New Limit of the $AdS_5 \times S^5$ Sigma Model

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and Work in progress with Cumrun Vafa

Pure spinor formalism can be used to quantize superstring in $AdS_5 \times S^5$ background with manifest $PSU(2,2|4)$ symmetry.

Cohomology arguments imply quantum theory is free of anomalies and has infinite set of non-local conserved currents. Recent support comes from perturbative computations of Mikhailov + Schäfer-Nameki (hep-th/0706.1525).

But pure spinor sigma model is just as complicated as Metsaev-Tseytlin GS sigma model, and computations are cumbersome.

Is there some limit which preserves $PSU(2,2|4)$ in which the pure spinor sigma model simplifies?

Yes! Consider pure spinor version of superparticle on $AdS_5 \times S^5$

($\alpha' \to 0$ limit of superstring)

Superparticle worldline action can be described by $\alpha' \to 0$ limit of a topological A-model with manifest $N=(2,2)$ worldsheet susy and global $PSU(2,2|4)$ invariance.

Topological A-model is constructed from 16 chiral and antichiral superfields

$$\Theta^\alpha = \theta^\alpha + \kappa Z^\alpha + \ldots$$

$$\bar{\Theta}^\alpha = \bar{\theta}^\alpha + \bar{\kappa} \bar{Z}^\alpha + \ldots$$

whose lowest components are $(\theta^\alpha, \bar{\theta}^\alpha)$. Bosonic superpartners $(Z^\alpha, \bar{Z}^\alpha)$ are twistor-like combinations of 10 x's of $AdS_5 \times S^5$ with 22 pure spinor ghosts.

Action is a gauged linear sigma model based on coset $\dfrac{PSU(2,2|4)}{SU(2,2) \times SU(4)}$.

The open string sector of this topological A-model appears to describe $N=4$ $d=4$ super-Yang-Mills.

There are many similarities with Gopakumar-Vafa duality which relates the closed topological A-model for resolved conifold with the open topological A-model for $d=3$ Chern-Simons.

Perhaps the worldsheet proof of Ooguri + Vafa for this open-closed duality can be extended to the Maldacena conjecture using the topological A-model described here?

Outline of Talk

1) Pure spinor superparticle in flat background

2) Pure spinor superparticle in $AdS_5 \times S^5$ background

3) Topological A-model based on $\frac{PSU(2,2|4)}{SU(2,2) \times SU(4)}$ coset

4) Open string sector and $N=4$ $d=4$ super-Yang-Mills

5) Similarities with open-closed duality of Gopakumar-Ooguri-Vafa

I. Superparticle in flat background

Brink-Schwarz superparticle with N=2 d=10 susy:

$$S = \int d\tau \left(\Pi^m P_m + e P^m P_m \right)$$

$$\Pi^m = \dot{x}^m - i\,\theta^\alpha_{(1)} \gamma^m_{\alpha\beta} \dot{\theta}^\beta_{(1)} - i\,\theta^\alpha_{(2)} \gamma^m_{\alpha\beta} \dot{\theta}^\beta_{(2)}$$

$$= \dot{x}^m - i\,\theta^\alpha \gamma^m_{\alpha\beta} \dot{\bar{\theta}}^\beta - i\,\bar{\theta}^\alpha \gamma^m_{\alpha\beta} \dot{\theta}^\beta$$

$$\theta^\alpha = \frac{1}{\sqrt{2}}\left(\theta^\alpha_{(1)} + i\,\theta^\alpha_{(2)}\right), \quad \bar{\theta}^\alpha = \frac{1}{\sqrt{2}}\left(\theta^\alpha_{(1)} - i\,\theta^\alpha_{(2)}\right)$$

$m = 0$ to 9, $\alpha = 1$ to 16, $\gamma^m_{\alpha\beta} = \gamma^m_{\beta\alpha}$

$p_\alpha \equiv \frac{\partial L}{\partial \dot{\theta}^\alpha}$ is not independent $\Rightarrow$ Dirac constraints

$$d_\alpha \equiv p_\alpha - i P_m (\gamma^m \bar{\theta})_\alpha \sim 0$$

$$\bar{d}_\alpha \equiv \bar{p}_\alpha - i P_m (\gamma^m \theta)_\alpha \sim 0$$

$\{ d_\alpha, \bar{d}_\beta \} = -2i P_m \gamma^m_{\alpha\beta} \Rightarrow$ 16 first-class and 16 second-class

Quantization in light-cone gauge implies Type IIB supergravity spectrum described by superfield $\Phi(x^m, \theta^a)$ with $a = 1$ to 8

Pure spinor superparticle has two major differences with Brink-Schwarz superparticle:

1) p_α and $\bar{p}_\alpha$ are independent variables.

2) κ-symmetry and reparam. inv. are "gauge-fixed" using "BRST" operator constructed from pure spinor bosonic ghosts $\lambda^\alpha, \bar{\lambda}^\alpha$.

$$S_{\substack{\text{pure}\\ \text{spinor}}} = \int d\tau \left(\dot{x}^m P_m + \dot{\theta}^\alpha p_\alpha + \dot{\bar{\theta}}^\alpha \bar{p}_\alpha - \frac{1}{2} P^m P_m + \dot{\lambda}^\alpha \omega_\alpha + \dot{\bar{\lambda}}^\alpha \bar{\omega}_\alpha \right)$$

$$= \int d\tau \left(\Pi^m P_m + \dot{\theta}^\alpha d_\alpha + \dot{\bar{\theta}}^\alpha \bar{d}_\alpha - \frac{1}{2} P^m P_m + \dot{\lambda}^\alpha \omega_\alpha + \dot{\bar{\lambda}}^\alpha \bar{\omega}_\alpha \right)$$

$$d_\alpha \equiv p_\alpha - i P_m (\gamma^m \bar{\theta})_\alpha \,, \quad \bar{d}_\alpha \equiv \bar{p}_\alpha - i P_m (\gamma^m \theta)_\alpha$$

$$\lambda^\alpha = \frac{1}{\sqrt{2}} \left(\lambda^\alpha_{(1)} + i \lambda^\alpha_{(2)} \right) \text{ and } \bar{\lambda}^\alpha = \frac{1}{\sqrt{2}} \left(\lambda^\alpha_{(1)} - i \lambda^\alpha_{(2)} \right)$$

where $\lambda^\alpha_{(1)}$ and $\lambda^\alpha_{(2)}$ are $d=10$ "pure spinors" satisfying

$$\boxed{\lambda^\alpha_{(1)} \gamma^m_{\alpha\beta} \lambda^\beta_{(1)} = \lambda^\alpha_{(2)} \gamma^m_{\alpha\beta} \lambda^\beta_{(2)} = 0}$$

Pure spinor condition $\lambda_{(1)}\gamma^m\lambda_{(1)} = \lambda_{(2)}\gamma^m\lambda_{(2)} = 0$ implies that $(\lambda^\alpha_{(1)}, \lambda^\alpha_{(2)})$ have 22 indep. components and that 10 components of $(\omega_{(1)\alpha}, \omega_{(2)\alpha})$ can be gauged away using

$$\delta\omega_{(1)\alpha} = \Lambda_m(\gamma^m\lambda_{(1)})_\alpha$$
$$\delta\omega_{(2)\alpha} = \tilde{\Lambda}_m(\gamma^m\lambda_{(2)})_\alpha .$$

Physical states are defined by cohomology at ghost-number +2 of BRST operator

$$\boxed{Q = \lambda^\alpha d_\alpha + \bar{\lambda}^\alpha \bar{d}_\alpha}$$

Q is nilpotent since

$$Q^2 = \lambda^\alpha \bar{\lambda}^\beta \{d_\alpha, \bar{d}_\beta\} = (\lambda\gamma^m\bar{\lambda})\, P_m$$
$$= \tfrac{1}{2}(\lambda_{(1)}\gamma^m\lambda_{(1)} + \lambda_{(2)}\gamma^m\lambda_{(2)})\, P_m = 0$$

Ghost-number +2 $\Rightarrow$ $V = \lambda^\alpha\bar{\lambda}^\beta A_{\alpha\beta}(x,\theta,\bar{\theta})$

$QV = 0$ and $\delta V = Q\Omega$ $\Rightarrow$ $A_{\alpha\beta}(x,\theta,\bar{\theta})$ describes linearized on-shell Type IIB supergravity

$$A_{\alpha\beta}(x,\theta,\bar{\theta}) = (g_{mn}(x) + b_{mn}(x) + \eta_{mn}\varphi(x))(\gamma^m\theta)_\alpha(\gamma^n\bar{\theta})_\beta + \dots$$

Superparticle action has 32 bosons $(x^m, \lambda^\alpha, \bar{\lambda}^\alpha)$ and 32 fermions $(\theta^\alpha, \bar{\theta}^\alpha)$.

$Q^2 = 0$ implies that Q can be interpreted as a (twisted) $N=2$ worldline susy generator. But in a flat background, it is difficult to construct a second worldline susy generator b satisfying $\{Q, b\} = H$.

However, in an $AdS_5 \times S^5$ background, both $N=2$ generators can be easily constructed and $N=2$ worldline susy can be made manifest! Worldline action can be interpreted as $\alpha' \to 0$ limit of topological A-model.

Similar to "doubly-supersymmetric" superembedding approach of Sorokin, Tonin, et al. But unlike superembedding approach, this model is easy to quantize.

II. Superparticle in $AdS_5 \times S^5$ background

As in Metsaev-Tseytlin, describe $AdS_5 \times S^5$ superspace using supercoset $g(x,\theta,\bar\theta) \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$

$$g(x,\theta,\bar\theta) = e^{\theta^\alpha Q_\alpha + \bar\theta^\alpha \bar Q_\alpha} e^{x^m P_m}$$

where $(P_m, Q_\alpha, \bar Q_\alpha)$ generate AdS susy algebra

g is identified with $g\,\Omega$ where $\Omega \in SO(4,1)\times SO(5)$

$g \to \Sigma\, g$ under $PSU(2,2|4)$ isometries of Σ.

$$g = \left(\begin{array}{c|c} g^A{}_B & g^A{}_J \\ \hline g^J{}_A & g^J{}_K \end{array} \right)$$

$A = 1$ to 4 is $SU(2,2)$ index

$J = 1$ to 4 is $SU(4)$ index

To lowest order, $g^A{}_B = \frac{SU(2,2)}{SO(4,1)} = AdS_5$

$$g^J{}_K = \frac{SU(4)}{SO(5)} = S^5$$

$$g^A{}_J = \theta^A{}_J = \theta^\alpha$$

$$g^J{}_A = \bar\theta^J{}_A = \bar\theta^\alpha$$

Convenient to define PSU(2,2|4)-invariant currents

$$J = g^{-1} \frac{\partial}{\partial \tau} g$$

Bosonic currents $J^A{}_B$ and $J^J{}_K$ decompose as

$$J^A{}_B = J^d (\sigma_d)^A{}_B + J^{[de]} (\sigma_{[de]})^A{}_B \qquad d = 0 \text{ to } 4$$

$$J^J{}_K = J^{d'} (\sigma_{d'})^J{}_K + J^{[d'e']} (\sigma_{[d'e']})^J{}_K \qquad d' = 5 \text{ to } 9$$

where $(\sigma_d)^A{}_B$ are SO(4,1) Pauli matrices and $(\sigma_{d'})^J{}_K$ are SO(5) Pauli matrices.

Brink-Schwarz superparticle action:

$$S = \int d\tau \left[J^m P_m + e P^m P_m \right]$$

$J^m = J^d$ when $m = 0$ to 4

$J^m = J^{d'}$ when $m = 5$ to 9

Pure spinor superparticle action:

$$S = \int d\tau \left[J^m P_m - \tfrac{1}{2} P^m P_m + J^\alpha d_\alpha + \bar{J}^\alpha \bar{d}_\alpha + w_\alpha \dot{\lambda}^\alpha + \bar{w}_\alpha \dot{\bar{\lambda}}^\alpha + \ldots \right]$$

where ... is determined by BRST invariance.

In $AdS_5 \times S^5$ background with R-R flux $F^{\alpha\beta}$,

$$\cdots = F^{\alpha\beta} d_\alpha \bar{d}_\beta + N^{mn} J_{[mn]} + R_{mnpq} N^{mn} N^{pq}$$

$$= (\gamma_{01234})^{\alpha\beta} d_\alpha \bar{d}_\beta + N^{mn} J_{[mn]} + N^{cd} N_{cd} - N^{c'd'} N_{c'd'}$$

where $N^{mn} = \omega \sigma^{mn} \lambda + \bar{\omega} \sigma^{mn} \bar{\lambda}$

After integrating out d_α, $\bar{d}_\alpha$ and P_m using their auxiliary equations of motion,

$$S_{\substack{\text{pure}\\ \text{spinor}}} = \int d\tau \Big[\tfrac{1}{2} J^m J_m + J^A{}_J J^J{}_A$$

$$+ w^J{}_A \dot{\lambda}^A{}_J + \bar{w}^A{}_J \dot{\bar{\lambda}}^J{}_A$$

$$+ N^{mn} J_{[mn]} + N^{cd} N_{cd} - N^{c'd'} N_{c'd'} \Big]$$

$$w^J{}_A = \omega_\alpha \quad , \quad \bar{w}^A{}_J = \bar{\omega}^\alpha$$

$$\lambda^A{}_J = \lambda^\alpha \quad , \quad \bar{\lambda}^J{}_A = \bar{\lambda}^\alpha$$

Although $S_{\substack{\text{pure}\\ \text{spinor}}}$ looks complicated, it can be simplified by performing a field-redefinition which combines 10 x's with 22 λ's.

III. Topological A-Model

To define new variables, first parameterize supercoset $g(x,\theta,\bar\theta) = e^{\theta^\alpha Q_\alpha + \bar\theta^\alpha \bar Q_\alpha} e^{x^c P_c} e^{x^{c'} P_{c'}}$

as $g(x,\theta,\bar\theta) = G(\theta,\bar\theta)\, H(x^c)\, \tilde H(x^{c'})$

$G(\theta,\bar\theta) \in \dfrac{PSU(2,2|4)}{SU(2,2)\times SU(4)}$, $H(x^c) \in \dfrac{SU(2,2)}{SO(4,1)}$

$\tilde H(x^{c'}) \in \dfrac{SU(4)}{SO(5)}$.

So x^m variables are described by bosonic cosets $H^A{}_B$ for AdS_5 and $\tilde H^J{}_K$ for S^5.

To combine $(\lambda^\alpha, w_\alpha)$ and $(\bar\lambda^\alpha, \bar w_\alpha)$ variables with x^m, define new <u>unconstrained</u> variables

$$Z^A{}_J = H^A_B(x^c)\, \tilde H^K_J(x^{c'})\, \lambda^B{}_K$$

$$\bar Z^J{}_A = (H^{-1}(x^c))^B{}_A\, (\tilde H^{-1}(x^{c'}))^J{}_K\, \bar\lambda^K{}_B$$

$$Y^J{}_A = (H^{-1}(x^c))^B{}_A\, (\tilde H^{-1}(x^{c'}))^J{}_K\, w^K{}_B$$

$$\bar Y^A{}_J = H^A_B(x^c)\, \tilde H^K_J(x^{c'})\, \bar w^B{}_K$$

In terms of fermionic coset $G(\theta, \bar\theta) \in \frac{PSU(2,2|4)}{SU(2,2)\times SU(4)}$

and <u>unconstrained</u> bosonic variables

$(Z^A{}_J, \bar Z^J{}_A, Y^J{}_A, \bar Y^A{}_J)$, worldline action is

$$S = \int d\tau \Big[(G^{-1}\dot G)^A{}_J (G^{-1}\dot G)^J{}_A + Y^J{}_A \dot Z^A{}_J + \bar Y^A{}_J \dot{\bar Z}^J{}_A$$

$$+ (G^{-1}\dot G)^A{}_B N^B{}_A + (G^{-1}\dot G)^J{}_K N^K{}_J$$

$$+ N^A{}_B N^B{}_A - N^J{}_K N^K{}_J \Big]$$

where $N^A{}_B = Z^A{}_J Y^J{}_B + \bar Y^A{}_J \bar Z^J{}_B$ are $SU(2,2)$ generators

$N^J{}_K = Y^J{}_A Z^A{}_K + \bar Z^J{}_A \bar Y^A{}_K$ are $SU(4)$ generators

and BRST operator is

$$Q = \lambda^\alpha d_\alpha + \bar\lambda^{\dot\alpha} \bar d_{\dot\alpha} = Z^A{}_J (G^{-1}\dot G)^J{}_A + \bar Z^J{}_A (G^{-1}\dot G)^A{}_J .$$

Second $N=2$ worldline susy generator is

$$b = Y^J{}_A (G^{-1}\dot G)^A{}_J + \bar Y^A{}_J (G^{-1}\dot G)^J{}_A$$

which satisfies $\{Q, b\} = H$.

Action can be written in $N=(2,2)$ superspace by combining $(\theta^A{}_J, Z^A{}_J, \bar{Y}^A{}_J)$ into chiral $N=(2,2)$ superfield $\Theta^A{}_J(\kappa, \kappa')$ as

$$\Theta^A{}_J(\kappa,\kappa') = \theta^A{}_J + \kappa Z^A{}_J + \kappa' \bar{Y}^A{}_J + \kappa\kappa' F^A{}_J$$

$$\bar{\Theta}^J{}_A(\bar{\kappa},\bar{\kappa}') = \bar{\theta}^J{}_A + \bar{\kappa} \bar{Z}^J{}_A + \bar{\kappa}' Y^J{}_A + \bar{\kappa}\bar{\kappa}' \bar{F}^J{}_A$$

where $F^A{}_J$ and $\bar{F}^J{}_A$ are auxiliary fields.

$N=(2,2)$ sigma model action is

$$S = \int d\tau \int d^2\kappa\, d^2\bar{\kappa}\; Tr\left[\log(1+\bar{\Theta}\Theta)\right].$$

Can trivially be generalized to A-model by allowing $\Theta^A{}_J$ and $\bar{\Theta}^J{}_A$ to depend on $(z,\bar{z})$.

$$S = \int dz\, d\bar{z} \int d^2\kappa\, d^2\bar{\kappa}\; Tr\left[\log(1+\bar{\Theta}\Theta)\right].$$

BRST operator transforms

$Q\,\theta^A{}_J = Z^A{}_J$ and $Q\,\bar{\theta}^J{}_A = \bar{Z}^J{}_A$,

so model has "A-twist".

Although action has $PSU(2,2|4)$ invariance, only bosonic subgroup $SU(2,2)\times SU(4)$ is manifest in non-linear sigma model action

$$\delta \textcircled{H}^A{}_J = \Omega^A{}_B \textcircled{H}^B{}_J + \Omega^K{}_J \textcircled{H}^A{}_K$$
$$+ \Omega^A{}_J + \textcircled{H}^A{}_K \Omega^K{}_B \textcircled{H}^B{}_J$$

To make $PSU(2,2|4)$ invariance manifest, introduce $U(4)$ gauge superfield V^J_K and chiral and antichiral scalar superfields $\Phi^J_K(\kappa,\kappa')$ and $\bar{\Phi}^J_K(\bar{\kappa},\bar{\kappa}')$. Action for gauged linear sigma model is

$$S = \int dz\, d\bar{z} \int d^2\kappa\, d^2\bar{\kappa}\; \mathrm{Tr}\left[\bar{\Phi} e^V \Phi + \bar{\textcircled{H}} e^V \textcircled{H} + t V\right]$$

where $\int d^2\kappa\, d^2\bar{\kappa}\; t\, \mathrm{Tr}\, V$ is Fayet-Illiopoulis term.

$$\delta \textcircled{H}^A{}_J = \Omega^A{}_B \textcircled{H}^B{}_J + \Omega^K{}_J \textcircled{H}^A{}_K + \Omega^A{}_K \Phi^K{}_J$$

In gauge $\Phi^J_K = \bar{\Phi}^J_K = \delta^J_K$, S reduces to

$$S = t \int dz\, d\bar{z} \int d^2\kappa\, d^2\bar{\kappa}\; \mathrm{Tr}\left[\log\left(1 + \bar{\textcircled{H}} \textcircled{H}\right)\right]$$

(M. Roček, private communication)

When $t \to \infty$, recover worldline action.

IV. Open string sector and N=4 d=4 sYM

For A-model, open string boundary conditions are $\Theta^A_J = \varepsilon^{AB}\delta_{JK}\bar{\Theta}^K_B$ at $z=\bar{z}$, $\kappa=\bar{\kappa}$, $\kappa'=\bar{\kappa}'$

where ε^{AB} is antisymmetric $Sp(4)$ metric and δ_{JK} is symmetric $SO(4)$ metric.

Boundary conditions break $PSU(2,2|4)$ to $OSp(4|4)$ which are symmetries of N=4 d=4 sYM on AdS_4.

Worldline action for Θ^A_J can be related to dimensional reduction on AdS_4 of N=1 d=10 pure spinor superparticle which describes super-Yang-Mills.

$$S_{N=1} = \int d\tau\,(\dot{x}^m P_m - \tfrac{1}{2}P^m P_m + \dot{\theta}^\alpha p_\alpha + \dot{\lambda}^\alpha w_\alpha)$$

Dimensional reduction: $\theta^\alpha \to \theta^A_J$, $\lambda^\alpha \to \lambda^A_J$ $\quad A=(\alpha,\dot{\alpha})$

$p_\alpha \to p_A^J$, $w_\alpha \to w_A^J$ $\quad J=1$ to 4

Just as $V = \lambda^\alpha \bar{\lambda}^\beta A_{\alpha\beta}(x,\theta,\bar{\theta})$ describes Type II sugra for $N=2$ $d=10$ superparticle, $V = \lambda^\alpha A_\alpha(x,\theta)$ describes super-Yang-Mills for $N=1$ $d=10$ superparticle. $N=1$ $d=10$ super-Yang-Mills action can be written in Chern-Simons-like form as

$$S = \frac{1}{g^2}\langle VQV + \tfrac{2}{3}V^3 \rangle$$ where

$Q = \lambda^\alpha d_\alpha$ and $\lambda^\alpha \gamma^m_{\alpha\beta} \lambda^\beta = 0$.

After dim. reduction to AdS_4, 16 bosonic variables are $H^A{}_B(x) \in \frac{SO(3,2)}{SO(3,1)}$ for AdS_4 and 12 pure spinor variables $\lambda^A{}_J, \omega_A{}^J$ satisfying $\lambda^a{}_J \lambda^{\dot{a}}{}_K \delta^{JK} = 0$. $A = (a, \dot{a})$, $a, \dot{a} = 1$ to 2.

Pure spinor variables are related to $\Theta^A{}_J$ by

$$\Theta^A{}_J = \theta^A{}_J + \kappa Z^A{}_J + \kappa' Y^A{}_J + \kappa\kappa' F^A{}_J$$

where $Z^A{}_J = H^A{}_B(x)\lambda^B{}_J$, $Y^A{}_J = (H^{-1}(x))^A{}_B\, \omega^B{}_J$

(See hep-th/0703282 for more details)

V. Conclusions and Open Questions

- Superparticle on $AdS_5 \times S^5$ is described by $\alpha' \to 0$ limit of closed topological A-model. 10 x's and 22 λ's combine to form worldsheet superpartners of 32 θ's. Action is linear gauged sigma model based on coset $\frac{PSU(2,2|4)}{SU(2,2) \times SU(4)}$.

- Open string sector of A-model describes $N=4$ $d=4$ super-Yang-Mills on AdS_4. Super-Yang-Mills action has Chern-Simons-like form $\mathcal{S} = \langle VQV + \frac{2}{3} V^3 \rangle$.

Questions:

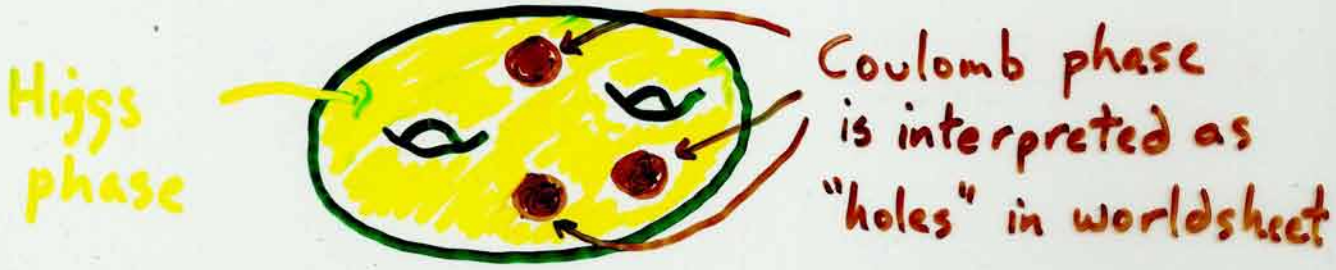
- Does closed A-model only describe $\alpha' \to 0$ limit of superstring (BPS states) or does it describe more?

- Can open-closed duality of Maldacena conjecture be proven using methods of Gopakumar-Ooguri-Vafa?

Gopakumar-Vafa ('99) and Ooguri-Vafa ('02) proved open-closed duality for resolved conifold and $d=3$ Chern-Simons using Higgs and Coulomb phases of gauged linear sigma model.

Resolved conifold described by closed A-model
Chern-Simons described by open A-model

Near $t=0$, both sides of duality can be computed perturbatively. To prove equivalence, compute closed amplitudes for resolved conifold using gauged linear sigma model with F.I. term $t\int d^4\kappa \,\mathrm{Tr}\, V$.



Near $t=0$, both Higgs and Coulomb phases exist. Using 't Hooft's large N prescription, Riemann surface can be interpreted as Feynman diagram in Chern-Simons gauge theory where "holes" become "faces" in the diagram.