Threshold Expansion for Higgs Boson Production at N3LO

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Higgs Production at N3LO

Uncharted territory in QCD - perturbation theory

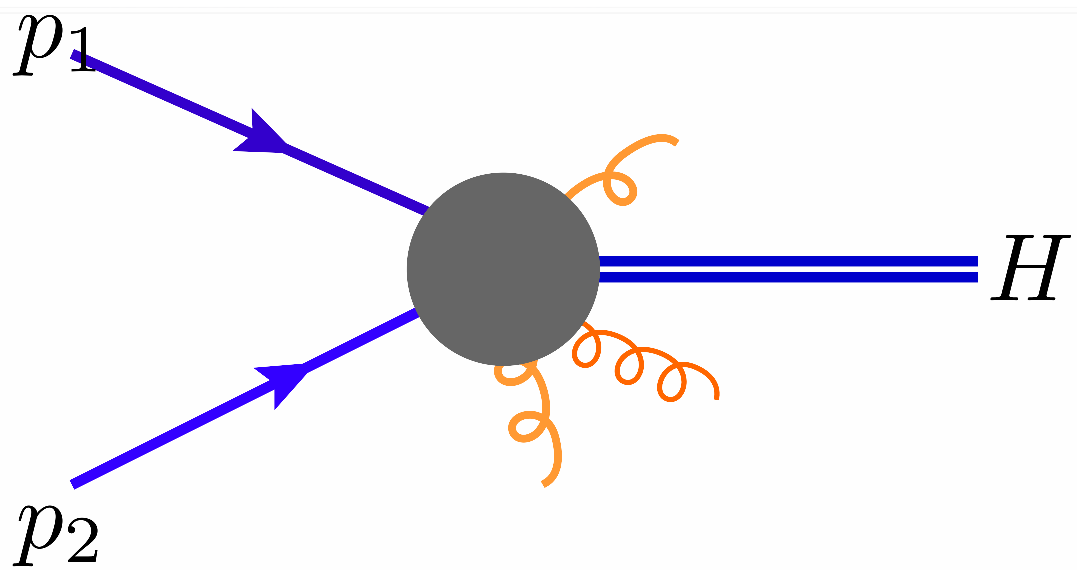
- Inclusive Gluon - Fusion Higgs production at N3LO in the Large Top Mass Limit

$$\hat{\sigma}(z) = \hat{\sigma}^{LO}(z) + \alpha_S \hat{\sigma}^{NLO}(z) + \alpha_S^2 \hat{\sigma}^{NNLO}(z) + \alpha_S^3 \hat{\sigma}^{N3LO}(z) + \mathcal{O}(\alpha_S^4)$$

First Calculation at this Order In QCD

- We need a Feasibility study
- We need Checks
- We need Boundary Conditions for Integrals

THRESHOLD EXPANSION



$$z = \frac{m_H^2}{\hat{s}} \sim 1$$

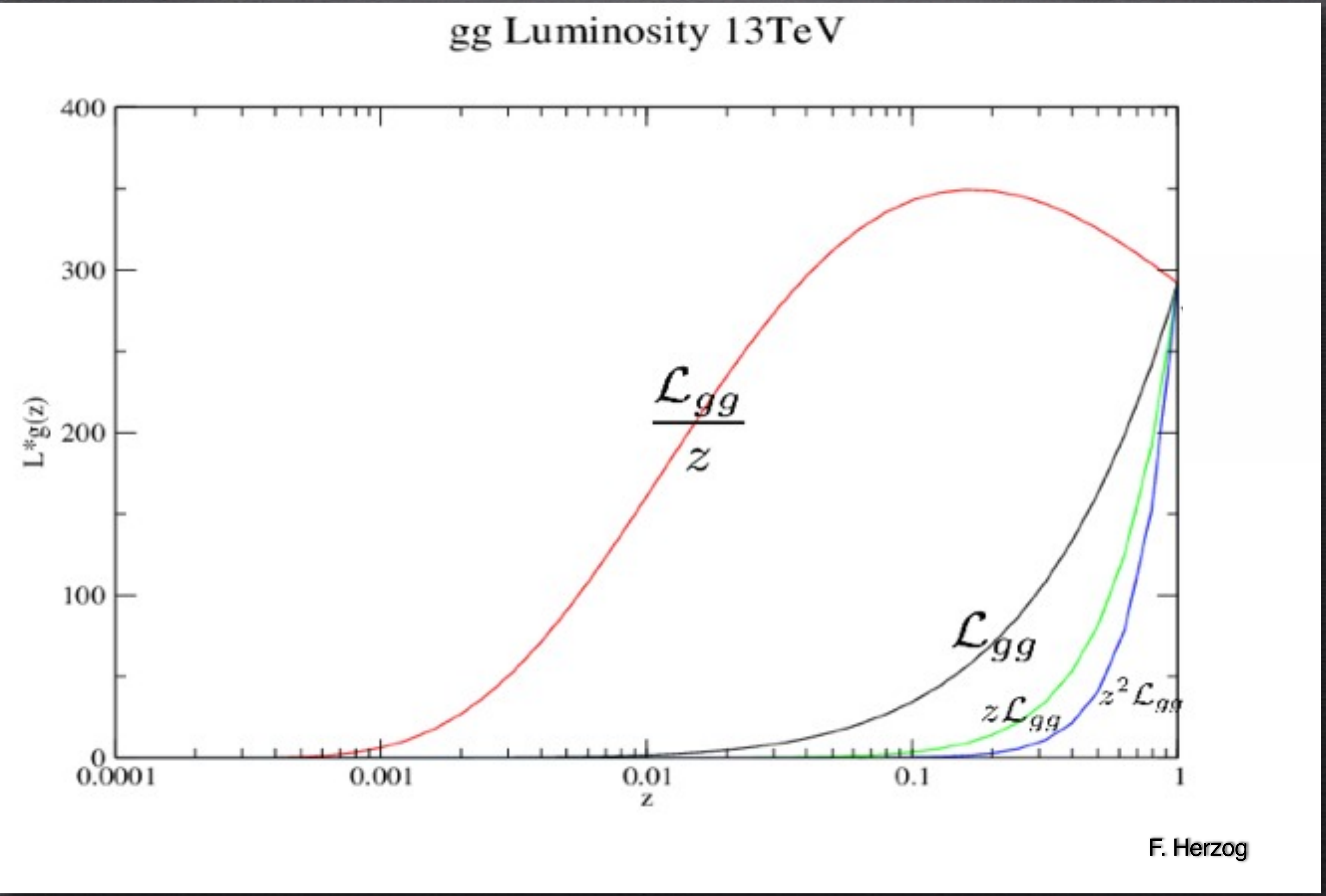
- Expand around production threshold of the Higgs boson

$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

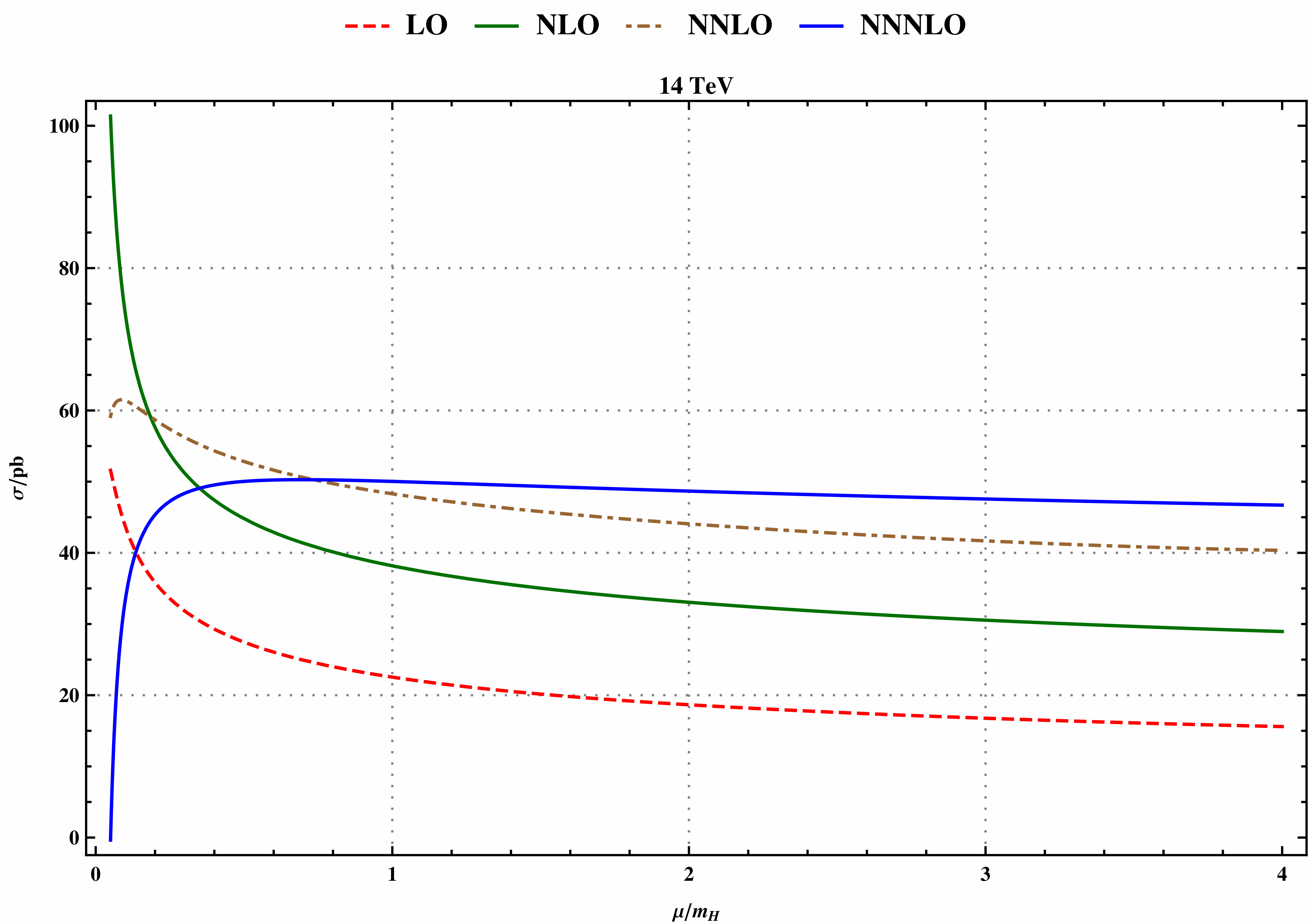
- Soft - Virtual term contains all 3-loop contributions + soft gluon radiation

GG-Luminosity

$$\sigma = \sum \int \frac{dz}{z} \mathcal{L}_{12}(\tau/z)\hat{\sigma}(z)$$



Soft-Virtual Cross-section



Soft-Virtual

We truncate the series after first Term

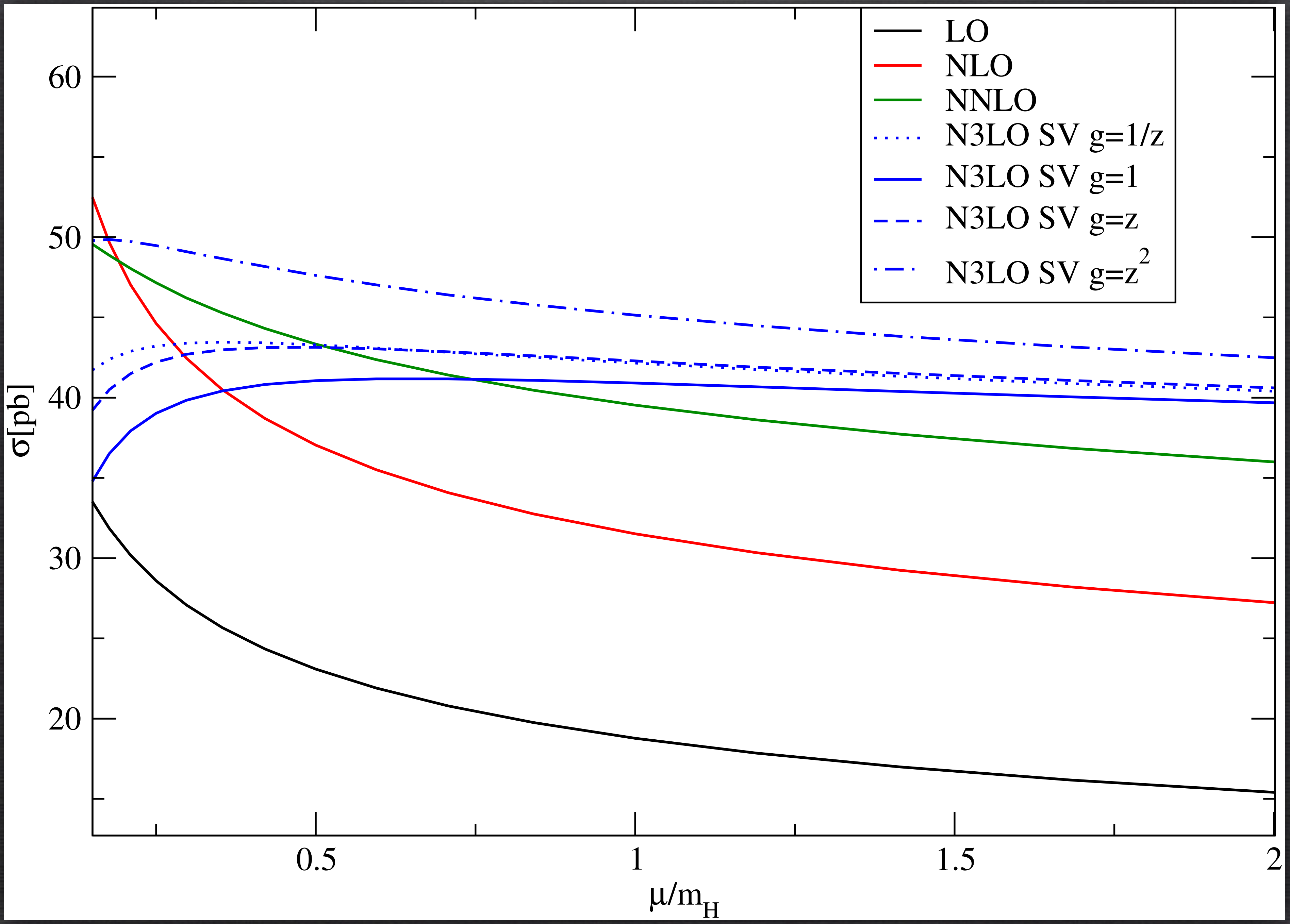
Soft-virtual term is ambiguous
- Let's estimate

$$\sigma = \int dx_1 dx_2 f(x_1) f(x_2) \left[z g(z)\right] \left[\frac{\hat{\sigma}(z)}{z g(z)}\right]_{\text{threshold}}$$

We can choose $g(z)$
as long as

$$\lim_{z \to 1} g(z) = 1$$

Soft-Virtual



Let's Calculate more

How to calculate

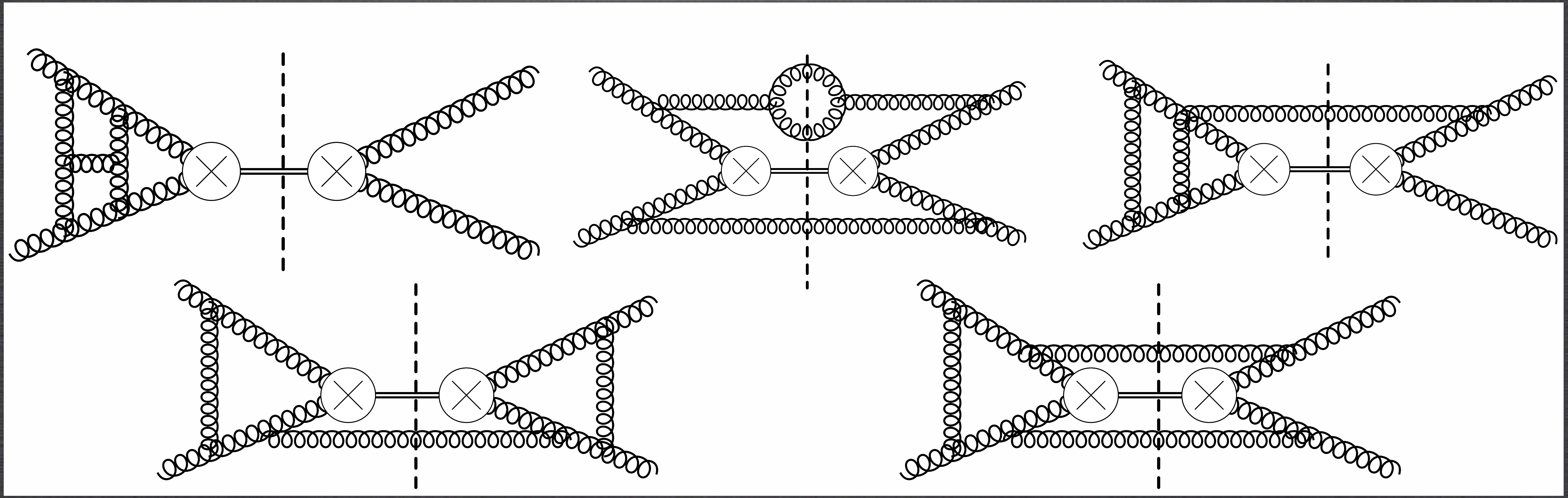
?

Feynman Diagrams

Combining real and virtual contributions

calculated with Feynman diagrams is the only way for analytic calculation at N3LO

@ N3LO: **~100 000** Interference Diagrams

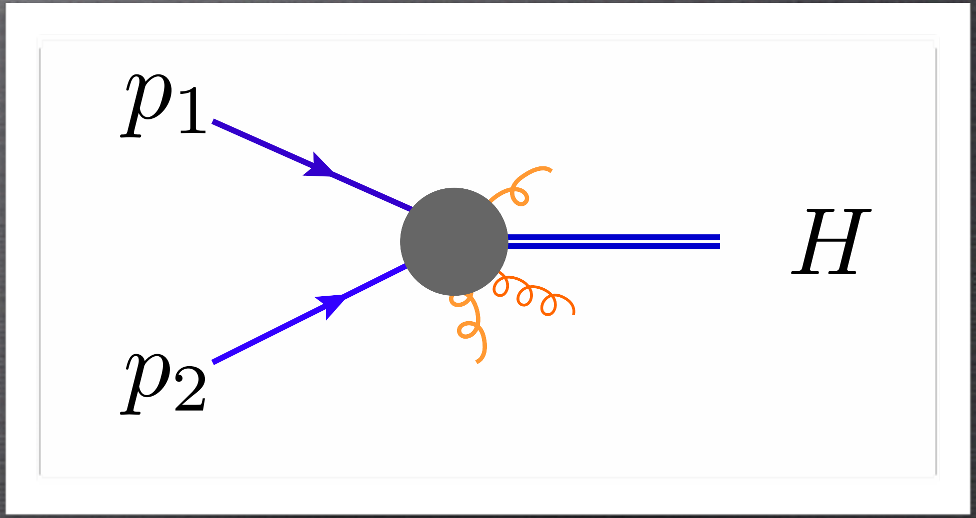


Automation is vital!

Threshold Expansion

General Idea:
Expand around z=1

All final state
radiation is soft



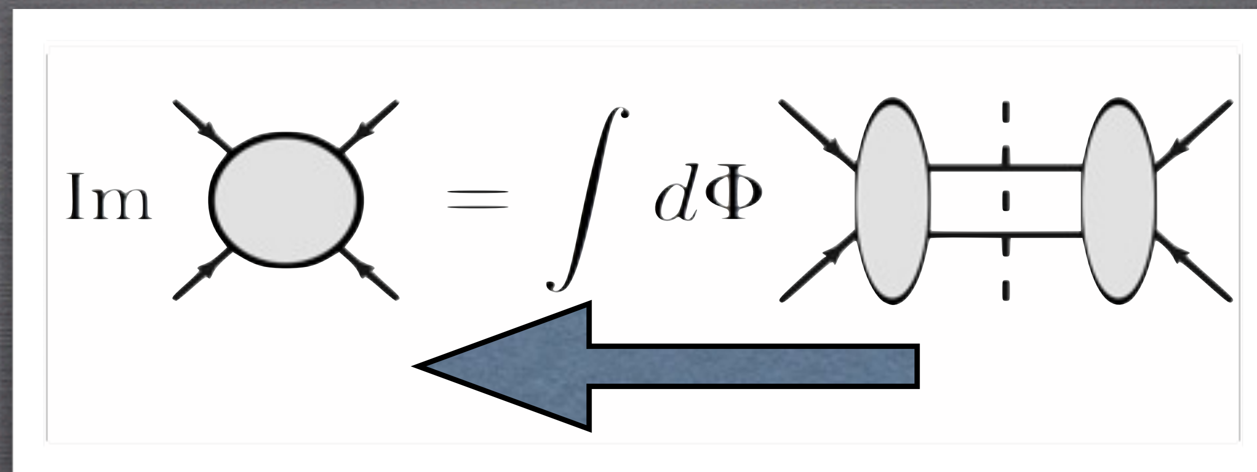
Re-parametrize all out-going parton momenta

$$p_f \to \bar{z} p_f \qquad \bar{z} = 1 - z$$

THRESHOLD EXPANSION

How to expand the Phase-Space Cuts?

Reverse Unitarity



Cutkosky's rule to relate on-shell constraints to cut -propagators

$$\delta^+(p^2) \to \left[\frac{1}{p^2}\right]_c \sim \frac{1}{p^2 + i\epsilon} - \frac{1}{p^2 - i\epsilon}$$

Allows to define derivatives and Expansion of cut-propagators

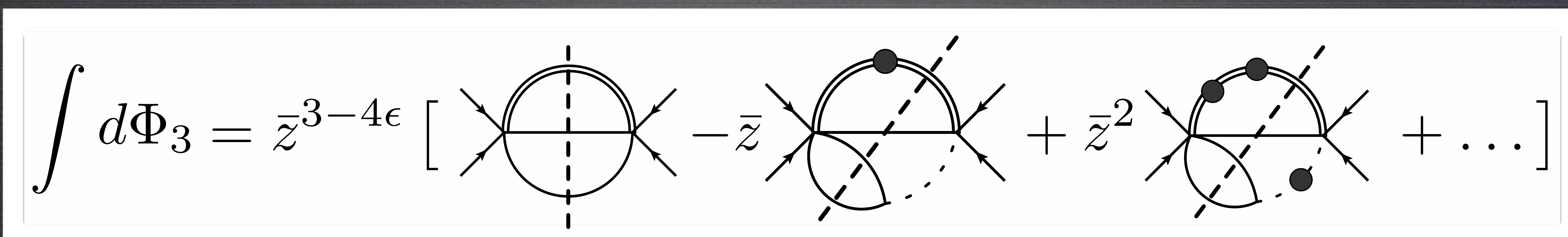
$$\left[\frac{1}{a + \bar{z} b}\right]_c = \left[\frac{1}{a}\right]_c - b\bar{z}\left[\frac{1}{a}\right]_c^2 + \ldots$$

THRESHOLD EXPANSION

$$\left[\frac{1}{a+\bar{z}b}\right]_c = \left[\frac{1}{a}\right]_c - b\bar{z}\left[\frac{1}{a}\right]_c^2 + \ldots$$

Expansion of (cut-)propagators yields soft (cut-)propagators

Example: Higgs+2 parton Phase-Space Volume

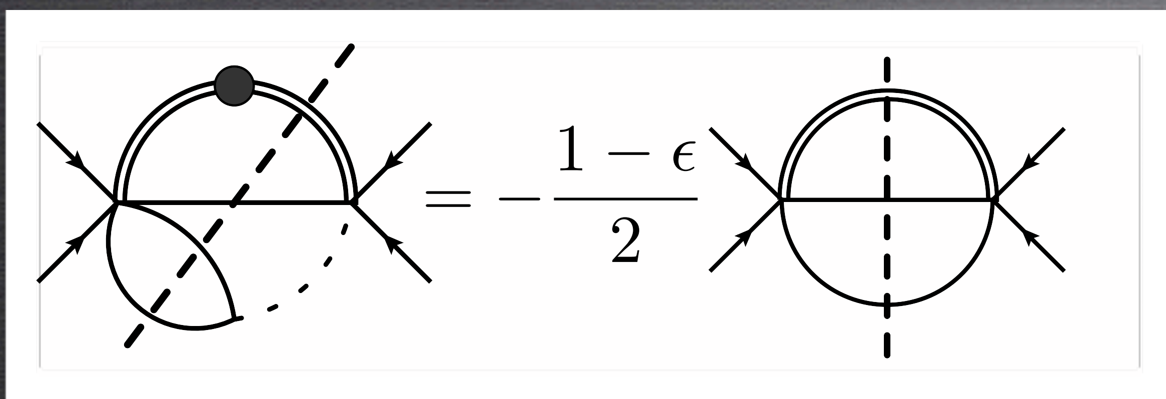


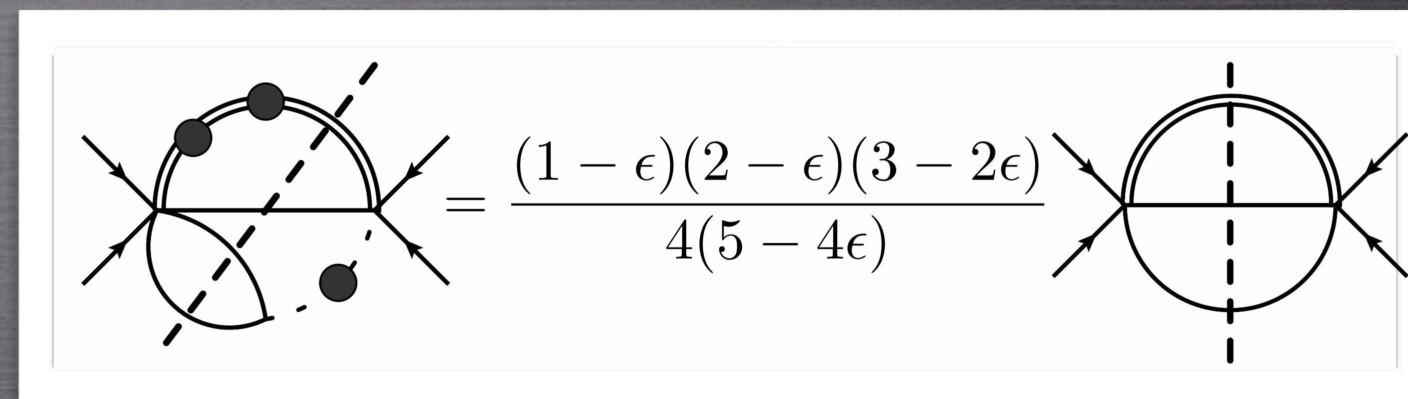
Apply Integration-By-Part (IBP) Identities

Relate Expanded Phase-Space Integrals To a Limited Set of ‘Master‘ Integrals

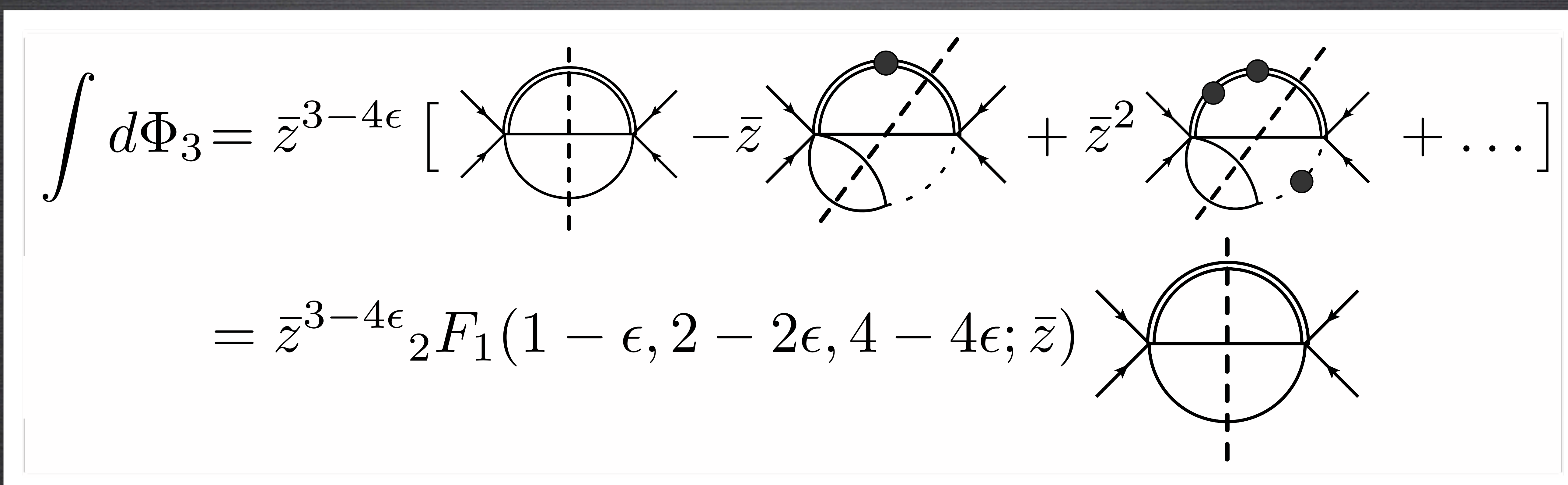
THRESHOLD EXPANSION

IBP Reduction yields





Compare with full Result



Threshold Expansion

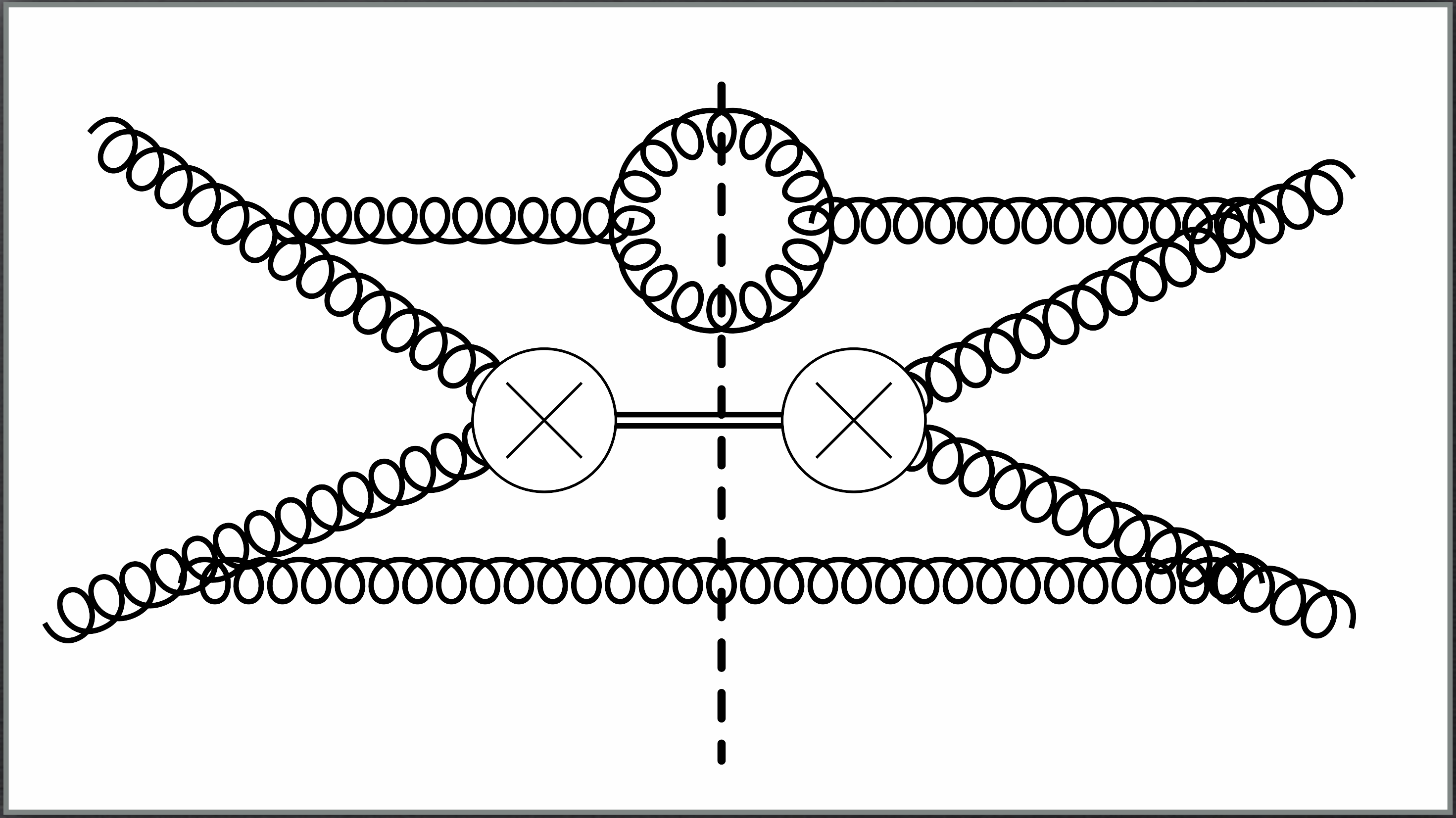
Ready for Triple Real!

Depends only on external momenta

$$p_f \to \bar{z} p_f$$

- Expand Integrand
- Expand Measure

✓

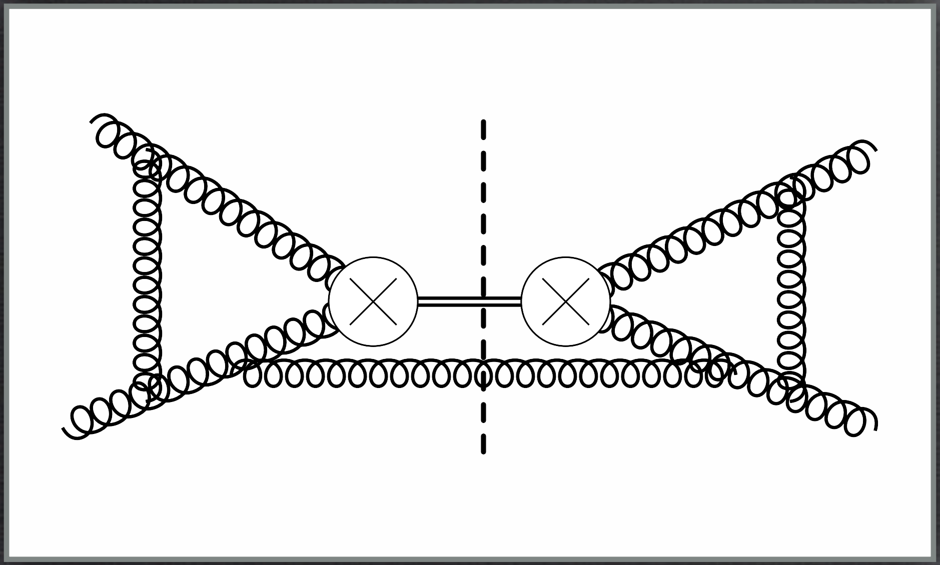


But, What about loops?

THRESHOLD EXPANSION

Loop-Integrals

$$\int \frac{d^d k}{(2\pi)^d}$$

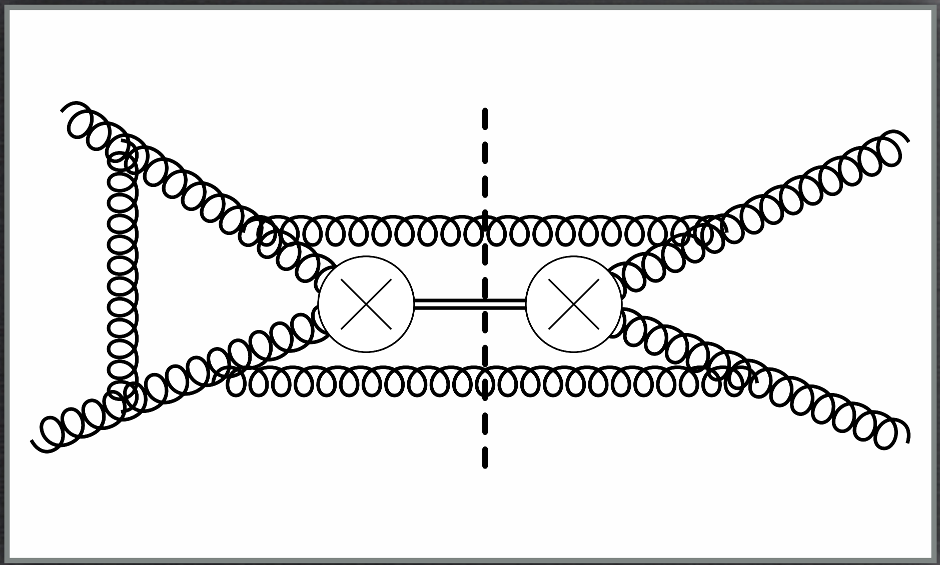


- Loop momentum is not fixed
- Follow the method of expansion by regions

Soft	Coll 1	Coll 2	Hard
$k \to \bar{z}k$	$k \to k \vert\vert p_1$	$k \to k \vert\vert p_2$	k

- Parametrize and expand systematically in every region
- Expand and Integrate Explicitly
- Sum of regions yields the full result

Double Real Virtual



Hard and Soft Work As Expected

Collinear is tricky!

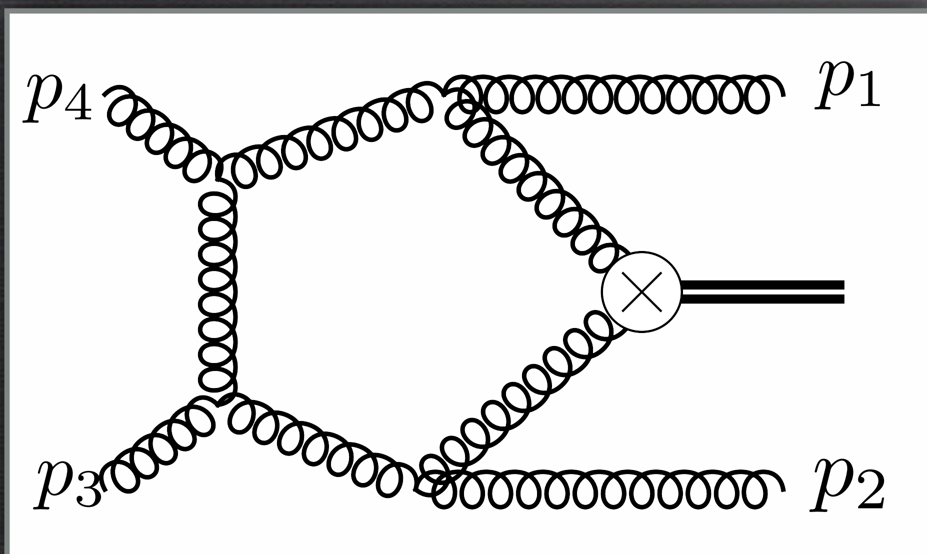
Soft	Coll 1	Coll 2	Hard
$k \to \bar{z}k$	$k \to k \vert\vert p_1$	$k \to k \vert\vert p_2$	k

$$k = \alpha p_1 + \beta p_2 + k_\perp$$

- Coll1: $\alpha \to \alpha, \quad k_\perp^2 \to \bar{z}k_\perp^2, \quad \beta \to \bar{z}\beta$
- Coll2: $\alpha \to \bar{z}\alpha, \quad k_\perp^2 \to \bar{z}k_\perp^2, \quad \beta \to \beta$

DOUBLE REAL VIRTUAL

Double-real-Virtual



$$k = \alpha p_1 + \beta p_2 + k_\perp$$

- Coll2: $\alpha \to \bar{z}\alpha, \quad k_\perp^2 \to \bar{z} k_\perp^2, \quad \beta \to \beta$

$$\int \frac{d^d k}{(2\pi)^2} \frac{1}{(k-p_2-p_3)^2(k-p_3)^2 k^2 (k+p_4)^2 (k+p_1+p_4)^2}$$

$$\frac{1}{(k-p_2-p_3)^2} \to \frac{1}{\bar{z}} \frac{1}{k^2 - 2kp_2 - 2kp_1 s_{23} + s_{23}} + \ldots$$

$$s_{ij} = 2p_i p_j$$

Double Real Virtual

$$\frac{1}{(k-p_2-p_3)^2} \to \frac{1}{\bar{z}} \frac{1}{k^2 - 2kp_2 - 2kp_1 s_{23} + s_{23}} + \ldots$$

Can't perform usual IBP-Reduction for combined Phase-Space and Loop-Integral!

1-Loop Reduction is possible!

All Collinear 1-Loop Integrals Reduce to Bubbles!

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+p)^2}$$

First: Reduce 1-Loop

Second: Reduce Phase-Space+1-Loop

→ Only 4 Collinear Master Integrals

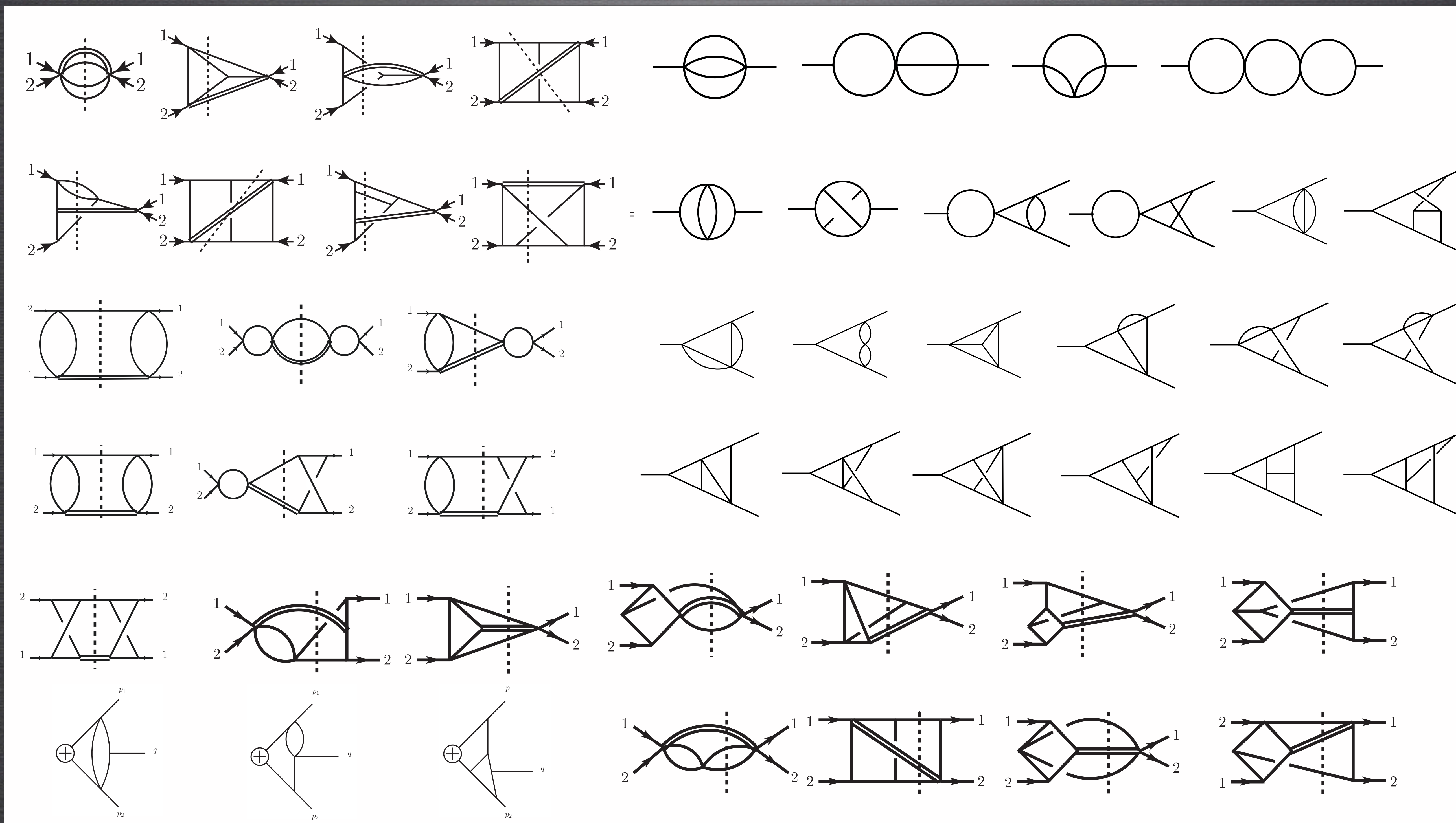
Threshold Expansion

- We found a method to systematically expand Matrix-Elements and Master Integrals
- We are able to apply IBP-Reduction AFTER performing the expansion
- We see a drastic simplification in the size of the Matrix-Elements
- We Observe a significant reduction of the Number of Master Integrals in the Expansion

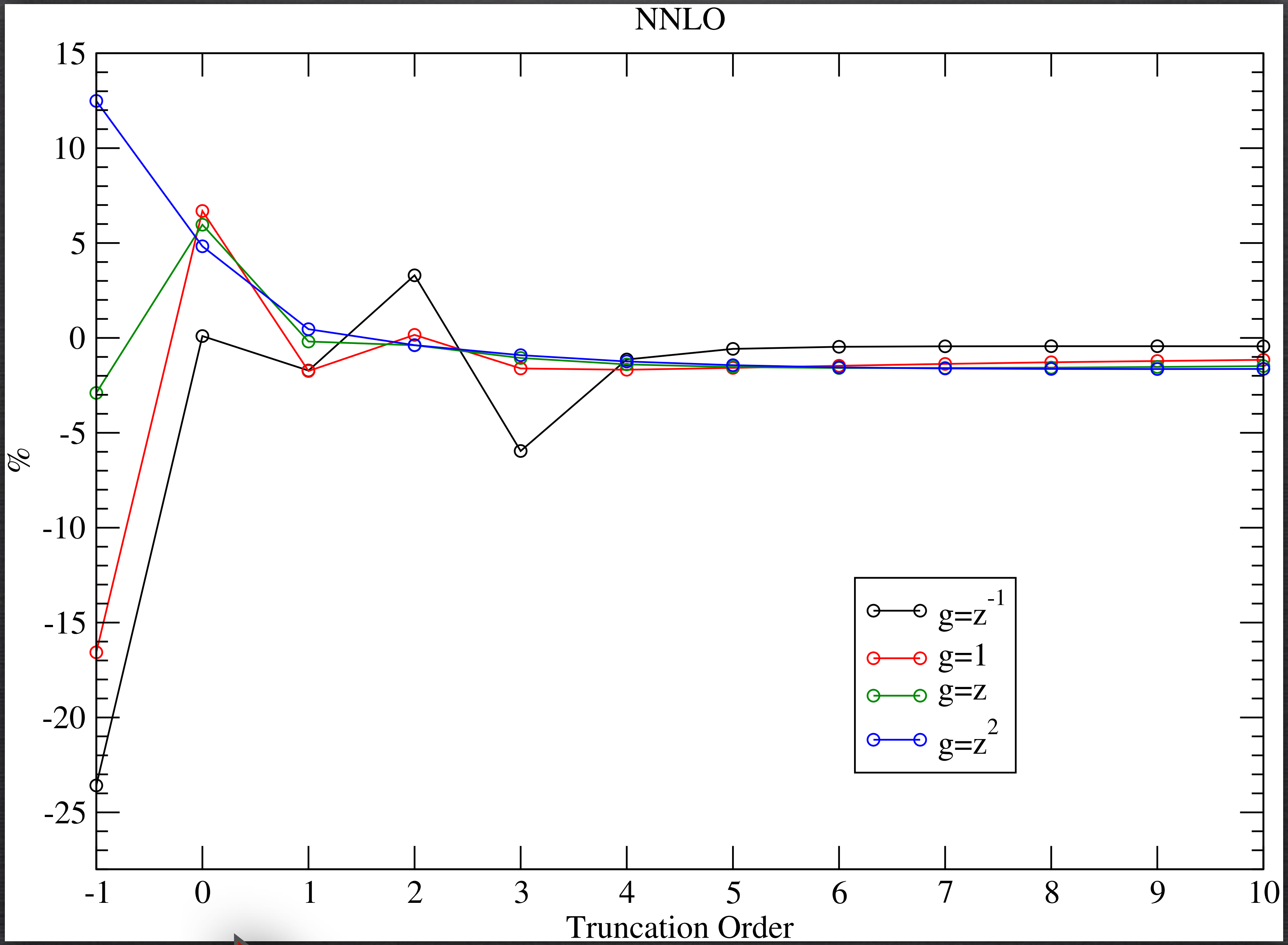
Double-Real Virtual

Full	Soft-Virtual
~350	11
Master Integrals	Master Integrals

Master Integrals



EXPANSION AT NNLO



Stay tuned

Conclusion/Outlook

- Systematic Expansion of Matrix Elements
- First Results: Soft-Virtual Cross-section
- Expansion as key ingredient for Full Kinematic Solution: boundary Condition
- Expansion as Check for Full Kinematic Cross-section

Thank you