Standard Model vacuum stability with a 125 GeV Higgs

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Outline

1. Standard Model vacuum stability
2. NNLO analysis: the gruesome details
3. NNLO analysis: the colorful plots

Outline

SM symmetry-breaking sector

Higgs potential

$$V(\phi) \sim \Lambda^4 - \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + Y_{ij} \bar{\psi}^i_L \psi^j \Phi + \frac{g^{ij}}{\Lambda} \psi^i_L \psi^{jT}_L \Phi \Phi^T$$

- Cosmological constant problem *(worst fine tuning problem ever!)*
- Quadratic sensitivity to regularization cut-off *(f.t. again... is it a true problem?)*
- Quadratic sensitivity to heavy dof's when matching onto UV theory *(do heavy dof's exist?)*
- Vacuum instability at large field values if $\lambda < 0 \leftrightarrow M_h$
- Loss of perturbativity if $\lambda > 4\pi \leftrightarrow M_h$
- SM flavor problem + M_ν:
  - large unexplained hierarchy $M_t/M_e \sim 3 \times 10^5$
  - $U(3)^5_F \xrightarrow[Y_{ij}]{} U(1)_B \otimes U(1)^{(3)}_L$

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The effective potential: single real scalar (1)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\,, \quad V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

- Minimum of $V(\phi)$ gives $\phi_c \equiv \langle\phi\rangle$ at the classical level
- we consider fluctuations around the minimum, $\phi \to \phi_c + \phi$
- $V(\phi)$ gives the lowest order (classical) 1PI vertices and propagator

Quantum corrections? [Coleman and E.Weinberg]

- V_{eff} is the order-zero term in the derivative expansion of the *effective action* (gen. of full 1PI functions)
- For constant ϕ_c, min of $V_{\text{eff}}(\phi)$ gives $\phi_c \equiv \langle\phi\rangle$, the true quantum minimum (constant $\leftrightarrow$ we don't want to break Poincaré)

The effective potential: single real scalar (2)

1-loop computation [Coleman and E.Weinberg, Jackiw] and renormalization (e.g. $\overline{\mathrm{MS}}$ or OS, μ is the 't Hooft mass or the subtraction point):

$$V_{\mathrm{eff}}(\phi_c) = \frac{m^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4 + \frac{(m^2 + 3\lambda\phi_c^2)^2}{64\pi^2} \ln \frac{m^2 + 3\lambda\phi_c^2}{\mu^2}$$

Consider e.g. $m^2 = 0$:

- $V(\phi) = \frac{\lambda}{4}\phi^4 \Rightarrow \phi = 0$ (min)
- $V_{\mathrm{eff}}(\phi_c) = \frac{\lambda}{4}\phi^4 + \frac{9\lambda^2\phi_c^4}{64\pi^2} \ln \frac{\phi_c^2}{\mu^2} \Rightarrow \begin{cases} \phi_c = 0 & \text{max} \\ \phi_c : \lambda \ln \frac{\phi_c}{\mu} \sim -\frac{8}{9}\pi^2 & \text{min} \end{cases}$

The min condition is for $\lambda \ln \frac{\phi_c}{\mu} \sim \mathcal{O}(1)$, but higher orders contribute to V_{eff} as $\lambda(\lambda \ln \frac{\phi_c}{\mu})^n$. A weapon: $\frac{dV_{\mathrm{eff}}}{d\mu} = 0 \Rightarrow$ resum logs with RGE

The SM effective potential

$$V_{\text{eff}}^{\text{RGI}}(\phi) \simeq \frac{m^2(\mu)}{2}\phi(\mu)^2 + \frac{\lambda(\mu)}{4}\phi(\mu)^4 \xrightarrow[\phi \gg v]{} \frac{\lambda(\mu)}{4}\phi(\mu)^4$$

- The choice $\mu \sim \phi$ helps minimizing the large logs
- The shape of $V_{\text{eff}}^{\text{RGI}}$ crucially depends on the running of λ

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\left[\overbrace{+24N_c\lambda^2}^{\text{scalar loop}} + \overbrace{\lambda(4N_cY_t - 9g^2 - 3g'^2)}^{\text{ext. leg corrections}} \underbrace{\overbrace{-2N_cY_t^4}^{\text{fermion loop}} + \overbrace{\frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2g'^2}^{\text{gauge bosons loop}}}_{\equiv B<0 \text{ at EW scale}} + \dots\right]$$

If $B = const$, V_{eff} unbounded from below at large ϕ, but B runs too!!

A few possibilities

- $B \sim 0$, M_h large: *Landau pole* (or triviality problem: probably consistent continuum limit for ϕ^4 theory $\Leftrightarrow \lambda_R = 0$)
- $B < 0$ at weak scale but does not run negative enough at large ϕ: V_{eff} *bounded from below* (SM vacuum *stable*)
- $B < 0$ at weak scale enough to stay negative at large ϕ: V_{eff} *unbounded from below* (SM vacuum *unstable*, need NP)
- $B < 0$ at weak scale but flips sign at large ϕ: V_{eff} develops *another min* (degenerate or lower) (SM vacuum *metastable*)

- All SM parameters known
- Assume no NP below M_{Pl}
- 3-loop RGE

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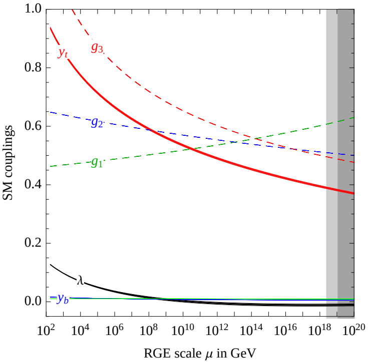

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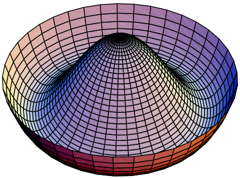
Beware of the dog bowl!

from A. Strumia

Illustrative

$\lambda(\mu) > 0$ up to M_{Pl}, i.e. stable

very unstable



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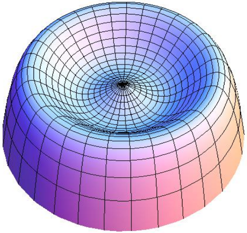
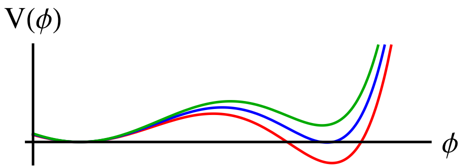


If your mexican hat turns out to be a dog bowl you have a problem...

Metastability



- ϕ_{EW} can be a false vacuum $\rightarrow$ quantum tunneling [Coleman; Callan, Coleman]
- compute bounce solution for Euclidean action ($\sim$ WKB)
- tunneling p $\sim \frac{\tau_U^4}{R^4} e^{-S_B(R)}$ for a bounce of size R, $S_B(R) = \frac{8\pi^2}{3\lambda(R^{-1})}$
- dominated by bounce that maximizes the action, i.e. $\beta_\lambda(R^{-1}) = 0$
- this scenario still ok if $\tau_{EW} \gg \tau_U$
- SM: p $\sim \left(\frac{e^{140}}{RM_{Pl}}\right)^4 e^{-\frac{2600}{|\lambda|/0.01}} \ll 1$ [Isidori, Ridolfi, Strumia 01]
- higher dim. operators (e.g. Planck scale physics) could change the transition probability [Branchina, Messina 13]

Analysis strategy

1) compute V_{eff} at n-loop level (not just $\lambda(\mu)\phi(\mu)^4/4$)

but one can't trust it at large field values, even in λ stays perturbative

2) improve it with $(n+1)$-loop beta-functions

now we can trust $V_{\text{eff}}^{\text{RGI}}$ up to large scale since λ stays perturbative

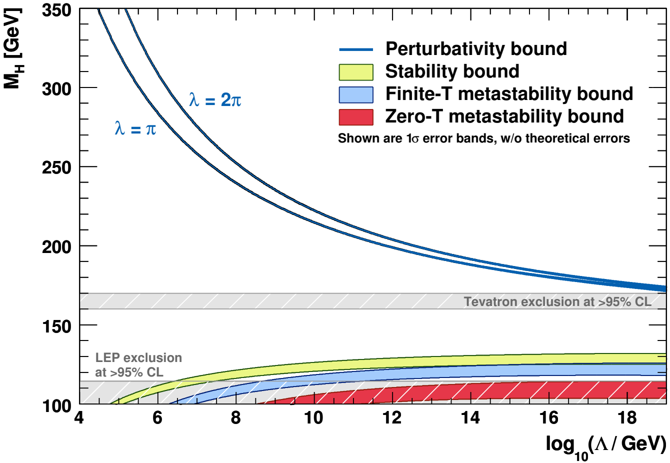
3) but ... how much are λ, y_t at Λ_{EW}? we know m_H, m_t!

$(n+1)$-loop running up to M_{Pl}, requires at least n-loop matching, can't use just the tree level $\lambda = G_\mu m_H^2/\sqrt{2}$ and $y_t^2 = 4G_\mu m_t^2/\sqrt{2}$

- lower and upper bound on m_h by requiring (meta)stability and perturbativity up to some scale Λ_I [pre-Higgs times, either H or NP ...]
- instability scale Λ_I as a function of m_h or m_t [gauge dependence ...]
- SM phase diag. in (m_h, m_t) plane: stable up to M_{Pl}? $\tau_{EW} \lesseqgtr \tau_U$?

Higgs mass bounds at NLO in 2009

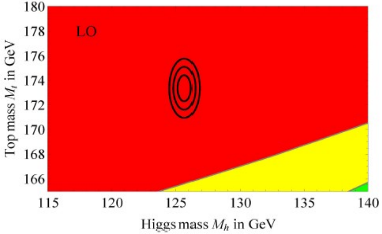
$$M_t = 173.1 \pm 1.3\,\mathrm{GeV}$$
$$\alpha_s(M_Z) = 0.1193 \pm 0.0028$$

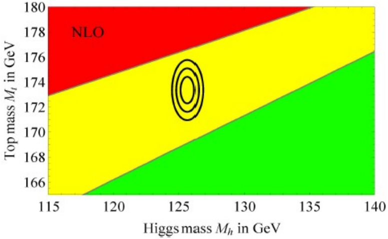


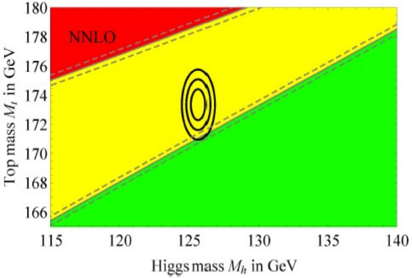
one-loop V_{eff} | two-loop running | one-loop matching [Ellis *et al.*09]

SM phase diagram: LO vs NLO vs NNLO

instability	$V_{\text{eff}} < 0$ before M_{Pl}, $\tau_{EW} < \tau_U$
metastability	$V_{\text{eff}} < 0$ before M_{Pl}, $\tau_{EW} > \tau_U$
stability	$V_{\text{eff}} > 0$ up to M_{Pl}, i.e. stable







Espinosa

Outline

State of the art: SM vacuum stability at NNLO

- Complete two-loop effective potential [Ford, Jack, Jones 92,97; Martin 02]
  - known since a long time but needed three-loop β's for RG improvement now three-loop known! [Martin 13]
- Complete three-loop beta-functions
  - g_i [Mihaila, Salomon, Steinhauser 12]
  - $Y_{t,b,\tau}, \lambda, \mu$ [Chetyrkin, Zoller 12,13; Bednyakov, Pikelner, Velizhanin 13]
- Two-loop matching conditions at the weak scale (large th. err, especially λ)

	1-loop	2-loop	3-loop
$g_{1,2}$	full	?	–
y_t	full	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha_s^3)$
λ	full	$\mathcal{O}(\alpha\alpha_s, \alpha^2)$	–

$\mathcal{O}(\alpha\alpha_s)$ [Bezrukov, Kalmykov, Kniehl, Shaposhnikov 12; Degrassi, Elias-Mirò, Espinosa, Giudice, Isidori, Strumia, DV 12]

$\mathcal{O}(\alpha^2)$ [Degrassi, Elias-Mirò, Espinosa, Giudice, Isidori, Strumia, DV 12]

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[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 13]

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we don't measure $hh \to hh$, need another way of determining $\lambda(\mu)$ from a physical observable

- $V_{\text{eff}} \Rightarrow \lambda(\mu)_{m_H^2=0}$ contribution
- a full OS framework $\sim$ [Sirlin, Zucchini 86]

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at what μ do we match? source of th. uncertainty

- going up with loops reduces μ dependence
- check how much matching at different EW scales alters the running

Higgs potential OS renormalization (1) $\sim$ [Sirlin, Zucchini 86]

We want the NNLO corrections in $\lambda(\mu) = \frac{G_\mu M_h^2}{\sqrt{2}} + \lambda^{(1)}(\mu) + \lambda^{(2)}(\mu)$

1. $V(H) = -m^2|H|^2 + \lambda|H|^4, \quad H = \begin{pmatrix} G^\pm \\ (v + h + iG^0)/\sqrt{2} \end{pmatrix}$
2. shift the bare parameters (m, λ, v): $x \to x - \delta x \Rightarrow V = V_r - \delta V_r$

$$\begin{aligned} V_r &= \lambda_r \left[G^+ G^- \left(G^+ G^- + h^2 + G_0 \right)^2 + \frac{1}{4} \left(h^2 + G_0^2 \right)^2 \right] \\ &\quad + \lambda_r v_r\, h \left[h^2 + G_0^2 + 2\, G^+ G^- \right] + \frac{1}{2} M_h^2\, h^2 \ , \end{aligned}$$

$$\boxed{M_h^2 \equiv 2\lambda_r v_r^2}$$

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$$\begin{aligned}
\delta V \;\; = \;\; & \delta\lambda \left[G^+ G^- \left(G^+ G^- + h^2 + G_0 \right) + \frac{1}{4} \left(h^2 + G_0^2 \right)^2 \right] \\
& + \left[\lambda_r \left(\frac{\delta v^2}{2\, v_r} + \frac{(\delta v^2)^2}{8\, v_r^3} \right) + v_r\, \delta\lambda \left(1 - \frac{\delta v^2}{2\, v_r^2} \right) \right] h \left[h^2 + G_0^2 + 2\, G^+ G^- \right] \\
& + \delta\tau \left(\frac{1}{2} G_0^2 + G^+ G^- \right) + \frac{1}{2} \delta M_h^2 h^2 + v_r\, \delta\tau \left(1 - \frac{\delta v^2}{2\, v_r^2} \right) h \, .
\end{aligned}$$

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$$
\begin{aligned}
\delta M_h^2 &\equiv 3\left[\lambda_r \delta v^2 + v_r^2 \delta\lambda \left(1 - \frac{\delta v^2}{v_r^2}\right)\right] - \delta m^2 \,, \\
\delta\tau &\equiv \lambda_r \delta v^2 + v_r^2 \delta\lambda \left(1 - \frac{\delta v^2}{v_r^2}\right) - \delta m^2 \,, \\
\sqrt{v_r^2 - \delta v^2} &\equiv v_r - \delta v
\end{aligned}
$$

Higgs potential OS renormalization (2) $\sim$ [Sirlin, Zucchini 86]

3. impose 3 renormalization conditions

- **tadpole cancellation** $\delta\tau\left(1-\frac{\delta v^2}{2\,v_r^2}\right)=-\frac{T}{v_r}$ $\Rightarrow v$ min. of full $V_{\rm eff}$
- **on-shell Higgs mass** $\delta M_h^2=\mathrm{Re}\,\Pi_{hh}(M_h^2)$ $\Rightarrow M_H\equiv 125.14$ GeV
- fix δv^2 from μ-decay, requiring that $v_r^2=(\sqrt{2}G_\mu)^{-1}$ from

$$\frac{G_\mu}{\sqrt{2}}=\frac{1}{2v_0^2}\left\{1-\frac{A_{WW}}{M_{W_0}^2}+V_W+M_{W_0}^2B_W+\left(\frac{A_{WW}}{M_W^2}\right)^2-\frac{A_{WW}V_W}{M_W^2}\right\}$$

4. solve previous relations for $\delta\lambda$ and then exploit

$$\lambda_0=\underbrace{\lambda_r-\delta\lambda}_{\text{OS}}=\underbrace{\lambda(\mu)-\delta\hat{\lambda}}_{\overline{\rm MS}}\Rightarrow \lambda(\mu)=\frac{G_\mu}{\sqrt{2}}M_h^2-\delta\lambda+\delta\hat{\lambda}$$

$\delta\lambda$ and $\delta\hat{\lambda}$ have the same pole structure, once we express everything in $\overline{\rm MS}$ $\Rightarrow$ finite Δ

$\lambda(\mu)$ matching condition

at two-loop level $\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta\lambda^{(1)}|_{\rm fin} - \delta\lambda^{(2)}|_{\rm fin} + \Delta$

$$\delta\lambda^{(1)} = -\frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \frac{A_{WW}^{(1)}}{M_W^2} - E^{(1)} - \frac{1}{M_h^2}\left[\mathrm{Re}\,\Pi_{hh}^{(1)}(M_h^2) + \frac{T^{(1)}}{v_r} \right] \right\}$$

[Sirlin, Zucchini 86]

$$\begin{aligned}
\delta\lambda^{(2)} = & -\frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \frac{A_{WW}^{(2)}}{M_W^2} - E^{(2)} - \frac{1}{M_h^2}\left[\mathrm{Re}\,\Pi_{hh}^{(2)}(M_h^2) + \frac{T^{(2)}}{v_r} \right] \right. \\
& + \left(\frac{A_{WW}^{(1)}}{M_W^2} - E^{(1)} \right) \left(\frac{A_{WW}^{(1)}}{M_W^2} - E^{(1)} - \frac{1}{M_h^2}\left[\mathrm{Re}\,\Pi_{hh}^{(1)}(M_h^2) + \frac{3}{2}\frac{T^{(1)}}{v_r} \right] \right) \\
& \left. + \frac{A_{WW}^{(1)}\,\delta^{(1)} M_W^2}{M_W^4} - \left(\frac{A_{WW}^{(1)}}{M_W^2} \right)^2 + \frac{A_{WW}^{(1)} V_W^{(1)}}{M_W^2} + \delta^{(1)} M_W^2 B_W^{(1)} \right\} .
\end{aligned}$$

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Evaluate **analytically** the NNLO correction in the gauge-less approx., i.e. neglect $g_{1,2}$ (beware that $\frac{A_{WW}^{(2)}}{M_W^2}$ has a contribution $G_\mu m_t^2$!!)

[Degrassi, Elias-Mirò, Espinosa, Giudice, Isidori, Strumia, DV 12]

$$\begin{aligned}\left(\delta\lambda^{(2)} - \Delta\right)_{g.l.} = & -\frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \frac{A_{WW}{}^{(2)}}{M_W^2} - \frac{1}{M_h^2}\left[\mathrm{Re}\,\Pi_{hh}{}^{(2)}(M_h^2) + \frac{T^{(2)}}{v_r} \right] \right. \\ & \left. + \frac{A_{WW}{}^{(1)}}{M_W^2}\left(\frac{A_{WW}{}^{(1)}}{M_W^2} - \frac{1}{M_h^2}\left[\mathrm{Re}\,\Pi_{hh}{}^{(1)}(M_h^2) + \frac{3}{2}\frac{T^{(1)}}{v_r} \right] \right) \right\}_{g.l.} - \Delta_{g.l.}\,,\end{aligned}$$

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Full NNLO correction [Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 13]

- need vertex and box corrections to μ-decay
- need to evaluate W and H self-energies on-shell (hard!)
- several masses in the loops (not solved analytically for self-energies $\Rightarrow$ **numerical** approach, `TSIL` [Maritin, Robertson 05])

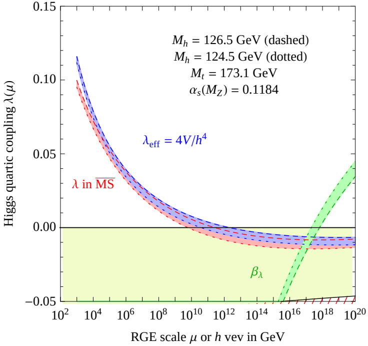
- Higgs tadpoles $\Rightarrow$ massive vacuum diagrams MVD
- W self-energies at $q^2 = m_W^2 = 0$ in the gauge-less limit $\Rightarrow$ MVD
- Higgs self-energies *on-shell* with scalar loops only $\Rightarrow$ Exact OS 1-scale propagators actually larger than y_t^6 contribution
- Higgs self-energies *on-shell* with top loops (~~thresholds~~) $\Rightarrow$ Taylor expand in $q^2 = M_h^2 \ll 4m_t^2$, MVD
- Higgs self-energies *on-shell* with top loops (thresholds) $\Rightarrow$ Asymptotic exp. for large m_t, MVD and 1-loop disc.

Outline

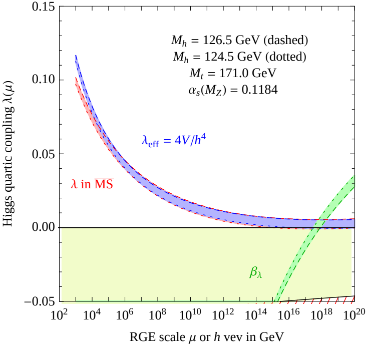
Stability and RGE evolution [Degrassi et al. 12]

At large ϕ

- one can approximate $\boxed{V_{\text{eff}} \simeq \lambda(\phi)\phi^4}$, but this means ignoring the non-logarithmic loop contrib still, it tells us that instability occurs around $10^{10} - 10^{11}$ GeV
- better: one can always write (choosing $\mu \sim \phi$), $\boxed{V_{\text{eff}} = \lambda_{\text{eff}}(\phi)\phi^4}$



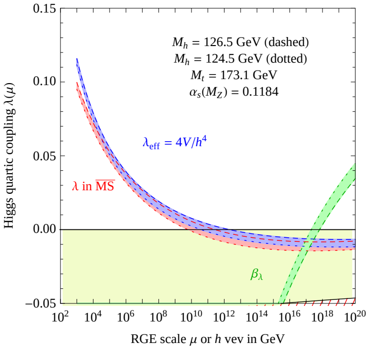
NNLO with prev. world average m_t



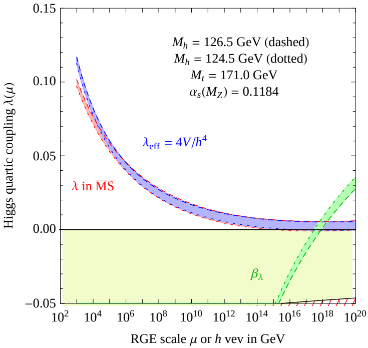
NNLO with m_t : $\lambda(M_{\text{Pl}}) = \beta_\lambda(M_{\text{Pl}})$

Stability and RGE evolution [Degrassi et al. 12]

- $\lambda(M_{Pl}) \lessgtr 0$ crucially depends on M_t no stability for central value. what about error bands?
- λ never runs too negative
- around M_{Pl} both λ and β_λ are ~ 0. any meaning? but no RGE fixed point

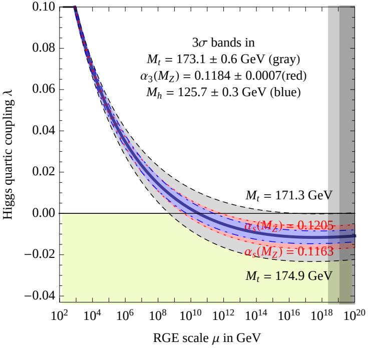


NNLO with prev. world average m_t



NNLO with $m_t : \lambda(M_{\rm Pl}) = \beta_\lambda(M_{\rm Pl})$

Stability condition and error budget [Degrassi et al. 12]

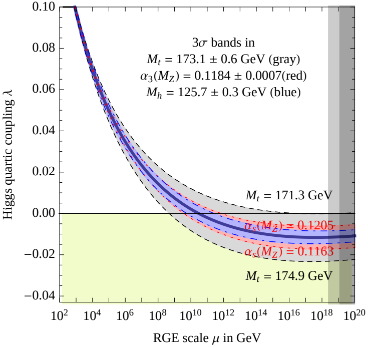


Err. Type	Err. estimate	Impact on M_h
M_t	expt. uncert. M_t	$\pm$1.4 GeV
α_s	expt. uncert. α_s	$\pm$0.5 GeV
Expt.	**Tot. combined in quadr.**	$\pm$1.5 GeV
λ	scale var. in λ	$\pm$0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\rm QCD})$ correction to M_t	$\pm$0.6 GeV
y_t	QCD threshold at 4 loops	$\pm$0.3 GeV
RGE	EW 3 loops + QCD 4 loops	$\pm$0.2 GeV
Theory	**Tot. combined in quadr.**	$\pm$1.0 GeV

SM absolute stability condition at NNLO

$$M_h\ [\text{GeV}] > 129.4 + 1.4\left(\frac{M_t\ [\text{GeV}]-173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z)-0.1184}{0.0007}\right) \pm 1.0_{\rm th}$$

Stability condition and error budget [Degrassi et al. 12]



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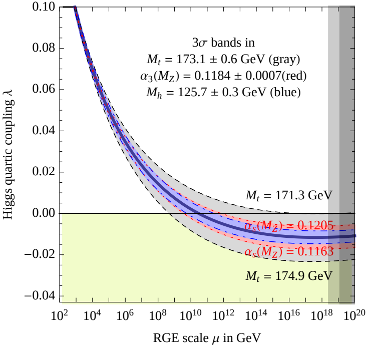
Theory	**Tot. combined in quadr.**	$\pm$1.0 GeV

NNLO shift w.r.t. NLO of about $+0.5\,\mathrm{GeV}$

$+\,0.6\,\mathrm{GeV}$ due to the QCD threshold corrections to λ;
$+\,0.2\,\mathrm{GeV}$ due to the Yukawa threshold corrections to λ;
$-\,0.2\,\mathrm{GeV}$ from RG equation at 3 loops;
$-\,0.1\,\mathrm{GeV}$ from the effective potential at 2 loops

Stability condition and error budget [Degrassi et al. 12]

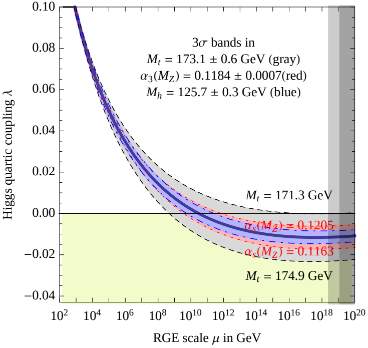


Err. Type	Err. estimate	Impact on M_h
M_t	expt. uncert. M_t	± 1.4 GeV
α_s	expt. uncert. α_s	± 0.5 GeV
Expt.	**Tot. combined in quadr.**	± 1.5 GeV
λ	scale var. in λ	± 0.7 GeV
y_t	$\mathcal{O}(\Lambda_{QCD})$ correction to M_t	± 0.6 GeV
y_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW 3 loops + QCD 4 loops	± 0.2 GeV
Theory	**Tot. combined in quadr.**	± 1.0 GeV

NNLO uncertainty reduction

λ matching: from $\pm 2.0\,\mathrm{GeV}$ (NLO) to $\pm 0.7\,\mathrm{GeV}$ (NNLO)
stability condition: from $\pm 3.0\,\mathrm{GeV}$ (NLO) to $\pm 1.0\,\mathrm{GeV}$ (NNLO)

Stability condition and error budget [Degrassi et al. 12]



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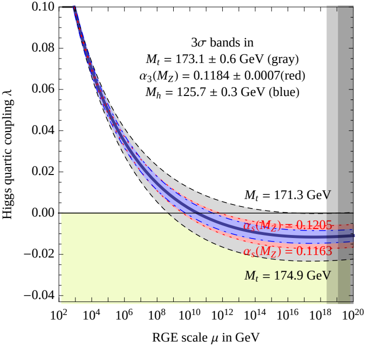
RGE	EW 3 loops + QCD 4 loops	$\pm$0.2 GeV
Theory	**Tot. combined in quadr.**	$\pm$1.0 GeV

full NNLO [Buttazzo et al. 13]

central value of M_h stability bound shifted by $+0.2\,\mathrm{GeV}$
total th. uncertainty reduced from $\pm 1.0\,\mathrm{GeV}$ to $\pm 0.7\,\mathrm{GeV}$ (NNLO)

Stability condition and error budget [Degrassi et al. 12]



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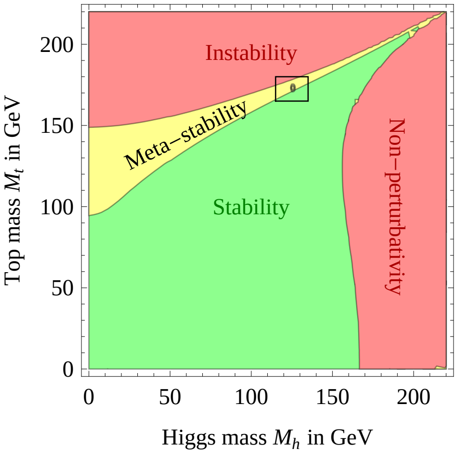
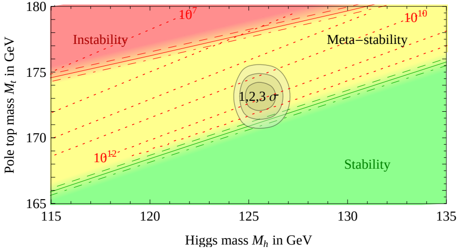
Theory	**Tot. combined in quadr.**	$\pm$1.0 GeV

full NNLO stability bound on m_t [Buttazzo et al. 13]

$$M_t < (171.53 \pm 0.15 \pm 0.23_{\alpha_S} \pm 0.15_{M_h})\,\mathrm{GeV} = (171.53 \pm 0.42)\,\mathrm{GeV}$$

SM Phase diagram [Degrassi et al. 12]

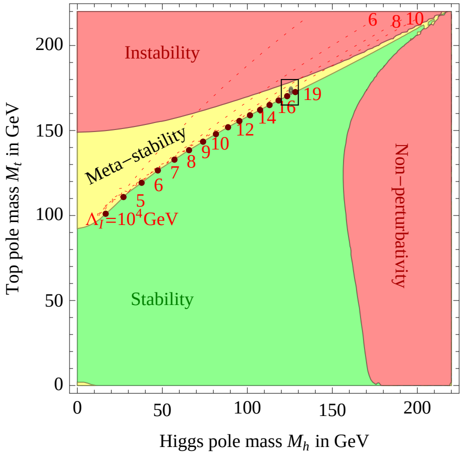


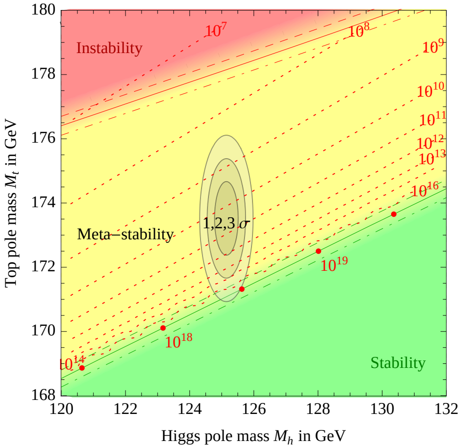


beware of possible Planck scale physics

modification of τ_{EW} [Branchina, Messina 13]

SM Phase diagram [Buttazzo et al. 13]



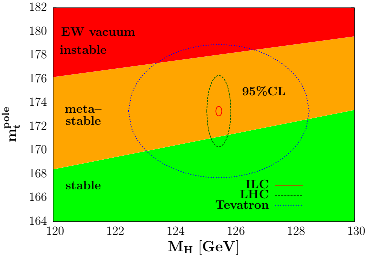


beware of possible Planck scale physics

modification of τ_{EW} [Branchina, Messina 13]

The m_t issue

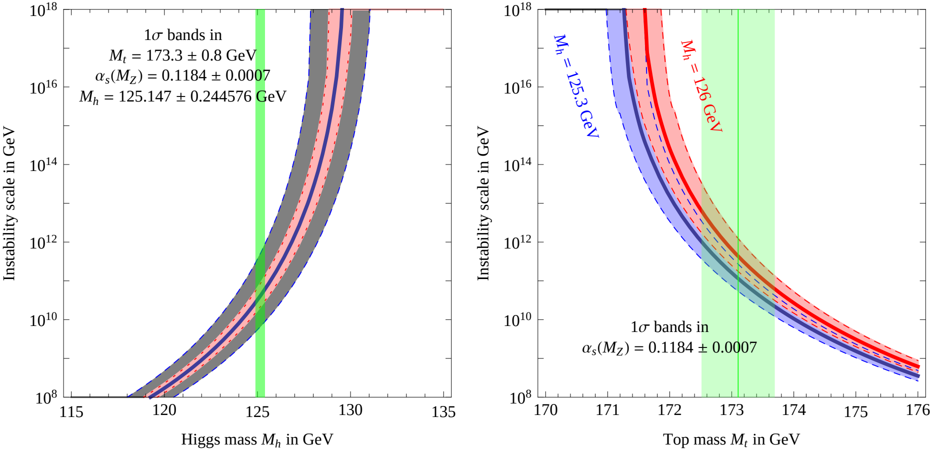
- position in the SM phase diag. $\leftrightarrow m_t$
- top mass used is the Tevatron+LHC average $m_t^{MC} = 173.34 \pm 0.76$ GeV
- m_t^{MC} extracted with template methods (Pythia mass) from decay products. Event modeling is delicate!
- we extract $y_t(\mu)$ from m_t^{pole}: $\mathcal{O}(\Lambda_{QCD})$ uncert. + is $m_t^{pole} = m_t^{MC}$?



[Alekhin, Djouadi, Moch 12]

- stay on the safe side: use $\overline{m}_t(m_t) = 162.3 \pm 2.3$ GeV from $t\bar{t}$ inclusive σ. But can't say much on the SM vacuum until ILC …
- exploit high precision in m_t^{MC} determination with new methods
- e.g. $m_t^{MC} \Rightarrow m_t^{pole} = 173.39^{+1.12}_{-0.98}$ GeV [Moch 14]

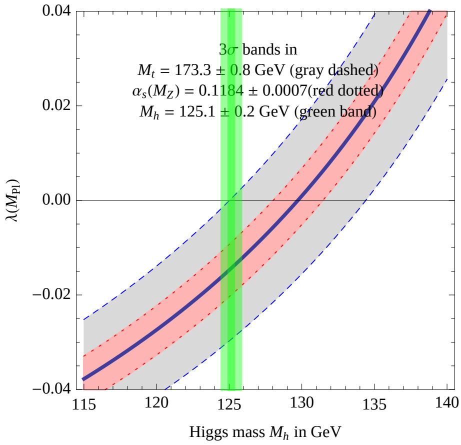
SM instability scale formula from [Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia 13]

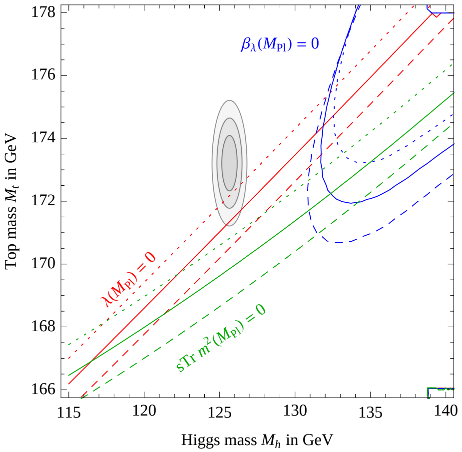


$$\log 10 \frac{\Lambda_V}{\mathrm{GeV}} =$$

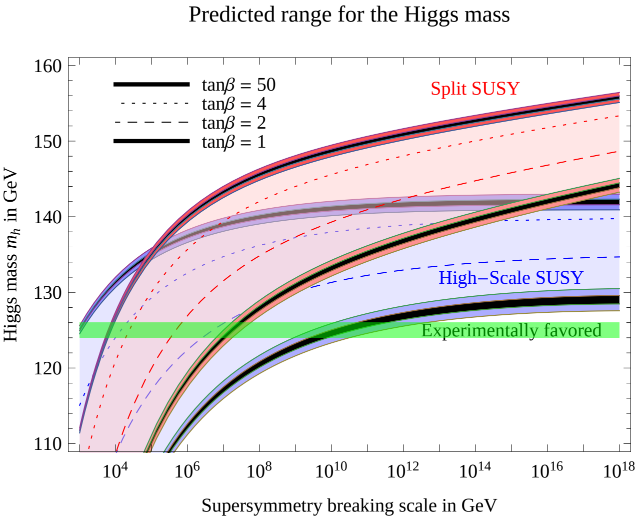
$$9.5 + 0.7\left(\frac{M_H}{\mathrm{GeV}} - 125.15\right) - 1.0\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) + 0.3\frac{\alpha_3(M_Z) - 0.1184}{0.0007}$$

Planck scale coupling





Split or high scale supersymmetry implications



- High-scale SUSY = all sparticles $\tilde{m}$
- Split-scale SUSY = all scalar sparticles $\tilde{m}$, all fermion sparticles EW scale mass

The (disappointing) conclusions

- A SM-like Higgs with $M_h \sim 125\,\mathrm{GeV}$ does not allow us to infer, in a model independent way, the scale of NP.
- The SM vacuum is probably metastable , but the tunneling is slow enough that the vacuum has a lifetime longer than the age of the universe.
- λ gets small at high energies . E.g. around $\mathcal{O}(10^{11}\,\mathrm{GeV})$ with the current m_t, around the Planck scale if $m_t \simeq 171\,\mathrm{GeV}$
- If MS is an EFT, we have to match it onto an UV model where the Higgs either
  - is weakly interacting if $\Lambda_{\mathrm{NP}} \simeq \Lambda_{\mathrm{EW}}$
  - has vanishing (?) λ if $\Lambda_{\mathrm{NP}} \simeq \Lambda_{\mathrm{Pl}}$
- Such reasonings strongly depend on m_t , M_h (and α_s) .
- If it's just SM. . . What about the naturalness problem?

The end

Thanks for your attention!

backup slides

More conservative analysis [Masina 12]