Vector boson production and decay in hadron collisions: q_T resummation at NNLL accuracy

Giancarlo Ferrera

Milan University & INFN Milan

In collaboration with:
G. Bozzi, S. Catani, D. de Florian & M. Grazzini

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Motivations

The Drell–Yan process [Drell,Yan('70)] is a benchmark process in hadron collider physics. Its study is well motivated:

- Large production rates and clean experimental signatures.
- Constraints for fits of PDFs.
- q_T spectrum: important for M_W measurement and Beyond the Standard Model analyses.
- Test of perturbative QCD predictions.

The above reasons and precise experimental data demands for accurate theoretical predictions ⇒ computation of higher-order QCD corrections.

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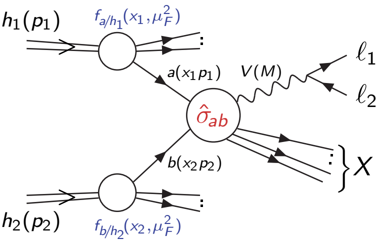
The Drell–Yan q_T distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M) + X \;\rightarrow\; \ell_1 + \ell_2 + X}$$

where $\quad V = \gamma^*, Z^0, W^\pm \quad$ and $\quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$

pQCD factorization formula:

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Standard fixed-order perturbative expansions ($Q_T \ll 1$):

$$\int_0^{Q_T^2} dq_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim 1 + \alpha_S\left[c_{12}\log^2\frac{M^2}{Q_T^2} + c_{11}\log\frac{M^2}{Q_T^2} + c_{10}\right]$$

$$+\alpha_S^2\left[c_{24}\log^4\frac{M^2}{Q_T^2} + \cdots + c_{21}\log\frac{M^2}{Q_T^2} + c_{20}\right] + \mathcal{O}(\alpha_S^3)$$

Fixed order calculation reliable only for $q_T \sim M$

For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$:

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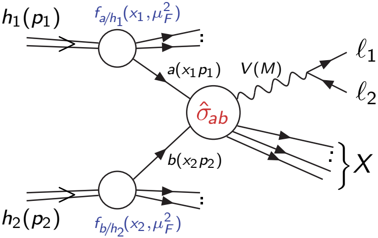

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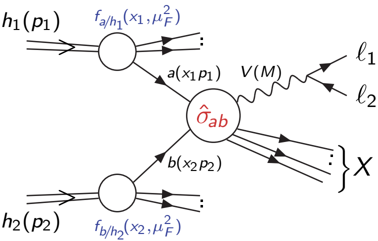

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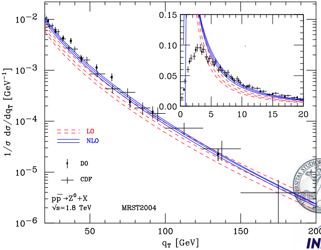

Idea of (analytic) resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S)$
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$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\alpha_S^n L^{2n-2}$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	next-to-dominant logs	$\cdots$	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$ i.e. when $\alpha_S L^2 \sim 1$ (and $\alpha_S \ll 1$).

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dynamics AND kinematics factorization
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions based on colour coherence. It is the analogous of the independent multiple soft-photon emission is QED:

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution of DY process it holds in the impact parameter space (Fourier transform).

$$\int d^2\mathbf{q_T}\, \exp(-i\mathbf{b}\cdot\mathbf{q_T})\, \delta\left(\mathbf{q_T} - \sum_{j=1}^{n} \mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot \sum_{j=1}^{n} \mathbf{q_{T_j}}) = \prod_{j=1}^{n} \exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space.
- Resummed result can then be properly combined with the fixed order result (*matching*) to have a good control of both the kinematical regions: $q_T \ll M$ and $q_T \sim M$.

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State of the art: transverse-momentum (q_T) resummation

- The method to perform the resummation of the large logarithms of q_T is known [Dokshitzer,Diakonov,Troian ('78)], [Parisi,Petronzio('79)], [Kodaira,Trentadue('82)],[Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)] [Catani,Grazzini('10)]
- Various phenomenological studies exist [Balasz,Qiu,Yuan('95)],[ResBos:Balasz, et al.],[Ellis et al.('97)], [Kulesza et al.('02)].
- More recently results for q_T resummation in the framework of Effective Theories developed [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)], [Mantry,Petriello('10)], [Becher,Neubert('10)],[Echevarria,Idilbi,Scimemi('11)].
- q_T distribution also studied by using transverse-momentum dependent (TMD) factorization and TMD parton densities [Roger,Mulders('10)],[Collins('11)],[D'Alesio,Echevarria,Melis, Scimemi('14)].

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Main distinctive features of the formalism [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)]:

- Resummation performed at partonic level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, customary study of μ_R and μ_F dependence.
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- Perturbative unitarity constraint:

$$\ln\left(Q^2 b^2\right) \rightarrow \widetilde{L} \equiv \ln\left(Q^2 b^2 + 1\right) \quad \Rightarrow \quad \exp\left\{\mathcal{G}_N(\alpha_S, \widetilde{L})\right\}\Big|_{b=0} = 1 \;\Rightarrow\; \int_0^\infty dq_T^2 \left(\frac{d\hat{\sigma}}{dq_T^2}\right)_{NLL+LO} = \hat{\sigma}_{NLO}^{(tot)},$$

  - avoids unjustified higher-order contributions in the small-b region.
  - recover *exactly* the total cross-section (upon integration on q_T)

DYqT: q_T-resummation at NNLL+NLO:

Bozzi,Catani,de Florian,G.F.,Grazzini('11)

- We have applied for Drell–Yan transverse-momentum distribution the resummation formalism developed by [Catani,de Florian, Grazzini('01)] already applied for the case of Higgs boson production [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)].
- We have performed the resummation up to NNLL+NLO. It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders;
  - NNLO corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T;
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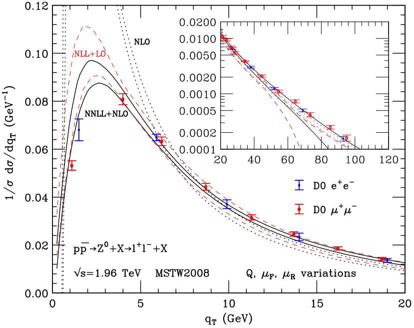
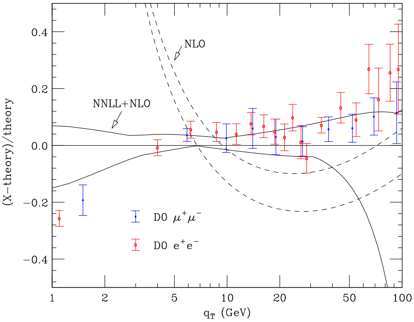
Resummed results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:
  $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
  to avoid large logarithmic contributions ($\sim \ln(\mu_F^2/\mu_R^2)$, $\ln(Q^2/\mu_R^2)$) in the evolution of the parton densities and in the the resummed form factor.
- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all q_T.
- Good convergence of resummed results: NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
  The perturbative uncertainty of the NNLL+NLO results is comparable with the experimental errors.

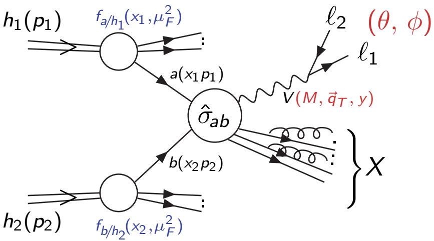
Resummed results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL+NLO, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL+NLO scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL+NLO bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL+NLO band.

q_T-resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.
- We have included the full dependence on the decay products variables: possible to apply cuts on vector boson and decay products.
- To construct the "finite" part we rely on the fully-differential NNLO result from the code DYNNLO [Catani,Cieri,de Florian,Ferrera,Grazzini('09)].
- Calculation implemented in a numerical program DYRes which includes spin correlations, $\gamma^* Z$ interference, finite-width effects and compute distributions in form of bin histograms: analogously to the HRes code.

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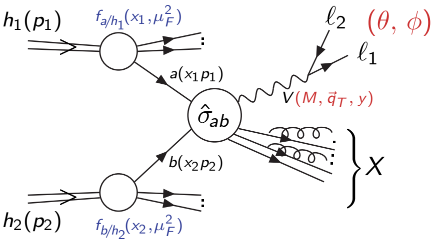

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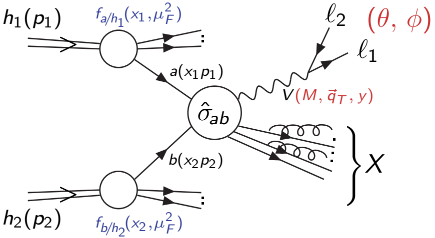

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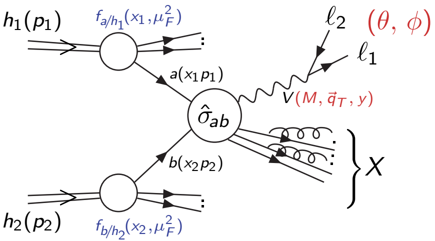

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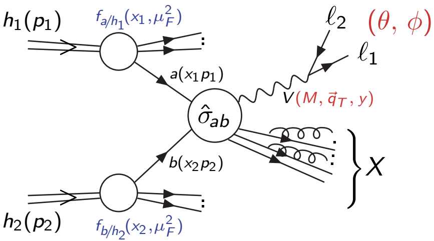

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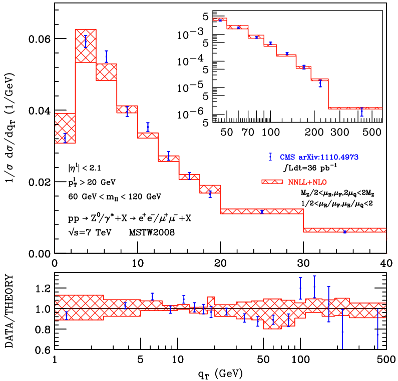
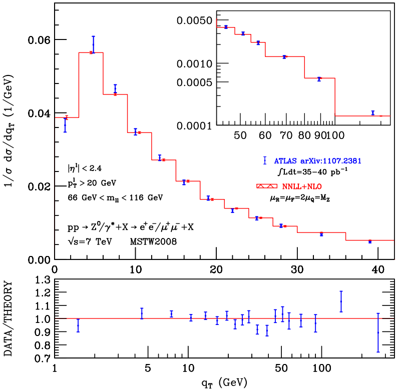
q_T-resummation with leptonic variables dependence



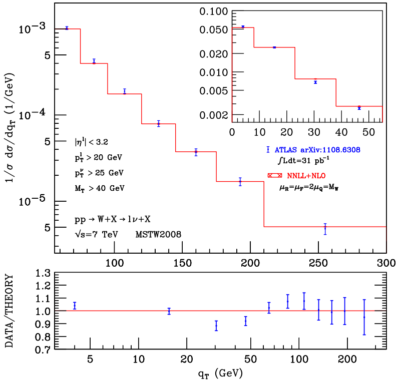
CMS data for the Z q_T spectrum compared with NNLL+NLO result. Scale variation:

$$1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R, 2Q/m_Z, Q/\mu_R\} \leq 2$$

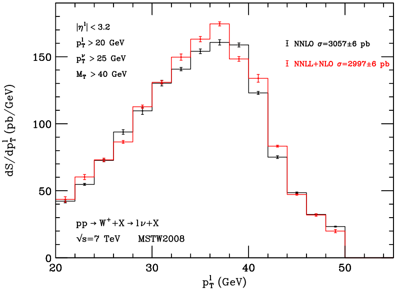


ATLAS data for the Z q_T spectrum compared with NNLL+NLO result.

q_T-resummation with decay variables dependence



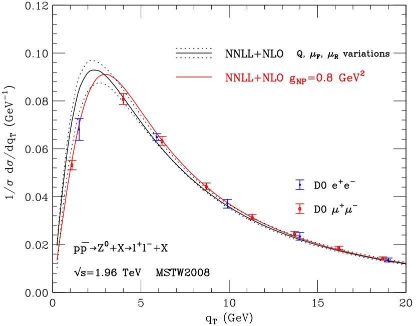
ATLAS data for the W q_T spectrum compared with NNLL+NLO result.



Lepton p_T spectrum from W^+ decay. NNLL+NLO result compared with the NNLO result.

Important spectrum for the measurement of M_W at the LHC.

Non perturbative Fermi motion effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \rightarrow \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

$$g_{NP} \simeq 0.8\ GeV^2 \quad \text{[Kulesza et al.('02)]}$$

- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

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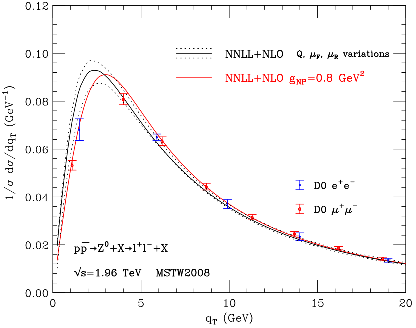

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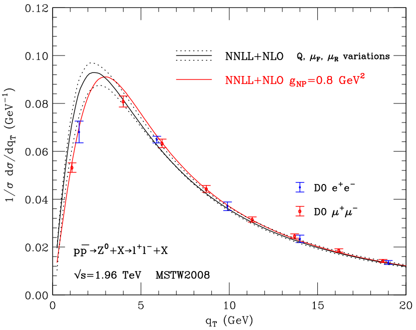

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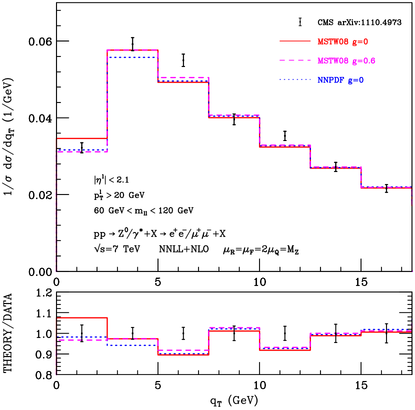

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Conclusions

- NNLL+NLO DY q_T-resummation [Bozzi,Catani,de Florian,G.F., Grazzini [arXiv:1007.2351]].
- A public version of the DYqT code is available. Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results consistent with the experimental data in a wide region of q_T.
- NEW: added full kinematical dependence on the vector boson and on the final state leptons.
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