LFV in EFT with dimension 6 operators: connecting low and high energy observables

(mainly based on arXiv:1408.3565, in collaboration with Adrian Signer)

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Outline

- Motivation: we all like lepton flavour violation
- A systematic bottom-up approach:
  - Search for lepton flavour violating decays of H at CMS
  - The SM with dimension 6 operators
- Automation and tools for low energy observables
  - FeynRules, from the Lagrangian to the FR
  - FeynArts, from the FR to the unintegrated amplitudes
- Calculating the Branching Ratio of $\tau \to \mu\gamma$
  - Tree level
  - One loop and renormalisation
- Connecting different scales
  - Extraction of the low energy limits (BaBar)
  - Translation into high energy limits and “comparison” (CMS)
- Conclusion

Flavour

It refers to the type of elementary particles (quarks/leptons) occurring in the Standard Model (SM) of particles.

The SM provides a flavour symmetry:

- GLOBAL: Quantum-Chromo-Dynamics (QCD) and Electro-Magnetic (EM);
- BROKEN: Electro-Weak (EW).

We already observed phenomenological violation of flavour both in the quark sector ($u/c/t \to d/s/b + W$) or in the neutrino sector ($\nu_{l_i} \to \nu_{l_j}$).

The latter already requires a consistent BSM description!!!

Lepton Flavour

Lepton sector comes in (at least) three flavours: e, μ and τ.

Is it a good quantum number (conserved quantity)?

- THEORY: yes.
- PHENO: we know that it is violated in the ν sector.

So far, no evidence of Lepton Flavour Violation (LFV) in the charged sector, while clear evidence of violation in the neutrino sector (PMNS mechanism at work) in solar/atmospheric/reactor/beam neutrino experiments.

Charged Lepton Flavour Violation

Simplest possible phenomenological realisations:

- $l_h^\pm \to \gamma + l_i^\pm$ where $h, i = e, \mu, \tau,$
- $l_h^\pm \to l_i^\pm l_j^\pm l_k^\mp$ where $h, i, j, k = e, \mu, \tau,$
- $Z \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau,$
- $H \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau.$

ADVERTISING SPACE

Muon sector investigated at the PSI:

- BR($\mu \to \gamma + e$)$< 5.7 \times 10^{-13}$ at the 90% C.L. (2013);

  MEG collaboration, Phys. Rev. Lett. **110** (2013) 201801
- BR($\mu \to 3e$)$< 10^{-16}$ at the 90% C.L. (2016).

  Mu3e collaboration, arXiv:1301.6113

SINDRUM Collaboration: BR($\mu \to 3e$)$< 10^{-12}$.

LFV *is* a BSM signal

Neutral sector

ν oscillation *is* a BSM signal, but what is the underlying picture? Several candidates, mechanisms, theories, but it is difficult to disentangle the various hypothesis through experiments.

Charged sector

We have searched for LFV and severely constrained the parameter space of LFV-BSM extensions, but no evidence so far. Moreover, we don't know what is really beyond.

Synergy among Low and High Energy Experiments

An extensive long-term programme is undergoing to push the experimental limits both at low and high energy scales.

- Low energy (from m_μ to m_b):
  - Muon: limit on $\mu \to e$ conversion (SINDRUM II), $\mu \to e + \gamma$ (MEG), $\mu \to 3e$ (SINDRUM), $\mu \to e + 2\gamma$ (LAMPF), etc.
  - Tau-lepton: $\tau \to e/\mu + \gamma$ (BaBar, Belle), $\tau \to l_i l_j l_k$ with $i, j, k = e, \mu$ (BaBar, Belle and LHCb).
- High energy (from the EW scale to LHC run 2)
  - Neutral current mediated: $Z \to l_i l_j$ with $i, j = e, \mu, \tau$ (ALEPH, DELPHI, L3, OPAL, UA1).
  - Higgs mediated: $H \to \tau\mu$ (CMS).

"Bottom-up" versus "top-down"

Top-down approach:

- breaking down of a system to gain insight into its compositional sub-systems;
- an overview of the system is formulated, specifying but not detailing any first-level subsystems;
- each subsystem is then refined in yet greater detail, sometimes in many additional subsystem levels, until the entire specification is reduced to base elements.

Bottom-up approach:

- piecing together of systems to give rise to grander systems, thus making the original systems sub-systems of the emergent system;
- the individual base elements of the system are first specified in great detail;
- these elements are then linked together to form larger subsystems, which then in turn are linked, sometimes in many levels, until a complete top-level system is formed.

CMS PAS HIG-14-005 (Abstract)

Search for lepton flavor violating decays of the Higgs boson

The CMS Collaboration

Abstract

The first direct search for Lepton Flavor Violating Decays of the recently discovered Higgs boson using 19.7 fb^{-1} of $\sqrt{s} = 8$ TeV data taken in 2012 using the $H \to \mu\tau_e$ and $H \to \mu\tau_{had}$ channels is described , where τ_{had} and τ_e are taus reconstructed in the hadronic and electronic decay channels respectively. The sensitivity of the search is an order of magnitude better than the existing indirect limits. A slight excess of signal events with a significance of 2.5σ is observed. The local p-value of this excess at $M_H = 126$ GeV is 0.007. Interpreted as a limit this results in a constraint of $B(H \to \mu\tau) < 1.57\%$ at 95% confidence level. The best fit branching fraction is $B(H \to \mu\tau) = (0.89^{+0.40}_{-0.37})\%$. The limit is subsequently used to constrain the $Y_{\mu\tau}$ Yukawa coupling.

CMS PAS HIG-14-005 (Introduction)

The discovery of the Higgs boson [1] has generated great interest in exploring its properties. In the standard model (SM) lepton flavor violating (LFV) decays are not allowed if the theory is to be renormalizable. If however, the requirement that it is renormalizable is relaxed, so that it is a theory valid only to a finite mass scale, then LFV couplings may be introduced. LFV decays can also occur naturally in models with more than one Higgs doublet without giving up on renormalizability [2]. They also arise in composite Higgs models [3, 4], models with flavor symmetries [5], Randall-Sundrum models [6] and many others.

The presence of LFV Higgs couplings would allow LFV effects in decays mediated by virtual Higgs. There are three possibilities $\mu \to e$, $\tau \to \mu$ and $\tau \to e$ transitions. The experimental constraints have been reviewed and translated into constraints on $B(H \to e\mu, \mu\tau, e\mu)$ in two recent papers [7, 8]. The $\mu \to e$ transition is strongly constrained by null searches for $\mu \to e\gamma$ [9], $B(H \to \mu e) < \mathcal{O}(10^{-8})$. The constraints on $\tau \to \mu$ and $\tau \to e$ are much less stringent. These come from searches for $\tau \to \mu\gamma$ and $\tau \to e\gamma$, muon and electron g-2 measurements. Exclusion limits on their electron dipole moments also provide complementary constraints. These lead to the much less restrictive limits: $B(H \to \mu\tau) < \mathcal{O}(10\%)$, $B(H \to e\tau) < \mathcal{O}(10\%)$.

[7] G. Blankenburg, J. Ellis, and G. Isidori, "Flavour-Changing Decays of a 125 GeV Higgs-like Particle", *Phys.Lett.* **B712** (2012) 386–390, doi:10.1016/j.physletb.2012.05.007, arXiv:1202.5704.

[8] R. Harnik, J. Kopp, and J. Zupan, "Flavor Violating Higgs Decays", *JHEP* **1303** (2013) 026, doi:10.1007/JHEP03(2013)026, arXiv:1209.1397.

CMS PAS HIG-14-005 (Interpretation)

The constraint on $B(H \to \mu\tau)$ can be interpreted in terms of LFV Higgs Yukawa couplings. The LFV decays $H \to e\mu, e\tau, \mu\tau$ arise at tree level from the assumed flavor violating Yukawa interactions where the relevant terms are explicitly

$$L_V \equiv -Y_{e\mu}\bar{e}_L\mu_R h - Y_{\mu e}\bar{\mu}_L e_R h - Y_{e\tau}\bar{e}_L\tau_R h - Y_{\tau e}\bar{\tau}_L e_R h - Y_{\mu\tau}\bar{\mu}_L\tau_R h - Y_{\tau\mu}\bar{\tau}_L\mu_R h$$

The branching fraction in terms of the Yukawa couplings are given by

$$B(H \to l^\alpha l^\beta) = \frac{\Gamma(H \to l^\alpha l^\beta)}{\Gamma(H \to l^\alpha l^\beta) + \Gamma_{SM}} \tag{1}$$

where $l^\alpha, l^\beta = e, \mu, \tau$ and $l^\alpha \neq l^\beta$. The decay width, in turn, is

$$\Gamma(H \to l^\alpha l^\beta) = \frac{m_h}{8\pi}(|Y_{l^\beta l^\alpha}|^2 + |Y_{l^\alpha l^\beta}|^2) \tag{2}$$

and SM Higgs width is $\Gamma_{SM} = 4.1$ MeV for a 125 GeV Higgs boson. It was assumed that at most one of non-standard decay mode of the Higgs is significant compared to the SM decay width.

The constraints on the Yukawa couplings derived from the limit $B(H \to \mu\tau) < 1.57\%$ are shown in Figure 6. This is compared to the constraints from previous indirect measurements. It can be seen that the direct search improves the constraint by roughly an order of magnitude.

Dimension 4 operator: the SM

The most general Yukawa coupling of a Higgs boson to leptons:

$$\mathcal{L}_{\mathrm{D4}} = -y_{pr}\left(\bar{l}_p e_r \varphi\right) + [\ldots]\,.$$

We diagonalise the interaction:

$$y = U_y D_y W_y^\dagger,$$

plus, we transform the gauge fields into the physical fields:

$$l \to U_y l, \qquad e \to W_y e.$$

Same transformation of the two components of the doublet implies that U_y and W_y disappear from the theory.

LEPTON FLAVOUR CONSERVATION!

A bottom-up approach: dim-n effective theory

Assumptions: SM is merely an effective theory, valid up to some scale Λ. It can be extended to a field theory that satisfy the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
- all the SM degrees of freedom must be incorporated;
- at low energies (i.e. when $\Lambda \to \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

Dimension 5 operator

Only one dimension 5 operator is allowed by gauge symmetry:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n \equiv (\widetilde{\varphi}^\dagger l_p)^T C(\widetilde{\varphi}^\dagger l_r).$$

After the EW symmetry breaking, it can generate neutrino masses and mixing (no other operator can do the job).

Its contribution to LFV has been widely studied in the late 70s:

- in the context of higher dimensional effective realisations;
  S. T. Petcov, Sov. J. Nucl. Phys. **25** (1977) 340 [Yad. Fiz. **25** (1977) 641]
- in connection with the "see-saw" mechanism.
  P. Minkowski, Phys. Lett. B **67**, 421 (1977)

It will not be considered in the current discussion.

Dimension 6 operators: tree level

Only one dim-6 term can produce $H \to l_i l_j$ at the tree level:

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

$$Q_{e\varphi} = (\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi),$$

that sums to the SM Yukawa sector:

$$\begin{aligned}
\mathcal{L}_{\mathrm{D4}} + \mathcal{L}_{e\varphi} =& \frac{v}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C_{e\varphi}^{pr}\right)\bar{e}_p e_r \\
&+ \frac{1}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C_{e\varphi}^{pr}\right)\bar{e}_p e_r h + \boxed{\frac{v^2}{\sqrt{2}\Lambda^2}C_{e\varphi}^{pr}}\,\bar{e}_p e_r h \\
&+ \frac{i}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C_{e\varphi}^{pr}\right)\bar{e}_p e_r \widehat{Z} \\
&+ i\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C_{e\varphi}^{pr}\right)\bar{e}_p \nu_r \widehat{W}^+ + [\ldots]\,.
\end{aligned}$$

Dimension 6 operators: one loop

2-leptons

$$
\begin{aligned}
Q_{eW} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu};\\
Q_{eB} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}.\\
Q^{(1)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)\\
Q^{(3)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)\\
Q_{\varphi e} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)
\end{aligned}
$$

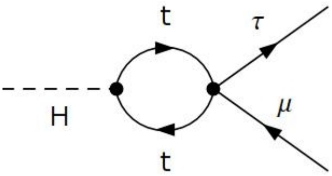
4-leptons

$$
\begin{aligned}
Q_{ll} &= (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)\\
Q_{ee} &= (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)\\
Q_{le} &= (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)
\end{aligned}
$$

4-fermions

$$
\begin{aligned}
Q^{(1)}_{lq} &= (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)\\
Q^{(3)}_{lq} &= (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)\\
Q_{eu} &= (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)\\
Q_{ed} &= (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{lu} &= (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)\\
Q_{ld} &= (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)\\
Q_{qe} &= (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)\\
Q_{ledq} &= (\bar{l}^j_p e_r)(\bar{d}_s q^j_t)\\
Q^{(1)}_{lequ} &= (\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)\\
Q^{(3)}_{lequ} &= (\bar{l}^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)
\end{aligned}
$$

3-point and 4-point fermionic insertions



$H \to \tau\mu$ at the one-loop level, induced by a 4-point fermionic insertion with intermediate t quarks. 9 diagrams of this kind, one for each SM fermion.

Furthermore, we have ~ 100 “triangles”, ~ 200 “bubbles” inserted in the external legs, *et cetera*.

Potentially, 1 operator at the tree level plus 18 at one loop.

Clearly, a single observable can only constrain a set of coefficients combined together in Wilson coefficients.

Neglecting quantum fluctuations

The BR induced by the tree-level $C_{e\varphi}$ coefficients reads:

$$\mathrm{BR}_{H\to\tau\mu} = \frac{\Gamma_{H\to\tau\mu}}{\Gamma_{TOT}} \sim \frac{|C_{e\varphi}^{\tau\mu}|^2 + |C_{e\varphi}^{\mu\tau}|^2}{\Lambda^4}\frac{v^6}{m_b^2} \lesssim 1.57\%.$$

Hence, the CMS measurement set the following constraint:

$$\frac{\sqrt{|C_{e\varphi}^{\tau\mu}|^2 + |C_{e\varphi}^{\mu\tau}|^2}}{\Lambda^2} \lesssim 10^{-8}/\mathrm{GeV}^2.$$

Now, the energy scale behaviour is explicit.

If the energy scale of the underlying theory is above 10 TeV, the effective coefficients are unbounded!

How to “compare” this bound with the indirect ones?

Toward a fully automatised approach

We have to deal with quantum fluctuations.

By calculating $\tau \to \mu\gamma$ at tree-level plus one-loop one finds:

- 1 operator at the tree level;
- 5 two-lepton operators at one loop (including $Q_{e\varphi}$);
- 3 four-lepton operators at one loop;
- 10 four-fermion operators at one loop;

19 operators in total, which results in hundreds of diagrams to be evaluated in the R_ξ gauge.

These new insertions give raise to $\sim \mathcal{O}(100)$ diagrams or more.

A non-automatised approach is tricky and tedious!

FeynRules

The generation of Feynman Rules was automatised by means of the FeynRules package.

Comput. Phys. Commun. **185** (2014) 2250 [arXiv:1310.1921 [hep-ph]]

At the end of the day, it was rather simple as we had great technical assistance (thanks to C. Duhr and C. Degrande).

The philosophy is straightforward:

- write your operator in a Mathematica notebook,
- press a button,
- print out your Feynman Rules.

Plus, it can also produce a FeynArts/FormCalc model file.

FeynRules coding

```
(* *****   effective Lagrangian   ***** *)

LQephi := Block[{sp,ii,jj,ff1,ff3,aux1,feynmangaugerules},
  feynmangaugerules = If[Not[FeynmanGauge], {G0|GP|GPbar ->0}, {}];
  aux1 = ExpandIndices[1/Lam^2 Qef[ff1, ff3] Phibar[jj] Phi[jj]
  LLbar[sp, ii, ff1].lR[sp, ff3] Phi[ii], FlavorExpand -> SU2D];
  aux1+HC[aux1]/.feynmangaugerules
];
```

Feynman Rules for LF violating interactions (Unitary Gauge):

$$\bar{\tau} - \mu - 1H \longrightarrow \frac{iv^2}{\sqrt{2}\Lambda^2}\left[C_{e\varphi}^{\tau\mu} P_R + C_{e\varphi}^{\mu\tau} P_L\right],$$

$$\bar{\tau} - \mu - 2H \longrightarrow \frac{i3v^2}{\sqrt{2}\Lambda^2}\left[C_{e\varphi}^{\tau\mu} P_R + C_{e\varphi}^{\mu\tau} P_L\right],$$

$$\bar{\tau} - \mu - 3H \longrightarrow \frac{i3v^2}{\sqrt{2}\Lambda^2}\left[C_{e\varphi}^{\tau\mu} P_R + C_{e\varphi}^{\mu\tau} P_L\right].$$

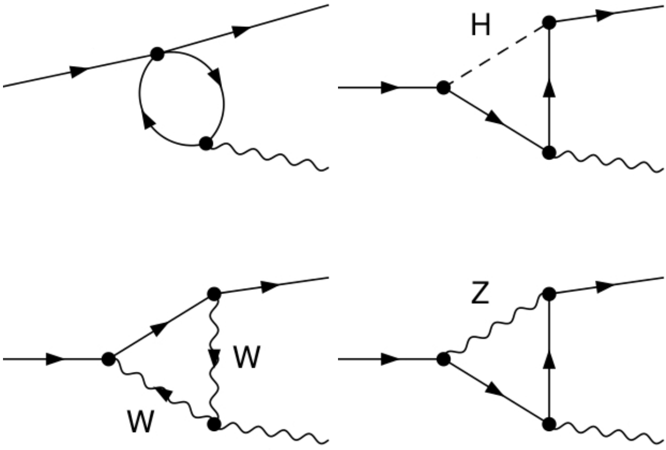
From FeynRules to FeynArts

FeynRules produces a FeynArts compatible model file.

Generation and visualisation of Feynman Diagrams and amplitudes is possible at a complete automatised level, via the following steps:

- generation of topologies (no adjacency restrictions),
- insertion of SM fields with extended coupling set,
- generation of Feynman Diagrams in visual format,
- generation of non-integrated amplitudes in the R_ξ-gauge,
- generation of a total-amplitude-evaluating FORM code.

$\tau \to \mu\gamma$: one-loop triangles



FORM code

```
************************************************************************

id numT01 = -(i_*ec3(Lor1)*intM(Den(q1, MH2), Den(-k2 + q1, ME2),
     Den(-k2 - k3 + q1, ME2))*Spinor(k2, ME, 1)*g_(100)*
     ((i_*gc17*g6_(100))/2 + (i_*gc17*g7_(100))/2)*
     (ME + g_(100, k2) - g_(100, q1))*
     (i_*gc704*g_(100, Lor1)*(g6_(100)/2) +
      i_*gc704*g_(100, Lor1)*(g7_(100)/2))*
     (ME + g_(100, k1) - g_(100, q1))*((i_*gc39*g6_(100))/2 +
      (i_*gc39*g7_(100))/2)*Spinor(k1, MM, 1))/(16*Pi^2);

id numT10 = (ec3(Lor1)*Spinor(k2, ME, 1)*g_(100)*
    (i_*gc212R*g_(100, Lor3)*(g6_(100)/2) +
     i_*gc212L*g_(100, Lor3)*(g7_(100)/2))*
    (ME + g_(100, k2) - g_(100, q1))*
    (i_*gc704*g_(100, Lor1)*(g6_(100)/2) +
     i_*gc704*g_(100, Lor1)*(g7_(100)/2))*
    (ME + g_(100, k1) - g_(100, q1))*
    (i_*gc217R*g_(100, Lor2)*(g6_(100)/2) +
     i_*gc217L*g_(100, Lor2)*(g7_(100)/2))*Spinor(k1, MM, 1)*
   (i_*Pi^2*met(Lor2, Lor3)*intM(Den(q1, MZ2), Den(-k2 + q1, ME2),
      Den(-k2 - k3 + q1, ME2)) -
    ((i_*Pi^2*intM(Den(q1, MZ2), Den(-k2 + q1, ME2),
         Den(-k2 - k3 + q1, ME2)) - i_*Pi^2*
        intM(Den(q1, GXi1*MZ2), Den(-k2 + q1, ME2),
         Den(-k2 - k3 + q1, ME2)))*q1(Lor2)*q1(Lor3))/MZ2))/(16*Pi^4);
```

Straight to the result?!

Unfortunately, even in a clean (from bugs) environment, at the moment is not possible to extract a reliable result in presence of 4-fermion vertices.

"**FeynArts cannot correctly build the fermion chains if vertices involving more than two fermions appear** because this information is simply not available from the Feynman rules. [...] the fermion fields must carry an additional kinematic index (e.g. a Dirac index) with which it is *afterwards* possible (i.e. *FeynArts* does not do this) to find the right concatenation."
FeynArts User's Guide - Thomas Hahn

A fully automatised calculation is not possible at the moment.

A lot of pros

We have the whole world in our hands, and we have also our hands full:

- we know the Feynman Rules of the effective theory;
- we know the exact number of diagrams involved;
- we can automatically extract the unintegrated amplitudes.

Here the automation stops, but on the other hand:

- the technology for one-loop calculations is well known,
- as well as the theory of $\tau \to \mu\gamma$,
- the probability of coding "mistakes" is scaled down.

Interaction and branching ratio

The partial width of the process $\tau \to \mu\gamma$ is given by

$$\Gamma_{\tau\to\mu\gamma} = \frac{1}{16\pi m_\tau} |\mathcal{M}|^2 \,.$$

The general photon-mediated FV interaction reads:

$$V^\mu = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left(C_{TL}\,\omega_L + C_{TR}\,\omega_R\right) \left(p_\gamma\right)_\nu \,.$$

The unpolarised squared matrix element is:

$$|\mathcal{M}|^2 = \frac{4\left(|C_{TL}|^2 + |C_{TR}|^2\right) m_\tau^4}{\Lambda^4},$$

and the branching ratio is

$$\mathrm{BR}(\tau \to \mu\gamma) = \frac{\Gamma_{\tau\to\mu\gamma}}{\Gamma_\tau} = \frac{m_\tau^3}{4\pi\Lambda^4\Gamma_\tau}\left(|C_{TL}|^2 + |C_{TR}|^2\right) \,.$$

Knocking out the way: $\tau \to \mu\gamma$ at tree level

Working in the physical basis rather than in the gauge basis, we consider:

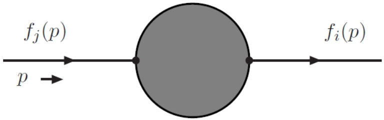
$$\begin{aligned} Q_{eB} &\to Q_{e\gamma} c_W - Q_{eZ} s_W\,, \\ Q_{eW} &\to -Q_{e\gamma} s_W - Q_{eZ} c_W\,, \end{aligned}$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle. The term

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi F_{\mu\nu} + \text{h.c.},$$

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor, is then the only term in the D-6 Lagrangian that induces a $\tau \to \mu\gamma$ transition at tree level.

Wave function renormalisation



The structure that corresponds to such transition consists of four possible coefficients:

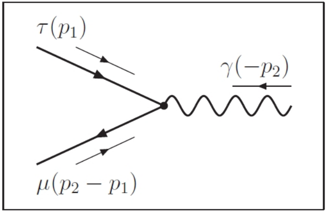
$$\Gamma^{f}_{ij}(p) = i\delta_{ij}(\not{p} - m_i) + i\left[\not{p}\,\omega_L\,\Sigma^{f,L}_{ij}(p^2) + \not{p}\,\omega_R\,\Sigma^{f,R}_{ij}(p^2) + \omega_L\,\Sigma^{f,l}_{ij}(p^2) + \omega_R\,\Sigma^{f,r}_{ij}(p^2)\right].$$

By applying the on-shell renormalisation conditions (à la Denner), one obtains:

$$\delta Z^L_{ij} = \frac{4}{m_i^2 - m_j^2}\left(m_j^2 \Sigma^{f,L}_{ij}(m_j^2) + m_i m_j \Sigma^{f,R}_{ij}(m_j^2) + m_j \Sigma^{f,r}_{ij}(m_j^2) + m_i \Sigma^{f,l}_{ij}(m_j^2)\right),$$

$$\delta Z^R_{ij} = \frac{4}{m_i^2 - m_j^2}\left(m_j^2 \Sigma^{f,R}_{ij}(m_j^2) + m_i m_j \Sigma^{f,L}_{ij}(m_j^2) + m_j \Sigma^{f,l}_{ij}(m_j^2) + m_i \Sigma^{f,r}_{ij}(m_j^2)\right).$$

Renormalised interaction



$$= \frac{1}{\Lambda^2}\left[\gamma^\mu \left(K_{VL}\,\omega_L + K_{VR}\,\omega_R\right) + \, i\sigma^{\mu\nu}\left(K_{TL}\,\omega_L + K_{TR}\,\omega_R\right)(p_2)_\nu\right].$$

$$\frac{K_{VL}}{\Lambda^2} = -\frac{e}{2}\left(\frac{1}{2}\delta Z^L_{\mu\tau} + \frac{1}{2}\left(\delta Z^L_{\mu\tau}\right)^\dagger\right) - \frac{ev^2}{4c_W s_W \Lambda^2}\left(C^{(1)}_{\varphi l} + C^{(3)}_{\varphi l}\right)\frac{1}{2}\delta Z_{ZA},$$

$$\frac{K_{VR}}{\Lambda^2} = -\frac{e}{2}\left(\frac{1}{2}\delta Z^R_{\mu\tau} + \frac{1}{2}\left(\delta Z^R_{\mu\tau}\right)^\dagger\right) - \frac{ev^2}{4c_W s_W \Lambda^2}C_{\varphi e}\frac{1}{2}\delta Z_{ZA},$$

$$\frac{K_{TL}}{\Lambda^2} = -\frac{v}{\sqrt{2}\Lambda^2}C^{\tau\mu}_{e\gamma}\left(1 + \frac{1}{2}\delta Z^L_{\tau\tau} + \frac{1}{2}\left(\delta Z^R_{\mu\mu}\right)^\dagger + \frac{1}{2}\delta Z_{AA} + \frac{\delta v}{v}\right) - \frac{v}{\sqrt{2}\Lambda^2}C^{\tau\mu}_{eZ}\frac{1}{2}\delta Z_{ZA},$$

$$\frac{K_{TR}}{\Lambda^2} = -\frac{v}{\sqrt{2}\Lambda^2}C^{\mu\tau}_{e\gamma}\left(1 + \frac{1}{2}\delta Z^R_{\tau\tau} + \frac{1}{2}\left(\delta Z^L_{\mu\mu}\right)^\dagger + \frac{1}{2}\delta Z_{AA} + \frac{\delta v}{v}\right) - \frac{v}{\sqrt{2}\Lambda^2}C^{\mu\tau}_{eZ}\frac{1}{2}\delta Z_{ZA}.$$

Fixed-order results

Operator	C_{TL} or $C_{TR}(\tau \longleftrightarrow \mu)$
$Q_{e\gamma}$	$-C_{e\gamma}\dfrac{\sqrt{2}m_W s_W}{e}$
Q_{eZ}	$-C_{eZ}\dfrac{e m_Z}{16\sqrt{2}\pi^2}\left(3-6c_W^2+4c_W^2\log\left[\dfrac{m_W^2}{m_Z^2}\right]+(12c_W^2-6)\log\left[\dfrac{m_Z^2}{\lambda^2}\right]\right)$
$Q_{\varphi l}^{(1)}$	$-C_{\varphi l}^{(1)}\dfrac{e m_\mu\left(1+s_W^2\right)}{24\pi^2}$
$Q_{\varphi l}^{(3)}$	$C_{\varphi l}^{(3)}\dfrac{e m_\mu\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{\varphi e}$	$C_{\varphi e}\dfrac{e m_\tau\left(3-2s_W^2\right)}{48\pi^2}$
$\boxed{Q_{e\varphi}}$	$C_{e\varphi}\dfrac{m_W s_W}{48\sqrt{2}m_H^2\pi^2}\left(4m_\mu^2+4m_\tau^2+3m_\mu^2\log\left[\dfrac{m_\mu^2}{m_H^2}\right]+3m_\tau^2\log\left[\dfrac{m_\tau^2}{m_H^2}\right]\right)$
$Q_{lequ}^{(3)}$	$-\dfrac{e}{2\pi^2}\sum_u m_u\left(C_{lequ}^{(3)}\right)^{\tau\mu uu}\log\left[\dfrac{m_u^2}{\lambda^2}\right]$

Operator	C_{TL}	C_{TR}
Q_{le}	$\dfrac{e}{16\pi^2}\left(m_e C_{le}^{\tau ee\mu}+m_\mu C_{le}^{\tau\mu\mu\mu}+m_\tau C_{le}^{\tau\tau\tau\mu}\right)$	$\dfrac{e}{16\pi^2}\left(m_e C_{le}^{\mu ee\tau}+m_\mu C_{le}^{\mu\mu\mu\tau}+m_\tau C_{le}^{\mu\tau\tau\tau}\right)$

Comparing processes at different energy scales

First of all, we have previously studied that two different bounds represent two complementary information to be merged in the same system of linear inequalities rather than two competitive constraints.

However, in the assumption that only one coefficient is at work (the Yukawa-like $C_{e\varphi}^{\tau\mu}$, according to the CMS paper), a further question arises...

Are we allowed to compare bounds from different processes at different scales?

1. Limit on $\mathrm{BR}(H \to \tau\mu)$ measured at $\lambda = m_H$.
2. Limit on $\mathrm{BR}(\tau \to \mu\gamma)$ measured at $\lambda = m_\tau$.

The answer is: “yes, under specific assumptions”

A priori, we are strictly not allowed.

However, the functional behaviour of the effective coefficient with respect to the energy scale can be investigated via the analysis of their anomalous dimensions.

By direct computation, one finds that the coefficients $C_{e\gamma}^{\tau\mu}$, $C_{eZ}^{\tau\mu}$, $C_{lequ}^{\tau\mu ii}$ (with $i = u, c, t$) mix together: for $\lambda > m_V$, one has

$$
\begin{aligned}
&16\pi^2 \frac{\partial C_{e\gamma}^{\tau\mu}}{\partial \log \lambda} \\
&= \left(e^2 \left(\frac{47}{3} + \frac{1}{4c_W^2} - \frac{9}{4s_W^2} \right) + 2\,Y_\mu^2 + \left(\frac{1}{2} + 2c_W^2 \right) Y_\tau^2 + \sum_l Y_l^2 + 3\sum_q Y_q^2 \right) C_{e\gamma}^{\tau\mu} \\
&+ \left(6e^2 \left(\frac{c_W}{s_W} - \frac{s_W}{c_W} \right) - 2c_W s_W Y_\tau^2 \right) C_{eZ}^{\tau\mu} + 16e \sum_u Y_u C_{\tau\mu uu}^{(3)}.
\end{aligned}
$$

Specific assumptions, again!

If $C_{lequ}^{\tau\mu ii}$ (with $i = u, c$) are very small in the energy range $m_\tau < \lambda < m_Z \sim m_H$, a somewhat simplified analysis can be performed.

For the running of $C_{e\gamma}(\lambda)$ below the electroweak scale, the leading contribution is the QED one:

$$16\pi^2 \frac{\partial C_{e\gamma}}{\partial \log \lambda} \simeq e^2 \left(10 + \frac{4}{3} \sum_q e_q^2(\lambda) \right) C_{e\gamma},$$

where $e_q(\lambda)$ denotes the electric charge of the fermion fields that are dynamical at the scale λ.

This is the pocket dictionary that we will bring with us in our travel from low energies to high energies.

Extracting the low energy limits

From LEP, the τ-lepton total width is inferred to be:

J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001

$$\Gamma_\tau = 2.3 \cdot 10^{-12}\ \mathrm{GeV}.$$

From the BaBar Collaboration, the direct limit reads:

B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **104** (2010) 021802

$$\mathrm{BR}(\tau^- \to \mu^- \gamma) \leq 4.4 \cdot 10^{-8}.$$

From this, the following LE limit is obtained:

$$\left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.7 \cdot 10^{-10}\,[\mathrm{GeV}]^{-1}\,.$$

Results at the $\lambda = m_\tau$ scale

$\tau \to \mu\gamma$			
3-P Coefficient	At fixed scale	4-P Coefficient	At fixed scale
$C_{e\gamma}^{\tau\mu}$	$2.7 \cdot 10^{-12} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$C_{le}^{\tau ee\mu}$	$4.8 \cdot 10^{-4} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$
$C_{eZ}^{\tau\mu}(m_Z)$	$1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$C_{le}^{\tau\mu\mu\mu}$	$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$1.7 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$C_{le}^{\tau\tau\tau\mu}$	$1.4 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		
$C_{\varphi e}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		
$\boxed{C_{e\varphi}^{\tau\mu}}$	$1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		

TABLE : Limits on the Wilson coefficients contributing to the $\tau \to \mu\gamma$ transition up to the one-loop level.

The LE limit evolved at the $\lambda = m_H$ scale

The limit evaluated at the EW scale reads

$$\sqrt{\frac{|C_{e\gamma}^{\tau\mu}(m_Z)|^2 + |C_{e\gamma}^{\mu\tau}(m_Z)|^2}{2}} \leq 2.0 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}.$$

The RG evolution relax the bound with an impact of $\sim \%$.

If we assume that only $C_{e\varphi}$ is contributing to the photon-mediated LFV (following the CMS interpretation), we can extract the following bounds:

$$C_{e\varphi}^{\tau\mu} \sim C_{e\varphi}^{\mu\tau} \lesssim 2.0 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}.$$

Now we can finally complete the picture with a comparison “against” the CMS result!

BaBar and CMS together (a serious one)

As a matter of fact, in a systematic bottom-up approach, the two experimental measures provided by BaBar and CMS represent complementary information in a broad system of linear inequalities, in which the variables are coefficients of higher dimensional operators.

The considered processes are phenomenologically different, as well as the energy scales at which they occur.

We learnt that the most diverse coefficients could produce flavour changing neutral currents in the leptonic sector (1 dim-5 plus 19 dim-6 different operators, each one coming with its matrix of coefficients).

BaBar "versus" CMS (a less serious one)

However, if we assume that:

- all of the coefficients are 0 except $C_{e\varphi}^{\tau\mu}$ or $C_{e\varphi}^{\mu\tau}$...
- ...at any energy scale...
- ...and $C_{e\varphi}$ does not mix when evolving from m_τ to m_H...
- ...and the NP live below 10 TeV (ahahah!)...

Then we are proud to announce that...

BaBar $C_{e\varphi}^{\tau\mu} \sim C_{e\varphi}^{\mu\tau} \lesssim \boxed{10^{-6}} \Lambda^2 / [\mathrm{GeV}]^2$.

CMS $C_{e\varphi}^{\tau\mu} \sim C_{e\varphi}^{\mu\tau} \lesssim \boxed{10^{-8}} \Lambda^2 / [\mathrm{GeV}]^2$.

CMS WINS!

Next round will be played at the BaBar stadium, and the coupling to probe will be $C_{e\gamma}$ at the $\lambda = m_H$ scale!

I'll sit on the fence...

Conclusion

- The motivation to study a dim-5 and dim-6 EFT containing LFV couplings was presented.
- The recent limits on the branching ratio of $H \to \tau\mu$ provided by CMS were discussed, as well as their interpretation (CMS PAS HIG-14-005).
- A systematic approach for the study of LFV observables was presented, and the benchmark process $\tau \to \mu\gamma$ was analysed at tree level and one loop.
- The limits from BaBar were interpreted as low energy limits on the various Wilson coefficients of the effective transition, and quantitative limits were provided under specific assumptions.
- The interpretation of LE constraints in terms of HE complementary limits was analysed by means of RGEs.

A very long to-do list

The dim-6 interpretation of experiments requires a programme:

- We started with the study of $l_i \to l_j \gamma$ in presence of dim-6 operators at tree level and one loop, but this represents a single condition to constrain 19 variables.
- The theoretical study of on-shell $Z \to l_i l_j$ (at LEP) and $H \to l_i l_j$ (at the LHC) is undergoing in collaboration with different groups.
- The off-shell case of the aforementioned channels will bring us to a complete treatment of LF violating 3-body decays, such as $\mu \to 3e$.
- The impact of 2 loops in the Yukawa sector on fixed order calculations and RGEs requires further analysis.
- Constraints from nuclear experiments (*e.g.*, conversion in nuclei) and flavour conserving observables (EDMs, $g-2$, *et cetera*) should definitely be part of the programme.

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