Universality of transverse-momentum and threshold resummations, and results up to N^{3}LO and N^{3}LL

Leandro Cieri

La Sapienza - Università di Roma

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Outline

- Introduction
- Motivation : process dependent hard factors
- Hard virtual factor (qT resummation)
- Some explicit results/examples (NNLO+NNLL)
- Threshold resummation
- Some explicit results/examples (N^3LO and N^3LL)

In collaboration with S. Catani, D. de Florian, G. Ferrera and M. Grazzini

Introduction

- We'll describe the inclusive scattering reaction

h1(p1) + h2(p2) → F({qi}) + X

with a final colourless state F(M^2,$\mathbf{q}_T$,y): such as lepton pairs (produced by DY mechanism (DY)), γγ, vector bosons, Higgs boson(s), and so forth.

Introduction

- We'll describe the inclusive scattering reaction

$$\mathbf{h1(p1) + h2(p2) \rightarrow F(\{qi\}) + X}$$

- As is well known, in the small-$\mathbf{q_T}$ region ($\mathbf{q_T} << M$) the convergence of the fixed order perturbative expansion in powers of the QCD coupling α_s is spoiled by the presence of large logarithmic terms of the type $\mathbf{Ln^n[M^2/q_T^2]}$. And it is known that the predictivity of perturbative QCD can be recovered through the summation of these logarithmically-enhanced contributions to all order in α_s.
- If $F(M^2,\mathbf{q_T},y)$ is colourless the large contributions can be sistematically resummed to all orders, and the structure of the resummed calculation can be organized in a process-independent form

Dokshitzer, Diakonov, Troian. (1978)
Parisi, Petronzio (1979)
Curci, Greco, Srivastava (1979)
Collins, Soper (1981)

Kodaira, Trentadue (1982)
Collins, Soper, Sterman (1985)
Catani, D'Emilio, Trentradue (1988)
de Florian, Grazzini (2000)
Catani, de Florian, Grazzini (2001)
Catani, Grazzini (2011)

Introduction

- Sketchy form of resummation formula

$$\boxed{\frac{d\sigma}{dq_T} \sim \sigma_F^{(0)} \otimes H^F \otimes (\text{“Log terms”})}$$

Born level

All-order hard factor

all-order process independent factor

Process dependent

- Closely related formulations based on transverse-momentum dependent (TMD) distributions (roughly speaking, they enconde the “log terms”)

Mantry, Petriello (2010)
Becher, Neubert (2011)

Echevarria, Idilbi, Scimemi (2012)
Collins, Rogers (2013)
Echevarria, Idilbi, Scimemi (2013)

Motivation

The Hard factors $H_c^{(n)F}$:

- Are a necessary ingredient of the transverse momentum q_T subtraction formalism to perform fully-exclusive perturbative calculations at full **next-to-next-to-leading-order** (NNLO)

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The q_T subtraction formalism has been applied to the NNLO computation of **Higgs boson** and **vector boson** production, **associated production** of the Higgs boson with a W boson, **diphoton production, ZZ,WW, Zγ** production

WW: Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)
ZZ: Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)

[See Rathlev's talk]

ZH: Ferrera, Grazzini, Tramontano (2014)
Zγ: Grazzini, Kallweit, Rathlev, Torre. (2013)
γγ: Catani, LC, de Florian, Ferrera, Grazzini. (2012)
WH: Ferrera, Grazzini, Tramontano. (2011)
DY: Catani, LC, Ferrera, de Florian, Grazzini, (2009)
Higgs: Catani, Grazzini. (2007)

[See Ferrera's talk]

Motivation

The Hard factors $H_c^{(n)F}$:

- Are a necessary ingredient of the transverse momentum q_T subtraction formalism to perform fully-exclusive perturbative calculations at full **next-to-next-to-leading-order** (NNLO)

- Control NNLO contributions in resummed calculations at full **next-to-next-to-leading logarithmic accuracy** (NNLL)

> This permits direct applications to NNLL+NNLO resummed calculations for colourless final states. As already was done for the cases of SM **Higgs boson, Drell-Yan (DY)** production, and Higgs boson production *via* **bottom quark** annihilation

Bozzi, Catani, de Florian, Grazzini (2006)
de Florian, Ferrera, Grazzini, Tommasini (2011)
Wang, C. Li, Z. Li, Yuan, H. Li. (2012)

[See Ferrera's talk]

Bozzi, Catani, Ferrera, de Florian, Grazzini (2011)
Guzzi, Nadolsky, Wang. (2013)
Harlander, Tripathi, Wiesemann (2014)

Motivation

The Hard factors $\mathbf{H_c^{(n)F}}$:

- Are a necessary ingredient of the transverse momentum $\mathbf{q_T}$ subtraction formalism to perform fully-exclusive perturbative calculations at full **next-to-next-to-leading-order** (NNLO)
- Control NNLO contributions in resummed calculations at full **next-to-next-to-leading logarithmic accuracy** (NNLL)
- Explicitly determine part of logarithmic terms at N^3LL accuracy

Motivation

The Hard factors $H_c^{(n)F}$:

- Are a necessary ingredient of the transverse momentum q_T subtraction formalism to perform fully-exclusive perturbative calculations at full **next-to-next-to-leading-order** (NNLO)
- Control NNLO contributions in resummed calculations at full **next-to-next-to-leading logarithmic accuracy** (NNLL)
- Explicitly determine part of logarithmic terms at N^3LL accuracy

The knowledge of the NNLO hard-virtual term completes the q_T resummation formalism in explicit form up to full NNLL+NNLO accuracy and it is a necessary ingredient for resummation at N^3LL accuracy

Small-qT resummation

- If $F(M^2,\mathbf{q_T},y)$ is colourless, the LO cross section is controlled by the partonic subprocess of quark-antiquark annihilation, and (or) gluon fusion.

- The all-order process-independent form of the resummed calculation has a factorized structure, whose resummation factors are the (quark and gluon) Sudakov form factor, process-independent collinear factors and a process-dependent hard or, more precisely, hard-virtual factor.

$$d\sigma_F = d\sigma_F^{(\mathrm{sing})} + d\sigma_F^{(\mathrm{reg})}$$

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma_{c\bar{c},F}^{(0)}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_c(M,b)$$
$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

Collins, Soper, Sterman (1985) Catani, de Florian, Grazzini (2001) Catani, Grazzini (2011)

Small-qT resummation

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\boxed{S_c(M,b)}$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

Collins, Soper, Sterman (1985)

$$\boxed{S_c(M,b)} = \exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$

Small-qT resummation

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\boxed{\left[d\sigma^{(0)}_{c\bar{c},F}\right]}\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\,\boxed{S_c(M,b)}$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$\boxed{S_c(M,b)} = \exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$

$$A_c(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}\quad,\qquad B_c(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

$$\boxed{\left[d\sigma^{(0)}_{c\bar{c},\,F}\right]} = \frac{d\hat{\sigma}^{(0)}_{c\bar{c},\,F}}{M^2\,d\mathbf{\Omega}}\left(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S(M^2)\right)$$

Small-qT resummation

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$S_c(M,b)=\exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[\boxed{A_c(\alpha_S(q^2))}\ln\frac{M^2}{q^2}+\boxed{B_c(\alpha_S(q^2))}\right]\right\}$$

$A^{(1)}_c$, $B^{(1)}_c$, $A^{(2)}_c$: Kodaira, Trentadue (1982); Catani, D'Emilio, Trentradue (1988)

$B^{(2)}_c$: Davies, Stirling (1984); Davies, Webber, Stirling (1985); de Florian, Grazzini (2000)

$A^{(3)}_c$: Becher, Neubert (2011)

Small-qT resummation

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\boxed{\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}}f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

Catani, de Florian, Grazzini (2001)

different scales

$$\boxed{\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2}} = H_q^F(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S(M^2))\;\;C_{q\,a_1}(z_1;\alpha_S(b_0^2/b^2))\;\;C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))$$

Process dependent

$$H_q^F(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S) = 1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}(x_1p_1,x_2p_2;\mathbf{\Omega})$$

Universal

$$C_{q\,a}(z;\alpha_S) = \delta_{q\,a}\;\;\delta(1-z)+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{q\,a}^{(n)}(z)\quad.$$

Small-qT resummation

Catani, de Florian, Grazzini (2001)

$$H_c^F(\alpha_S) \to H_c^F(\alpha_S)\,[h_c(\alpha_S)]^{-1}\,,$$
$$B_c(\alpha_S) \to B_c(\alpha_S) - \beta(\alpha_S)\frac{d\ln h_c(\alpha_S)}{d\ln\alpha_S}\,,$$
$$C_{cb}(\alpha_S) \to C_{cb}(\alpha_S)\,[h_c(\alpha_S)]^{1/2}\,,$$

These relations imply: the resummation factors $\mathbf{C}_{qa}$, $\mathbf{S}_c$, $\mathbf{H}^F$ are not separately defined (and, thus, computable) in an unambiguous way. Equivalently, each of these separate factors can be precisely defined only by specifying a **resummation scheme**.

- The ***hard scheme :*** the $\mathbf{C}^{(n)}_{ab}$ coefficients do not contain any $\delta(1-z)$ term

- This implies that all the process-dependent virtual corrections to the Born level subprocess are embodied in the resummation coefficient $\mathbf{H}^F_c$

Small-qT resummation

Process initiated at the Born level by the gluon fusion channel

- The physics of the small-qT cross section has a richer structure which is the consequence of collinear correlations that are produced by the evolution of the colliding hadrons into gluon partonic states. Catani, Grazzini (2011)

Collinear radiation from the colliding gluons leads to spin and azimuthal correlations

$$[H^F C_1 C_2]_{gg;a_1 a_2} = H^F_{g;\mu_1\,\nu_1,\mu_2\,\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S(M^2)) \times C^{\mu_1\,\nu_1}_{g\,a_1}(z_1; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2))\; C^{\mu_2\,\nu_2}_{g\,a_2}(z_2; p_1, p_2, \mathbf{b}; \alpha_S(b_0^2/b^2))$$

Depends on spins of the colliding gluons

$$H_g^{F\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}; \alpha_S) = H_g^{F(0)\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega}) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_g^{F(n)\mu_1\nu_1,\mu_2\nu_2}(x_1 p_1, x_2 p_2; \mathbf{\Omega})$$

The small-qT cross section depends on φ(qT) plus a contribution in function of cos[2 φ(qT)], sin[2φ(qT)], cos[4φ(qT)] and sin[4 φ(qT)]

$$C^{\mu\nu}_{g\,a}(z; p_1, p_2, \mathbf{b}; \alpha_S) = d^{\mu\nu}(p_1, p_2)\, C_{g\,a}(z; \alpha_S) + D^{\mu\,\nu}(p_1, p_2; \mathbf{b})\, G_{g\,a}(z; \alpha_S)$$

$$d^{\mu\nu}(p_1, p_2) = -g^{\mu\nu} + \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2}$$

$$D^{\mu\nu}(p_1, p_2; \mathbf{b}) = d^{\mu\nu}(p_1, p_2) - 2\,\frac{b^\mu\, b^\nu}{\mathbf{b}^2}$$

Process-independent coefficients

(hard scheme)

Davies, Stirling (1984);
Davies, Webber, Stirling (1985);
de Florian, Grazzini (2000);
Kauffman (1992);
Yuan (1992)

$$C_{qq}^{(1)}(z) = \frac{1}{2} C_F (1-z) \ ,$$

$$C_{gq}^{(1)}(z) = \frac{1}{2} C_F \, z \ ,$$

$$C_{qg}^{(1)}(z) = \frac{1}{2} z(1-z) \ ,$$

$$C_{gg}^{(1)}(z) = C_{q\bar{q}}(z) = C_{qq'}(z) = C_{q\bar{q}'}(z) = 0$$

Process-independent coefficients

(hard scheme)

Catani, LC, de Florian, Ferrera, Grazzini (2009); Catani, LC, de Florian, Ferrera, Grazzini (2012)

$$2C_{qq}^{(2)}(z) = \mathcal{H}_{q\bar{q}\leftarrow q\bar{q}}^{DY(2)}(z)|_{\text{no }\delta(1-z)} - \frac{C_F^2}{4}\left[\left(2\pi^2 - 18\right)(1-z) - (1+z)\ln z\right]$$

Catani, LC, de Florian, Ferrera, Grazzini (2009); Catani, LC, de Florian, Ferrera, Grazzini (2012)

$$C_{qg}^{(2)}(z) = \mathcal{H}_{q\bar{q}\leftarrow qg}^{DY(2)}(z) - \frac{C_F}{4}\left[z\ln z + \frac{1}{2}(1-z^2) + (\pi^2-8)z(1-z)\right]$$

$$C_{gq}^{(2)}(z) = \mathcal{H}_{gg\leftarrow gq}^{H(2)}(z) + C_F^2\frac{3}{4}z + C_F C_A \frac{1}{z}\left[(1+z)\ln z + 2(1-z) - \frac{5+\pi^2}{4}z^2\right]$$

Catani, Grazzini (2007); Catani, Grazzini (2012)

$$2C_{gg}^{(2)}(z) = \mathcal{H}_{gg\leftarrow gg}^{H(2)}(z)|_{\text{no }\delta(1-z)} + C_A^2\left(\frac{1+z}{z}\ln z + 2\,\frac{1-z}{z}\right)$$

Process-independent coefficients

(hard scheme)

Catani, LC, de Florian, Ferrera, Grazzini (2009); Catani, LC, de Florian, Ferrera, Grazzini (2012)

$$2C_{qq}^{(2)}(z) = \mathcal{H}_{q\bar{q}\leftarrow q\bar{q}}^{DY(2)}(z)|_{\text{no }\delta(1-z)} - \frac{C_F^2}{4}\left[\left(2\pi^2 - 18\right)(1-z) - (1+z)\ln z\right]$$

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Catani, Grazzini (2007); Catani, Grazzini (2012)

$$2C_{gg}^{(2)}(z) = \mathcal{H}_{gg\leftarrow gg}^{H(2)}(z)|_{\text{no }\delta(1-z)} + C_A^2\left(\frac{1+z}{z}\ln z + 2\,\frac{1-z}{z}\right)$$

Cumbersome part hidden in the notation $\mathcal{H}$ (z), e.g :

$$\begin{aligned}
\mathcal{H}^{DY(2)}_{q\bar q\leftarrow q\bar q}(z) = C_A C_F \Big\{ & \left(\frac{7\zeta_3}{2} - \frac{101}{27}\right)\left(\frac{1}{1-z}\right)_+ + \left(\frac{59\zeta_3}{18} - \frac{1535}{192} + \frac{215\pi^2}{216} - \frac{\pi^4}{240}\right)\delta(1-z) \\
& + \frac{1+z^2}{1-z}\Big(- \frac{\mathrm{Li}_3(1-z)}{2} + \mathrm{Li}_3(z) - \frac{\mathrm{Li}_2(z)\log(z)}{2} - \frac{1}{2}\mathrm{Li}_2(z)\log(1-z) - \frac{1}{24}\log^3(z) \\
& - \frac{1}{2}\log^2(1-z)\log(z) + \frac{1}{12}\pi^2\log(1-z) - \frac{\pi^2}{8}\Big) + \frac{1}{1-z}\Big(- \frac{1}{4}\left(11-3z^2\right)\zeta_3 \\
& - \frac{1}{48}\left(-z^2+12z+11\right)\log^2(z) - \frac{1}{36}\left(83z^2-36z+29\right)\log(z) + \frac{\pi^2 z}{4}\Big) \\
& + (1-z)\Big(\frac{\mathrm{Li}_2(z)}{2} + \frac{1}{2}\log(1-z)\log(z)\Big) + \frac{z+100}{27} + \frac{1}{4}z\log(1-z)\Big\} \\
& + C_F n_F \Big\{\frac{14}{27}\left(\frac{1}{1-z}\right)_+ + \frac{1}{864}\left(192\zeta_3 + 1143 - 152\pi^2\right)\delta(1-z) \\
& + \frac{(1+z^2)}{72(1-z)}\log(z)(3\log(z)+10) + \frac{1}{108}(-19z-37)\Big\} \\
& + C_F^2\Big\{\frac{1}{4}\left(-15\zeta_3 + \frac{511}{16} - \frac{67\pi^2}{12} + \frac{17\pi^4}{45}\right)\delta(1-z) \\
& + \frac{1+z^2}{1-z}\Big(\frac{\mathrm{Li}_3(1-z)}{2} - \frac{5\mathrm{Li}_3(z)}{2} + \frac{1}{2}\mathrm{Li}_2(z)\log(1-z) + \frac{3\mathrm{Li}_2(z)\log(z)}{2} \\
& + \frac{3}{4}\log(z)\log^2(1-z) + \frac{1}{4}\log^2(z)\log(1-z) - \frac{1}{12}\pi^2\log(1-z) + \frac{5\zeta_3}{2}\Big) \\
& + (1-z)\left(-\mathrm{Li}_2(z) - \frac{3}{2}\log(1-z)\log(z) + \frac{2\pi^2}{3} - \frac{29}{4}\right) + \frac{1}{24}(1+z)\log^3(z) \\
& + \frac{1}{1-z}\Big(\frac{1}{8}\left(-2z^2+2z+3\right)\log^2(z) + \frac{1}{4}\left(17z^2-13z+4\right)\log(z)\Big) - \frac{z}{4}\log(1-z)\Big\} \\
& + C_F\Big\{\frac{1}{z}(1-z)\left(2z^2-z+2\right)\left(\frac{\mathrm{Li}_2(z)}{6} + \frac{1}{6}\log(1-z)\log(z) - \frac{\pi^2}{36}\right) \\
& + \frac{1}{216z}(1-z)\left(136z^2-143z+172\right) - \frac{1}{48}\left(8z^2+3z+3\right)\log^2(z) \\
& + \frac{1}{36}\left(32z^2-30z+21\right)\log(z) + \frac{1}{24}(1+z)\log^3(z)\Big\} ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}^{DY(2)}_{q\bar q\leftarrow q\bar q'}(z) = C_F\Big\{ & \frac{1}{12z}(1-z)\left(2z^2-z+2\right)\left(\mathrm{Li}_2(z) + \log(1-z)\log(z) - \frac{\pi^2}{6}\right) \\
& + \frac{1}{432z}(1-z)\left(136z^2-143z+172\right) + \frac{1}{48}(1+z)\log^3(z) \\
& - \frac{1}{96}\left(8z^2+3z+3\right)\log^2(z) + \frac{1}{72}\left(32z^2-30z+21\right)\log(z)\Big\} ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}^{DY(2)}_{q\bar q\leftarrow qq}(z) = C_F\left(C_F - \frac{1}{2}C_A\right)\Big\{ & \frac{1+z^2}{1+z}\Big(\frac{3\mathrm{Li}_3(-z)}{2} + \mathrm{Li}_3(z) + \mathrm{Li}_3\left(\frac{1}{1+z}\right) - \frac{\mathrm{Li}_2(-z)\log(z)}{2} \\
& - \frac{\mathrm{Li}_2(z)\log(z)}{2} - \frac{1}{24}\log^3(z) - \frac{1}{6}\log^3(1+z) + \frac{1}{4}\log(1+z)\log^2(z) \\
& + \frac{\pi^2}{12}\log(1+z) - \frac{3\zeta_3}{4}\Big) + (1-z)\left(\frac{\mathrm{Li}_2(z)}{2} + \frac{1}{2}\log(1-z)\log(z) + \frac{15}{8}\right) \\
& - \frac{1}{2}(1+z)\big(\mathrm{Li}_2(-z) + \log(z)\log(1+z)\big) + \frac{\pi^2}{24}(z-3) + \frac{1}{8}(11z+3)\log(z)\Big\} \\
& + C_F\Big\{\frac{1}{12z}(1-z)\left(2z^2-z+2\right)\left(\mathrm{Li}_2(z) + \log(1-z)\log(z) - \frac{\pi^2}{6}\right) \\
& + \frac{1}{432z}(1-z)\left(136z^2-143z+172\right) - \frac{1}{96}\left(8z^2+3z+3\right)\log^2(z) \\
& + \frac{1}{72}\left(32z^2-30z+21\right)\log(z) + \frac{1}{48}(1+z)\log^3(z)\Big\} ,
\end{aligned}$$

$$\mathcal{H}^{DY(2)}_{q\bar q\leftarrow qq'}(z) = \mathcal{H}^{DY(2)}_{q\bar q\leftarrow q\bar q'}(z) ,$$

$$\begin{aligned}
\mathcal{H}^{DY(2)}_{q\bar q\leftarrow qg}(z) = C_A\Big\{ & -\frac{1}{12z}(1-z)\left(11z^2-z+2\right)\mathrm{Li}_2(1-z) \\
& + \left(2z^2-2z+1\right)\left(\frac{\mathrm{Li}_3(1-z)}{8} - \frac{1}{8}\mathrm{Li}_2(1-z)\log(1-z) + \frac{1}{48}\log^3(1-z)\right) \\
& + \left(2z^2+2z+1\right)\Big(\frac{3\mathrm{Li}_3(-z)}{8} + \frac{\mathrm{Li}_3\left(\frac{1}{1+z}\right)}{4} - \frac{\mathrm{Li}_2(-z)\log(z)}{8} - \frac{1}{24}\log^3(1+z) \\
& + \frac{1}{16}\log^2(z)\log(1+z) + \frac{1}{48}\pi^2\log(1+z)\Big) + \frac{1}{4}z(1+z)\mathrm{Li}_2(-z) + z\mathrm{Li}_3(z) \\
& - \frac{1}{2}z\mathrm{Li}_2(1-z)\log(z) - z\mathrm{Li}_2(z)\log(z) - \frac{3}{8}\left(2z^2+1\right)\zeta_3 - \frac{149z^2}{216} \\
& - \frac{1}{96}\left(44z^2-12z+3\right)\log^2(z) + \frac{1}{72}\left(68z^2+6\pi^2 z-30z+21\right)\log(z) + \frac{\pi^2 z}{24} + \frac{43z}{48} \\
& + \frac{43}{108z} + \frac{1}{48}(2z+1)\log^3(z) - \frac{1}{2}z\log(1-z)\log^2(z) - \frac{1}{8}(1-z)z\log^2(1-z) \\
& + \frac{1}{4}z(1+z)\log(1+z)\log(z) + \frac{1}{16}(3-4z)z\log(1-z) - \frac{35}{48}\Big\} \\
& + C_F\Big\{\left(2z^2-2z+1\right)\Big(\zeta_3 - \frac{\mathrm{Li}_3(1-z)}{8} - \frac{\mathrm{Li}_3(z)}{8} + \frac{1}{8}\mathrm{Li}_2(1-z)\log(1-z) \\
& + \frac{\mathrm{Li}_2(z)\log(z)}{8} - \frac{1}{48}\log^3(1-z) + \frac{1}{16}\log(z)\log^2(1-z) + \frac{1}{16}\log^2(z)\log(1-z)\Big) \\
& - \frac{3z^2}{8} - \frac{1}{96}\left(4z^2-2z+1\right)\log^3(z) + \frac{1}{64}\left(-8z^2+12z+1\right)\log^2(z) \\
& + \frac{1}{32}\left(-8z^2+23z+8\right)\log(z) + \frac{5}{24}\pi^2(1-z)z + \frac{11z}{32} + \frac{1}{8}(1-z)z\log^2(1-z) \\
& - \frac{1}{4}(1-z)z\log(1-z)\log(z) - \frac{1}{16}(3-4z)z\log(1-z) - \frac{9}{32}\Big\} .
\end{aligned}$$

Hard virtual coefficients

In the hard scheme, this coefficient contains all the information on the process-dependent virtual corrections

- We consider the partonic elastic-production process

$$c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\})$$

The renormalized all-loop amplitude has the perturbative (loop) expansion:

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left(\alpha_S(\mu_R^2)\,\mu_R^{2\epsilon}\right)^k \left[\mathcal{M}^{(0)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)\mathcal{M}^{(1)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^2 \mathcal{M}^{(2)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R)\right]$$

The structure of the hard-virtual term

In the hard scheme, this coefficient contains all the information on the process-dependent virtual corrections

All the remaining contributions to H_c^F are:

- **factorized**
- **universal (process independent)**

Introduce auxiliary hard-virtual amplitude $\tilde{\mathcal{M}}$ and subtraction operator $\tilde{I}_c$:

$$\widetilde{\mathcal{M}}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left[1-\tilde{I}_c(\epsilon,M^2)\right]\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\})$$

$$\tilde{I}_c(\epsilon,M^2) = \frac{\alpha_S(\mu_R^2)}{2\pi}\,\tilde{I}_c^{(1)}(\epsilon,M^2/\mu_R^2) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^2 \tilde{I}_c^{(2)}(\epsilon,M^2/\mu_R^2) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n \tilde{I}_c^{(n)}(\epsilon,M^2/\mu_R^2)$$

Then:

$$\alpha_S^{2k}(M^2)\,H_q^F(x_1p_1,x_2p_2;\mathbf{\Omega};\alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1,x_2p_2;\{q_i\})|^2}\,,$$

The structure of the hard-virtual term

The subtraction operartor (1- $\tilde{I}_c$)

- **contains IR divergences (ε-poles) + IR finite terms**
- **originates from universal soft-collinear factorization formulae of scattering amplitudes** Catani, Grazzini (2000); Bern, Del Duca, Kilgore, Shmidt (1999); Catani, Grazzini (2000), Campbell, Glover (1998), Kosower,Uwer (1999)

In the hard scheme

- ***Collinear terms*** **: only ε-poles from collinear counterterm of PDF (virtual part of AP splitting functions)**
- ***Soft terms*** **: both ε-poles and IR finite part**

simply proportional to C_c [up to $O(\alpha^3_S)$]

$C_c \; \begin{cases} C_F \\ C_A \end{cases}$

a single UNIVERSAL COEFFICIENT at each perturbative order $R^{fin(1)}_{qT}$, $R^{fin(2)}_{qT}$

- **ε poles known from their universality**

Catani (1998)
Dixon, Magnea, Sterman (2008)
Becher, Neubert (2009)

Hard virtual coefficients

In the hard scheme, this coefficient contains all the information on the process-dependent virtual corrections

The (IR divergent and finite) terms are removed from $M_{cc\to F}$ originate from real emission contributions to the cross section, with the IR subtraction operators $\boldsymbol{I}^{(n)}_c$

$$\widetilde{\mathcal{M}}^{(0)}_{c\bar{c}\to F} = \mathcal{M}^{(0)}_{c\bar{c}\to F}\,,$$

$$\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to F} = \mathcal{M}^{(1)}_{c\bar{c}\to F} - \tilde{I}^{(1)}_c(\epsilon, M^2/\mu_R^2)\;\mathcal{M}^{(0)}_{c\bar{c}\to F}\,,$$

$$\widetilde{\mathcal{M}}^{(2)}_{c\bar{c}\to F} = \mathcal{M}^{(2)}_{c\bar{c}\to F} - \tilde{I}^{(1)}_c(\epsilon, M^2/\mu_R^2)\;\mathcal{M}^{(1)}_{c\bar{c}\to F} - \tilde{I}^{(2)}_c(\epsilon, M^2/\mu_R^2)\;\mathcal{M}^{(0)}_{c\bar{c}\to F}$$

With:

$$1 - \tilde{I}^{q_T}_c(\epsilon, M^2) = \exp\left\{R^{q_T}_c(\epsilon, \alpha_S(M^2)) - i\Phi_c(\epsilon, \alpha_S(M^2)) + \mathcal{O}(\epsilon)\right\}$$

$$-i\Phi_c(\epsilon, \alpha_S) = \frac{i\pi\, C_c}{2\epsilon}\left\{\left(\frac{\alpha_S}{\pi}\right) + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{1}{2}\left(\gamma^{(1)}_{\rm cusp} - \frac{\beta_0\pi}{\epsilon}\right) + \left(\frac{\alpha_S}{\pi}\right)^3 \frac{1}{3}\left(\gamma^{(2)}_{\rm cusp} - \frac{1}{\epsilon}\gamma^{(1)}_{\rm cusp}\,\beta_0\pi + \frac{1}{\epsilon}\pi^2\left(\frac{\beta_0^2}{\epsilon} - \beta_1\right)\right)\right\} + \mathcal{O}(\alpha_S^4)$$

Hard virtual coefficients

$$R_c^{q_T}(\epsilon,\alpha_S) = R_{c\ ,q_T}^{\rm soft}(\epsilon,\alpha_S) + R_c^{\rm coll}(\epsilon,\alpha_S)$$

And we have :

$$R_{c\ ,q_T}^{\rm soft}(\epsilon,\alpha_S) = C_c\left(\frac{\alpha_S}{\pi}R_{q_T}^{{\rm soft}(1)}(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^2 R_{q_T}^{{\rm soft}(2)}(\epsilon)\right) + \mathcal{O}(\alpha_S^3)$$

$$R_c^{\rm coll}(\epsilon,\alpha_S) = \frac{\alpha_S}{\pi}R_c^{{\rm coll}(1)}(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^2 R_c^{{\rm coll}(2)}(\epsilon) + \mathcal{O}(\alpha_S^3)$$

At the first order in α_s :

$$R_{q_T}^{{\rm soft}(1)}(\epsilon) = \frac{1}{2\epsilon^2} + R_{q_T}^{{\rm fin}(1)}\ ,$$
$$R_c^{{\rm coll}(1)}(\epsilon) = \frac{\gamma_c}{2\epsilon}\ ,$$

$$R_{q_T}^{{\rm fin}(1)} = -\frac{\pi^2}{24}$$

Hard virtual coefficients

At the second order in α_s :

$$R_{q_T}^{\mathrm{soft}(2)}(\epsilon) = -\frac{3}{8}\frac{\beta_0\pi}{\epsilon^3} + \frac{1}{8\epsilon^2}\gamma_{\mathrm{cusp}}^{(1)} - \frac{1}{16\epsilon}d_{(1)} + R_{q_T}^{\mathrm{fin}(2)} \ ,$$
$$R_c^{\mathrm{coll}(2)}(\epsilon) = -\frac{\beta_0\pi}{4\epsilon^2}\gamma_c + \frac{1}{8\epsilon}\gamma_c^{(1)} \ ,$$

$$R_{q_T}^{\mathrm{fin}(2)} = C_A\left(-\frac{77}{144}\zeta_3 + \frac{\pi^4}{288} - \frac{67}{576}\pi^2 + \frac{607}{648}\right) + n_F\left(\frac{7}{72}\zeta_3 + \frac{5}{288}\pi^2 - \frac{41}{324}\right)$$

Soft UNIVERSAL coefficients

The explicit determination of $R^{fin(2)}_{qT}$ requires a detailed calculation

- Such a calculation can be explicitly performed in a general process-independent form. (which is based on NNLO soft/collinear factorization formulae)

extending the analysis in: [de Florian, Grazzini (2001)]

- Alternatively, we can exploit our proof of the universality of $R^{fin(2)}_{qT}$ and, therefore, we can determine the value of $R^{fin(2)}_{qT}$ from the NNLO calculation of a single specific process. (We have followed this approach)

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In the case of the DY process

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}\,,$$

Explicit value from the NNLO computation of the DY cross section at small values of q_T.

[Catani, LC, de Florian, Ferrera Grazzini (2009)]

linearly depends on $R^{fin(2)}_{qT}$

The scattering amplitude $\mathcal{M}_{qq\to DY}$ for the DY process was computed long ago up to the two-loop level

[Gonsalves(1983);Kramer ,Lampe(1987);Matsuura, van Neerven (1988); Matsuura, van der Marck, van Neerven (1989)]

$$\widetilde{\mathcal{M}}^{(1)}_{c\bar{c}\to F} = \mathcal{M}^{(1)}_{c\bar{c}\to F} - \tilde{I}^{(1)}_c(\epsilon, M^2/\mu_R^2)\ \mathcal{M}^{(0)}_{c\bar{c}\to F}\,,$$

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In the case of the Higgs boson production

- The same procedure can be applied to extract the value of $R^{fin(2)}_{qT}$ from Higgs boson production by gluon fusion. Because $\mathbf{H}^{H(2)}_{g}$ is known [Catani, Grazzini (2012)] and also the two-loops matrix elements [Harlander (2000); Ravindran, Smith, van Neerven (2005)]

- Using these results, **we confirm** the value of $R^{fin(2)}_{qT}$ that we have extracted from the DY process → **highly non-trivial check!**

Since we are considering two processes that are controlled by the quark-antiquark annihilation channel and the gluon fusion channel ($R^{fin(2)}_{qT}$ is instead independent of the specific channel).

Transverse momentum dependent (TMD) factorization

Gehrmann, Lubbert, Yang (2012),(2014)

- Similar results were obtained in a completely different approach to qT-resummation, based on a different factorization into individual contributions
- The **building blocks** of the resummed cross section can not be compared one-by-one between the two approaches

they are scheme-dependent

- Both approaches must agree on the scheme-independent ("physical") expression for the resummed cross section

In our case

$$\mathcal{H}^F_{ab\leftarrow jk}(z,\alpha_s) = \int_0^1 dz_1 \int_0^1 dz_2\, \delta(z - z_1 z_2)\, [H^F C_1 C_2]$$

In TMD

$$\mathcal{H}^{DY}_{q\bar{q}\leftarrow jk}(z,\alpha_s) = \left|C_V(-q^2,\sqrt{q^2})\right|^2 I_{q/j}(z, x_T^2, \mu_x) \otimes I_{\bar{q}/k}(z, x_T^2, \mu_x)$$

$$\mathcal{H}^H_{gg\leftarrow jk}\left(z,\alpha_s,\log\frac{m_t^2}{m_h^2}\right) = H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2, m_h^2, m_h)\, I^{\mu_1\nu_1}_{g/j}(z, x_\perp, \mu_x) \otimes I^{\mu_2\nu_2}_{g/k}(z, x_\perp, \mu_x)$$

$$H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2, m_h^2, m_h) = C_t^2(m_t^2, m_h)\left|C_S(-m_h^2, m_h)\right|^2 g_{\mu_1\mu_2} g_{\nu_1\nu_2}$$

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These results provide a remarkable and fully independent check of our results in a completely different approach

Results/examples

In the case of the DY process (production of a vector boson V=γ*,W±,Z, and the subsequent leptonic decay)

Catani, LC, Ferrera, de Florian, Grazzini, (2012)

$$H_q^{DY(1)} = C_F\left(\frac{\pi^2}{2} - 4\right)$$

$$H_q^{DY(2)} = C_F C_A\left(\frac{59\zeta_3}{18} - \frac{1535}{192} + \frac{215\pi^2}{216} - \frac{\pi^4}{240}\right) + \frac{1}{4}C_F^2\left(-15\zeta_3 + \frac{511}{16} - \frac{67\pi^2}{12} + \frac{17\pi^4}{45}\right) + \frac{1}{864}C_F N_f\left(192\zeta_3 + 1143 - 152\pi^2\right)\,.$$

In the case of the Higgs boson production (through the gluon fusion channel)

Catani, Grazzini, (2011)

$$c_H(m_Q) \longrightarrow \frac{5C_A - 3C_F}{2} = \frac{11}{2}$$

$$H_g^{H(1)} = C_A\pi^2/2 + c_H(m_Q)$$

$$H_g^{H(2)} = C_A^2\left(\frac{3187}{288} + \frac{7}{8}L_Q + \frac{157}{72}\pi^2 + \frac{13}{144}\pi^4 - \frac{55}{18}\zeta_3\right) + C_A C_F\left(-\frac{145}{24} - \frac{11}{8}L_Q - \frac{3}{4}\pi^2\right) + \frac{9}{4}C_F^2 - \frac{5}{96}C_A - \frac{1}{12}C_F - C_A N_f\left(\frac{287}{144} + \frac{5}{36}\pi^2 + \frac{4}{9}\zeta_3\right) + C_F N_f\left(-\frac{41}{24} + \frac{1}{2}L_Q + \zeta_3\right)$$

$$L_Q = \ln(M^2/m_Q^2)$$

Results/examples

In the case of the diphoton production: Catani, LC, Ferrera, de Florian, Grazzini, (2013)

The $H_q^{\gamma\gamma(1)}$ was known: Balazs, Berger, Mrenna, Yuan (1998)

$$H_q^{\gamma\gamma(1)}(v) = \frac{C_F}{2}\Bigg\{(\pi^2-7) + \frac{1}{(1-v)^2+v^2}\Bigg[\Big((1-v)^2+1\Big)\ln^2(1-v) + v(v+2)\ln(1-v) + \Big(v^2+1\Big)\ln^2 v + (1-v)(3-v)\ln v\Bigg]\Bigg\} .$$

Catani, LC, Ferrera, de Florian, Grazzini, (2013)

$$H_q^{\gamma\gamma(2)}(v) = \frac{1}{4\mathcal{A}_{LO}(v)}\left[\mathcal{F}^{0\times 2}_{inite,q\bar{q}\gamma\gamma;s} + \mathcal{F}^{1\times 1}_{inite,q\bar{q}\gamma\gamma;s}\right] + 3\zeta_2\, C_F H_q^{\gamma\gamma(1)}(v) - \frac{45}{4}\zeta_4\, C_F^2 + C_F C_A\left(\frac{607}{324} + \frac{1181}{144}\zeta_2 - \frac{187}{144}\zeta_3 - \frac{105}{32}\zeta_4\right) + C_F N_f\left(-\frac{41}{162} - \frac{97}{72}\zeta_2 + \frac{17}{72}\zeta_3\right) ,$$

$$\mathcal{A}_{LO}(v) = 8\, N_c\, \frac{1-2v+2v^2}{v(1-v)}$$

$v=-u/s=-u/M^2$

Results/examples

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$$H_q^{\gamma\gamma(2)}(v) = \frac{1}{4\mathcal{A}_{LO}(v)}\left[\mathcal{F}^{0\times 2}_{inite,q\bar{q}\gamma\gamma;s} + \mathcal{F}^{1\times 1}_{inite,q\bar{q}\gamma\gamma;s}\right] + 3\zeta_2\, C_F\, H_q^{\gamma\gamma(1)}(v) - \frac{45}{4}\zeta_4\, C_F^2 + C_F C_A\left(\frac{607}{324} + \frac{1181}{144}\zeta_2 - \frac{187}{144}\zeta_3 - \frac{105}{32}\zeta_4\right) + C_F N_f\left(-\frac{41}{162} - \frac{97}{72}\zeta_2 + \frac{17}{72}\zeta_3\right) ,$$

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$v=-u/s=-u/M^2$

Anastasiou, Glover , Tejeda-Yeomans (2002)

Results/examples

In the case of $b\bar{b} \to H$: **Harlander, Tripathi, Wiesemann (2014)**

$$H^H_{b,\mathrm{hard}}(\alpha_s) = \left|\widetilde{F}^h_b(\alpha_s)\right|^2$$

$$\widetilde{F}^h_b = 1 + \frac{\alpha_s}{\pi} C_F \left(\frac{\pi^2}{4} - \frac{1}{2}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[C_A C_F \left(\frac{37\zeta_3}{72} + \frac{83}{144} + \frac{125\pi^2}{432} - \frac{\pi^4}{480}\right)$$
$$+ C_F^2 \left(-\frac{15\zeta_3}{8} + \frac{3}{8} + \frac{\pi^2}{24} + \frac{23\pi^4}{1440}\right) + C_F N_f \left(\frac{\zeta_3}{9} + \frac{1}{36} - \frac{5\pi^2}{108}\right)$$
$$+ i\pi \left(C_A C_F \left(\frac{13\zeta_3}{8} - \frac{121}{216} - \frac{11\pi^2}{288}\right) + C_F^2 \left(\frac{\pi^2}{8} - \frac{3\zeta_3}{2}\right) + \left(\frac{7}{54} + \frac{\pi^2}{144}\right) C_F N_f \right) \Bigg]$$

And also, the same universal formula was used in the following cases:

ZZ, WW, Zγ production at NNLO

[See RATHLEV's talk]

Universality and threshold resummation, results up to N^{3}LO and N^{3}LL

Universality and threshold resummation

The total cross section for the production of the system F has the form

Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1 a_2}(\hat{s} = z_1 z_2 s; M^2; \alpha_S(M^2)) \; f_{a_1/h_1}(z_1, M^2) \; f_{a_2/h_2}(z_2, M^2) \;\; ,$$

$$\hat{\sigma}^F_{c\bar{c}}(\hat{s}; M^2; \alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \sum_{n=0}^{\infty} \left(\frac{\alpha_S(M^2)}{\pi}\right)^n \; z \; g^{F(n)}_{c\bar{c}}(z)$$

The Mellin transform of the partonic cross section is defined as:

$$\hat{\sigma}^F_{a_1 a_2,\,N}(M^2;\alpha_S(M^2)) \equiv \int_0^1 dz \; z^{N-1} \; \hat{\sigma}^F_{a_1 a_2}(\hat{s} = M^2/z; M^2; \alpha_S(M^2))$$

$$\hat{\sigma}^F_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \hat{\sigma}^{F(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) \left[1 + \mathcal{O}(1/N)\right]$$

Universality and threshold resummation

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Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1 a_2}(\hat{s}=z_1 z_2 s; M^2; \alpha_S(M^2)) \; f_{a_1/h_1}(z_1,M^2) \; f_{a_2/h_2}(z_2,M^2) \;\;,$$

The Mellin transform of the partonic cross section is defined as:

$$\hat{\sigma}^F_{a_1 a_2,\,N}(M^2;\alpha_S(M^2)) \equiv \int_0^1 dz \; z^{N-1} \; \hat{\sigma}^F_{a_1 a_2}(\hat{s}=M^2/z; M^2; \alpha_S(M^2))$$

$$\hat{\sigma}^F_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) \left[1+\mathcal{O}(1/N)\right]$$

and has an universal all-order structure

Sterman (1987); Catani, Trentadue (1989); Catani, de Florian, Grazzini, Nason (2003); Moch, Vermaseren, Vogt (2005)

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \; C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2)) \; \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2\int_{M^2}^{(1-z)^2 M^2} \frac{dq^2}{q^2} A_c^{\mathrm{th}}(\alpha_S(q^2)) + D_c(\alpha_S((1-z)^2 M^2)) \right] \right\}$$

Universality and threshold resummation

The total cross section for the production of the system F has the form

Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1a_2}(\hat{s}=z_1z_2s;M^2;\alpha_S(M^2)) \; f_{a_1/h_1}(z_1,M^2) \; f_{a_2/h_2}(z_2,M^2) \;\;,$$

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \; C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2)) \; \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2\int_{M^2}^{(1-z)^2M^2} \frac{dq^2}{q^2} A_c^{\mathrm{th}}(\alpha_S(q^2)) + D_c(\alpha_S((1-z)^2M^2)) \right] \right\}$$

$$\boxed{A_c^{\mathrm{th}}(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{\mathrm{th}\,(n)} \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D_c^{(2)} + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D_c^{(n)}$$

For n < 3: $A_c^{\mathrm{th}} = A_c$ Catani,Trentadue (1989); Catani,Webber (1989);

For n = 3: $A_c^{\mathrm{th}(3)} \neq A_c^{(3)}$ Moch, Vermaseren, Vogt (2004) and (2005)

For n = 4: $A_c^{\mathrm{th}(4)}$ Moch, Vermaseren, Vogt (2005)

Numerical approximations indicate that this coefficient can have a small quantitative effect in practical applications of threshold resummation.

Universality and threshold resummation

The total cross section for the production of the system F has the form

Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1 a_2}(\hat{s}=z_1 z_2 s; M^2; \alpha_S(M^2)) \; f_{a_1/h_1}(z_1,M^2) \; f_{a_2/h_2}(z_2,M^2) \;\; ,$$

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \; C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2)) \; \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2\int_{M^2}^{(1-z)^2 M^2} \frac{dq^2}{q^2} A^{\mathrm{th}}_c(\alpha_S(q^2)) + D_c(\alpha_S((1-z)^2 M^2)) \right] \right\}$$

$$\boxed{A^{\mathrm{th}}_c(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{\mathrm{th}\,(n)}_c \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D^{(2)}_c + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D^{(n)}_c$$

$D_c^{(1)} = 0$ Vogt (2001); Catani, de Florian, Grazzini (2001);

$D_c^{(2)}, D_c^{(3)}$ → Moch, Vogt (2005); Laenen, Magnea (2006)

Universality and threshold resummation

We can write in a factorized form:

$$\alpha_S^{2k}(M^2)\, C^{\rm th}_{c\bar{c}\to F}(\alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}^{\rm th}_{c\bar{c}\to F}|^2}{|\mathcal{M}^{(0)}_{c\bar{c}\to F}|^2}\,,$$

$$\widetilde{\mathcal{M}}^{\rm th}_{c\bar{c}\to F} = \left[1 - \tilde{I}^{\rm th}_c(\epsilon, M^2)\right] \mathcal{M}_{c\bar{c}\to F}$$

in the same way that we did in the case of the q_T resummation formalism

$$\tilde{I}^{\rm th}_c(\epsilon, M^2) = \frac{\alpha_S(\mu_R^2)}{2\pi}\, \tilde{I}^{\rm th(1)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right) + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^2 \tilde{I}^{\rm th(2)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right) + \sum_{n=3}^{\infty}\left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n \tilde{I}^{\rm th(n)}_c\left(\epsilon, \frac{M^2}{\mu_R^2}\right)$$

$$1 - \tilde{I}^{\rm th}_c(\epsilon, M^2) = \exp\left\{R^{\rm th}_c(\epsilon, \alpha_S(M^2)) - i\Phi_c(\epsilon, \alpha_S(M^2)) + \mathcal{O}(\epsilon)\right\}$$

$$R^{\rm th}_c(\epsilon, \alpha_S) = R^{\rm soft}_{c\ ,th}(\epsilon, \alpha_S) + R^{\rm coll}_c(\epsilon, \alpha_S)$$

$$R^{\rm soft}_{c\ th}(\epsilon, \alpha_S) = C_c\left(\frac{\alpha_S}{\pi} R^{\rm soft(1)}_{th}(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^2 R^{\rm soft(2)}_{th}(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^3 R^{\rm soft(3)}_{th}(\epsilon)\right) + \mathcal{O}(\alpha_S^4)$$

$$R^{\rm coll}_c(\epsilon, \alpha_S) = \frac{\alpha_S}{\pi} R^{\rm coll(1)}_c(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^2 R^{\rm coll(2)}_c(\epsilon) + \left(\frac{\alpha_S}{\pi}\right)^3 R^{\rm coll(3)}_c(\epsilon) + \mathcal{O}(\alpha_S^4)\,.$$

Universality and threshold resummation

To extend the results to the third order, we introduce:

$$1 - \tilde{I}_c^{\,\mathrm{th}}(\epsilon, M^2) = \exp\left\{ R_c^{\mathrm{th}}(\epsilon, \alpha_S(M^2)) - i\Phi_c(\epsilon, \alpha_S(M^2)) + \mathcal{O}(\epsilon) \right\}$$

$$R_{th}^{\mathrm{soft}(1)}(\epsilon) = \frac{1}{2\epsilon^2} + R_{th}^{\mathrm{fin}(1)}$$

$$R_c^{\mathrm{coll}(1)}(\epsilon) = \frac{\gamma_c}{2\epsilon}$$

$$R_{th}^{\mathrm{soft}(2)}(\epsilon) = -\frac{3}{8}\frac{\beta_0\pi}{\epsilon^3} + \frac{1}{8\epsilon^2}\gamma_{\mathrm{cusp}}^{(1)} - \frac{1}{16\epsilon}d_{(1)} + R_{th}^{\mathrm{fin}(2)}$$

$$R_c^{\mathrm{coll}(2)}(\epsilon) = -\frac{\beta_0\pi}{4\epsilon^2}\gamma_c + \frac{1}{8\epsilon}\gamma_c^{(1)} \,,$$

$$R_{th}^{\mathrm{soft}(3)}(\epsilon) = \frac{11\beta_0^2 - 8\beta_1\,\epsilon}{36\epsilon^4}\pi^2 - \frac{5}{36\epsilon^3}\beta_0\pi\gamma_{\mathrm{cusp}}^{(1)} + \frac{1}{18\epsilon^2}\gamma_{\mathrm{cusp}}^{(2)} + \frac{1}{24\epsilon^2}\beta_0\pi d_{(1)} - \frac{1}{48\epsilon}d_{(2)} + R_{th}^{\mathrm{fin}(3)}$$

$$R_c^{\mathrm{coll}(3)}(\epsilon) = \frac{\gamma_c}{6\epsilon^2}\left(\frac{(\beta_0\pi)^2}{\epsilon} - \beta_1\pi^2\right) - \beta_0\pi\frac{\gamma_c^{(1)}}{12\epsilon^2} + \frac{1}{24\epsilon}\gamma_c^{(2)}$$

$$R_{th}^{\mathrm{fin}(1)} = -\frac{\pi^2}{8} \,,$$

Catani, LC, de Florian, Ferrera, Grazzini (2013)

$$R_{th}^{\mathrm{fin}(2)} = C_A\left(\frac{607}{648} - \frac{469}{1728}\pi^2 + \frac{\pi^4}{288} - \frac{187}{144}\zeta_3\right) + n_F\left(-\frac{41}{324} + \frac{35}{864}\pi^2 + \frac{17}{72}\zeta_3\right)$$

Universality and threshold resummation

To extend the results to the third order, we introduce:

$$R_{th}^{\text{soft}(1)}(\epsilon) = \frac{1}{2\epsilon^2} + R_{th}^{\text{fin}(1)}$$

$$R_c^{\text{coll}(1)}(\epsilon) = \frac{\gamma_c}{2\epsilon}$$

$$R_{th}^{\text{soft}(2)}(\epsilon) = -\frac{3}{8}\frac{\beta_0\pi}{\epsilon^3} + \frac{1}{8\epsilon^2}\gamma_{\text{cusp}}^{(1)} - \frac{1}{16\epsilon}d_{(1)} + R_{th}^{\text{fin}(2)}$$

$$R_c^{\text{coll}(2)}(\epsilon) = -\frac{\beta_0\pi}{4\epsilon^2}\gamma_c + \frac{1}{8\epsilon}\gamma_c^{(1)} \ ,$$

$$R_{th}^{\text{fin}(2)} - R_{q_T}^{\text{fin}(2)} = C_A\left(-\frac{55}{72}\zeta_3 - \frac{67}{432}\pi^2\right) + n_F\left(\frac{5}{36}\zeta_3 + \frac{5}{216}\pi^2\right)$$

$$R_{th}^{\text{fin}(1)} = -\frac{\pi^2}{8} \ ,$$

Catani, LC, de Florian, Ferrera, Grazzini (2013)

$$R_{th}^{\text{fin}(2)} = C_A\left(\frac{607}{648} - \frac{469}{1728}\pi^2 + \frac{\pi^4}{288} - \frac{187}{144}\zeta_3\right) + n_F\left(-\frac{41}{324} + \frac{35}{864}\pi^2 + \frac{17}{72}\zeta_3\right)$$

Universality and threshold resummation

Consider general expression of the N^{3}LO term in σ_{cc}^{RES}

Then use:

+) soft virtual N^{3}LO result for Higgs boson production (gg → H)

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (2014)

+) the gluon form factor up to 3-loop level

P.A. Baikov, K.G. Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser (2009)
R.N. Lee, A.V. Smirnov, V.A. Smirnov (2010)
T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus (2010)

We obtain:

$$\begin{aligned}
R_{th}^{\mathrm{fin}(3)} = & \left(\frac{5211949}{1679616} - \frac{578479}{559872}\pi^2 + \frac{9457}{311040}\pi^4 + \frac{19}{326592}\pi^6 - \frac{64483}{7776}\zeta_3 + \frac{121}{192}\pi^2\zeta_3 + \frac{67}{72}\zeta_3^2 \right. \\
& \left. - \frac{121}{144}\zeta_5\right) C_A^2 + \left(-\frac{412765}{839808} + \frac{75155}{279936}\pi^2 - \frac{79}{9720}\pi^4 + \frac{154}{81}\zeta_3 - \frac{11}{288}\pi^2\zeta_3 - \frac{1}{24}\zeta_5\right) C_A\, n_F \\
& + \left(-\frac{42727}{62208} + \frac{605}{6912}\pi^2 + \frac{19}{12960}\pi^4 + \frac{571}{1296}\zeta_3 - \frac{11}{144}\pi^2\zeta_3 + \frac{7}{36}\zeta_5\right) C_F\, n_F \\
& + \left(-\frac{2}{6561} - \frac{101}{7776}\pi^2 + \frac{37}{77760}\pi^4 - \frac{185}{1944}\zeta_3\right) n_F^2 \ .
\end{aligned}$$

Catani, LC, de Florian, Ferrera, Grazzini (2014)

Universality and threshold resummation

As an application of our general formalism and results, we can consider the production of a vector boson V (V = Z, $W^{\pm}$) by the DY process $q\bar{q} \to V$. Using the subtraction operator I^{th}_{c}, and the results for the quark form factor up to three-loop order,

P.A. Baikov, K.G. Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser (2009)
R.N. Lee, A.V. Smirnov, V.A. Smirnov (2010)
T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus (2010)

we can compute the coefficients $C^{th}_{c\bar{c}\to V}$ up to order $O(\alpha_S^3)$.

$$C^{\mathrm{th}(1)}_{q\bar{q}\to V} = C_F\left(-4+\frac{\pi^2}{3}\right)$$

$$C^{\mathrm{th}(2)}_{q\bar{q}\to V} = C_F^2\left(\frac{511}{64}-\frac{35}{48}\pi^2+\frac{\pi^4}{40}-\frac{15}{4}\zeta_3\right) + C_F C_A\left(-\frac{1535}{192}+\frac{37}{54}\pi^2-\frac{\pi^4}{240}+\frac{7}{4}\zeta_3\right)+C_F n_F\left(\frac{127}{96}-\frac{7}{54}\pi^2+\frac{1}{2}\zeta_3\right)$$

Universality and threshold resummation

As an application of our general formalism and results, we can consider the production of a vector boson V (V = Z, $W^{\pm}$) by the DY process $q\bar{q} \rightarrow V$. Using the subtraction operator I^{th}_{c}, and the results for the quark form factor up to three-loop order, we can compute the coefficients $C^{th}_{c\bar{c}\rightarrow V}$ up to order $O(\alpha_s^3)$.

$$
\begin{aligned}
C^{\mathrm{th}(3)}_{q\bar{q}\to V} = C_F^3 &\left(-\frac{5599}{384} - \frac{65}{576}\pi^2 - \frac{17}{320}\pi^4 + \frac{803}{136080}\pi^6 - \frac{115}{16}\zeta_3 + \frac{5}{24}\pi^2\zeta_3 + \frac{1}{2}\zeta_3^2 + \frac{83}{4}\zeta_5\right) \\
&+ C_F^2 C_A \left(\frac{74321}{2304} - \frac{6593}{5184}\pi^2 + \frac{94}{1215}\pi^4 - \frac{2309}{272160}\pi^6 - \frac{8653}{432}\zeta_3 + \frac{53}{54}\pi^2\zeta_3 + \frac{37}{12}\zeta_3^2 - \frac{689}{72}\zeta_5\right) \\
&+ C_A^2 C_F \Bigg(- \frac{1505881}{62208} + \frac{281}{128}\pi^2 + \frac{14611}{311040}\pi^4 + \frac{829}{272160}\pi^6 + \frac{82385}{5184}\zeta_3 - \frac{221}{288}\pi^2\zeta_3 \\
&- \frac{25}{12}\zeta_3^2 - \frac{51}{16}\zeta_5\Bigg) + C_A C_F n_F \left(\frac{110651}{15552} - \frac{7033}{7776}\pi^2 - \frac{1439}{77760}\pi^4 - \frac{94}{81}\zeta_3 + \frac{13}{72}\pi^2\zeta_3 - \frac{\zeta_5}{8}\right) \\
&+ C_F^2 n_F \left(-\frac{421}{192} + \frac{329}{1296}\pi^2 - \frac{223}{19440}\pi^4 + \frac{869}{216}\zeta_3 - \frac{7}{27}\pi^2\zeta_3 - \frac{19}{18}\zeta_5\right) \\
&+ C_F n_F^2 \left(-\frac{7081}{15552} + \frac{151}{1944}\pi^2 + \frac{\pi^4}{486} - \frac{79}{324}\zeta_3\right) \\
&+ C_F N_{F,V} \left(\frac{N_c^2 - 4}{N_c}\right)\left(\frac{1}{8} + \frac{5}{96}\pi^2 - \frac{\pi^4}{2880} + \frac{7}{48}\zeta_3 - \frac{5}{6}\zeta_5\right) ,
\end{aligned}
$$

Catani, LC, de Florian, Ferrera, Grazzini (2014)

$N_{F,V}$ is a factor originating by diagrams where the virtual gauge boson does not couple directly to the initial state quarks, and is proportional to the charge weighted sum of the quark flavours

Universality and threshold resummation

$$
\begin{aligned}
C^{\mathrm{th}(3)}_{q\bar{q}\to V} = C_F^3 &\left(-\frac{5599}{384} - \frac{65}{576}\pi^2 - \frac{17}{320}\pi^4 + \frac{803}{136080}\pi^6 - \frac{115}{16}\zeta_3 + \frac{5}{24}\pi^2\zeta_3 + \frac{1}{2}\zeta_3^2 + \frac{83}{4}\zeta_5\right) \\
&+ C_F^2 C_A \left(\frac{74321}{2304} - \frac{6593}{5184}\pi^2 + \frac{94}{1215}\pi^4 - \frac{2309}{272160}\pi^6 - \frac{8653}{432}\zeta_3 + \frac{53}{54}\pi^2\zeta_3 + \frac{37}{12}\zeta_3^2 - \frac{689}{72}\zeta_5\right) \\
&+ C_A^2 C_F \Big(- \frac{1505881}{62208} + \frac{281}{128}\pi^2 + \frac{14611}{311040}\pi^4 + \frac{829}{272160}\pi^6 + \frac{82385}{5184}\zeta_3 - \frac{221}{288}\pi^2\zeta_3 \\
&- \frac{25}{12}\zeta_3^2 - \frac{51}{16}\zeta_5\Big) + C_A C_F n_F \left(\frac{110651}{15552} - \frac{7033}{7776}\pi^2 - \frac{1439}{77760}\pi^4 - \frac{94}{81}\zeta_3 + \frac{13}{72}\pi^2\zeta_3 - \frac{\zeta_5}{8}\right) \\
&+ C_F^2 n_F \left(-\frac{421}{192} + \frac{329}{1296}\pi^2 - \frac{223}{19440}\pi^4 + \frac{869}{216}\zeta_3 - \frac{7}{27}\pi^2\zeta_3 - \frac{19}{18}\zeta_5\right) \\
&+ C_F n_F^2 \left(-\frac{7081}{15552} + \frac{151}{1944}\pi^2 + \frac{\pi^4}{486} - \frac{79}{324}\zeta_3\right) \\
&+ C_F N_{F,V} \left(\frac{N_c^2-4}{N_c}\right)\left(\frac{1}{8} + \frac{5}{96}\pi^2 - \frac{\pi^4}{2880} + \frac{7}{48}\zeta_3 - \frac{5}{6}\zeta_5\right),
\end{aligned}
$$

Catani, LC, de Florian, Ferrera, Grazzini (2014)

With the coefficients $C^{\mathrm{th}(n)}_{cc\to V}$, $A^{(n)}_c$ and $D^{(n)}_c$ up to order $O(\alpha_S^3)$, we obtained the explicit expression of the soft virtual N^{3}LO cross section for the DY process. Which is in agreement with the result: Ahmed, Mahakhud, Rana, Ravindran (2014)

Summary

- We have shown that $\mathbf{H}_{c}^{F}$ ($\mathbf{C}^{th}_{cc\to F}$) is directly related in a universal way to the IR finite part of the all order virtual amplitude $\mathcal{M}_{cc\to F}$

- Therefore, the all-order scattering amplitude $\mathcal{M}_{cc\to F}$ is the *sole* process-dependent information that is eventually required by the all-order resummation formula

- The relation between $\mathbf{H}_{c}^{F}$ ($\mathbf{C}^{th}_{cc\to F}$) and $\mathcal{M}_{cc\to F}$ follows from an universal all-order factorization formula that originates from factorization properties of *soft (*and *collinear*) parton radiation

- The presented results complete the **qT** subtraction formalism in explicit form up to full NNLL and NNLO accuracy. The results constitute a necessary ingredient for resummation at N^3LL accuracy

- Similar reasoning and analysis apply to threshold resummation: universal structure of related hard factor $\mathbf{C}^{th}_{cc\to F}$ explicitly determined up to N^3LO and N^3LL accuracy

Backup slides

Transverse momentum dependent (TMD) factorization

Gehrmann, Lubbert, Yang (2012),(2014)

In our case

$$\mathcal{H}^F_{ab\leftarrow jk}(z,\alpha_s) = \int_0^1 dz_1 \int_0^1 dz_2\, \delta(z-z_1z_2)\,[H^F C_1 C_2]$$

In TMD

matching kernel

$$\mathcal{H}^{DY}_{q\bar{q}\leftarrow jk}(z,\alpha_s) = |C_V(-q^2,\sqrt{q^2})|^2 I_{q/j}(z,x_T^2,\mu_x) \otimes I_{\bar{q}/k}(z,x_T^2,\mu_x)$$

in full agreement with the results in Catani, LC, de Florian, Ferrera, Grazzini (2009);

gluon matching tensor

$$\mathcal{H}^H_{gg\leftarrow jk}(z,\alpha_s,\log\frac{m_t^2}{m_h^2}) = H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2,m_h^2,m_h)\, I^{\mu_1\nu_1}_{g/j}(z,x_\perp,\mu_x) \otimes I^{\mu_2\nu_2}_{g/k}(z,x_\perp,\mu_x)$$

$$H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2,m_h^2,m_h) = C_t^2(m_t^2,m_h)\left|C_S(-m_h^2,m_h)\right|^2 g_{\mu_1\mu_2}g_{\nu_1\nu_2}$$

in full agreement with the results in Catani, Grazzini (2012)

Transverse momentum dependent (TMD) factorization

Gehrmann, Lubbert, Yang (2012),(2014)

In our case

$$\mathcal{H}^F_{ab\leftarrow jk}(z,\alpha_s) = \int_0^1 dz_1 \int_0^1 dz_2\, \delta(z - z_1 z_2)\, [H^F C_1 C_2]$$

In TMD

$$\mathcal{H}^{DY}_{q\bar{q}\leftarrow jk}(z,\alpha_s) = \left|C_V(-q^2,\sqrt{q^2})\right|^2 I_{q/j}(z,x_T^2,\mu_x) \otimes I_{\bar{q}/k}(z,x_T^2,\mu_x)$$

in full agreement with the results in Catani, LC, de Florian, Ferrera, Grazzini (2009);

$$\mathcal{H}^H_{gg\leftarrow jk}\left(z,\alpha_s,\log\frac{m_t^2}{m_h^2}\right) = H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2,m_h^2,m_h)\, I^{\mu_1\nu_1}_{g/j}(z,x_\perp,\mu_x) \otimes I^{\mu_2\nu_2}_{g/k}(z,x_\perp,\mu_x)$$

$$H^H_{\mu_1\nu_1,\,\mu_2\nu_2}(m_t^2,m_h^2,m_h) = C_t^2(m_t^2,m_h)\left|C_S(-m_h^2,m_h)\right|^2 g_{\mu_1\mu_2} g_{\nu_1\nu_2}$$

in full agreement with the results in Catani, Grazzini (2012)

These results constitute a fully independent validation of them in a completely different calculational approach

The Normalization H

Expand to the fixed order in α_s

$$\mathcal{H}^F = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \dots \qquad \sim \delta(q_T^2)$$

LO NLO NNLO

Normalization of $\sigma_{tot}^{(N)NLO}$ → computational effort comparable to $\sigma_{tot}^{(N)NLO}$

$$p_T^2 \ll Q^2 \qquad \int_0^{p_T^2} dq_T^2 \frac{d\sigma^F}{dq_T^2} \equiv \sigma_{LO}^F R^F(p_T/Q)$$

The coefficients appear in the constant term

$$R^{F(1)} = l_0^2\,\Sigma^{F(1;2)} + l_0\,\Sigma^{F(1;1)} + \mathcal{H}^{F(1)} + \mathcal{O}(p_T^2/Q^2)$$

$$l_0 = \ln\frac{Q^2}{p_T^2}$$

$$R^{F(2)} = l_0^4\,\Sigma^{F(2;4)} + l_0^3\,\Sigma^{F(2;3)} + l_0^2\,\Sigma^{F(2;2)}$$
$$+ l_0\,(\Sigma^{F(2;1)} - 16\zeta_3\Sigma^{F(2;4)}) + \mathcal{H}^{F(2)} - 4\zeta_3\Sigma^{F(2;3)} + \mathcal{O}(p_T^2/Q^2)$$

Very hard to reach that accuracy... but...

$$\int_0^{p_T^2} dq_T^2 \, \frac{d\sigma^F}{dq_T^2} \equiv \sigma_{tot}^{(N)NLO} - \int_{p_T^2}^{\infty} dq_T^2 \, \frac{d\sigma^{F+jet(N)LO}}{dq_T^2}$$

Inclusive

(analytic) distribution

Integral can be carried out in 4-dimensions

known for Drell-Yan and Higgs!

Method used to obtain $\mathcal{H}^{F(2)}$ for Higgs and Drell-Yan

Universality and threshold resummation

We consider the N^3LO contribution of the threshold resummation formula (z space) , in the z → 1 limit.

The terms that are explicitly denoted here, define the soft-virtual (SV) approximation of the N^3LO $g^{F(3)}_{cc}$ contribution to the partonic cross section.

$$
\begin{aligned}
g_{c\bar{c}}^{F(3)}(z) = {} & 8\left(A_c^{(1)}\right)^3 \mathcal{D}_5 - \frac{40}{3}\beta_0\pi\left(A_c^{(1)}\right)^2 \mathcal{D}_4 \\
& + \left(-\frac{32}{3}\pi^2\left(A_c^{(1)}\right)^3 + 8C^{\mathrm{th}(1)}_{c\bar{c}\to F}\left(A_c^{(1)}\right)^2 + 16A_c^{(1)}A_c^{(2)} + \frac{16}{3}(\beta_0\pi)^2 A_c^{(1)}\right)\mathcal{D}_3 \\
& + \Bigg(160\zeta_3\left(A_c^{(1)}\right)^3 - 4\beta_0\pi\, A_c^{(1)}C^{\mathrm{th}(1)}_{c\bar{c}\to F} + 8\beta_0\pi^3\left(A_c^{(1)}\right)^2 - 8\beta_0\pi\, A_c^{(2)} \\
& + 6A_c^{(1)}D_c^{(2)} - 4A_c^{(1)}\beta_1\pi^2\Bigg)\mathcal{D}_2 \\
& + \Bigg(4\left(A_c^{(3)} + A_c^{(2)}C^{\mathrm{th}(1)}_{c\bar{c}\to F} + A_c^{(1)}C^{\mathrm{th}(2)}_{c\bar{c}\to F}\right) - \frac{16}{3}A_c^{(1)}A_c^{(2)}\pi^2 \\
& - \frac{8}{3}\left(A_c^{(1)}\right)^2 C^{\mathrm{th}(1)}_{c\bar{c}\to F}\pi^2 - \frac{4}{9}\pi^4\left(A_c^{(1)}\right)^3 - 4\beta_0\,\pi\left(D_c^{(2)} + 24\left(A_c^{(1)}\right)^2\zeta_3\right)\Bigg)\mathcal{D}_1 \\
& + \Bigg(\left(192\zeta_5 - \frac{64}{3}\pi^2\zeta_3\right)\left(A_c^{(1)}\right)^3 + 16A_c^{(1)}\zeta_3\left(2A_c^{(2)} + A_c^{(1)}C^{\mathrm{th}(1)}_{c\bar{c}\to F}\right) + \frac{4}{9}\left(A_c^{(1)}\right)^2\beta_0\pi^5 \\
& + C^{\mathrm{th}(1)}_{c\bar{c}\to F}D_c^{(2)} + D_c^{(3)} - \frac{2}{3}A_c^{(1)}D_c^{(2)}\pi^2\Bigg)\mathcal{D}_0 \\
& + \Bigg(C^{\mathrm{th}(3)}_{c\bar{c}\to F} - \frac{2}{45}A_c^{(1)}A_c^{(2)}\pi^4 - \frac{1}{45}\left(A_c^{(1)}\right)^2 C^{\mathrm{th}(1)}_{c\bar{c}\to F}\pi^4 + \left(\frac{160}{3}\zeta_3^2 - \frac{116}{2835}\pi^6\right)\left(A_c^{(1)}\right)^3 \\
& + 4A_c^{(1)}D_c^{(2)}\zeta_3 + \frac{16}{3}\left(A_c^{(1)}\right)^2\beta_0\pi\left(\pi^2\zeta_3 - 12\zeta_5\right)\Bigg)\delta(1-z) + \ldots \quad ,
\end{aligned}
$$

Universality and threshold resummation

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (2014)

In the case of the Higgs boson production ($gg \to H$), the SV N^{3}LO expression of $g^{F(3)}_{cc}$ exactly corresponds to the result of the explicit computation performed in a recent calculation.

$$
\begin{aligned}
\hat\eta^{(3)}(z) = \delta(1-z)\Big\{ & C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right)\\
&+N_F\Big[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right)\\
&+C_AC_F\left(\frac{5}{2}\zeta_5+3\zeta_3\zeta_2+\frac{11}{72}\zeta_4+\frac{13}{2}\zeta_3-\frac{71}{36}\zeta_2-\frac{63991}{5184}\right)+C_F^2\left(-5\zeta_5+\frac{37}{12}\zeta_3+\frac{19}{18}\right)\Big]\\
&+N_F^2\Big[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\Big]\Big\}\\
+\left[\frac{1}{1-z}\right]_+\Big\{ & C_A^3\left(186\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right)\\
&+N_F\Big[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_AC_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\Big]\Big\}\\
+\left[\frac{\log(1-z)}{1-z}\right]_+\Big\{ & C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right)\\
&+N_F\Big[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_AC_F\left(6\zeta_3-\frac{63}{8}\right)\Big]\Big\}\\
+\left[\frac{\log^2(1-z)}{1-z}\right]_+\Big\{ & C_A^3\left(181\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\Big[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_AC_F\Big]-\frac{10}{27}N_F^2C_A\Big\}\\
+\left[\frac{\log^3(1-z)}{1-z}\right]_+\Big\{ & C_A^3\left(-56\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_FC_A^2+\frac{4}{27}N_F^2C_A\Big\}\\
+\left[\frac{\log^4(1-z)}{1-z}\right]_+ & \left(\frac{20}{9}N_FC_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+ 8C_A^3\,.
\end{aligned}
$$

Universality and threshold resummation

In the case of the Higgs boson production ($gg \to H$), the SV N^{3}LO expression of $g^{F(3)}_{cc}$ exactly corresponds to the result of the explicit computation performed in a recent calculation. Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (2014)

$$
\begin{aligned}
C^{\mathrm{th}(3)}_{gg\to H} = C_A^3 &\left(\frac{215131}{5184} + \frac{16151}{7776}\pi^2 - \frac{1765}{15552}\pi^4 + \frac{1}{2160}\pi^6 - \frac{15649}{432}\zeta_3 - \frac{77}{144}\pi^2\zeta_3 + \frac{3}{2}\zeta_3^2 + \frac{869}{144}\zeta_5\right) \\
&+ C_A^2 n_F \left(-\frac{98059}{5184} - \frac{35}{243}\pi^2 + \frac{2149}{38880}\pi^4 + \frac{29}{8}\zeta_3 - \frac{29}{72}\pi^2\zeta_3 + \frac{101}{72}\zeta_5\right) \\
&+ C_A C_F n_F \left(-\frac{63991}{5184} - \frac{71}{216}\pi^2 + \frac{11}{6480}\pi^4 + \frac{13}{2}\zeta_3 + \frac{1}{2}\pi^2\zeta_3 + \frac{5}{2}\zeta_5\right) \\
&+ C_F^2 n_F \left(\frac{19}{18} + \frac{37}{12}\zeta_3 - 5\zeta_5\right) + C_A n_F^2 \left(\frac{2515}{1728} - \frac{133}{1944}\pi^2 - \frac{19}{3240}\pi^4 + \frac{43}{108}\zeta_3\right) \\
&+ C_F n_F^2 \left(\frac{4481}{2592} - \frac{23}{432}\pi^2 - \frac{1}{3240}\pi^4 - \frac{7}{6}\zeta_3\right) . \qquad (4
\end{aligned}
$$

Hard virtual coefficients

In the hard scheme, this coefficient contains all the information on the process-dependent virtual corrections

At the second order in α_s :

$$\tilde{I}_a^{(2)}(\epsilon, M^2/\mu_R^2) = -\frac{1}{2}\left[\tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right]^2 + \left\{\frac{2\pi\beta_0}{\epsilon}\left[\tilde{I}_a^{(1)}(2\epsilon, M^2/\mu_R^2) - \tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right] + \boxed{K}\,\tilde{I}_a^{(1)\,\mathrm{soft}}(2\epsilon, M^2/\mu_R^2) + \boxed{\widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2)}\right\}$$

$$\widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2) = \widetilde{H}_a^{(2)\,\mathrm{coll}}(\epsilon, M^2/\mu_R^2) + \widetilde{H}_a^{(2)\,\mathrm{soft}}(\epsilon, M^2/\mu_R^2)$$
$$= \frac{1}{4\epsilon}\left(\frac{M^2}{\mu_R^2}\right)^{-2\epsilon}\left(\frac{1}{4}\gamma_{a\,(1)} + C_a\, d_{(1)} + \epsilon\, C_a\, \delta_{(1)}^{q_T}\right)$$

$$d_{(1)} = \left(\frac{28}{27} - \frac{1}{3}\zeta_2\right)N_f + \left(-\frac{202}{27} + \frac{11}{6}\zeta_2 + 7\zeta_3\right)C_A$$

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{5}{9}N_f$$

$$\delta_{(1)}^{q_T} = \frac{20}{3}\zeta_3\pi\beta_0 + \left(-\frac{1214}{81} + \frac{67}{18}\zeta_2\right)C_A + \left(\frac{164}{81} - \frac{5}{9}\zeta_2\right)N_f$$

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$$\tilde{I}_a^{(2)}(\epsilon, M^2/\mu_R^2) = -\frac{1}{2}\left[\tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right]^2 + \left\{\frac{2\pi\beta_0}{\epsilon}\left[\tilde{I}_a^{(1)}(2\epsilon, M^2/\mu_R^2) - \tilde{I}_a^{(1)}(\epsilon, M^2/\mu_R^2)\right] + K\,\tilde{I}_a^{(1)\,\mathrm{soft}}(2\epsilon, M^2/\mu_R^2) + \widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2)\right\}$$

$$\widetilde{H}_a^{(2)}(\epsilon, M^2/\mu_R^2) = \widetilde{H}_a^{(2)\,\mathrm{coll}}(\epsilon, M^2/\mu_R^2) + \widetilde{H}_a^{(2)\,\mathrm{soft}}(\epsilon, M^2/\mu_R^2) = \frac{1}{4\epsilon}\left(\frac{M^2}{\mu_R^2}\right)^{-2\epsilon}\left(\frac{1}{4}\gamma_{a\,(1)} + C_a\,d_{(1)} + \epsilon\,C_a\,\delta_{(1)}^{q_T}\right)$$

$$d_{(1)} = \left(\frac{28}{27} - \frac{1}{3}\zeta_2\right)N_f + \left(-\frac{202}{27} + \frac{11}{6}\zeta_2 + 7\zeta_3\right)C_A$$

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$$\gamma_{q\,(1)} = \gamma_{\bar{q}\,(1)} = (-3 + 24\zeta_2 - 48\zeta_3)\,C_F^2 + \left(-\frac{17}{3} - \frac{88}{3}\zeta_2 + 24\zeta_3\right)C_F C_A + \left(\frac{2}{3} + \frac{16}{3}\zeta_2\right)C_F N_f$$

$$\gamma_{g\,(1)} = \left(-\frac{64}{3} - 24\zeta_3\right)C_A^2 + \frac{16}{3}C_A N_f + 4\,C_F N_f$$

Universality and threshold resummation

The total cross section for the production of the system F has the form:

Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2\;\; \hat{\sigma}^F_{a_1a_2}(\hat{s}=z_1z_2s;M^2;\alpha_S(M^2))\; f_{a_1/h_1}(z_1,M^2)\; f_{a_2/h_2}(z_2,M^2)\;\; ,$$

Sterman (1987); Catani, Trentadue (1989); Catani, de Florian, Grazzini, Nason (2003); Moch, Vermaseren, Vogt (2005)

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2))\, \boxed{C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2))}\, \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \left[2\int_{M^2}^{(1-z)^2M^2} \frac{dq^2}{q^2} \boxed{A^{\mathrm{th}}_c(\alpha_S(q^2))} + \boxed{D_c(\alpha_S((1-z)^2M^2))} \right] \right\}$$

$$\boxed{A^{\mathrm{th}}_c(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{\mathrm{th}\,(n)}_c \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D^{(2)}_c + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D^{(n)}_c$$

For n < 3: $A_c^{\mathrm{th}} = A_c$

For n = 3: $A_c^{\mathrm{th}(3)} \neq A_c^{(3)}$

Catani,Trentadue (1989); Catani,Webber (1989); Moch, Vermaseren, Vogt (2004) and (2005)

Universality and threshold resummation

The total cross section for the production of the system F has the form:

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Sterman (1987); Catani, Trentadue (1989); Catani, de Florian, Grazzini, Nason (2003); Moch, Vermaseren, Vogt (2005)

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \; \boxed{C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2))} \; \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2 \int_{M^2}^{(1-z)^2 M^2} \frac{dq^2}{q^2} \boxed{A^{\mathrm{th}}_c(\alpha_S(q^2))} + \boxed{D_c(\alpha_S((1-z)^2 M^2))} \right] \right\}$$

$$\boxed{A^{\mathrm{th}}_c(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{\mathrm{th}\,(n)}_c \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D^{(2)}_c + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D^{(n)}_c$$

For n = 4: $A_c^{th(4)}$ Moch, Vermaseren, Vogt (2005)

Numerical approximations indicate that this coefficient can have a small quantitative effect in practical applications of threshold resummation.

Universality and threshold resummation

The total cross section for the production of the system F has the form:

Sterman (1987); Catani, Trentadue (1989)

$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1a_2}(\hat{s}=z_1z_2s;M^2;\alpha_S(M^2)) \; f_{a_1/h_1}(z_1,M^2) \; f_{a_2/h_2}(z_2,M^2) \;\; ,$$

Sterman (1987); Catani, Trentadue (1989); Catani, de Florian, Grazzini, Nason (2003); Moch, Vermaseren, Vogt (2005)

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \boxed{C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2))} \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2\int_{M^2}^{(1-z)^2M^2} \frac{dq^2}{q^2} \boxed{A^{\mathrm{th}}_c(\alpha_S(q^2))} + \boxed{D_c(\alpha_S((1-z)^2M^2))} \right] \right\}$$

$$\boxed{A^{\mathrm{th}}_c(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{\mathrm{th}\,(n)}_c \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D^{(2)}_c + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D^{(n)}_c$$

$D_c^{(1)} = 0$

$D_c^{(2)}, D_c^{(3)}$ → Vogt (2001); Catani, de Florian, Grazzini (2001); Moch, Vogt (2005); Laenen, Magnea (2006)

Universality and threshold resummation

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$$\sigma_F(p_1,p_2;M^2) = \sum_{a_1,a_2} \int_0^1 dz_1 \int_0^1 dz_2 \;\; \hat{\sigma}^F_{a_1a_2}(\hat{s} = z_1 z_2 s; M^2; \alpha_S(M^2)) \; f_{a_1/h_1}(z_1, M^2) \; f_{a_2/h_2}(z_2, M^2) \;\; ,$$

Sterman (1987); Catani, Trentadue (1989); Catani, de Florian, Grazzini, Nason (2003); Moch, Vermaseren, Vogt (2005)

$$\hat{\sigma}^{F\,(\mathrm{res})}_{c\bar{c},\,N}(M^2;\alpha_S(M^2)) = \sigma^{(0)}_{c\bar{c}\to F}(M^2;\alpha_S(M^2)) \boxed{C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S(M^2))} \Delta_{c,\,N}(M^2)$$

$$\Delta_{c,\,N}(M^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left[2 \int_{M^2}^{(1-z)^2 M^2} \frac{dq^2}{q^2} \boxed{A^{\mathrm{th}}_c(\alpha_S(q^2))} + \boxed{D_c(\alpha_S((1-z)^2 M^2))} \right] \right\}$$

$$\boxed{A^{\mathrm{th}}_c(\alpha_S)} = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A^{\mathrm{th}\,(n)}_c \quad , \qquad \boxed{D_c(\alpha_S)} = \left(\frac{\alpha_S}{\pi}\right)^2 D^{(2)}_c + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n D^{(n)}_c$$

$$\boxed{C^{\,\mathrm{th}}_{c\bar{c}\to F}(\alpha_S)} = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C^{\,\mathrm{th}\,(n)}_{c\bar{c}\to F}$$

Universality and threshold resummation

$$A_c^{(1)} = C_c \quad ,$$

$$A_c^{(2)} = \frac{1}{2} K\, C_c \quad , \qquad K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_F \ ,$$

$$A_c^{(3)} = C_c \Bigg(\left(\frac{245}{96} - \frac{67}{216}\pi^2 + \frac{11}{720}\pi^4 + \frac{11}{24}\zeta_3 \right) C_A^2 + \left(-\frac{209}{432} + \frac{5}{108}\pi^2 - \frac{7}{12}\zeta_3 \right) C_A\, n_F$$

$$+ \left(-\frac{55}{96} + \frac{1}{2}\zeta_3 \right) C_F\, n_F - \frac{1}{108} n_F^2 \Bigg) \ ,$$

Universality and threshold resummation

$$A_c^{(1)} = C_c \ ,$$

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$$A_c^{(3)} = C_c \Bigg(\left(\frac{245}{96} - \frac{67}{216}\pi^2 + \frac{11}{720}\pi^4 + \frac{11}{24}\zeta_3 \right) C_A^2 + \left(-\frac{209}{432} + \frac{5}{108}\pi^2 - \frac{7}{12}\zeta_3 \right) C_A\, n_F$$
$$+ \left(-\frac{55}{96} + \frac{1}{2}\zeta_3 \right) C_F\, n_F - \frac{1}{108} n_F^2 \Bigg) \ ,$$

$$D_c^{(2)} = C_c \left(C_A \left(-\frac{101}{27} + \frac{11}{18}\pi^2 + \frac{7}{2}\zeta_3 \right) + n_F \left(\frac{14}{27} - \frac{1}{9}\pi^2 \right) \right) \ ,$$

$$D_c^{(3)} = C_c \Bigg(C_A^2 \left(-\frac{297029}{23328} + \frac{6139}{1944}\pi^2 - \frac{187}{2160}\pi^4 + \frac{2509}{108}\zeta_3 - \frac{11}{36}\pi^2\zeta_3 - 6\zeta_5 \right)$$
$$+ C_A\, n_F \left(\frac{31313}{11664} - \frac{1837}{1944}\pi^2 + \frac{23}{1080}\pi^4 - \frac{155}{36}\zeta_3 \right)$$
$$+ C_F\, n_F \left(\frac{1711}{864} - \frac{1}{12}\pi^2 - \frac{1}{180}\pi^4 - \frac{19}{18}\zeta_3 \right) + n_F^2 \left(-\frac{58}{729} + \frac{5}{81}\pi^2 + \frac{5}{27}\zeta_3 \right) \Bigg)$$

Universality and threshold resummation

$$A_c(\alpha_S) = C_c \left(\frac{\alpha_S}{\pi}\right)\left(1 + \left(\frac{\alpha_S}{\pi}\right)\gamma^{(1)}_{\rm cusp} + \left(\frac{\alpha_S}{\pi}\right)^2 \gamma^{(2)}_{\rm cusp}\right) + \left(\frac{\alpha_S}{\pi}\right)^4 A_c^{(4)} + \mathcal{O}(\alpha_S^5)$$

$$\gamma_q = \frac{3}{2} C_F \,,$$

$$\begin{aligned}
\gamma_q^{(1)} &= \left(\frac{3}{8} - \frac{1}{2}\pi^2 + 6\zeta_3\right) C_F^2 + \left(\frac{17}{24} + \frac{11}{18}\pi^2 - 3\zeta_3\right) C_F C_A + \left(-\frac{1}{12} - \frac{1}{9}\pi^2\right) C_F n_F \,, \\
\gamma_q^{(2)} &= C_F^3 \left(\frac{29}{16} + \frac{3}{8}\pi^2 + \frac{\pi^4}{5} + \frac{17}{2}\zeta_3 - \frac{2}{3}\pi^2\zeta_3 - 30\zeta_5\right) \\
&+ C_F^2 C_A \left(\frac{151}{32} - \frac{205}{72}\pi^2 - \frac{247}{1080}\pi^4 + \frac{211}{6}\zeta_3 + \frac{1}{3}\pi^2\zeta_3 + 15\zeta_5\right) \\
&+ C_A^2 C_F \left(-\frac{1657}{288} + \frac{281}{81}\pi^2 - \frac{\pi^4}{144} - \frac{194}{9}\zeta_3 + 5\zeta_5\right) \\
&+ C_F^2 n_F \left(-\frac{23}{8} + \frac{5}{36}\pi^2 + \frac{29}{540}\pi^4 - \frac{17}{3}\zeta_3\right) + C_F n_F^2 \left(-\frac{17}{72} + \frac{5}{81}\pi^2 - \frac{2}{9}\zeta_3\right) \\
&+ C_F C_A n_F \left(\frac{5}{2} - \frac{167}{162}\pi^2 + \frac{\pi^4}{360} + \frac{25}{9}\zeta_3\right) \,, \qquad (4
\end{aligned}$$

$$\begin{aligned}
\gamma_g &= \frac{11}{6} C_A - \frac{1}{3} n_F \,, \\
\gamma_g^{(1)} &= \left(\frac{8}{3} + 3\zeta_3\right) C_A^2 - \frac{2}{3} C_A n_F - \frac{1}{2} C_F n_F \,, \\
\gamma_g^{(2)} &= C_A^3 \left(\frac{79}{16} + \frac{\pi^2}{18} + \frac{11}{432}\pi^4 + \frac{67}{3}\zeta_3 - \frac{1}{3}\pi^2\zeta_3 - 10\zeta_5\right) + C_A^2 n_F \left(-\frac{233}{144} - \frac{\pi^2}{18} - \frac{\pi^4}{216} - \frac{10}{3}\zeta_3\right) \\
&+ \frac{1}{8} C_F^2 n_F - \frac{241}{144} C_A C_F n_F + \frac{29}{144} C_A n_F^2 + \frac{11}{72} C_F n_F^2 \,. \qquad (4
\end{aligned}$$

Universality and threshold resummation

$$d_{(1)} = \left(\frac{28}{27} - \frac{1}{18}\pi^2\right) n_F + \left(-\frac{202}{27} + \frac{11}{36}\pi^2 + 7\zeta_3\right) C_A \,,$$

$$\begin{aligned}
d_{(2)} = {} & C_A^2 \left(-\frac{136781}{5832} + \frac{6325}{1944}\pi^2 - \frac{11}{45}\pi^4 + \frac{329}{6}\zeta_3 - \frac{11}{9}\pi^2\zeta_3 - 24\zeta_5\right) \\
& + C_A\, n_F \left(\frac{5921}{2916} - \frac{707}{972}\pi^2 + \frac{\pi^4}{15} - \frac{91}{27}\zeta_3\right) + C_F\, n_F \left(\frac{1711}{216} - \frac{\pi^2}{12} - \frac{\pi^4}{45} - \frac{38}{9}\zeta_3\right) \\
& + n_F^2 \left(\frac{260}{729} + \frac{5}{162}\pi^2 - \frac{14}{27}\zeta_3\right) \,.
\end{aligned}$$