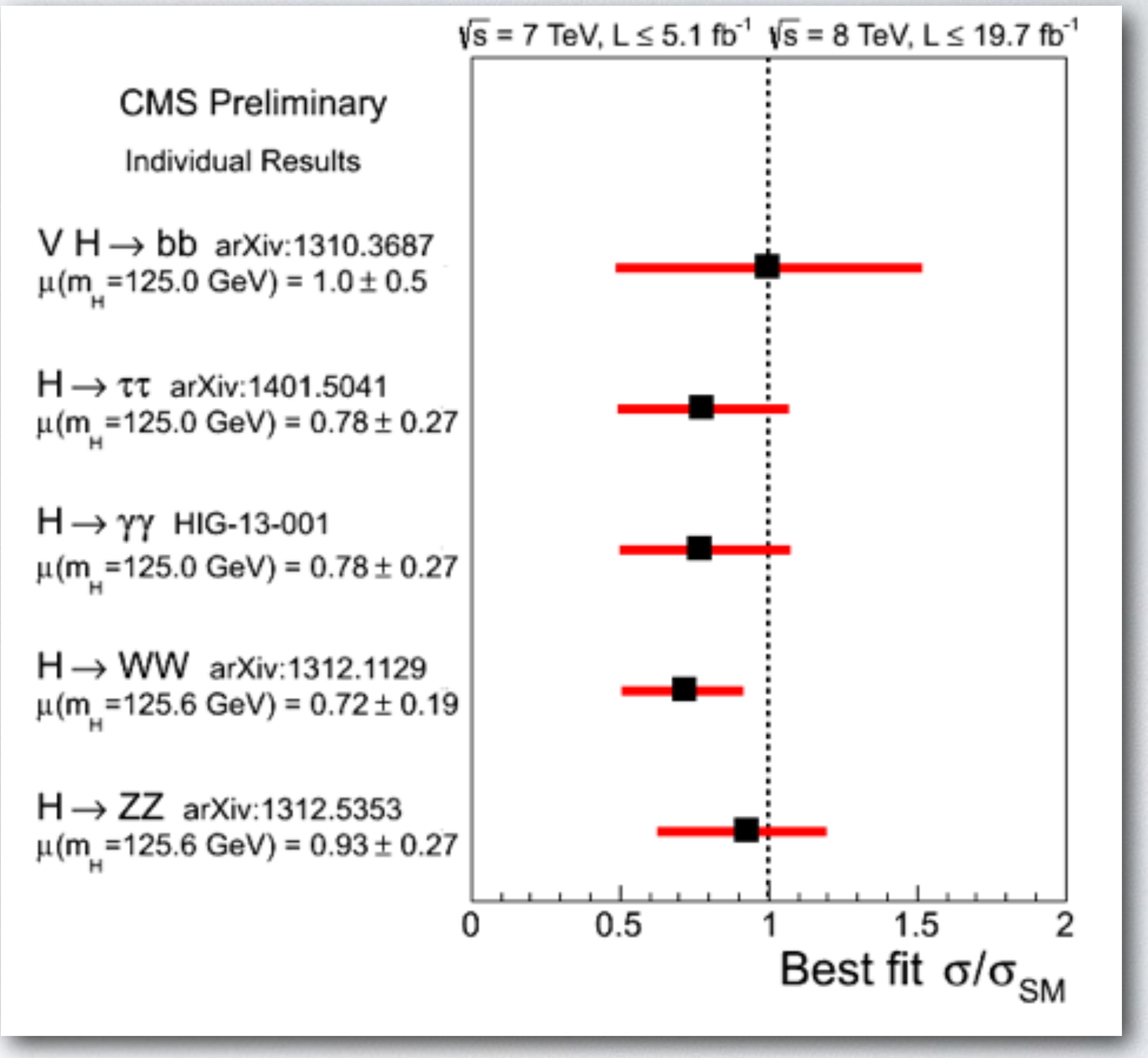
Motivation

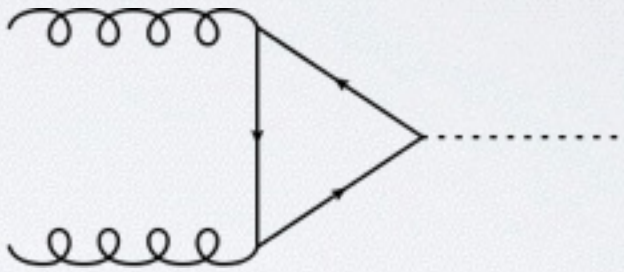
- Discovery marks the beginning of the experimental era of Higgs physics
- Determination of the properties of the Higgs will be a challenge for years to come
- Requires precision measurements and predictions



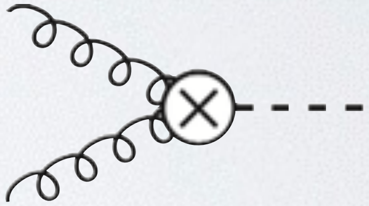
Great challenge for the theory community

The gluon fusion cross section

- The dominant Higgs production mode at the LHC is gluon fusion
    - Loop-induced process



- The Higgs boson is light compared to the top quark
- The top loop can be integrated out → effective theory



- The tree-level coupling of the gluons to the Higgs is described by a dimension five operator

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} - \frac{1}{4v} C_1 H G^a_{\mu\nu} G_a^{\mu\nu}$$

The gluon fusion cross section

- The gluon fusion cross-section in perturbation theory is

$$\sigma\left(pp \to H + X\right) = \tau \sum_{ij} \int_{\tau}^{1} dz \mathcal{L}_{ij}(z) \hat{\sigma}_{ij}\left(\frac{\tau}{z}\right)$$

- We compute the inclusive partonic cross section

- The partonic cross section is a function of

$$z = \frac{m_h^2}{\hat{s}} \quad \to \quad \bar{z} = \frac{\hat{s} - m_h^2}{\hat{s}} \qquad\qquad \tau = \frac{m_h^2}{E_{cm}^2}$$

- In perturbation theory the partonic cross section can be expanded

$$\hat{\sigma}(z) = \hat{\sigma}^{\mathrm{LO}}(z) + \alpha_s \hat{\sigma}^{\mathrm{NLO}}(z) + \alpha_s^2 \hat{\sigma}^{\mathrm{NNLO}}(z) + \alpha_s^3 \boxed{\hat{\sigma}^{\mathrm{N3LO}}(z)} + \ldots$$

The gluon fusion cross section

- The lower orders of the gluon fusion cross section have been computed

fixed order only

	σ [8 TeV]	$\delta\sigma$ [%]
LO	9.6 pb	~ 25%
NLO	16.7 pb	~ 20%
NNLO	19.6 pb	~ 7 - 9%
N3LO	???	~ 4 - 8%

  - NLO (full theory)

    [Dawson; Djouadi, Spira, Zerwas]

  - NNLO (effective theory and sub-leading top-mass corrections)

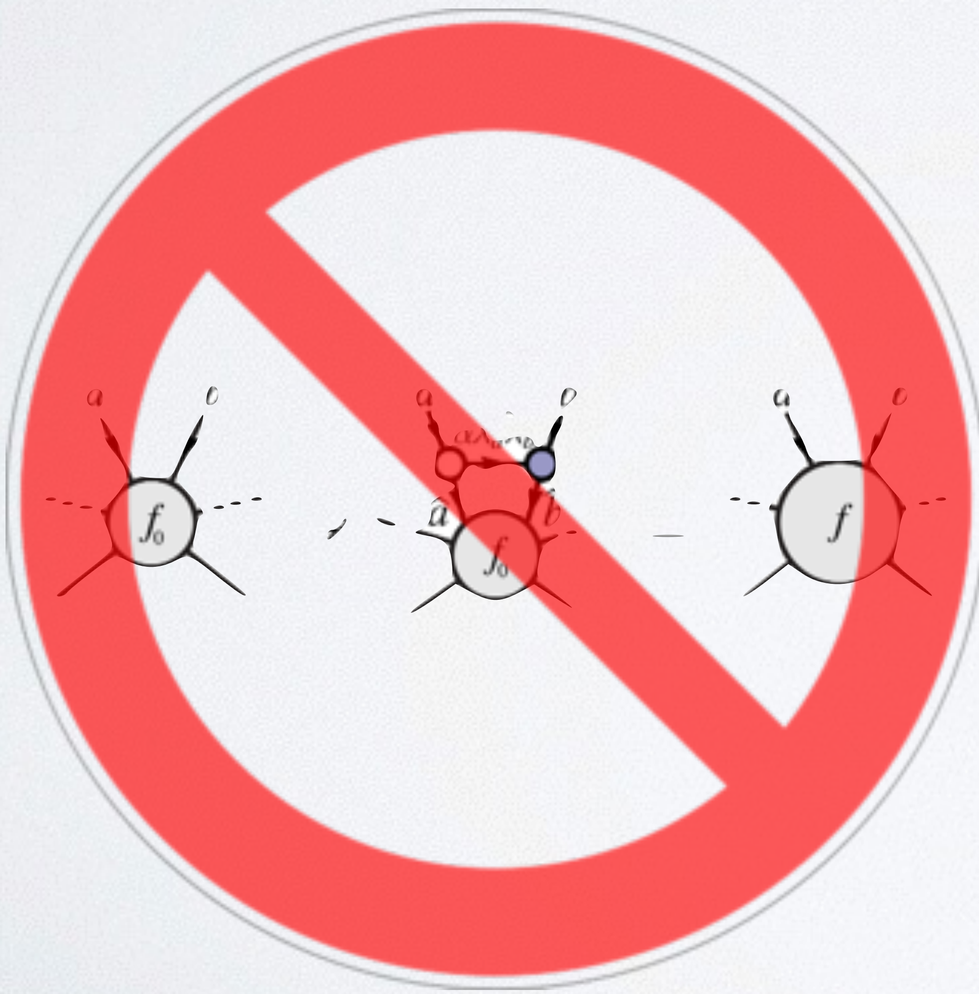
    [Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

- We want to push the calculation one order higher

- Uncharted territory in perturbation theory

The calculation

- Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point

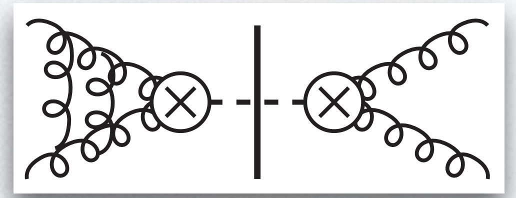
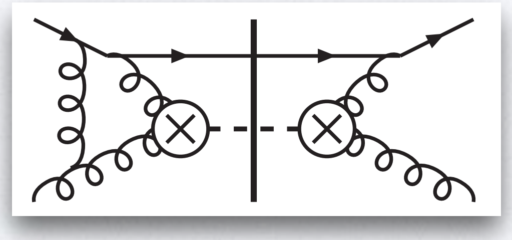
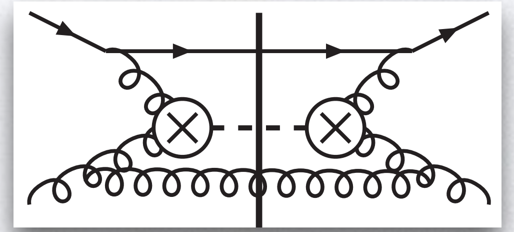


The calculation

- Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point

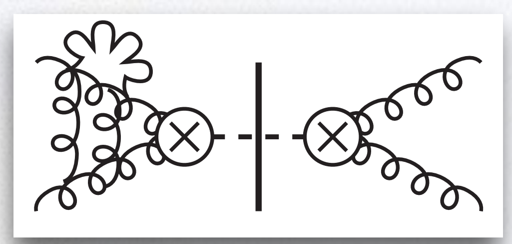
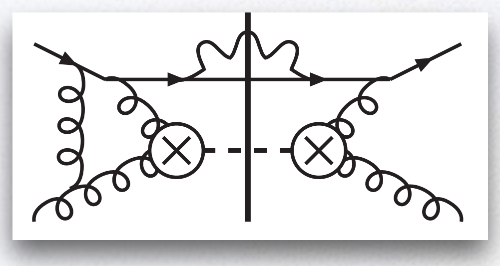
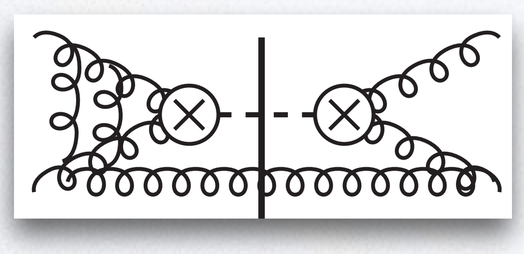
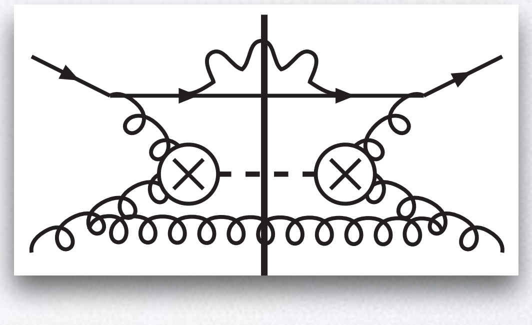
- Lots of Feynman diagrams

- At NNLO: ~1000 interference diagrams

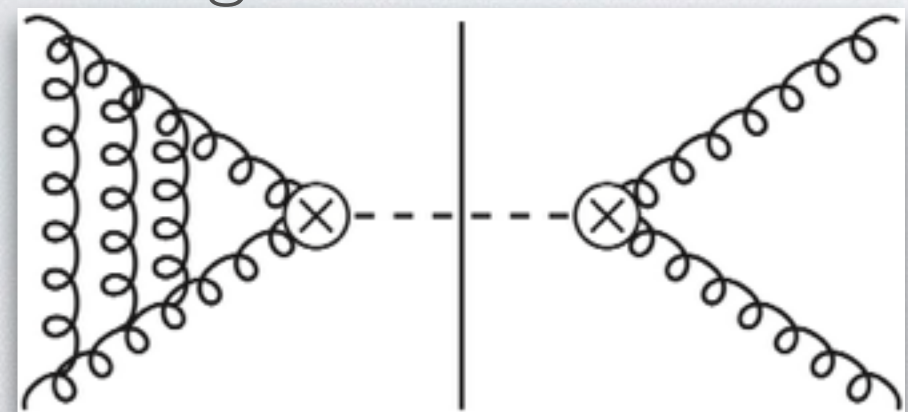
The calculation

- Combination of loop corrections and real emissions computed using Feynman diagrams is the only way for analytic computations at N3LO at this point
- Lots of Feynman diagrams
- At **N3LO: ~100000** interference diagrams

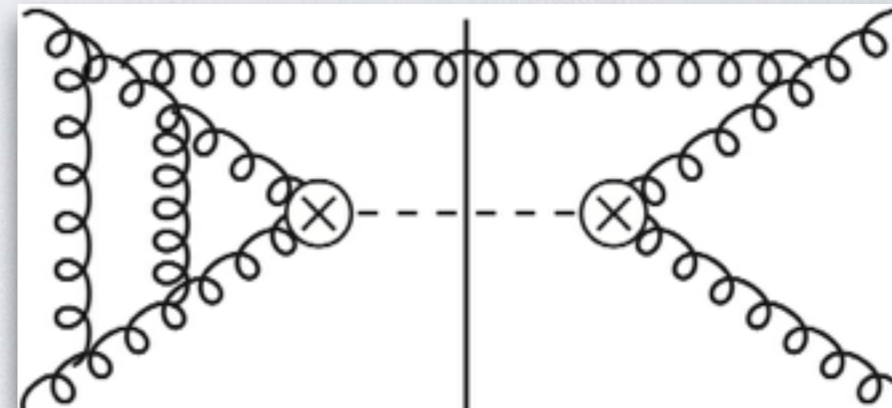





The gluon fusion cross section

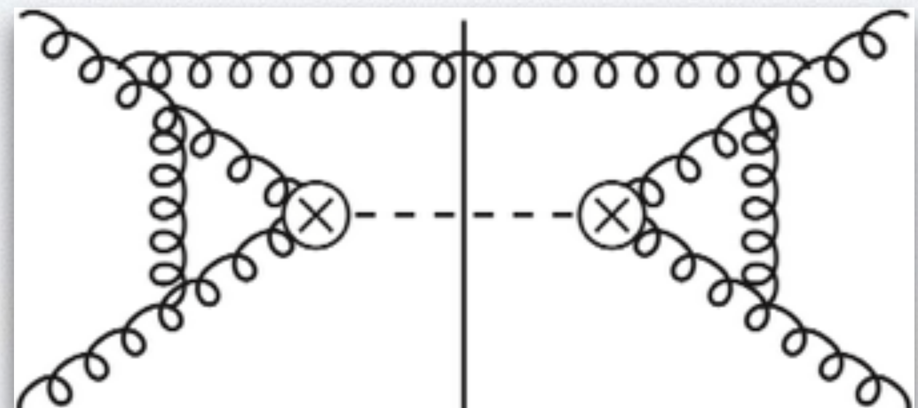
- Diagrammatic contributions at N3LO



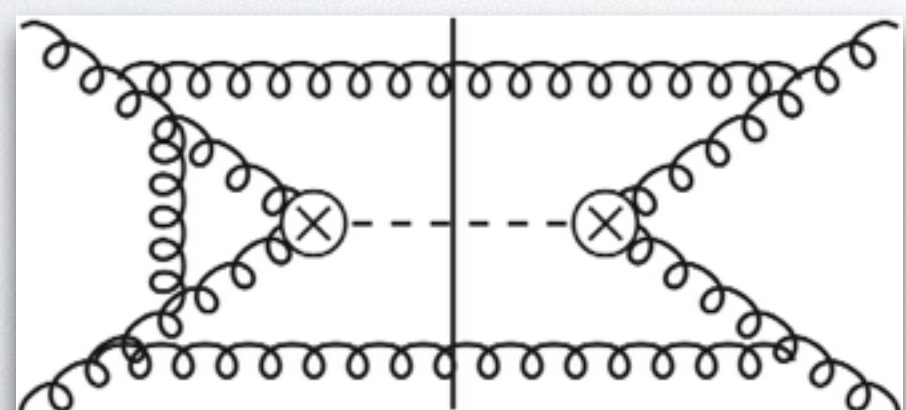
triple virtual



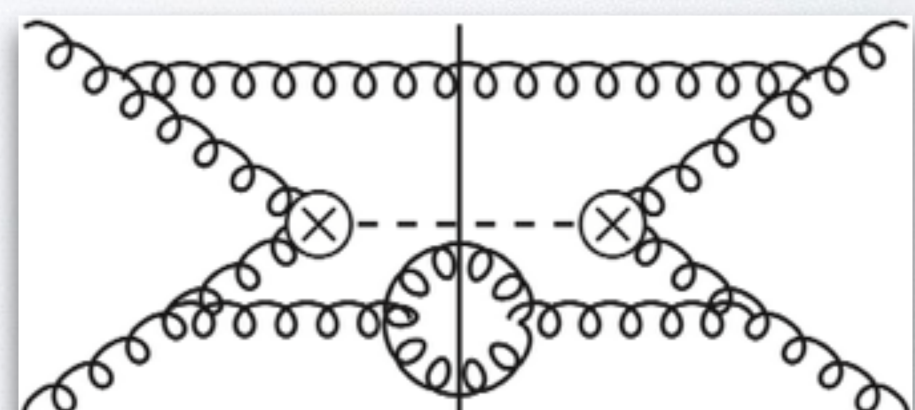
double virtual real



real virtual squared



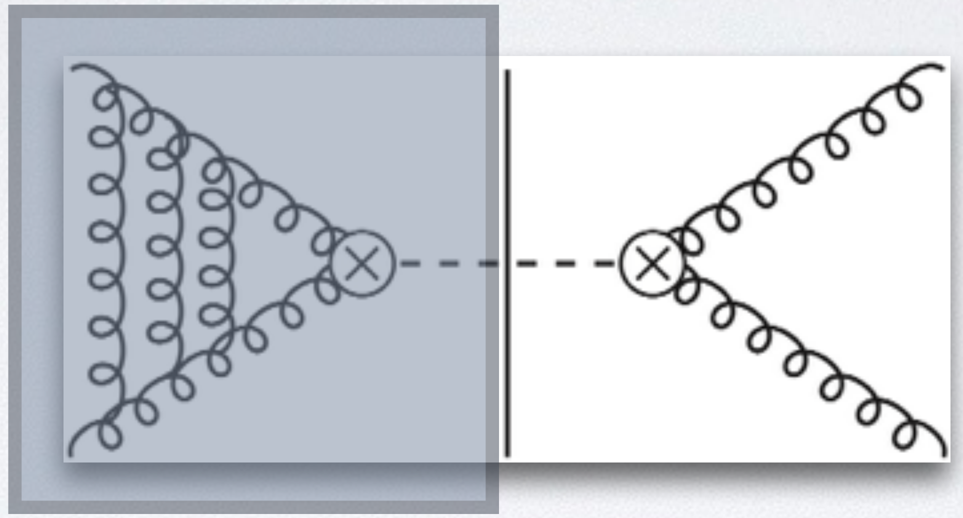
double real virtual



triple real

The triple virtual

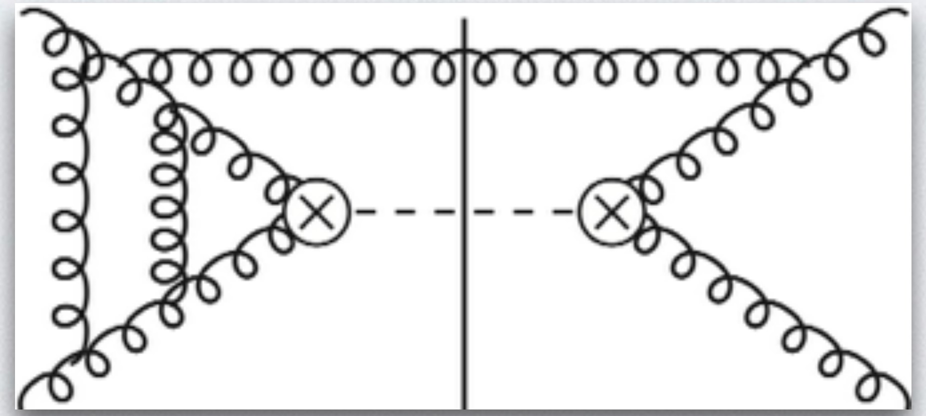
- The triple virtual is directly related to the three loop QCD form factor



- The QCD form factor is well known
    - at one loop
    - at two loops [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maitre]
    - at three loops [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
- The pure loop contributions are not a problem in the calculation

The double-virtual real

- Let us focus on the double-virtual real correction to the cross section



[Duhr, Gehrmann; FD, Mistlberger]

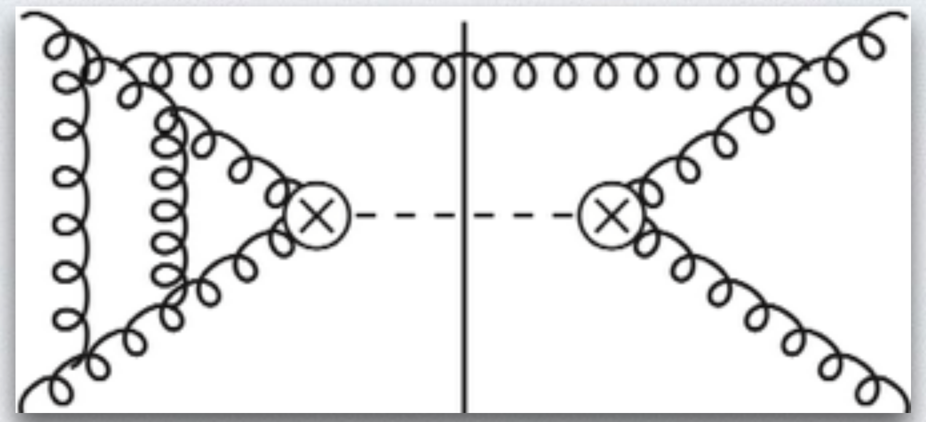
- Two-loop correction to Higgs+jet

- 2 loop master integrals are known

[Gehrmann, Remiddi]

- But not to high enough order in the dimensional parameter

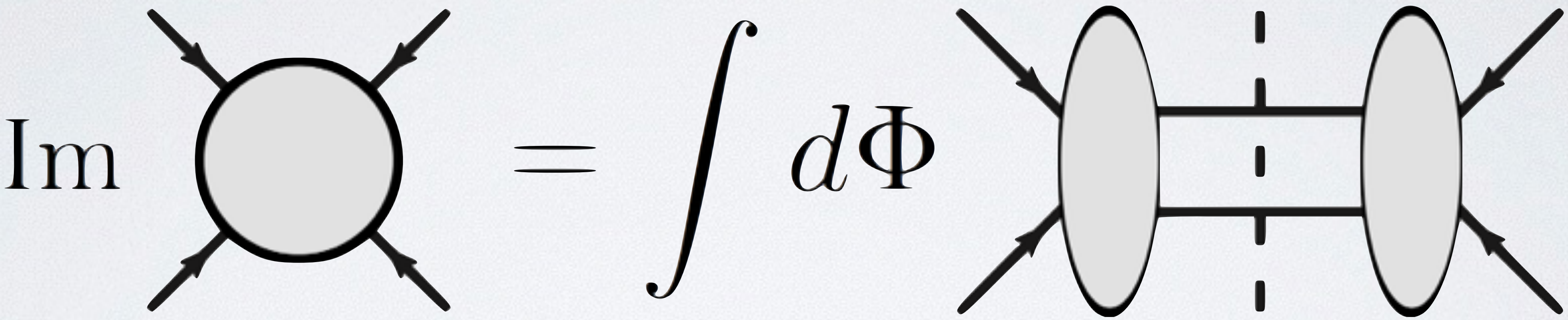
The double-virtual real



- Two possibilities
  - Recompute 2-loop masters and do phase space integrals over them
    - Well known, requires subtractions, not feasible for complicated phase spaces, manual
  - Compute loops and phase space in one go
    - New method, uniform treatment of loops and phase space, very automatic
    - Easily generalises to more complicated phase spaces

Unitarity

- Optical theorem:



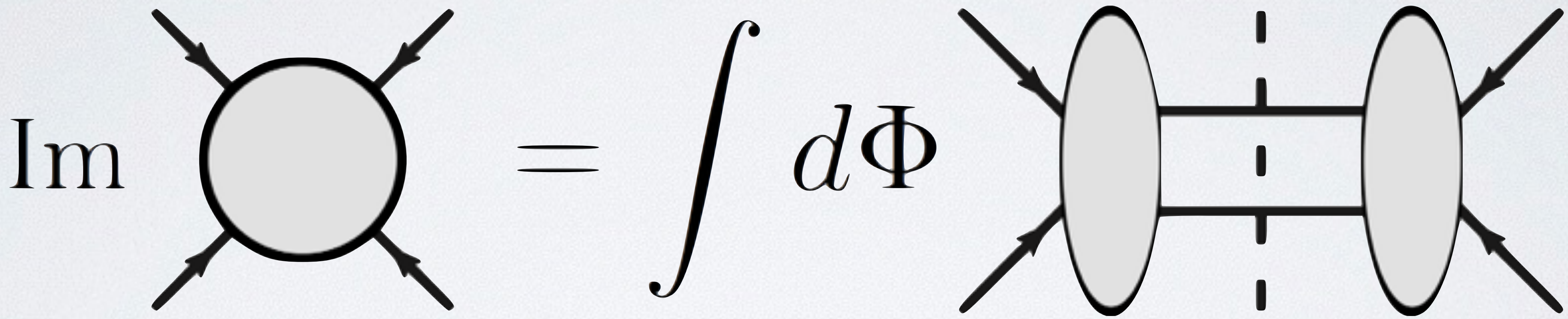
  - Discontinuities of loop integrals are phase space integrals

- Discontinuities of loop integrals are given by Cutkosky's rule:

$$\frac{1}{p^2 - m^2 + i\epsilon} \to \delta^+(p^2 - m^2) = \delta(p^2 - m^2)\theta(p^0)$$

Reverse unitarity

- Optical theorem:



- The optical theorem can be read ‘backwards’

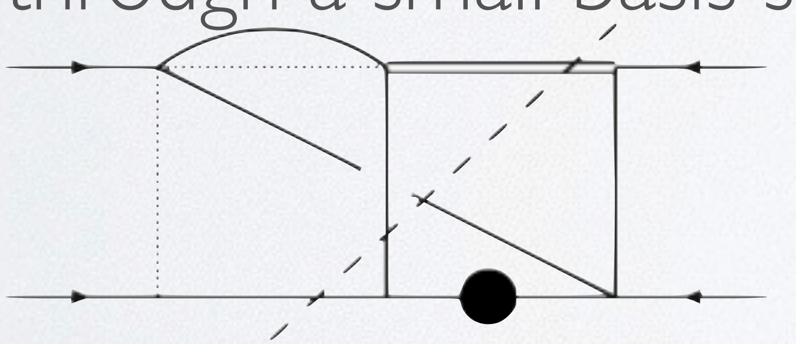
- This way, phase space integrals can be expressed as unitarity cuts of loop integrals [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

- We can compute loop integrals with cuts instead of phase space integrals

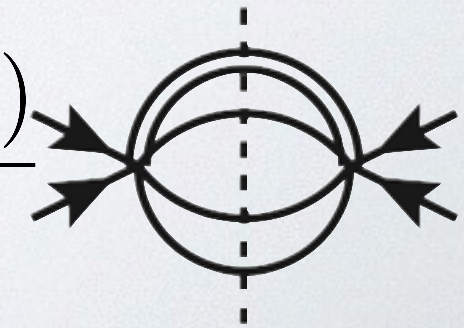
- This makes the rich technology developed for loop integrals available

IBPs and master integrals

- Loop integrals are in general not independent but related by Integration-by-parts identities (IBPs)
- The IBPs form a system of equations for a given class of loop integrals
- The system can be solved algorithmically expressing all integrals through a small basis set of integrals (master integrals)



$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$

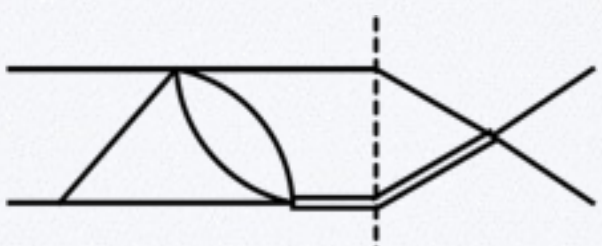


IBPs and differential equations

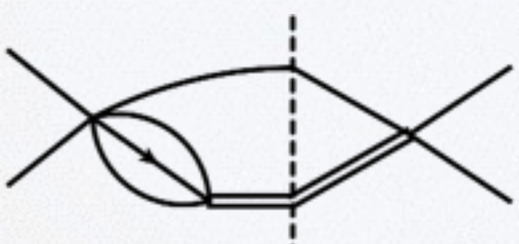
- Having access to IBP technology allows us to derive differential equations for master integrals
- The derivative of a master integral w.r.t. kinematic invariants can be expressed as a linear combination of master integrals
- Leads to a coupled system of linear differential equations for the master integrals

$$\bar{z} = 1 - z = \frac{s - m_h^2}{s}$$

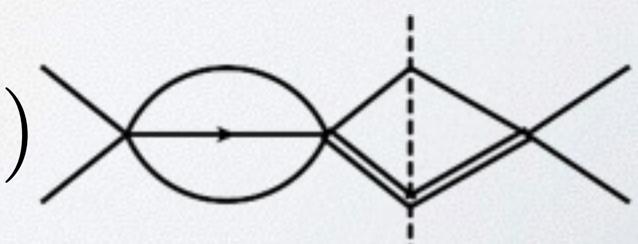
$$\Big[\partial_{\bar{z}} - 3\epsilon\ \mathrm{dlog}(1-\bar{z})\Big]$$



$$= \epsilon\ \mathrm{dlog}(1-\bar{z})$$



$$-3\epsilon\ \mathrm{dlog}(1-\bar{z})$$



Differential equations

- It is possible to transform the system of differential equations to a canonical form [Gehrmann, Remiddi; Henn]

$$d\vec{I}(\bar{z}) = \epsilon \underbrace{\sum_k A_k \omega_k}_{=A} \vec{I}(\bar{z}) \qquad \omega_k = dlog(\bar{z} - \bar{z}_k)$$

- Integral can have branch cuts at $\bar{z} = \bar{z}_k$

- The formal solution of this system is

$$\vec{I} = L\vec{\alpha}$$

$$L = \mathcal{P} e^{\epsilon \int A}$$

Multiple polylogarithms

- The formal path ordered exponential can be performed order by order in ϵ

- The expansion corresponds exactly to the definition of the multiple polylogarithms

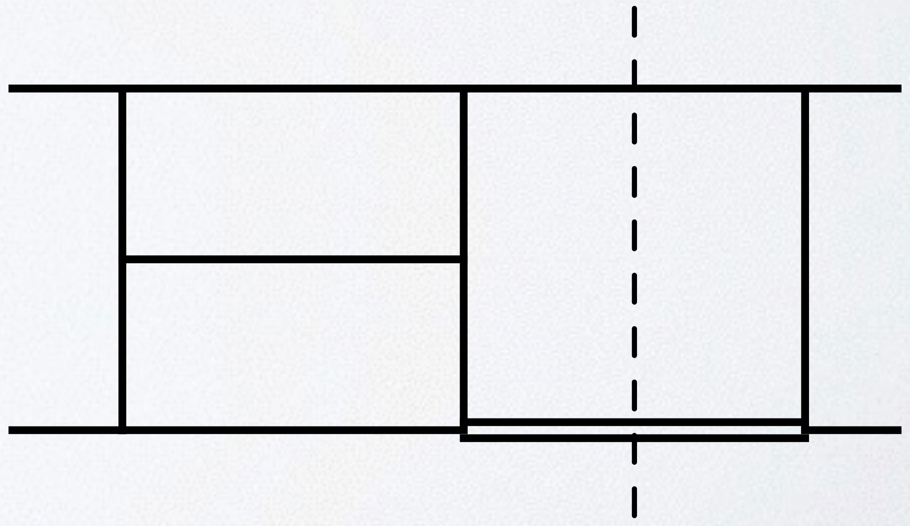
$$G\left(a_1,\ldots,a_n;z\right)=\int_0^z\frac{dt}{t-a_1}G(a_2,\ldots,a_n;t)\;\Big|\;\mathrm{Li}_n(z)=\int_0^z\frac{dt}{t}\mathrm{Li}_{n-1}(t)$$

- Multiple polylogarithms are a generalisation of the classical polylogarithms and the HPLS, 2dHPLs, …

- Multiple polylogarithms are very well understood by now: Symbol formalism, Hopf algebra structure,… [Goncharov; Brown; Duhr]

Boundary conditions

- What remains is fixing the boundary vector
- Naively: We need to fix one constant per integral by evaluating the integral at a specific point
- Evaluate the integral means: Feynman parametrise and analytically calculate the integral in the limit $\bar{z} = \bar{z}_k$
- We have 72 masters
- Need a better method



Boundary conditions

- Another way of fixing the boundary vector is using analyticity

[Gehrmann, Remiddi]

- We demand that $L\vec{\alpha}$ has the correct branch structure

- Eg. no u-channel cut in planar 4-point integrals

- This only works for pure loop integrals, since they have a well defined cut structure

- The branch structure of phase space integrals is not known in general

- This method does not work for phase space integrals

Boundary decomposition

- Is there a different method to constrain the boundary vector?
- The solutions of the differential equations have the general structure

$$\vec{I} = \sum_k \bar{z}^{-k\epsilon} \vec{h}_k(\bar{z})$$

- Can we find a system of differential equations for the $\vec{h}_k(\bar{z})$?
- What are the possible values of k ?

Boundary decomposition

- Recall the system matrix $A = \sum_k A_k dlog(\bar{z} - \bar{z}_k)$
- For small $\bar{z}$ the term with $dlog(\bar{z})$ dominates
- The system matrix in the limit is then

$$\tilde{A} = A_0 dlog(\bar{z})$$

- The solution in the limit is

$$\vec{I}_0 = \bar{z}^{A_0 \epsilon} \vec{c}_0$$

Boundary decomposition

- The general solution

$$\vec{I} = \sum_k \bar{z}^{-k\epsilon} \vec{h}_k(\bar{z})$$

$$\Updownarrow$$

- The limit solution

$$\vec{I}_0 = \bar{z}^{A_0 \epsilon} \vec{c}_0$$

- How to connect the two?

- Diagonalise A_0 and go to the eigenbasis

- However: Diagonalisation is not possible in general

- Next-best thing: Jordan normal form

Boundary decomposition

- Computing the Jordan decomposition of A_0 yields
  - A matrix containing the eigenvalues of A_0 on its diagonal
    - This fixes the possible exponents k to be eigenvalues of A_0

Jordan normal form

- Take a matrix

$$M = \begin{pmatrix} -2 & 0 & 0 \\ 5 & 0 & -2 \\ 3 & 1 & -3 \end{pmatrix}$$

- The Jordan normal form is

$$J = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- The rotation matrix is

$$R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

- So that $M = RJR^{-1}$

Jordan normal form

- Jordan normal form has b blocks on the diagonal
- If the matrix is diagonalisable the Jordan decomposition will just diagonalise it
- In our example: 2 blocks

$$J = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Boundary decomposition

- Computing the Jordan decomposition of A_0 yields
  - A matrix containing the eigenvalues of A_0 on its diagonal
    - This fixes the possible exponents k to be eigenvalues of A_0
  - A rotation matrix that takes us to the Jordan normal form
    - This allows us to rotate the differential equations to determine the $\vec{h}_k(\bar{z})$
    - It relates the $\vec{h}_k(\bar{z})$ to the $\vec{c}_0$

Boundary decomposition

- Using the Jordan decomposition we write the solution in the limit

$$\lim_{\bar{z}\to 0} I = \sum_i \bar{z}^{i\epsilon} \left(\alpha_{i0} + \log(\bar{z})\alpha_{i1} + \log(\bar{z})^2\alpha_{i2} + \ldots\right)$$

- Every boundary condition α appears in combination with exactly one specific exponent i

- This connects to expansion by regions

- The entries of α are the different regions

- By using expansion by regions we can compute α

Boundary decomposition

- How does this reduce the amount of integrals that we have to compute?
- There can at most be b independent entries in α
- Terms with explicit logs do not appear in our case
- Some eigenvalues can be unphysical
    - The only allowed exponents in our case are -2, -3, -4, -5, -6
    - The boundaries corresponding to other eigenvalues vanish

Boundary decomposition

- Computing the limit of every master: **72** integrals
- Boundary decomposition: **15** integrals

- These 15 boundaries appear in different masters
- We can pick the simplest master to compute a boundary

Conclusion

- We have developed a new method to solve systems of differential equations for combined loop and phase space integrals
- We have used it to compute the double-virtual real corrections to Higgs boson production at N3LO
- The method scales to more complicated phase spaces
- It can be used to compute the double-real virtual contributions to Higgs at N3LO
- We are on track to computing the full Higgs cross section at N3LO

Thank you for your attention