Soft-gluon resummation for gluon-induced Higgs-Strahlung

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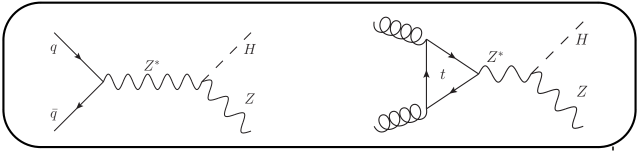
Importance of $gg \to HZ$

- A Higgs boson found with a mass of $125\ \mathrm{GeV}$
- Precision study needed to determine if it is SM Higgs
- One process is Higgs-strahlung (H+Z final state)
- At LO $pp \to HZ$ is described by $q\bar{q} \to HZ$
- Drell-Yan corrections up to NNLO *[Hamberg, Neerven, Matsuura, '91] [Harlander, Kilgore, '02] [Brein, Djouadi, Harlander, '04]*
- $gg \to HZ$ at NLO *[Altenkamp, Dittmaier, Harlander, Rzehak, Zirke, '12]*
  - Large corrections (factor of 2)
  - Still has significant scale dependence



Results NLO $gg \to HZ$

[Altenkamp, Dittmaier, Harlander, Rzehak, Zirke, '12]

$\sqrt{s}$ [TeV]	m_H [GeV]	σ_{gg}^{LO} [fb]	σ_{gg}^{NLO} [fb]
8	115	$19.8^{+61\%}_{-34\%}$	$39.3^{+32\%}_{-24\%}$
8	120	$18.7^{+61\%}_{-34\%}$	$37.2^{+32\%}_{-24\%}$
8	125	$17.7^{+61\%}_{-34\%}$	$35.1^{+32\%}_{-24\%}$
8	130	$16.7^{+61\%}_{-34\%}$	$33.1^{+32\%}_{-24\%}$
14	115	$79.1^{+51\%}_{-31\%}$	$152^{+27\%}_{-21\%}$
14	120	$75.1^{+51\%}_{-31\%}$	$144^{+27\%}_{-21\%}$
14	125	$71.1^{+51\%}_{-31\%}$	$136^{+27\%}_{-21\%}$
14	130	$67.2^{+51\%}_{-31\%}$	$129^{+27\%}_{-21\%}$

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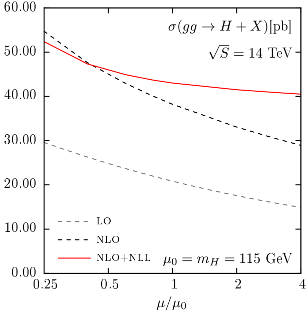
Importance of Resummation

- Resummation up to NNLL already improved Higgs production results *[Catani, de Florian, Grazzini, Nason, '03] [de Florian, Grazzini, '09] [de Florian, Grazzini, '12]*
- $gg \to HZ$ similar loop induced process $\Rightarrow$ threshold resummation could help further improve results



Agrees with *[Catani, de Florian, Grazzini, Nason, '03]*

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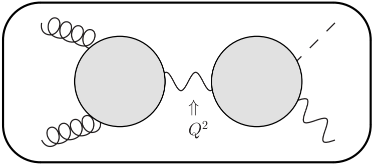
Definition of Threshold

Q-approach



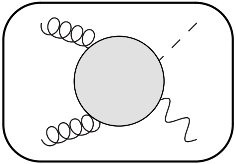
Threshold variable $\hat{\tau}_Q = \frac{Q^2}{\hat{s}}$

Q^2: the invariant mass final state particles

$$1 - \hat{\tau}_Q = 1 - \frac{Q^2}{\hat{s}}$$

$$\sim \frac{\text{energy of the emitted gluons}}{\text{total available energy}}$$

M-approach (absolute threshold)



Threshold variable $\hat{\tau}_M = \frac{M^2}{\hat{s}}$

$M = m_H + m_Z$

$$1 - \hat{\tau}_M = 1 - \frac{M^2}{\hat{s}}$$

$$\sim \frac{\text{maximum energy of the emitted gluons}}{\text{total available energy}}$$

$\sqrt{\hat{s}}$: the partonic center of mass energy

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Logarithms

Q-approach

The IR divergences lead to logarithms:

$$\alpha_s^n \left(\frac{\log^m(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q} \right)_+ \equiv \alpha_s^n D_{Q,m}(\hat{\tau}_Q), \; m \leq 2n-1$$

In general logarithms of $1-\hat{\tau}_Q$

M-approach

For $2 \to 2$ process: logarithms of $1-\hat{\tau}_M$:

$$\alpha_s^n \log^m(1-\hat{\tau}_M) \equiv \alpha_s^n D_{M,m-1}(\hat{\tau}_M), \; m \leq 2n$$

Logarithms become large in threshold: $\hat{\tau} \to 1$

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Mellin Transform

Mellin transform is used with respect to τ (needed for factorization of phase space):

$$\tilde{\Sigma}_{pp\to HZ}(N) \equiv \int_0^1 d\tau\, \tau^{N-1}\Sigma_{pp\to HZ}(\tau, m_Z, m_H, \mu_R, \mu_F)$$
$$= \sum_{i,j} \tilde{f}_{i/p}(N+1,\mu_F)\tilde{f}_{j/p}(N+1,\mu_F)\tilde{\hat{\Sigma}}_{ij\to HZ}(N,\mu_R,\mu_F)$$

- $\tilde{f}_{i/p}(N+1,\mu_F)$: Mellin transform with respect to x
- $\tilde{\hat{\Sigma}}_{ij\to HZ}(N,\mu_R,\mu_F)$: Mellin transform with respect to $\hat{\tau}$
  - $\Sigma_{ij\to HZ} = \frac{\mathrm{d}\sigma_{ij\to HZ}}{\mathrm{d}Q^2}$ in Q-approach
  - $\Sigma_{ij\to HZ} = \sigma_{ij\to HZ}$ in M-approach

$D_n\left(\hat{\tau}\right) \Rightarrow \log^{n+1} N$ and threshold $\hat{\tau} \to 1 \sim N \to \infty$

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Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in α_s and L:

[Kodaira, Trentadue, '82][Sterman, '87][Catani, d'Emilio, Trentadue, '88][Catani, Trentadue, '89]

$$\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times \mathcal{C}(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right]$$

With orders of precision: LL ⇓ $\alpha_s^n \log^{n+1}(N)$; NLL ⇓ $\alpha_s^n \log^n(N)$; NNLL ⇓ $\alpha_s^{n+1} \log^n(N)$

Exponential functions are well known and the same as for $gg \to H$

[Catani, de Florian, Grazzini, Nason, '03]

Hard Matching Coefficient (Schematically)

$$\mathcal{C}(\alpha_s) = 1 + \frac{\alpha_s}{\pi}\mathcal{C}^{(1)} + \cdots$$

Originates from NLO calculation. Using terms proportional to:

$$\Rightarrow \sigma_{LO}, \quad \sigma_{LO} D_{M,0}, \quad \sigma_{LO} D_{M,1}$$

OR

$$\Rightarrow \sigma_{LO}\delta(Q^2 - \hat{s}), \quad \sigma_{LO} D_{Q,0}, \quad \sigma_{LO} D_{Q,1}$$

Mellin transform leads to:

$$\frac{\alpha_s}{\pi}[\mathcal{C}^{(1)}\tilde{\Sigma}_{LO} + \mathcal{O}(\tilde{\Sigma}_{LO}\log(N), \tilde{\Sigma}_{LO}\log^2(N)) + \cdots]$$

$$\Downarrow$$

Expansion of exponential

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Hard Matching Coefficient

$$\hat{\Sigma}^{\text{NLO}} = \hat{\Sigma}^{\text{R}} + \hat{\Sigma}^{\text{V}} + \hat{\Sigma}^{\text{C}}$$

$$= \int_3 \left[\mathrm{d}\hat{\Sigma}^{\text{R}}|_{\epsilon=0} - \mathrm{d}\hat{\Sigma}^{\text{A}}|_{\epsilon=0} \right] + \int_2 \left[\mathrm{d}\hat{\Sigma}^{\text{V}} + \int_1 \mathrm{d}\hat{\Sigma}^{\text{A}} \right]_{\epsilon=0} + \hat{\Sigma}^{\text{C}}$$

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Supressed in threshold limit

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$\Downarrow$

Supressed in threshold limit

$\Rightarrow \mathcal{C}^{(1)}$ calculated by: $\hat{\Sigma}^{\text{V}} + \hat{\Sigma}^{\text{A}} + \hat{\Sigma}^{\text{C}}$

In agreement with: *[Catani, Cieri, de Florian, Ferrera, Grazzini, '13]*

Hard Matching Coefficient (Result)

$$\mathcal{C}^{(1)} = \frac{\hat{\sigma}_{\mathsf{virt}}}{\hat{\sigma}_{\mathsf{LO}}} \frac{\pi}{\alpha_s} + \left[\frac{2}{3} T_R\, n_l - \left(\frac{11}{6} - 2\gamma_E\right) C_A\right] \log\left(\frac{\mu^2}{W^2}\right) - \left(\frac{50}{9} - \frac{2\pi^2}{3} - 2\gamma_E^2\right) C_A + \frac{16}{9} T_R\, n_l$$

- Q-approach: Absolute threshold expansion $\hat{\sigma}_{virt}$ and $\hat{\sigma}_{\mathsf{LO}}$, $W^2 = Q^2$
- W-approach: $W^2 = M^2$

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Matching to Fixed Order

Resummed Cross Section

$$\Sigma^{(\mathrm{NLO+NLL})}_{gg\to HZ}(\tau) = \Sigma^{(\mathrm{NLO})}_{gg\to HZ}(\tau) + \int_{\mathrm{CT}} \frac{dN}{2\pi i} \tau^{-N} \tilde{f}_{g/p}(N+1) \tilde{f}_{g/p}(N+1) \times \left[\tilde{\hat{\Sigma}}^{(\mathrm{NLL})}_{gg\to HZ}(N) \; - \; \tilde{\hat{\Sigma}}^{(\mathrm{NLL})}_{gg\to HZ}(N)\big|_{(\mathrm{NLO})} \right]$$

Matching to fixed order required to avoid double counting.

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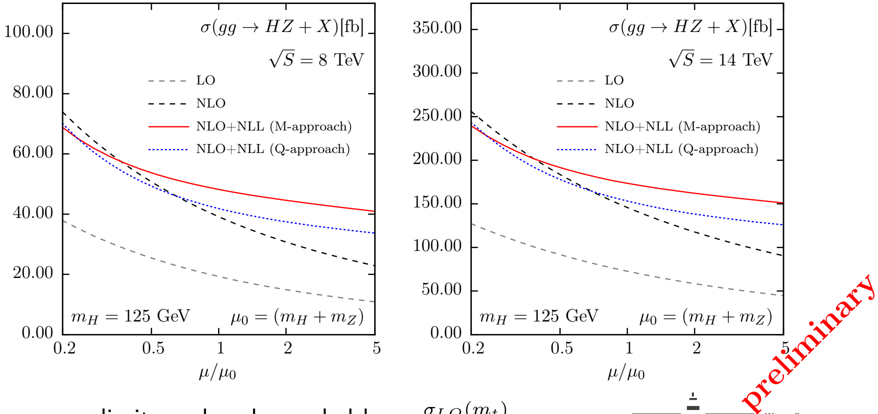
Results

M-approach(NLL resummation)

[Harlander, Kulesza, VT, Zirke, in preparation]

PDFs used: MSTW2008NNLO



$m_t \to \infty$ limit used and rescaled by $\frac{\sigma_{LO}(m_t)}{\sigma_{LO}^{\text{thr.}}(m_t \to \infty)}$

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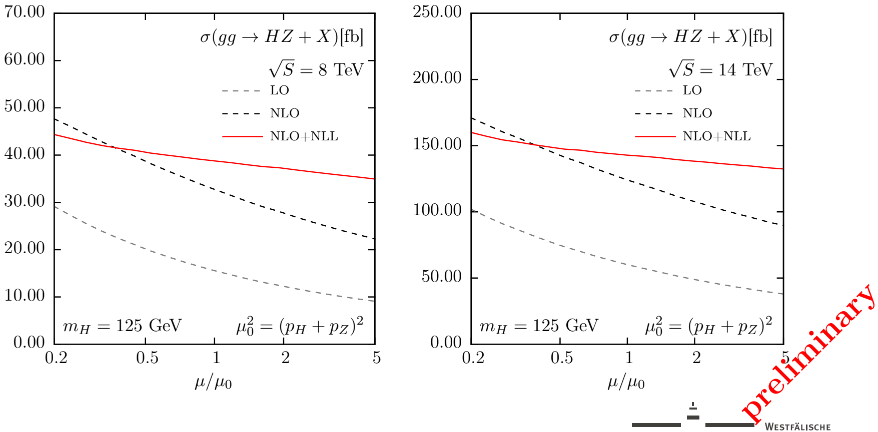

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Summary

Conclusions

- Improvement in scale dependence:

  $\sigma^{\text{NLO}} = 32.7^{+31\%}_{-24\%}$ fb and $\sigma_Q^{\text{NLO+NLL}} = 38.8^{+8.3\%}_{-6.9\%}$ fb for 8 TeV

  $\sigma^{\text{NLO}} = 124^{+26\%}_{-21\%}$ fb and $\sigma_Q^{\text{NLO+NLL}} = 143^{+6.9\%}_{-5.1\%}$ fb for 14 TeV

  Error determined at $Q^2/3$ and $3Q^2$
- Sizable correction: $\frac{\sigma^{\text{NLO+NLL}}}{\sigma^{\text{NLO}}} = 1.18\ (1.15)$ for 8 (14) TeV

Outlook

- NNLL resummation
- Combine into $pp \to HZ$ results

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Thank you for your attention

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