Introduction to Multiple Parton Interactions

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KEYWORDS:

one hadron-hadron collision - two (or more) parton-parton collisions partonic "pile-up"

- MPI = additional source of multi-jet production source of information about the internal structure of the proton
- DPI = two hard (semi-hard) parton interactions in one event
- Hidden Reefs of the MPI Analysis
- The role of perturbative parton correlations (1 x 2 processes)
- Look for MPI. Where and How?

(Homework)

Multi-Parton Interactions

- prehistory: Daniele Treleani & Co (1982-)
- new life: Tevatron experiments CDF (1997) & D0 (2009, 2011) (dijet together with a photon-jet pair)
- LHC epoch: intensive MPI studies in various channels (two pairs of jets, W/Z bosons, heavy quark pairs, etc.)

On the theory side, MPI are being pursued by a number of teams.

Will follow the line of reasoning developed in

B. Blok, Yu. Dokshitzer, L. Frankfurt and M. Strikman :

The Four jet production at LHC and Tevatron in QCD Phys. Rev. D83: 071501, 2011; e-Print: arXiv:1009.2714 [hep-ph]

pQCD Physics of Multiparton Interactions Eur.Phys.J. C72 (2012) 1963; e-Print: arXiv:1106.5533 [hep-ph]

Perturbative QCD correlations in multi-parton collisions Eur.Phys.J. C74 (2014) 2926; e-Print: arXiv:1306.3763 [hep-ph]

Double Parton Interactions

Establishing adequate QFT means for describing MPI The origin of *Generalized Double Parton Distributions* (2GPD)

Modeling intra-hadron 2-parton correlations (*limited but restrictive*)

- Examining the role of pQCD parton-parton correlations in DPI
- Giving numerical estimates for Tevatron and LHC experiments

2-parton collision

The standard approach to the multi-jet production is the QCD improved parton model.

It is based on the assumption that the cross section of a hard hadron-hadron interaction is calculable in terms of the convolution of parton distributions within colliding hadrons with the cross section of a hard *two-parton collision*.

$$\sigma_2 = \int d^2 \rho_1 d^2 B f(x_1, \vec{\rho_1}, p^2) f(x_2, \vec{B} - \vec{\rho_1}, p^2) \frac{d\sigma^h}{d\hat{t}} d\hat{t}$$
parton probability density :
$$f(x, \vec{\rho}, p^2) = \psi^+(x, \vec{\rho}, p^2) \psi(x, \vec{\rho}, p^2)$$

Result of the impact parameter integration - squaring of the amplitude in the momentum space:

$$\int \frac{d^2 k_{\perp}}{(2\pi)^2} \psi(x,k_{\perp}) \int \frac{d^2 k'_{\perp}}{(2\pi)^2} \psi^{\dagger}(x,k'_{\perp}) \times \int d^2 \rho \ e^{i\vec{\rho}\cdot(\vec{k}_{\perp}-\vec{k}'_{\perp})} \quad \Longrightarrow \int \frac{d^2 k_{\perp}}{(2\pi)^2} \ \psi(x,k_{\perp}) \times \psi^{\dagger}(x,k_{\perp})$$

Hard collision of two partons produces, typically, *two* large transverse momentum *jets*.

An application of the standard picture to the processes with production of, e.g., *four jets* implies that all jets in the event are produced in a hard collision of *two* initial state partons.

Recent data of the CDF and D0 Collaborations provide evidence that there exists a kinematical domain where a more complicated mechanism becomes important :

double hard interaction

of two partons in one hadron with two partons in the second hadron.



Let us see, what difference does it make to our formulae

multi-partons

Multi-parton wave function

$$\psi_n \ (x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \ldots) = \int \prod_{i=1}^{i=n} \frac{d^2 k_i}{(2\pi)^2} \exp(i \sum_{i=1}^{i=n} \vec{k}_i \vec{\rho}_i) \ \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, \ldots) (2\pi)^2 \delta(\sum \vec{k}_i)$$

Inclusive 2-parton probability distribution in the impact parameter space :

$$D (x_1, x_2, \vec{\rho_1}, \vec{\rho_2}) = \sum_{n=3}^{n=\infty} \int \prod_{i\geq 3}^{i=n} \left[dx_i d^2 \rho_i \right] \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \psi_n^+(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}, \dots) \delta(\sum_{i=1}^{i=n} x_i \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_1}, \dots, x_i, \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, x_2, \vec{\rho_2}, \dots, x_i, \vec{\rho_i}) \psi_n(x_1, \vec{\rho_1}, \dots, x_i, \vec{\rho_i}) \psi_n(x_1, \vec{\rho_i}, \dots, x_i, \vec{\rho_i$$

Independent impact parameter integration \longrightarrow equality of parton momenta in ψ and ψ^{\dagger} $k_{\perp}~=~k_{\perp}'$

$$\rho_{1} + \rho_{2} \longrightarrow k_{1}' - k_{1} = -(k_{2}' - k_{2}) \equiv \Delta$$

$$\rho_{3} + \rho_{4} \longrightarrow k_{3}' - k_{3} = -(k_{4}' - k_{4}) \equiv \widetilde{\Delta}$$

$$(\rho_{1} - \rho_{2}) + (\rho_{3} - \rho_{4}) \longrightarrow \Delta = -\widetilde{\Delta}$$

$$\delta((\rho_{1} - \rho_{2}) - (\rho_{3} - \rho_{4})) \longrightarrow \widetilde{\Delta} \text{ arbitrary}$$



4-parton collision



In order to be able to trace the *relative distance between the partons*, one has to use the mixed *longitudinal momentum – impact parameter* representation which, in the momentum language, reduces to introduction of a **mismatch** between the transverse momentum of the parton in the *amplitude* and that of the same parton in the *amplitude conjugated*.

We have examined the *transverse momentum* structure of the interaction amplitude

Now, have a look at the *longitudinal momenta* of participating partons ...

A hidden reef of the MPI analysis

3-parton collision

Have a look at a peculiar multi-parton process two hard collisions

btw two partons (3,4) from one hadron with the offspring (1,2) of a perturbative splitting of a single parton (0) stemming from another hadron

quasi-on-mass-shell partons

virtual lines

0

2



A tree Feynman diagram. Momenta of internal parton lines are fixed ...

not anymore

Singularities in the physical region of parton momenta !

Return to a good old single hard interaction picture :

 Q^2

In DIS we trace the fate of **1** but *integrate* over "histories" of the accompanying parton **2**.

Now we want #2 to enter 2nd hard interaction.

In the DIS picture this may happen "in the next room" ...

We, however, want the two hard interactions to occur in the same place !

In fact, partons **3** and **4** *cannot be* represented by plainly independent *plane waves*: they belong to *one hadron*, and therefore, are *localized within the hadron pancake*...

Remedy: introduce wave packet smearing (longitudinal momentum fraction integral). Importantly, this has to be done at the *amplitude level* !

 $k_{3+} + k_{4+}$ fixed by hard scattering kinematics $k_{3+} - k_{4+}$ arbitrary **T**

The fake singularity disappears

mind your head

From theory to experiment

General formalism for DPI:



NO familiar factorization !

The product of probability distributions $f_{i_1j_1}(x_1,y_1,\mu_F)f_{i_2j_2}(x_2,y_2,\mu_F)$

gets replaced by a convolution :

 $= \int d^2 \Delta f_{i_1 j_1}(x_1, y_1, Q_x, Q_y; \vec{\Delta}) f_{i_2 j_2}(x_2, y_2, Q_x, Q_y; \vec{\Delta})$

Generalized two-parton distributions

4-parton cross section

$$\frac{1}{S} = \frac{\int \frac{d^2 \vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2; \vec{\Delta}) D_b(x_3, x_4; -\vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)}$$

S - effective parton interaction area

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 \, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \frac{1}{S}$$

D - the generalized double parton distribution
- a new object we know very little (hardle anything) about.

Can one model it, for lack of anything better ?

a model

has been developed based on an assumption that the partons are *uncorrelated* at the level of the *non-perturbative* proton wave function.

Such a model cannot be true all over the range of parton momentum fractions.

It has a limited range of applicability : 0.1 > x > 0.001 which, however, covers the Tevatron - as well as the main LHC - kinematics.

Good news: the model has predictive power!

- Physical input from the HERA physics
- Oops: Independent parton approximation underestimates the DPI Xsection
- In the Tevatron kinematics the PT parton correlations can explain the missing factor 2 enhancement

 Non-PT intra-hadron 2-parton correlations should emerge at x < 0.001 based on the analysis of *inelastic diffraction* in the framework of the Gribov-Regge Pomeron picture

in: Origins of Parton Correlations in Nucleon and Multi-Parton Collisions B.Blok et al e-Print: arXiv:1206.5594 [hep-ph]



2GPD vs 2 GPDs

Such an amplitude describes exclusive photo-(/electro-) production of vector mesons at HERA !

Note : the analogy is *imperfect*. **OK** for *high enough energies*: $A \simeq i \, \text{Im} A$ Imaginary part of the "skewed" amplitude vs. that of non-diagonal "elastic" transition ...

Generalized parton distribution :

$$G_N(x,Q^2,\vec{\Delta}) = G_N(x,Q^2)F_{2g}(\Delta)$$

- **G** the usual 1-parton distribution (determining DIS structure functions)
- F the two-gluon form factor of the nucleon

the dipole fit : $F_{2g}(\Delta) \simeq \frac{1}{\left(1 + \Delta^2/m_a^2\right)^2}$

$$m_g^2(x \sim 0.03, Q^2 \sim 3 \text{GeV}^2)$$

 $\simeq 1.1 \text{GeV}^2$



If partons were *uncorrelated*, we would write

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta})G(x_2, p_2^2, \vec{\Delta})$$

and use the dipole fit to get the estimate

$$\frac{D(x_1, x_2, -\overrightarrow{\Delta})D(x_3, x_4, \overrightarrow{\Delta})}{D(x_1)D(x_2)D(x_3)D(x_4)} \simeq F_{2g}^4(\Delta)$$

The "interaction area" :

$$\longrightarrow \int \frac{d^2 \Delta}{(2\pi)^2} F_g^2(\Delta^2) \times F_g^2(\Delta^2) = \frac{m_g^2}{28\pi}$$

Another mechanism : 2 partons from a short-range PT correlation No Δ -dependence from the upper side ! $\longrightarrow \int \frac{d^2\Delta}{(2\pi)^2} F_g^2(\Delta^2) = \frac{m_g^2}{12\pi}$ 1 x 2 contribution vs. 2 x 2 is enhanced by a factor $2 \times \frac{7}{3} \simeq 5$

power counting

4-parton interaction is a "higher twist effect"



Always a *small contribution* to the *total 4-jet production cross section* End of story?... Not at all

What distinguishes "double hard collisions" is the differential spectrum

back-to-back kinematics



2 -> 4 processes

What if *both parton pairs* originate from PT splittings ?

No Δ — dependence whatsoever...

The integral *diverges* ?..

This is *not* an amplitude of a *4-parton collision* but a one-loop correction to the *2-parton collision*



A large ("*leading twist*") contribution but *not* an MPI (DPI) No enhancement in the back-to-back kinematical region!



Both these regimes are present in differential momentum distributions due to Double-Parton Interactions

reminder :

Drell-Yan process

Massive lepton pair production cross section

$$\frac{d\sigma}{dq^2 dq_{\perp}^2} = \frac{d\sigma_{\text{tot}}}{dq^2} \quad \times \frac{\partial}{\partial q_{\perp}^2} \left\{ D_a^q \left(x_1, q_{\perp}^2 \right) D_b^q \left(x_2, q_{\perp}^2 \right) S_q^2 \left(q^2, q_{\perp}^2 \right) \right\}$$

Quark form factor:
$$S_q(Q^2, \kappa^2) = \exp\left\{-\int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz P_q^q(z)\right\}$$

Gluon form factor:
$$S_g(Q^2, \kappa^2) = \exp\left\{-\int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz \left[zP_g^g(z) + n_f P_g^q(z)\right]\right\}$$

Parton splitting probabilities

$$P_q^q(z) = C_F \frac{1+z^2}{1-z}, \qquad P_q^g(z) = P_q^q(1-z),$$

 $P_g^q(z) = T_R[z^2 + (1-z)^2],$

$$P_g^g(z) = C_A \frac{1 + z^4 + (1 - z)^4}{z(1 - z)}$$

4-jet diff. spectrum

Generalization of the DDT-formula for back-to-back 4-jet production spectrum

$$\pi^{2} \frac{d\sigma^{(4 \to 4)}}{d^{2} \delta_{13} d^{2} \delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_{1} d\hat{t}_{2}} \cdot \frac{\partial}{\partial \delta_{13}^{2}} \frac{\partial}{\partial \delta_{24}^{2}} \Big\{ {}_{[2]} D_{a}^{1,2}(x_{1}, x_{2}; \delta_{13}^{2}, \delta_{24}^{2}) \times {}_{[2]} D_{b}^{3,4}(x_{3}, x_{4}; \delta_{13}^{2}, \delta_{24}^{2}) \\ \times S_{1} \left(Q^{2}, \delta_{13}^{2} \right) S_{3} \left(Q^{2}, \delta_{13}^{2} \right) \times S_{2} \left(Q^{2}, \delta_{24}^{2} \right) S_{4} \left(Q^{2}, \delta_{24}^{2} \right) \Big\}$$

Not forgetting the Δ —integration and short-range correlations :

$$[2] D_a \times [2] D_b + [2] D_a \times [1] D_b + [1] D_a \times [2] D_b$$

Additional 1 x 2 contribution :

$$\frac{\pi^2 \, d\sigma_2^{(3\to4)}}{d^2 \delta_{13} \, d^2 \delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1 \, d\hat{t}_2} \cdot \frac{\alpha_{\text{s}}(\delta^2)}{2\pi \, \delta^2} \sum_c P_c^{1,2} \left(\frac{x_1}{x_1 + x_2}\right)$$

 $S_1(Q^2,\delta^2) S_2(Q^2,\delta^2) \frac{\partial}{\partial \delta'^2} \left\{ S_c(\delta^2,\delta'^2) \frac{G_a^c(x_1+x_2;\delta'^2,Q_0^2)}{x_1+x_2} S_3(Q^2,\delta'^2) S_4(Q^2,\delta'^2) \times_{[2]} D_b^{3,4}(x_3,x_4;\delta'^2,\delta'^2) \right\}$

effective interaction area

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 \, d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \frac{1}{S}$$

two contributions :

$$\sigma_{\text{eff}}^{-1} = \sigma_4^{-1} + \sigma_3^{-1}$$

$2 \otimes 2$

$$\frac{\prod_{i=1}^{4} D(x_i)}{\sigma_4} = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} \,_{[2]} D_{h_1}(x_1, x_2, Q_1^2, Q_2^2; \vec{\Delta}) \,_{[2]} D_{h_2}(x_3, x_4, Q_1^2, Q_2^2; -\vec{\Delta})$$

$$1 \otimes 2$$

$$\begin{split} \frac{\prod_{i=1}^{4}D(x_{i})}{\sigma_{3}} = & \int \!\! \frac{d^{2}\vec{\Delta}}{(2\pi)^{2}} \Big[_{[2]}\!D_{h_{1}}\!(x_{1},x_{2},Q_{1}^{2},Q_{2}^{2};\vec{\Delta})_{[1]}\!D_{h_{2}}\!(x_{3},x_{4},Q_{1}^{2},Q_{2}^{2}) \\ &+ [1]\!D_{h_{1}}\!(x_{1},x_{2},Q_{1}^{2},Q_{2}^{2})_{[2]}\!D_{h_{2}}\!(x_{3},x_{4},Q_{1}^{2},Q_{2}^{2};\vec{\Delta}) \Big] \end{split}$$

At TEVATRON energies, pQCD 1×2 contributions explain, quite naturally, the factor 2 enhancement of the DPI rate At TEVATRON energies, 1×2 contributions explain, quite naturally, the factor 2 enhancement of the DPI rate

Results of Numerical Analysis

Perturbative QCD correlations in multi-parton collisions Eur.Phys.J. C74 (2014) 2926; e-Print: arXiv:1306.3763 [hep-ph] The 1 x 2 contribution in terms of the ratio

$$R \equiv \frac{\sigma_{1 \otimes 2}}{\sigma_{2 \otimes 2}} = \frac{\sigma_4}{\sigma_3}$$

For the effective interaction area ("cross section")

 $\sigma_{\rm eff} = \frac{28\pi}{m_g^2} \cdot \frac{1}{1+R} \simeq \frac{35\,{\rm mb}}{m_g^2\,[{\rm GeV}]} \cdot \frac{1}{1+R} \simeq \frac{32\,{\rm mb}}{1+R}$ (using the pheno value (HERA) $m_g^2 = 1.1\,{\rm GeV}^2$)

- 1. $u(\bar{u})$ quark and three gluons which is relevant for "photon plus 3 jets" CDF and D0 experiments,
- 2. four gluons (two pairs of hadron jets),
- 3. $u\bar{d}$ plus two gluons, illustrating W^+jj production.
- 4. $u\bar{d}$ plus $d\bar{u}$, corresponding to the W^+W^- channel.

for **Tevatron**

 $\sqrt{s} = 1.8 \div 1.96 \,\mathrm{TeV}$

and LHC energies $\sqrt{s}=7\,{
m TeV}$

CDF: 20 GeV photon & jet plus a pair of 5 GeV jets



DO: effective interaction area for 70 GeV photon & jet as a function of the transverse momentum of jets in an additional jet pair



mild squeezing with hardness increasing, consistent with data

LHC: two pairs of (back-to-back) 50 GeV jets



pQCD correlation: local, increases in fwd regions

LHC: dependence on the hardnesses of 4-gluon collisions $\mathbf{1}+\mathbf{R}$



Peculiarities of the 1×2 MPI mechanism

is bound to bring in a visible **x**-*dependence* of $\sigma_{
m eff}$

in particular, in "pre-forward" kinemo ($x_1, x_2 \gg x_3, x_4$) where 1×2 is *large*

- should cause *asymmetry in rapidity* of accompanying multiplicity density
- introduces *specific correlation* between jet-pair *transverse momentum imbalances*

punchline

- Multi-parton collisions contribute substantially to 4 jet production in the back-to-back kinematics
- 2 x 2 and 1 x 2 parton subprocesses are both enhanced in the back-to-back region, while "double perturbative parton splittings" generate effectively 1 x 1, which is not
- To describe multi-parton collisions one has to introduce and explore a new object – Generalized Double-Parton Distributions

$${}_{[2]}D_h^{a,b}(x_1,x_2;q_1^2,q_2^2;\vec{\Delta})$$

the parameter $\vec{\Delta}$ encodes the information about the impact-parameter-space correlation between the two partons from one hadron

- experimentally observed enhancement of a 4-jet cross section indicates the presence of short range two-parton correlations in the nucleon parton wave function, as determined by the range of integral over $\vec{\Delta}$
- I'd rather experimental studies employed QCD-motivated jet finding algorithms and concentrated on correlations in transverse momenta rather than angles

A new subject

Theoretically complicated **Experimentally challenging**

Conclusions

Theorists:

think harder

Q: can one get away within the *probabilistic picture*, in some approximation, or interferences ("cross-talk") are unavoidable ?

MPI

MC builders : think twice

Q: how do you make sure that the two partons originate, space-time-wise, from one and the same hadron?

Experimenters: do it (but mind your head, now and then) A.