Non-perturbative Dynamics of Modified Gravity

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Plan:

- Motivation
- Model
- Non-perturbative dynamics
  - Weak fields!!!
  - Mass screening.
- Master equation
  - (De)coupling limit
  - Global constraints
- Selfaccelerated solution

$$G_{\mu\nu} + m_c\left(K_{\mu\nu} - g_{\mu\nu}K\right) = T_{\mu\nu}$$

trace:

$$R + 3m_c K = -T$$

- Cosmology: homog. & isotrop. T
- Localized soures/perturbations

(Deffayet)

(Deffayet, Dvali, G.G.

• Cosmology:

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{8\pi G_N}{3}\rho + \frac{m_c^2}{4}} \pm \frac{m_c}{2}\right)^2$$

• Conventional branch:

4D evolution $\longrightarrow$ 5D evolution

(slower deceleration)

$w_{eff} < -1$

• Selfaccelerated branch:

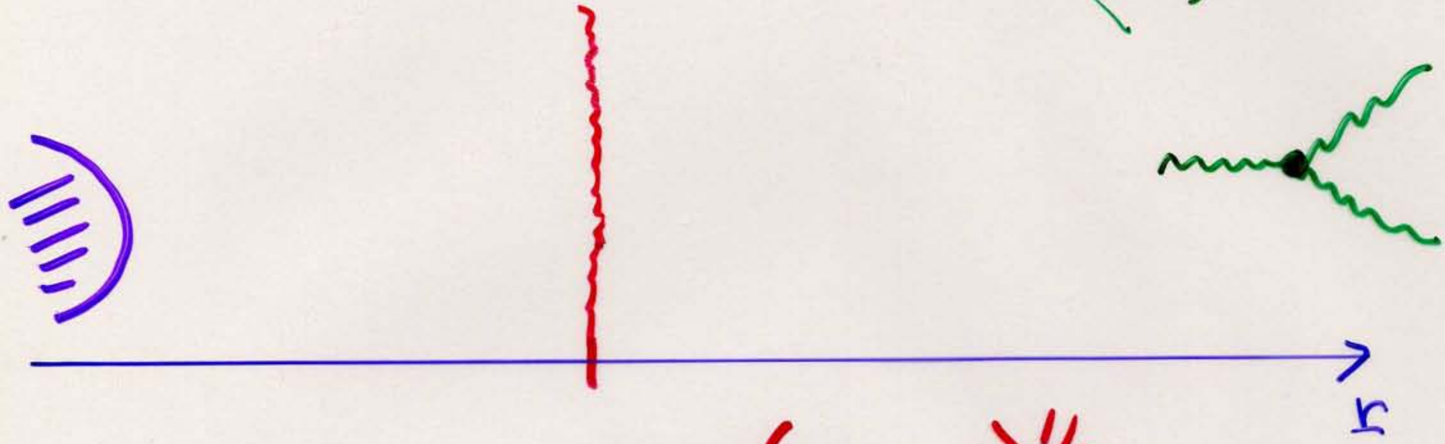
4D evolution $\longrightarrow$ selfacceleration

due to modified gravity!

Scale : $r_c = 1/m_c$

- Breakdown of perturbation theory

(Deffayet, Dvali
G.G., Vainshtein)



$$r_* \sim \left(r_g r_c^2\right)^{1/3}$$

$$R \sim m_c^2$$

(Luty, Porrati
Rattazzi)

Sun: $r_* \sim$ Kpc

Galaxy: $r_* \sim$ Mpc

- What happens at $r \lesssim r_*$?

- Physics at $r \gtrsim r_*$?

- Matching at $r = r_*$?

- $r \lesssim r_*$

Newton potential

$$g_{00} = 1 - \frac{r_g}{r} \mp m_c^2 r^2 g\left(r/r_*\right)$$

↳ (G.G., Iglesias)

(Gruzinov)

Compare with (A)dS-Schwarz.

$$g \gg 1$$

in the $r \ll r_*$ regime!

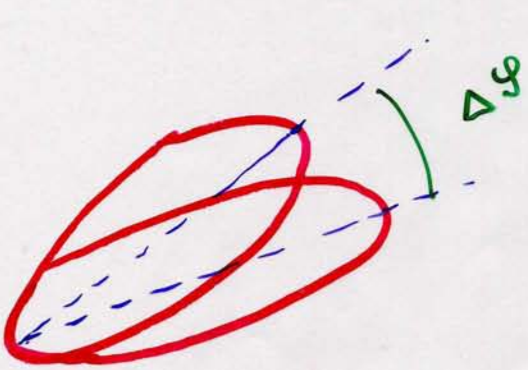
Stronger effects

Additional perihelion precession

$$r \ll r_*$$

$$-2\Phi = \frac{r_g}{r} - \alpha m_c^2 r^2 \left(\frac{r_*}{r}\right)^{3/2-\beta}$$

$\alpha \simeq \pm 0.84 \qquad \beta \simeq 0.04$



(Dvali, Gruzinov, Zaldarriaga)

(Lue, Starkman)

(G.G., Iglesias)

precession per orbit

$$2\pi + \underbrace{\frac{3\pi r_M}{r}}_{\text{GR}} \mp \underbrace{\frac{3\pi|\alpha|}{4}\left(\frac{r}{r_*}\right)^{3/2+0.04}}_{\text{modified gravity}}$$

Earth – Moon : $\mp 0.7\ 10^{-12}$ (exp $2.4\ 10^{-11}$)

Sun – Mars : $\mp 1.3\ 10^{-11}$ (exp 9×10^{-11})

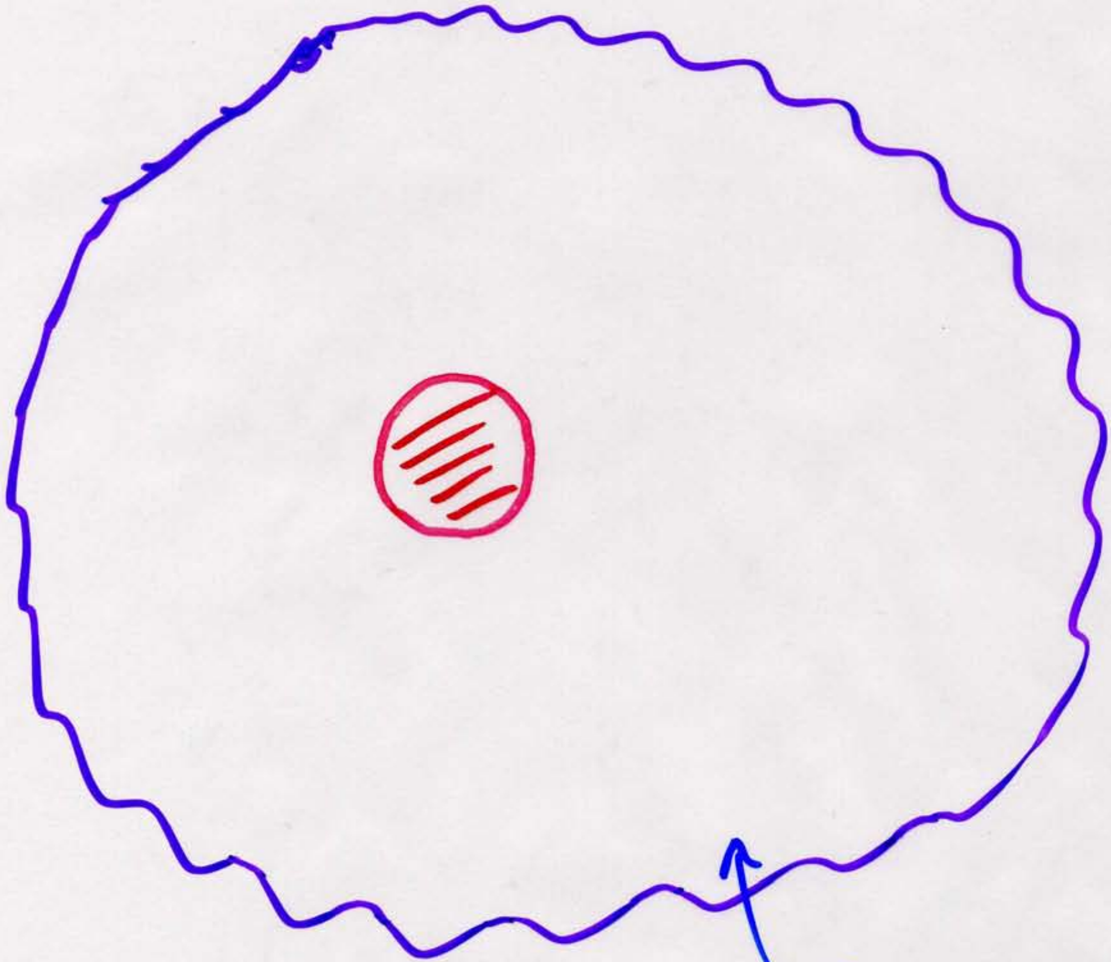
precession per unit time

$$\Gamma = \mp \underbrace{\frac{3}{8 r_c}}_{\text{Lue \& Starkman}} \times \underbrace{\frac{|\alpha|}{\sqrt{2}}\left(\frac{r}{r_*}\right)^{0.04}}_{\text{due to non-pert. solution}}$$

Increasing Astronomical Unit? (Iorio, 05)

- $r \sim r_*$ and beyond

(G.G., Iglesias)



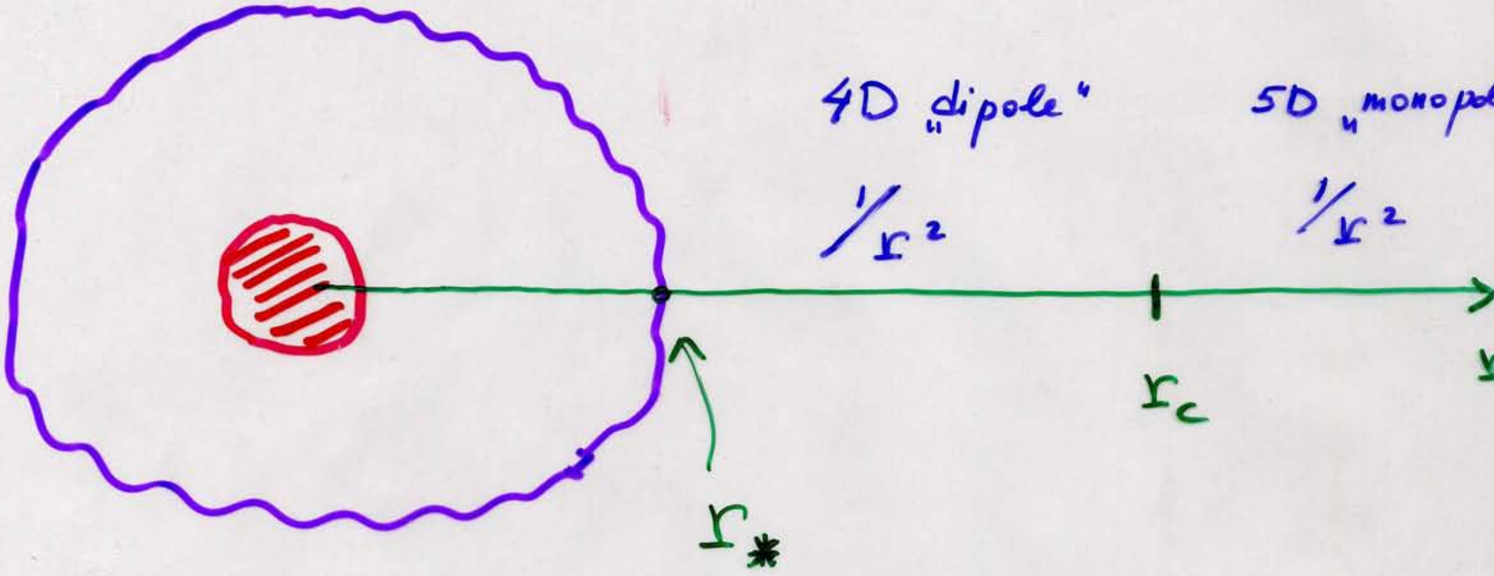
$R \neq 0$

$R(r \sim r_*) \simeq m_c^2$

$\Delta M \sim M$!

Order of magnitude estimate

(G.G.; A. Iglesi



$$ds^2 = -e^{-\lambda}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2 + \underline{\underline{\gamma\, dr\, dy}} + e^{\sigma}dy^2$$

$$ds^2\big|_{4D} = -e^{-\lambda}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$$

$$\gamma\big| = 0$$

γ does not contribute to 4D mass

but it does contribute to 5D mass.

$$M_{5D} \sim M\left(\frac{r_*}{r_c}\right)^{1/3}$$

- Master equation

$$R = \frac{R^2 - 3R_{\mu\nu}^2}{3m_c^2}$$

Consequence of the bulk + junction eqs.

Bulk: (intrinsic) ~ (extrinsic)^2

Junction: (intrinsic) ~ (extrinsic)

due to brane induced term

- π - lagrangian

(Luty, Porrati, Rattazzi

$$\mathcal{L} = (\partial\pi)^2 - \frac{(\partial\pi)^2\Box\pi}{\Lambda^3}$$

$$+ \text{(free tensor)}$$

$\Lambda^3 \equiv M_p m_c^2 \qquad M_p \to \infty \qquad m_c \to 0$

$$\boxed{3\Box\pi = \frac{(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2}{\Lambda^3}}$$

(Nicolis, Rattazzi)

(Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi)

- $\overline{\text{Master}}$ equation

$$\bar{R} \equiv M_p R$$

$$\bar{R} = \frac{\bar{R}^2 - 3\bar{R}_{\mu\nu}^2}{\Lambda^3}$$

$$\bar{h}_{\mu\nu} = \eta_{\mu\nu}\,\pi$$

$$3\Box\pi = \frac{(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2}{\Lambda^3}$$

$$+ \tilde{h}_{\mu\nu}$$

$$+ \tilde{R}$$

$$+ \frac{\tilde{R}\,\Box\pi - 2\tilde{R}^{\mu\nu}\partial_\mu\partial_\nu\pi}{\Lambda^3} + \frac{\tilde{R}^2 - 3\tilde{R}_{\mu\nu}^2}{3\Lambda^3}$$

- Super Master Equation

$$\boxed{\bar{G}_{\mu\nu} = \frac{\bar{B}_{\mu\nu}}{\Lambda^3} - \bar{E}_{\mu\nu}}$$

$$\bar{B}_{\mu\nu} \equiv -\bar{G}_{\mu\alpha} G^{\alpha}_{\nu} + \frac{1}{3}\bar{G} G_{\mu\nu} + $$

$$+ \frac{1}{2}\eta_{\mu\nu}\bar{G}_{\alpha\beta}\bar{G}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu}\bar{G}^2$$

tensor equation that survives in the (de)coupling limit

- The π-Lagrangian does not capture properties of DGP
- Long-range tensor interactions survive, different analytic properties.

(Deffayet, G.G., Iglesias)

- Global constraints on lineariz.

$$R = R_1(h_1) + R_1(h_2) + R_2(h_1) + \cdots$$

$$R_1(h_1) = \Box h_1 - \partial\partial h_1 = 0$$

$$\cancel{R_1(h_1)} + R_1(h_2) + R_2(h_1) = \frac{R_1^2\cancel{(h_1)} - 3\, R_{\mu\nu,1}^2(h_1)}{3 m_c^2} + \cdots$$

- total derivative
- $h_2 \sim 1/r^2$ or faster.

$$\int \left[3 m_c^2 R_2(\underline{\underline{h_1}}) + R_{\mu\nu,1}^2(\underline{\underline{h_1}}) \right] =$$

$$= \int \frac{1}{M_p^2} \left(6 R_{\mu\nu}(\underline{\underline{h_1}}) T_1^{\mu\nu} - R_1(\underline{\underline{h_1}}) T_1 \right) - \frac{1}{M_p^4} \left(3 (T_{\mu\nu}')^2 - (T')^2 \right)$$