Testing General Relativity with Atom Interferometry

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with

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Testing Large Distance GR

Cosmological Constant Problem suggests

Our understanding of GR is incomplete

(unless there are ~10^{500} universes!)

CCP+DM inspired proposals for IR modifications:

Damour-Polyakov
DGP
ADDG (non-locality)
Ghost condensation
...

MOND
Beckenstein
...
Brans-Dicke
Bimetric
...

Precision long distance tests

GR: Principle of Equivalence tested to 3×10^{-13}

most other tests ~ 10^{-3} to 10^{-5}

time delay (Cassini tracking)	10^{-5}
light deflection (VLBI)	10^{-3}
perihelion shift	10^{-3}
Nordtvedt effect	10^{-3}
Lense-Thirring (GPB)	

QED: 10 digit accuracy

g-2, EDMs, etc

Precision GR tests mostly use:

Planets and photons over astronomical distances

Can we study GR using atoms over short distances (meters)?

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Can we study GR using atoms over short distances (meters)?

Yes, thanks to the tremendous advances in
Atom Interferometry

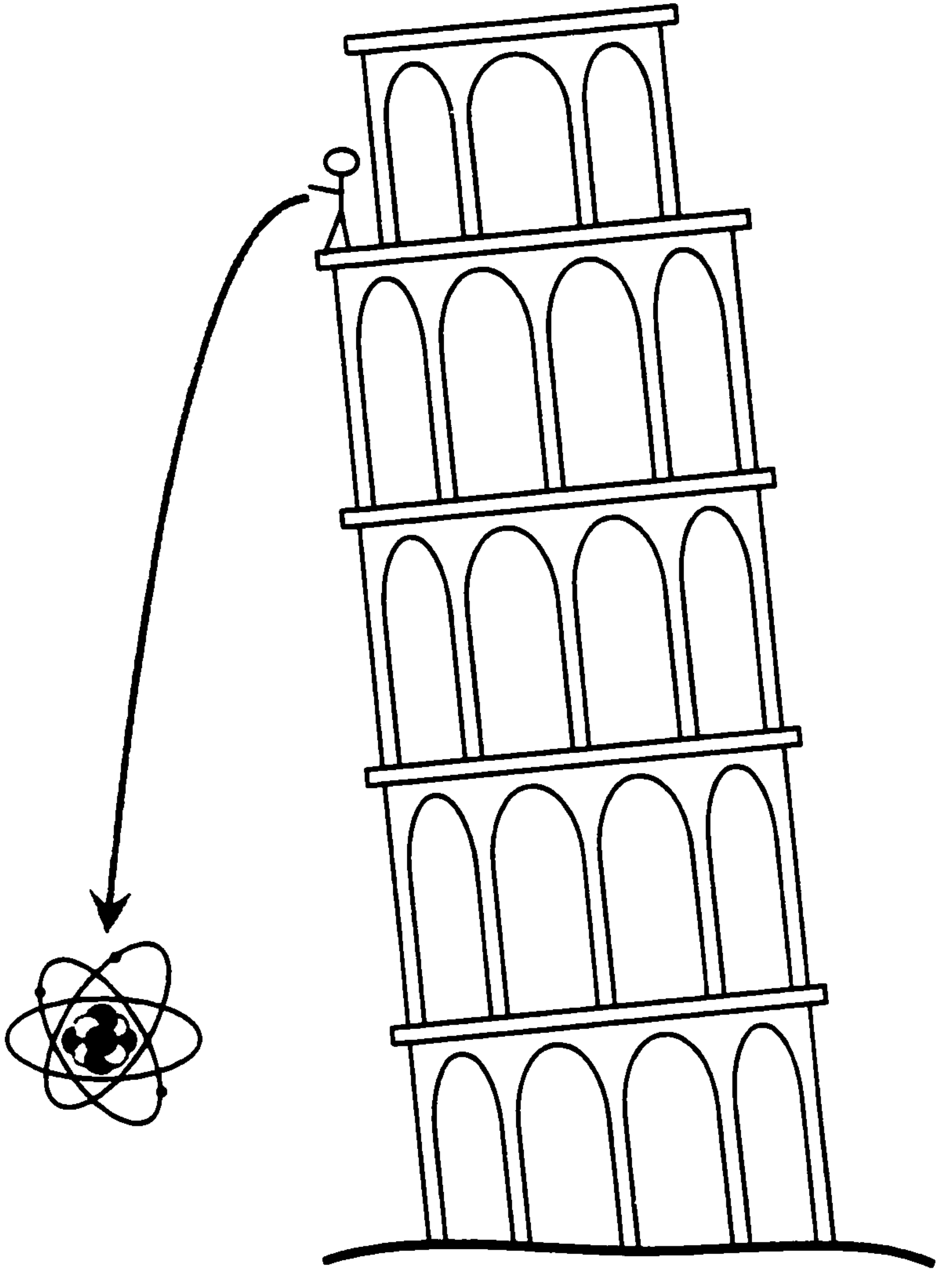
- Unprecedented Precision

  (see Nobel Lectures ’97, ’01, ’05)

- Several control variables (v, t, ω, h)

We are at crossroads where atoms may compete with astrophysical tests of GR

An old idea



A.Peters

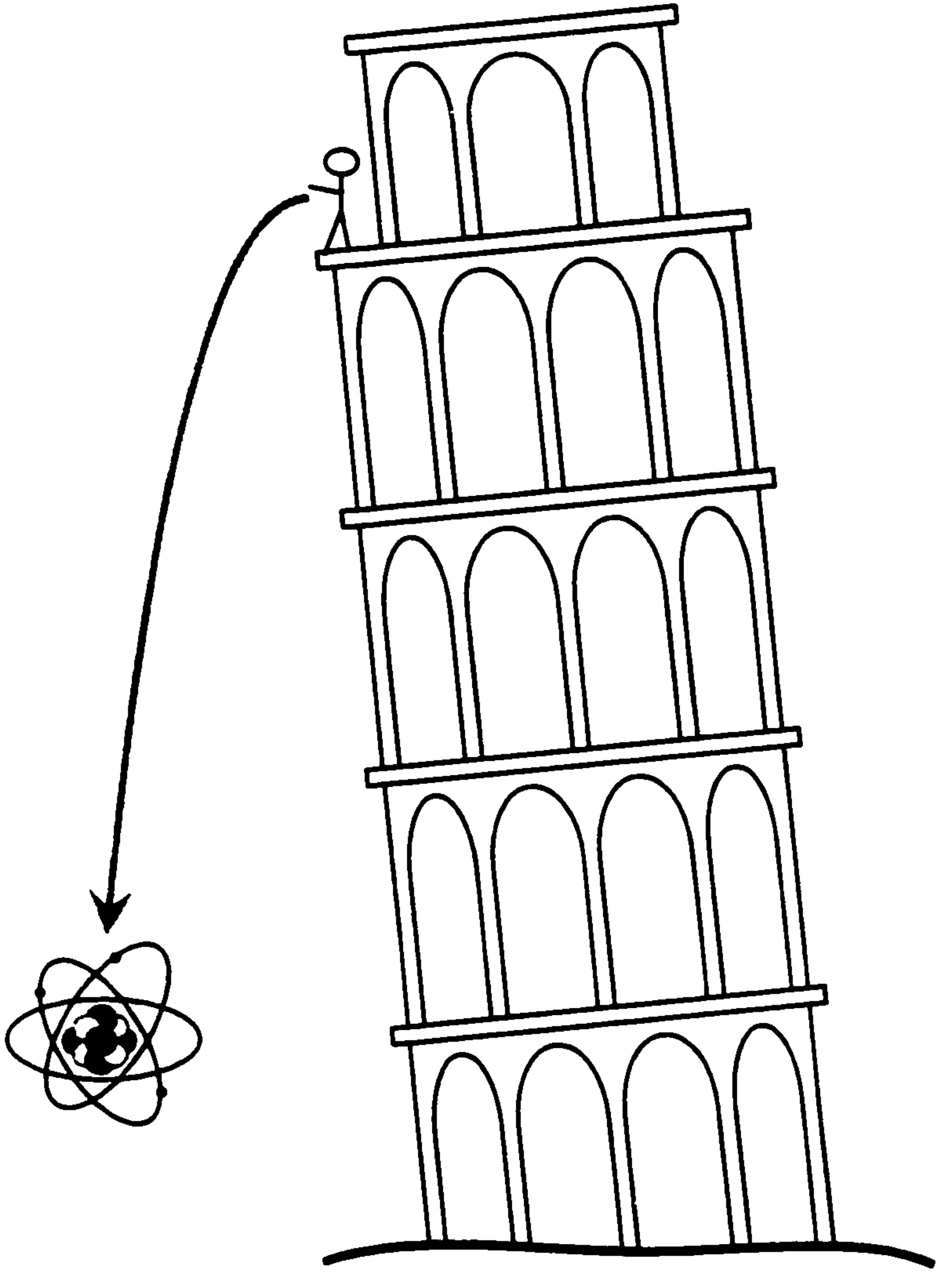
Atom Interferometry can measure minute forces

Galileo $\sim g$

Current $\sim 10^{-11} g$

Future $\sim 10^{-17} g$

An old idea



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Atom Interferometry can measure minute forces

Galileo $\sim g$

Current $\sim 10^{-11} g$

Future $\sim 10^{-17} g$

$$\frac{dv}{dt} = -\nabla\phi + \boxed{\text{GR}}$$

$$\phi = G_N \frac{M_e}{R_e}$$

Outline

- Post Newtonian General Relativity
- Atom Interferometry
- Preliminary estimates

Post-Newtonian Approximation

Expansion in potential and velocity

Small Numbers

Atom velocity: $$v_{\mathrm{atoms}} \sim 10\frac{m}{sec} \sim 3 \times 10^{-8}$$

Earth's potential: $$\phi = \frac{\mathrm{G_N M_{earth}}}{\mathrm{R_{earth}}} \sim \frac{1}{2} \times 10^{-9}$$

Gradient: $$\frac{\mathrm{height}}{\mathrm{R_{earth}}} \sim \frac{10\ \mathrm{m}}{6 \times 10^{6}\ \mathrm{m}} \sim \frac{1}{6} \times 10^{-5}$$

Particle equation of motion

ϕ	Newtonian Gravitational Potential
ψ	Kinetic Energy Gravitational Potential
ζ	Rotational Energy Gravitational Potential

$$\frac{d\vec{v}}{dt} = -\nabla(\phi + 2\phi^2 + \psi)$$

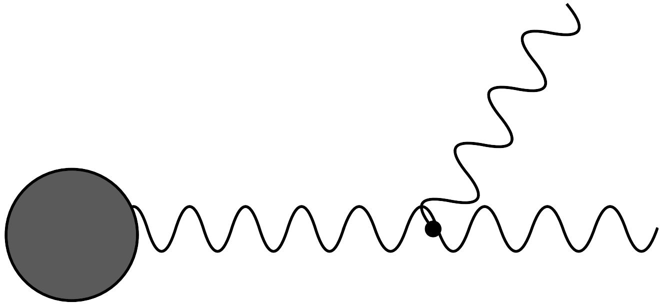
"scalar potential"

$$-\frac{\partial\vec{\zeta}}{\partial t} + \vec{v} \times (\nabla \times \vec{\zeta})$$

"vector potential"

$$+3\vec{v}\frac{\partial\phi}{\partial t} + 4\vec{v}(\vec{v} \cdot \nabla)\phi - \vec{v}^2\nabla\phi$$

Non-abelian gravity



In empty space

Newton $\nabla \cdot \vec{g} = \nabla^2\phi = 4\pi \mathrm{G}_{\mathrm{N}}\rho = 0$

Einstein $\nabla^2\delta\phi = (\nabla\phi)^2 \sim \nabla^2\phi^2$

$$\Rightarrow \delta\phi \sim \phi^2$$

$$\Longrightarrow \text{“}\nabla \cdot \vec{g} \neq 0\text{”}$$

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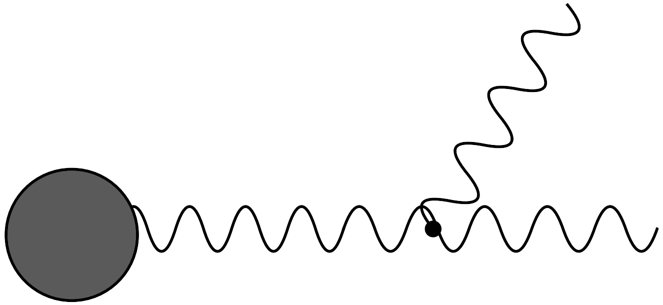

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Effect $\sim 10^{-9} g$

only gradient measurable → $10^{-15} g$

$$\frac{d\vec{v}}{dt} = -\nabla(\phi + \boxed{2\phi^2} + \psi) - \frac{\partial\vec{\zeta}}{\partial t} + \vec{v} \times (\nabla \times \vec{\zeta})$$

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“Kinetic Energy Gravitates”

$$-\vec{v}^2\nabla\phi + 4\vec{v}(\vec{v}\cdot\nabla)\phi$$

$$\text{Effect} \sim v_{\text{atoms}}^2 g \sim 10^{-15} g$$

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General Relativity effects on equation of motion

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1 $\qquad 10^{-9}$ $\qquad \sim 0$ $\qquad 10^{-13}$

10^{-15}

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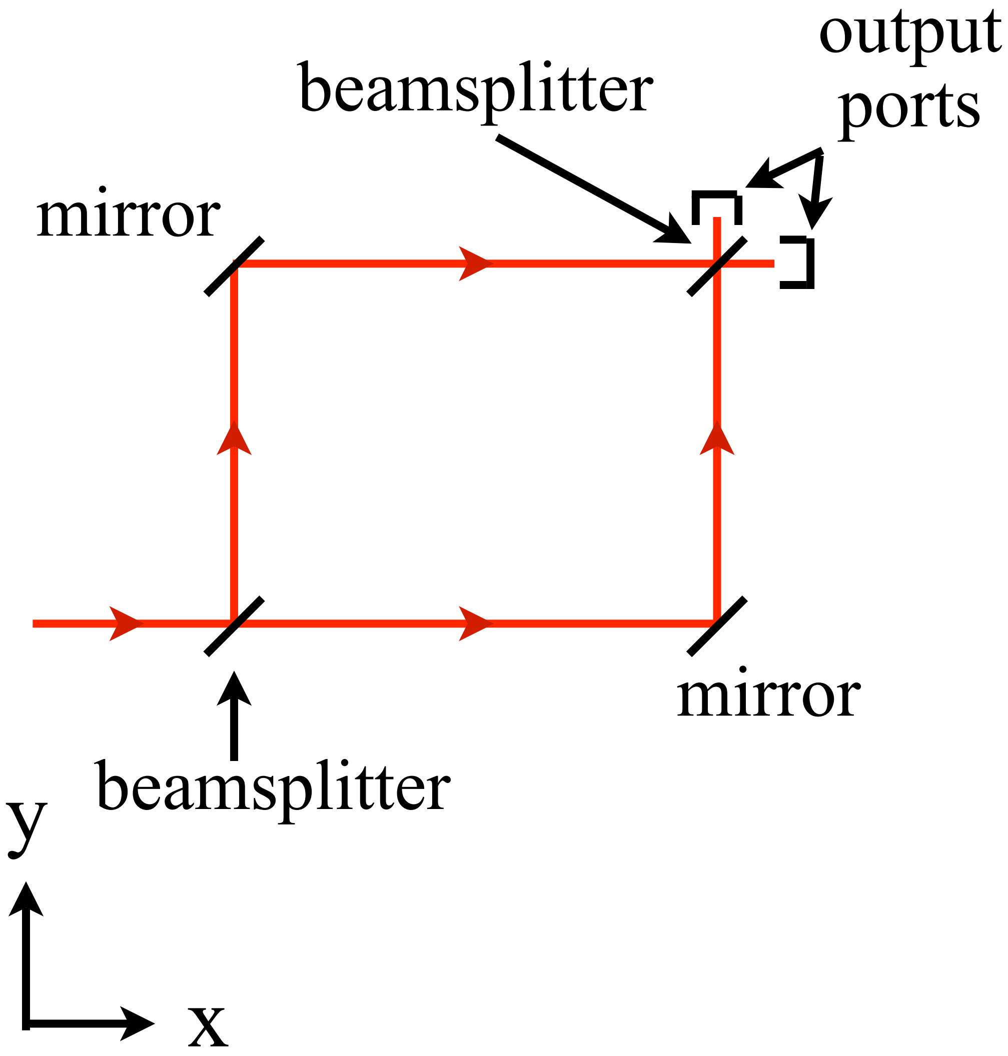
Can these terms be measured in the lab?

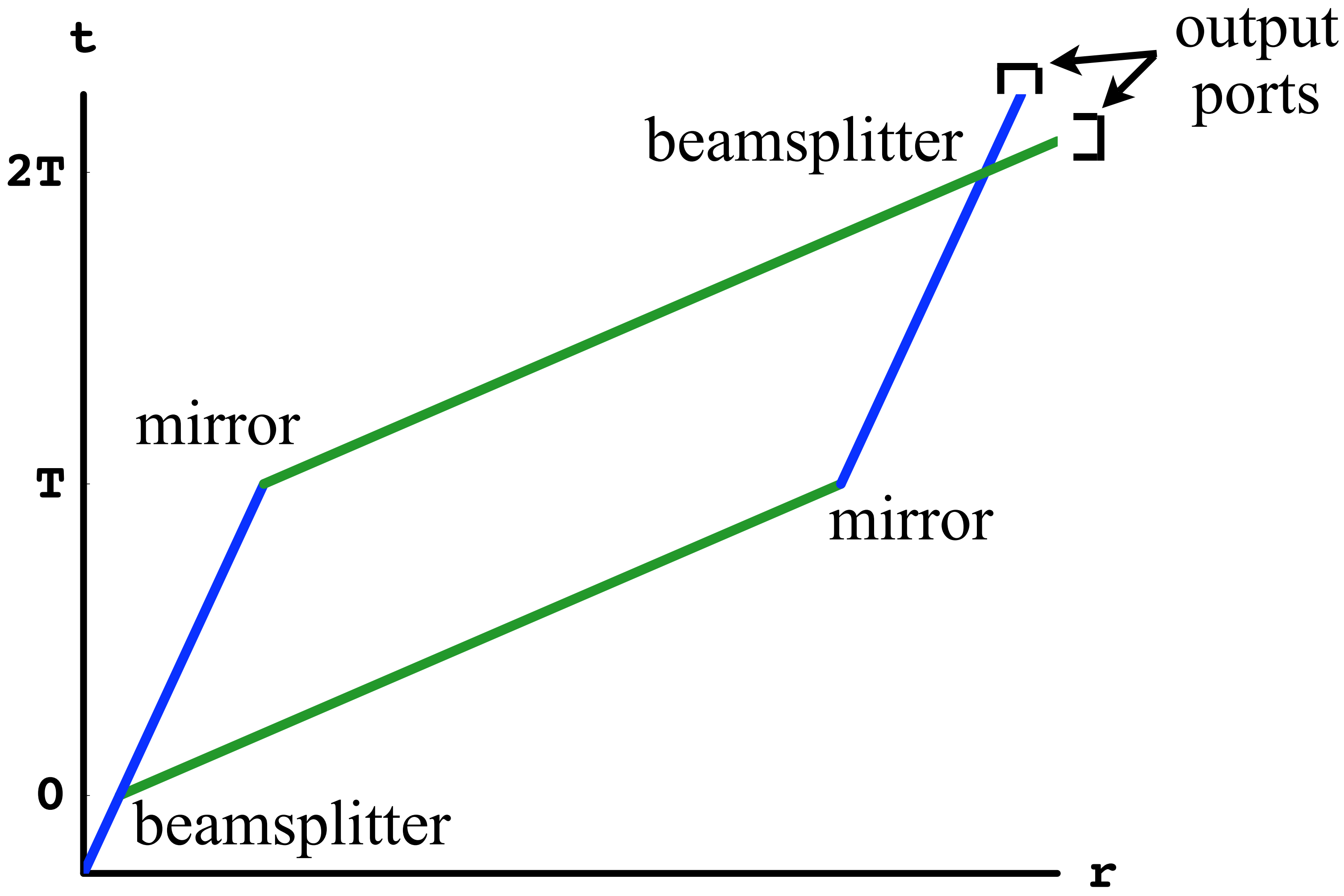
Light Interferometry



accurate measurement of

$$\frac{\Delta L}{L} \sim \frac{\lambda}{L} \times \text{(phase resolution)}$$

Atom Interferometry



similar to light interferometer but arms are separated in space-time instead of space-space

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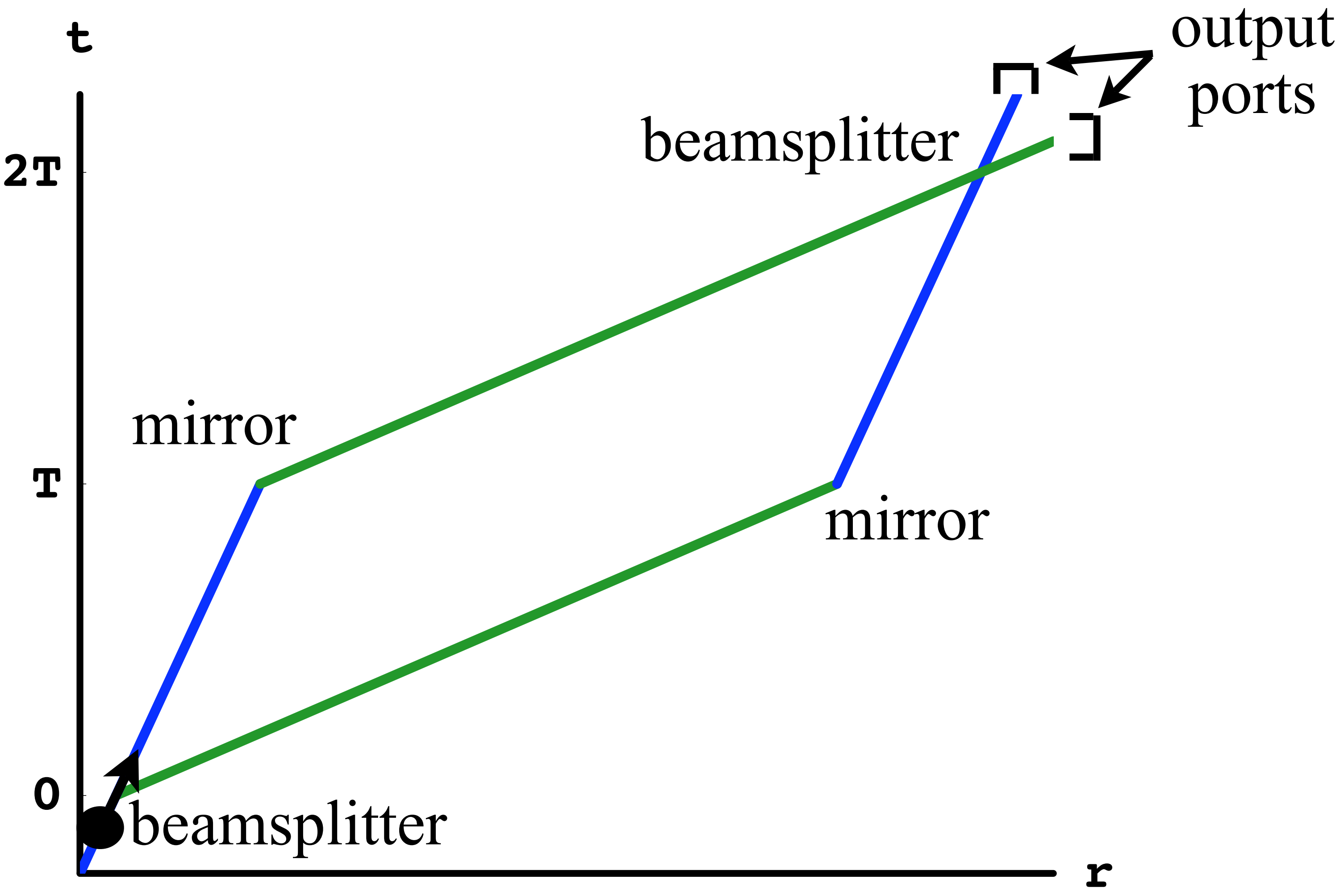

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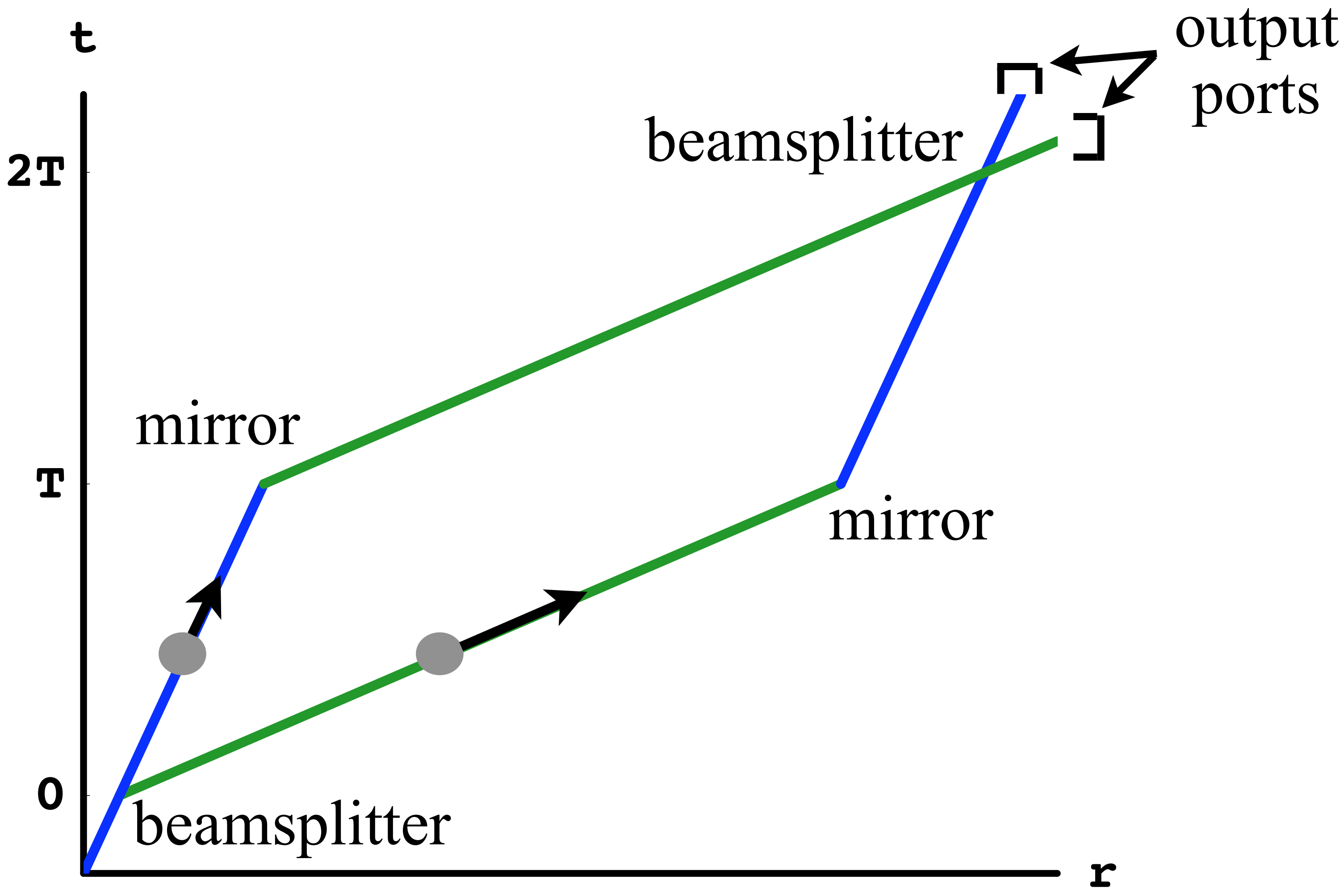

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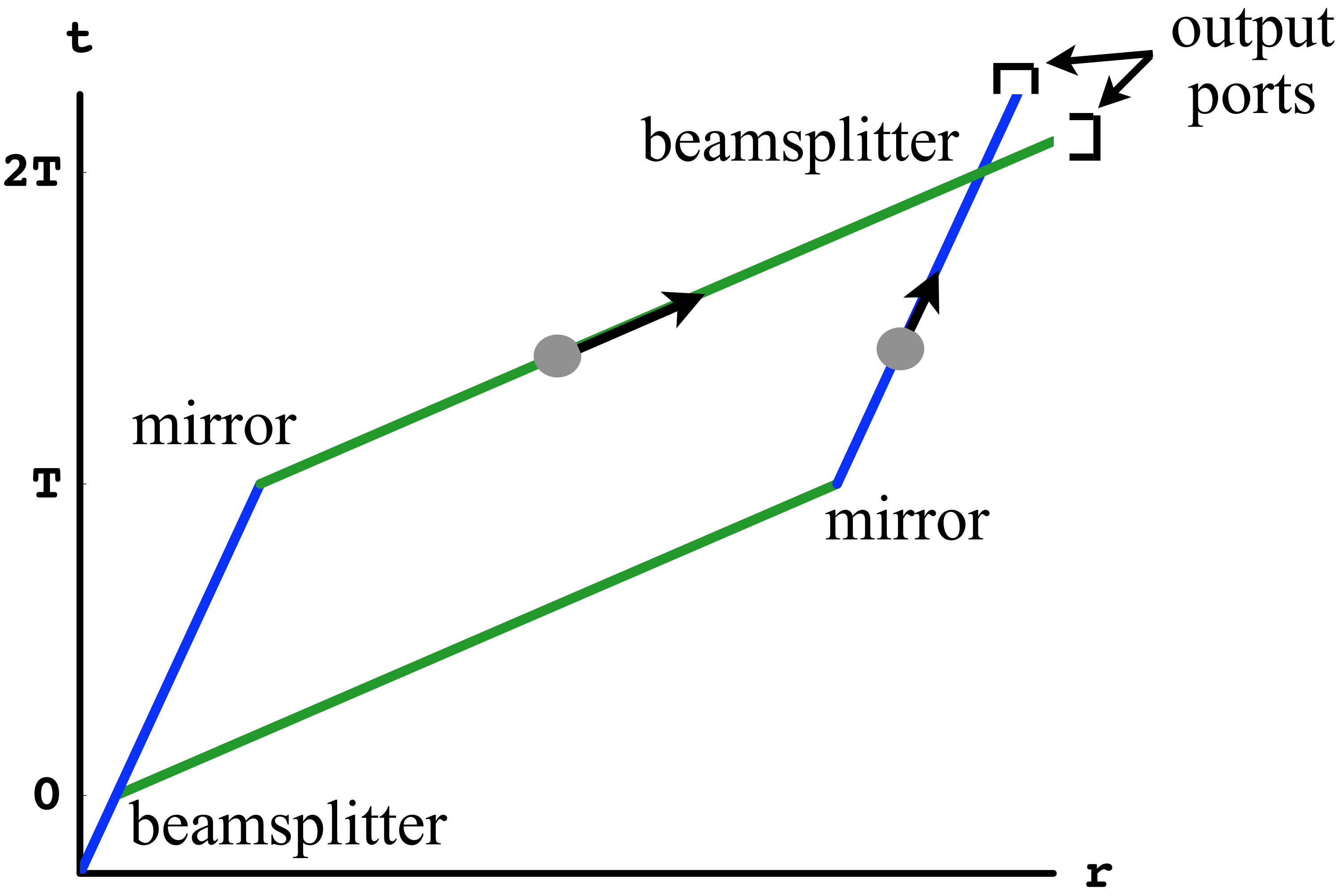

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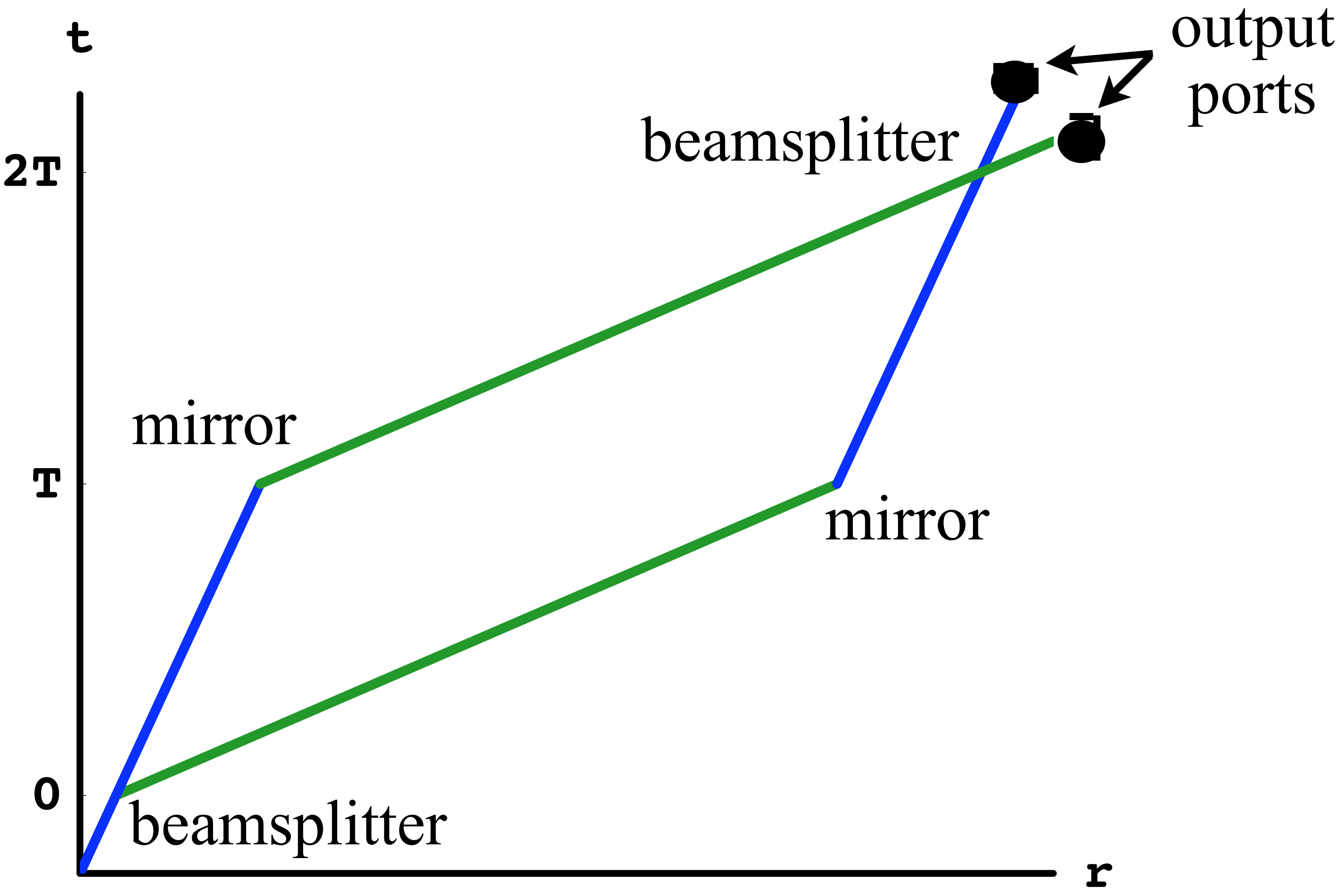

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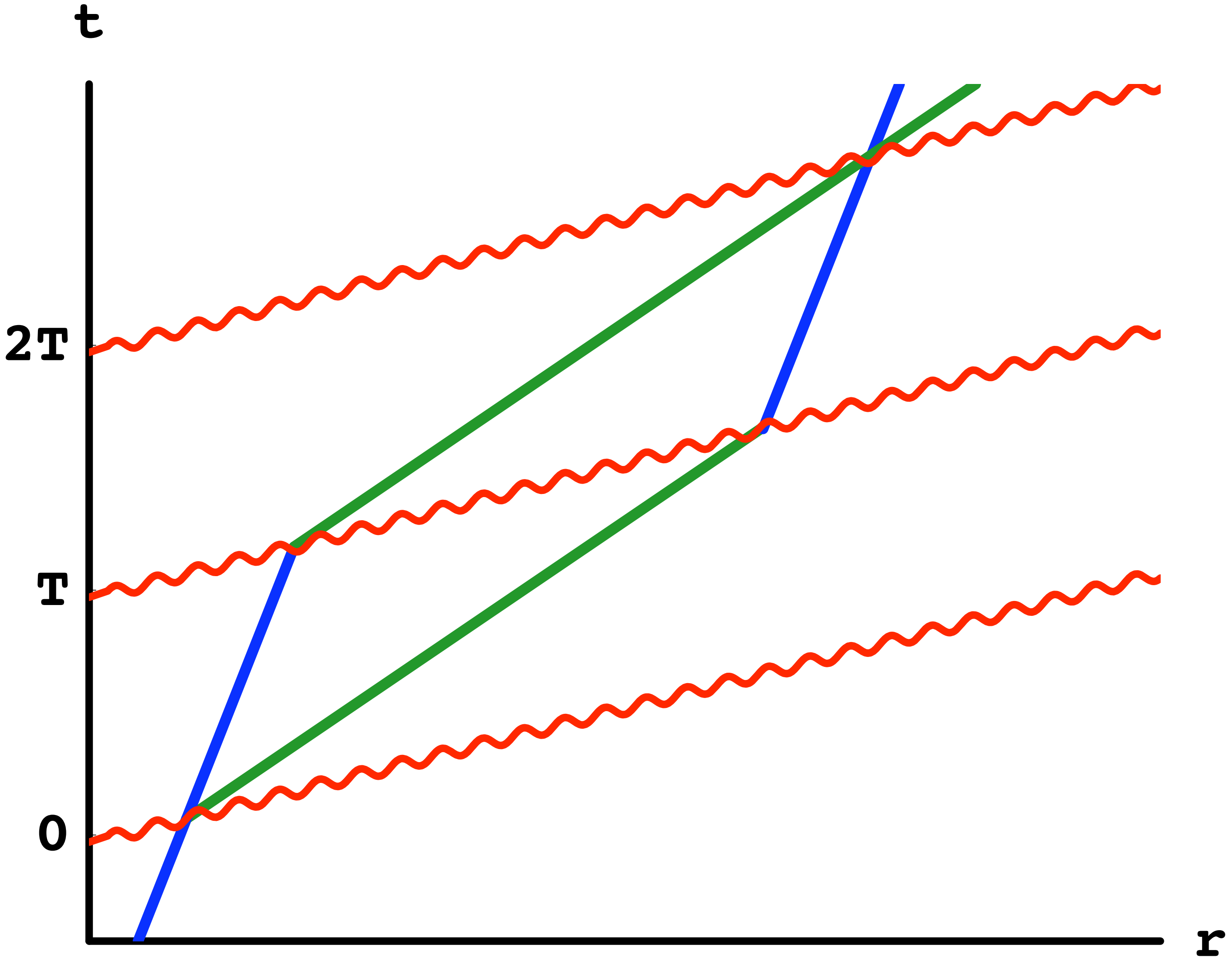
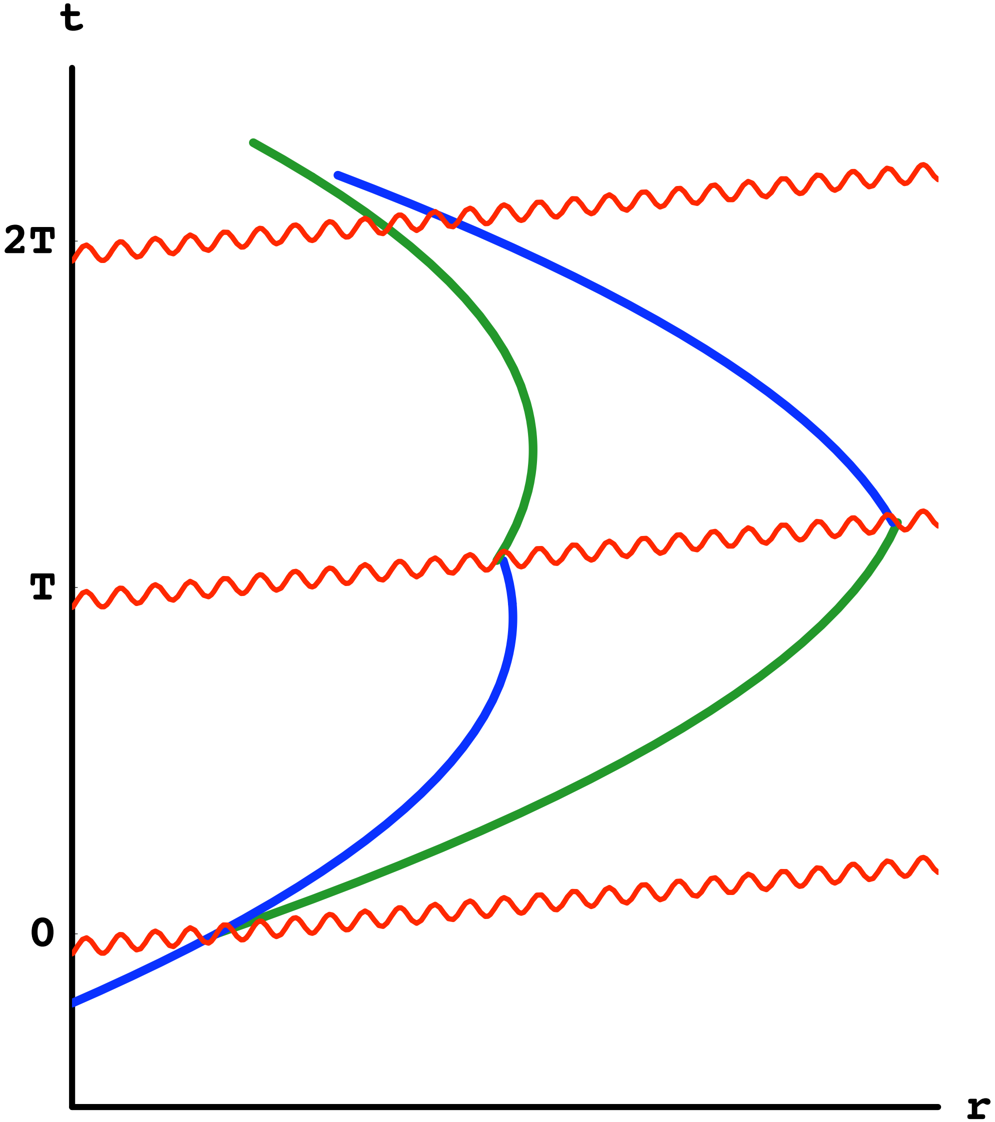


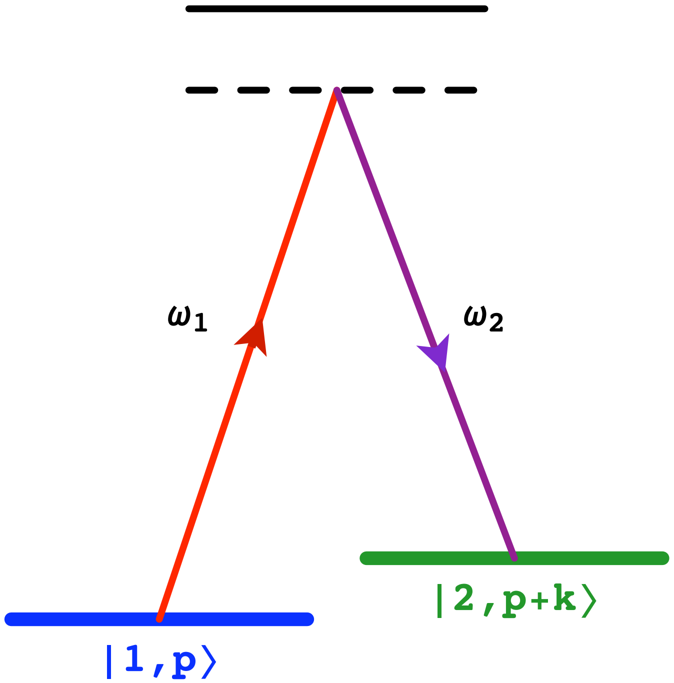
use lasers as beamsplitters and mirrors

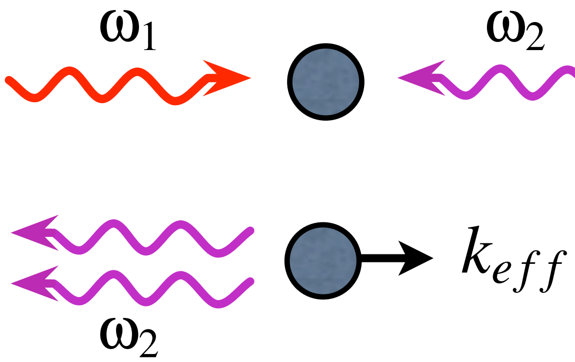
Atom Interferometry



slow atoms fall more under gravity and
the interferometer can be as long as 1 sec ~ earth-moon distance!

Raman Transition



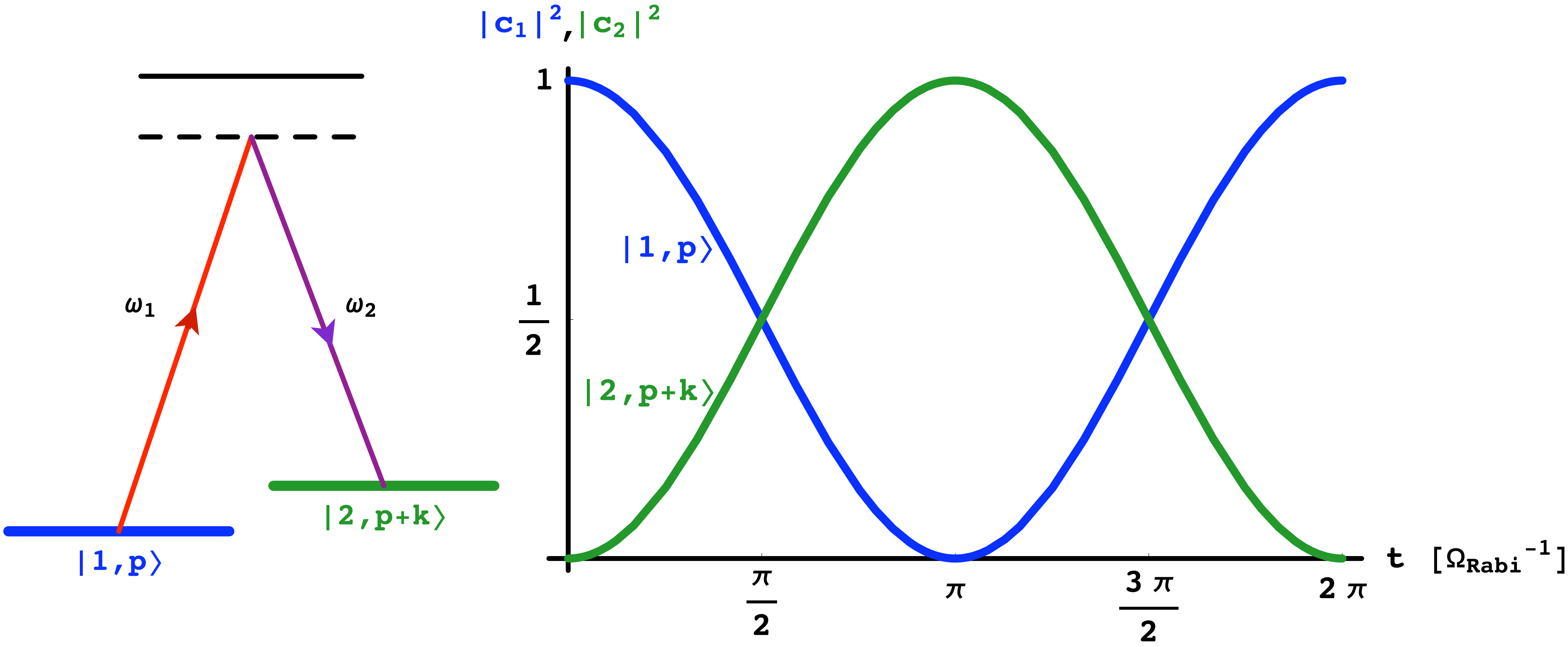


$$k_{eff} = \omega_1 + \omega_2 \sim 1\ \mathrm{eV}$$

$$\omega_{eff} = \omega_1 - \omega_2 \sim 10^{-5}\ \mathrm{eV}$$

Raman Transition

$$\psi = c_1|1,p\rangle + c_2|2,p+k\rangle$$



$\pi/2$ pulse is a beamsplitter

π pulse is a mirror

AI Phase Shifts

Total phase difference comes from three sources:

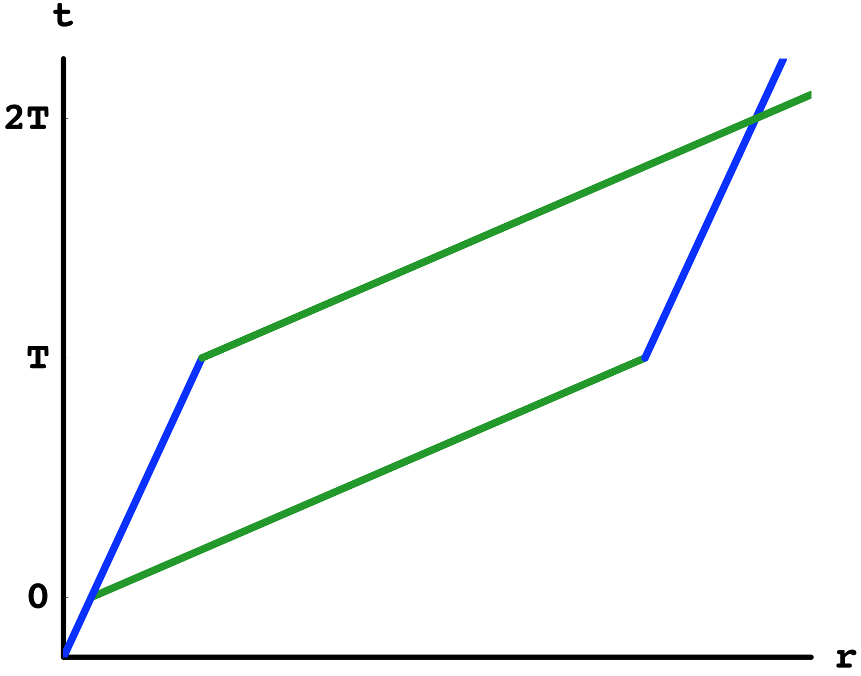
$$\Delta\phi_{\rm tot} = \Delta\phi_{\rm propagation} + \Delta\phi_{\rm laser} + \Delta\phi_{\rm separation}$$

Propagation Phase

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{laser}} + \Delta\phi_{\text{separation}}$$

$$\phi_{\text{propagation}} = \int m d\tau = \int L dt = \int p_\mu dx^\mu$$

integral taken over each arm of interferometer



Laser Phase

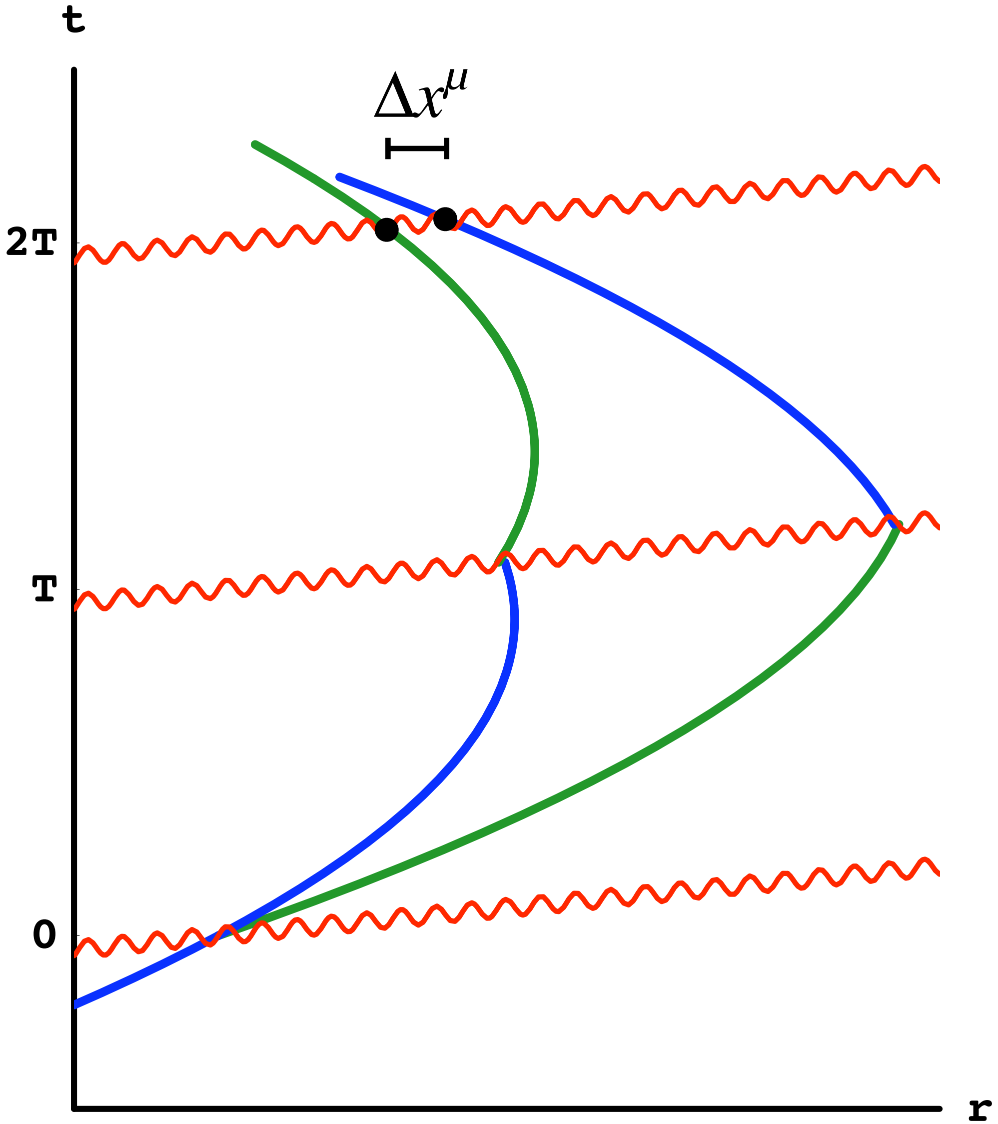
$$\Delta\phi_{\mathrm{tot}} = \Delta\phi_{\mathrm{propagation}} + \Delta\phi_{\mathrm{laser}} + \Delta\phi_{\mathrm{separation}}$$

$$\phi_{\mathrm{laser}} = \sum_{\mathrm{vertices}} (\text{phase of laser})$$

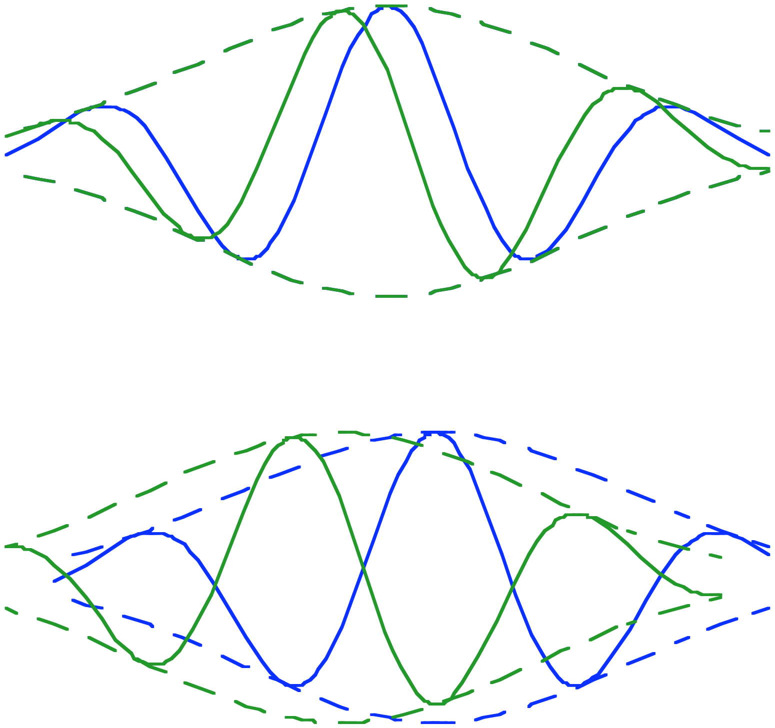
$$\langle \mathrm{out}|H_{\mathrm{int}}|\mathrm{in}\rangle = \langle \mathrm{out}|\vec{\mu}\cdot\vec{E}_0 e^{i\vec{k}\cdot\vec{x}}|\mathrm{in}\rangle$$

the laser imparts a phase to the atom just as a mirror or beamsplitter imparts a phase to light

Separation Phase

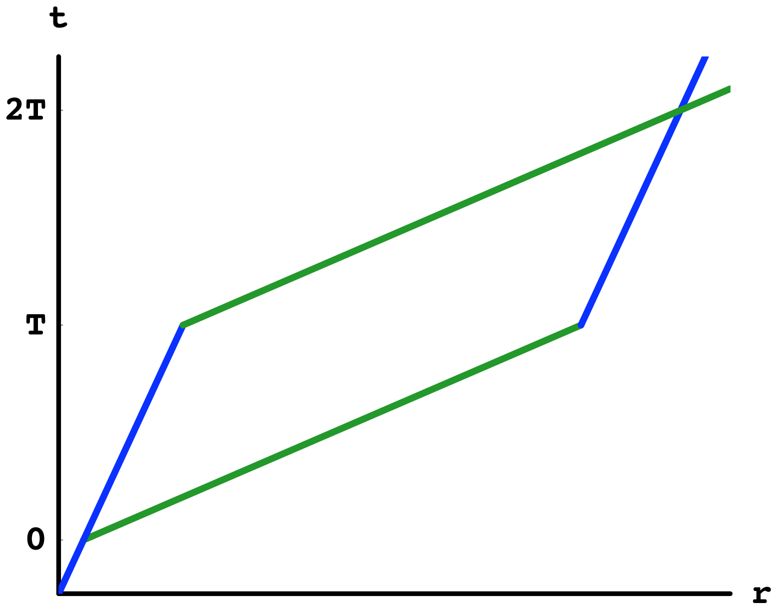


$$\Delta\phi_{\text{separation}} = \int_{\Delta x^\mu} p_\nu dx^\nu$$



Measuring Gravity

a constant gravitational field produces a phase shift:



$$\phi_{\text{propagation}} = \int \left(\frac{1}{2} m v^2 - m g h \right) dt$$

$$\Delta\phi_{\text{propagation}} = mg \times (\text{area}) = mg \times \frac{k}{m} T \times T$$

$$\Delta\phi_{\text{tot}} = k g T^2 \sim 10^8 \text{ radians}$$

Gravity Phases

$\frac{GkMT^2}{R_e^2}$	$1. \times 10^{8}$
$-\frac{2GkMT^3v_L}{R_e^3}$	$-2. \times 10^{3}$
$-\frac{GMT^2\omega}{R_e^2}$	$-1. \times 10^{3}$
$\frac{GMT^2\omega_A}{R_e^2}$	$1. \times 10^{3}$
$\frac{7G^2kM^2T^4}{6R_e^5}$	1.16667×10^{2}
$\frac{3GkMT^2v_L}{R_e^2}$	$3. \times 10^{1}$
$-\frac{3G^2kM^2T^3}{R_e^4}$	$-3.$
$-\frac{Gk^2MT^3}{mR_e^3}$	$-1.$
$\frac{7GkMT^4v_L^2}{2R_e^4}$	3.5×10^{-2}
$\frac{2GMT^3\omega v_L}{R_e^3}$	$2. \times 10^{-2}$
$-\frac{2GMT^3v_L\omega_A}{R_e^3}$	$-2. \times 10^{-2}$
$\frac{3Gk^2MT^2}{2mR_e^2}$	1.5×10^{-2}
$\frac{G^2kM^2T^2}{R_e^3}$	$1. \times 10^{-2}$
$-\frac{11G^2kM^2T^5v_L}{2R_e^6}$	-5.5×10^{-3}
$-\frac{7G^2M^2T^4\omega}{6R_e^5}$	-1.16667×10^{-3}
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$-\frac{3GMT^2\omega v_L}{R_e^2}$	$-3. \times 10^{-4}$
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non-relativistic constant g

NR gravity gradient

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Gravity Phases

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$-\frac{2GkMT^3v_L}{R_e^3}$	$-2. \times 10^3$	NR gravity gradient
$-\frac{GMT^2\omega}{R_e^2}$	$-1. \times 10^3$	
$\frac{GMT^2\omega_A}{R_e^2}$	$1. \times 10^3$	
$\frac{7G^2kM^2T^4}{6R_e^5}$	1.16667×10^2	
$\frac{3GkMT^2v_L}{R_e^2}$	$3. \times 10^1$	Doppler shift
$-\frac{3G^2kM^2T^3}{R_e^4}$	$-3.$	
$-\frac{Gk^2MT^3}{mR_e^3}$	$-1.$	
$\frac{7GkMT^4v_L^2}{2R_e^4}$	3.5×10^{-2}	
$\frac{2GMT^3\omega v_L}{R_e^3}$	$2. \times 10^{-2}$	
$-\frac{2GMT^3v_L\omega_A}{R_e^3}$	$-2. \times 10^{-2}$	
$\frac{3Gk^2MT^2}{2mR_e^2}$	1.5×10^{-2}	
$\frac{G^2kM^2T^2}{R_e^3}$	$1. \times 10^{-2}$	GR $\nabla\phi^2$
$-\frac{11G^2kM^2T^5v_L}{2R_e^6}$	-5.5×10^{-3}	
$-\frac{7G^2M^2T^4\omega}{6R_e^5}$	-1.16667×10^{-3}	
$\frac{7G^2M^2T^4\omega_A}{6R_e^5}$	1.16667×10^{-3}	
$-\frac{8GkMT^3v_L^2}{R_e^3}$	$-8. \times 10^{-4}$	
$-\frac{3GMT^2\omega v_L}{R_e^2}$	$-3. \times 10^{-4}$	
$\frac{35G^2kM^2T^4v_L}{2R_e^5}$	1.75×10^{-4}	
$\frac{5GkMT^2v_L^2}{R_e^2}$	$5. \times 10^{-6}$	GR $-\vec{v}^2\nabla\phi + 4\vec{v}(\vec{v}\cdot\nabla)\phi$
$-\frac{11G^2k^2M^2T^5}{4mR_e^6}$	-2.75×10^{-6}	
$-\frac{15G^2kM^2T^3v_L}{R_e^4}$	-1.5×10^{-6}	

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non-relativistic constant g

NR gravity gradient

Doppler shift

GR $\nabla\phi^2$

GR $-\vec{v}^2\nabla\phi + 4\vec{v}(\vec{v}\cdot\nabla)\phi$

GR $\nabla\phi^2$

Gravity Phases

kgT^2	10^8	
$-2\,kgT^2\,\frac{v_L T}{R_e}$	-2×10^3	
$3\,kgT^2\,v_L$	3×10^1	← doppler shift
$kgT^2\,\phi$	10^{-2}	
$5\,kgT^2\,v_L^2$	5×10^{-6}	GR terms
$-15\,kgT^2\,\frac{v_L T}{R_e}\phi$	10^{-6}	

experimentally controllable parameters are: $k,\ v_L,\ T$

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same scalings, measure with gradient

experimentally controllable parameters are: $k,\ v_L,\ T$

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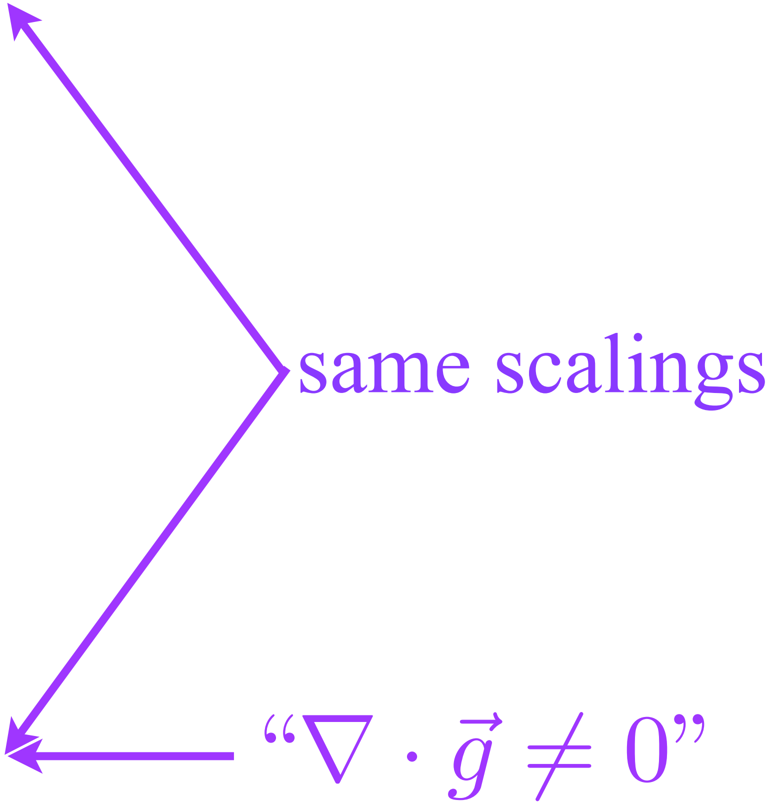

same scalings

"$\nabla\cdot\vec{g}\neq 0$"

experimentally controllable parameters are: $k,\ v_L,\ T$

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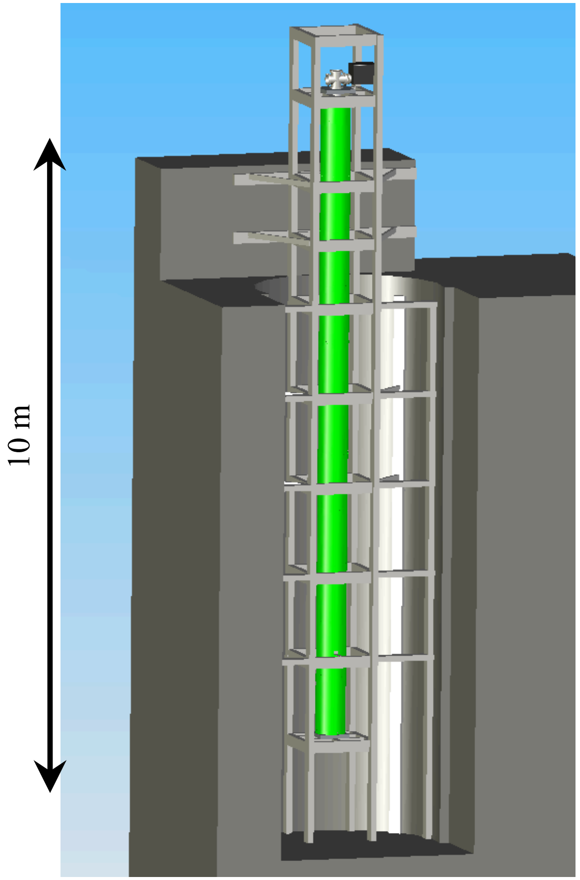
← unique scaling

experimentally controllable parameters are: $k,\ v_L,\ T$

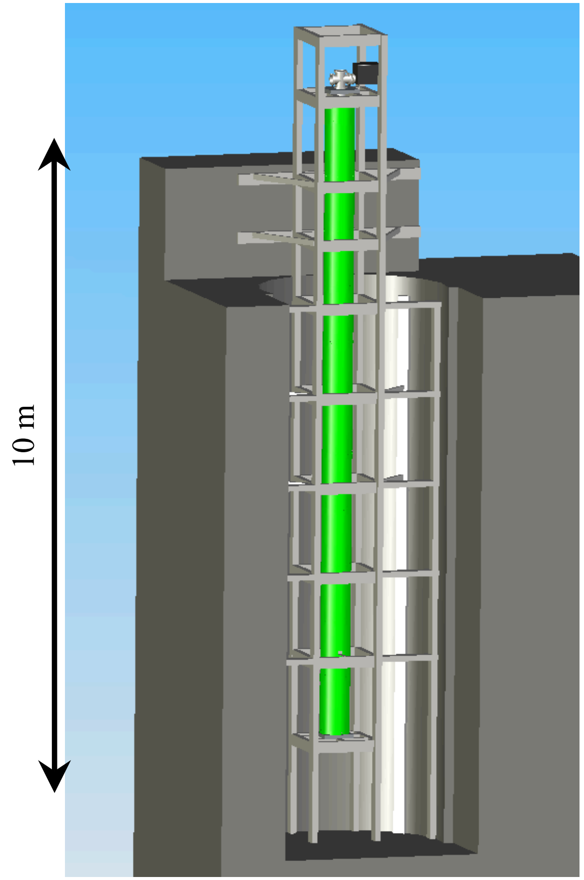
Atomic Interferometer



10 m atom drop tower.

currently under
construction
at Stanford

Atomic Equivalence Principle Test



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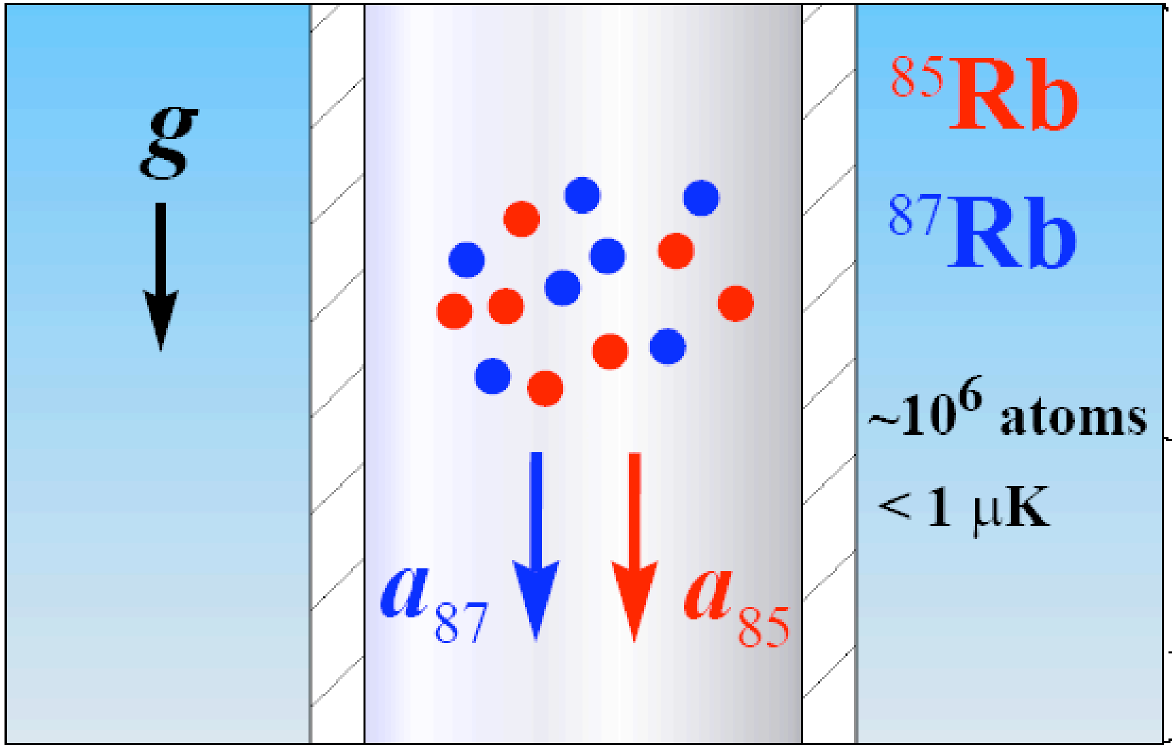
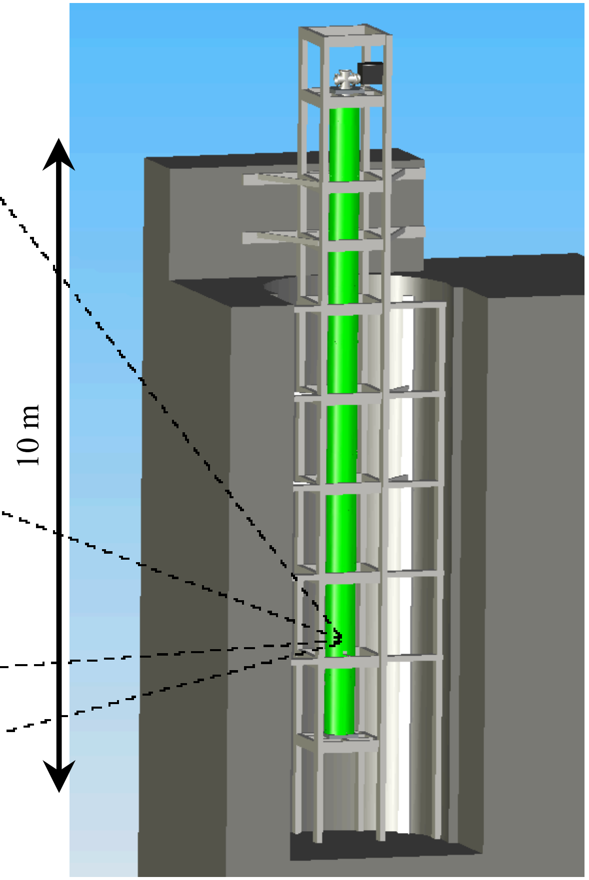
Atomic Equivalence Principle Test



Co-falling ^{85}Rb and ^{87}Rb ensembles

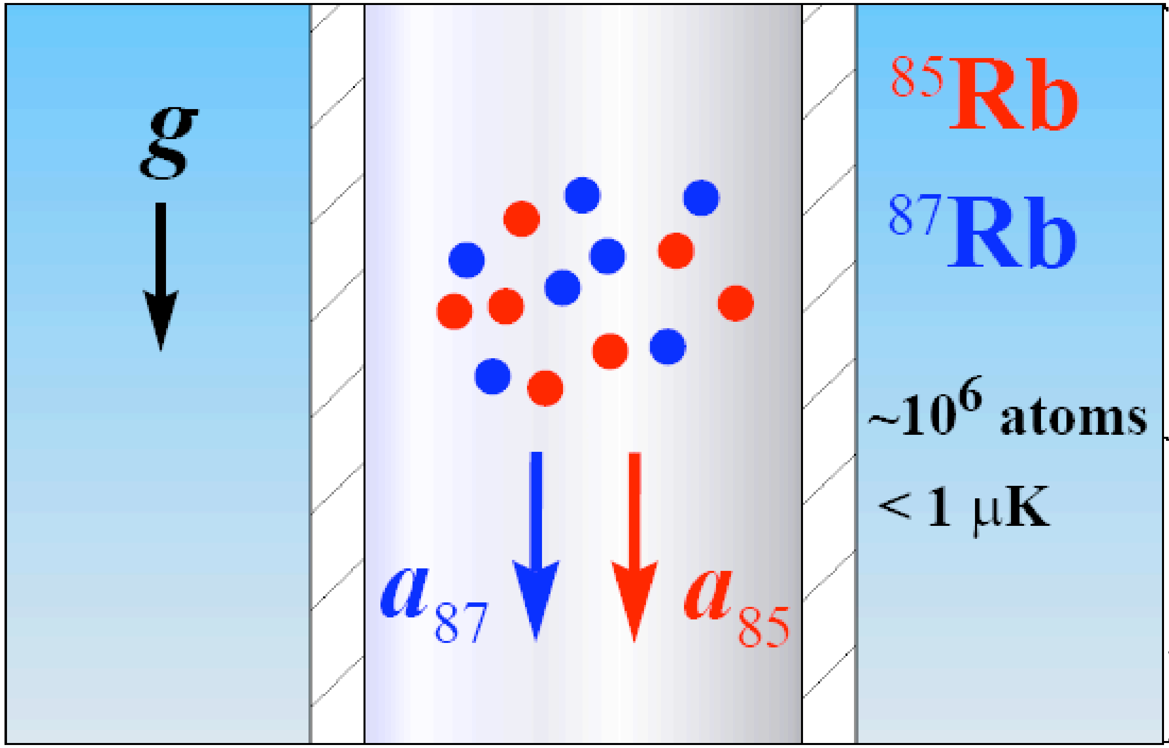


10 m atom drop tower.

Will reach accuracy $\sim 10^{-16}$

Compared to Lunar Laser Ranging $\sim 3 \times 10^{-13}$

Atomic Equivalence Principle Test



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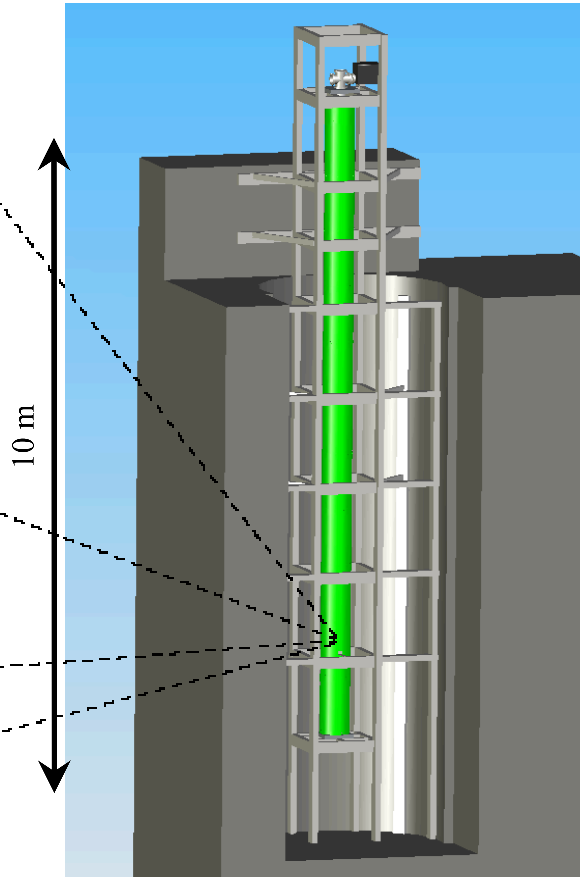

10 m atom drop tower.

Will reach accuracy $\sim 10^{-16}$

Compared to Lunar Laser Ranging $\sim 3 \times 10^{-13}$

Then will test PN GR

Other equivalence principle measurements

Atomic Equivalence Principle Test at Stanford	10^{-15} g	2008
MICROSCOPE	10^{-15} g	2011
Galileo Galilei	10^{-17} g	launch 2009?
STEP	10^{-17} g	?

Can we measure H ?

Pioneer anomaly ?

Radio ranging of Pioneer ↔ Laser ranging of atoms

BUT equivalence principle says only tides measurable

and Riemann $R \sim H^2$ way too small

Can we measure H ?

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Similarly DM is not measurable

But, Sun's radiation pressure is measurable at the $10^{-17}g$ level, and causes the earth not to be an inertial frame

GR Experimentation

1916 - 1920	Precession of Mercury and light bending
1920 - 1960	Hibernation
1960 - Now	Golden Era, many astronomical tests
New epoch?	High precision atom interferometry allows for greater control and ability to isolate and study individual effects in GR such as 3-graviton coupling and gravitation of kinetic energy

Good to Go!