N=8 AND N=2 EXTREMAL BLACK-HOLE ATTRACTORS AND THEIR CLASSICAL MODULI SPACE

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"Attractor mechanism", was first considered in the framework of $N=2$ supergravity in $D=4$ dimensions

S.F., Kallosh, Strominger
S.F., Kallosh; Strominger
S.F. Gibbons, Kallosh

Extremal Black-Holes ($T=0$)

$$\phi^i(r) \underset{r\to\infty}{\longrightarrow} \phi^i_\infty \in \mathcal{M}$$

$$\phi^i(r) \underset{r\to r_H}{\longrightarrow} \phi^i_H(e_\Lambda, m^\Lambda)$$

The flow is non singular provided

$$\left.\frac{\partial V_{BH}}{\partial \phi^i}\right|_{\phi^i=\phi^i_H} = 0$$

The Bekenstein-Hawking entropy formula

$$S = \frac{A_H}{4} = \pi \left. V_{BH}\right|_{\phi^i=\phi^i_H}$$

Calabi-Yau B-H (Attractor varieties) G. Moore...

Calabi-Yau extremal black-holes

(S.F., Kallosh, Strominger)

IIB:

$$\int_{S^2 \times \gamma_3} F_5 = (p, q) \qquad 2h_{12}+2$$

D_3 branes

$$\int_{S^2 \times CY_6} F_5 \wedge \Omega = Z \quad \text{(central charge)}$$

IIA

$$\int_{S_2} F_2 = p^0 \quad 1 \qquad \int_{S_2 \times CY} F_8 = q_0 \quad 1$$

$$\int_{S_2 \times \gamma_2} F_4 = p^A \quad h_{11} \qquad \int_{S_2 \times \gamma_4} F_6 = q_A \quad h_{11}$$

D_0 D_6 D_2 D_4 branes

wrapped on C.Y. cycles.

(p^Λ, q_Λ) also (m^Λ, e_Λ)

Recent advances:

Extremal non-BPS black-holes, attractors and critical points

$$M^2_{ADM}\Big|_{\text{Horizon}} > |Z|^2_H$$

Horizon — Central charge

Kallosh, S.F. hep-th/0603247

Giryavets, Kallosh, S.F. hep-th/0606211

Bellucci, Gunaydin, Marrani, S.F. hep-th/0606209

Marrani, S.F. arXiv:0705.3866
arXiv:0706.1674

Tripathy-Trivedi

Goldstein, Iizuka, Jena, Trivedi

Kallosh. De Wit et al (Higher derivative corrections..)

Sen,

Dabholkar, Sen, Trivedi Kraus, Larsen (black rings, beyond Einstein)

Saraikin, Vafa

Cerchiai, Dall'Agata

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O.S.V. (Ooguri, Strominger, Vafa) (top. part.fun.)

Sen (Entropy function formalism)

For asymptotically flat extremal black-holes the black-hole potential is given in terms of the complex symmetric matrix

$$\mathcal{N}_{\Lambda\Sigma} = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} + i\,\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}\,, \quad \mathrm{Im}\,\mathcal{N} < 0$$

($\Lambda, \Sigma = 1 \dots n_V$ vector fields of the theory)

$\mathcal{N}(\phi^i)$ (over the moduli space)

$$\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}\, F^{\Lambda}_{\mu\nu} F^{\mu\nu\Sigma} + \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}\, F^{\Lambda}_{\mu\nu} \tilde{F}^{\mu\nu\Sigma}$$

and of the background charges

$$\frac{1}{4\pi}\int_{S_2} F^{\Lambda} = m^{\Lambda}\,, \quad \frac{1}{4\pi}\int_{S_2} \frac{\delta \mathcal{L}}{\delta F^{\Lambda}} = G_{\Lambda} = e_{\Lambda}$$

(S.F., Gibbons, Kallosh)

$$V_{BH}(\phi^i, e, m) = -\frac{1}{2}\left(e_{\Lambda} - \mathcal{N}_{\Lambda\Sigma} m^{\Sigma}\right)\left(\mathrm{Im}\,\mathcal{N}\right)^{-1\,\Lambda\Delta}\left(e_{\Delta} - \overline{\mathcal{N}}_{\Delta\Gamma} m^{\Gamma}\right)$$

This formula is valid for any theory coupling Einstein gravity to Scalars and Maxwell vector fields

For supergravity theories we can express V_{BH} in terms of dressed-charges which appear in the "fermion" transformation rules in a B-H background

$$\delta_\epsilon \lambda^I \Big|_{B\text{-}H} = \dots Z^I(\phi, e, m)\,\epsilon$$

then $V_{BH} = \dots |Z^I|^2$

Hence in N-extended supergravity

$$V_{BH} = \frac{1}{2}|Z_{AB}|^2 + |Z^I|^2$$

$Z_{AB} = -Z_{BA}$ central charge matrix

Z^I matter charges

$$\delta_\epsilon \psi_{\mu A} = \dots Z_{AB}\gamma_\mu \epsilon^B$$

$$\delta_\epsilon \lambda^I_A = \dots Z^I \epsilon_A$$

In $N=8$ Superparticles

$$V_{BH} = \frac{1}{2} |Z_{AB}|^2 \qquad A,B = 1 \ldots 8$$

$$Z_{AB} = L^{\Lambda}_{AB}(\phi) Q_\Lambda \qquad Q = (e, m)$$

$$L \in E_{7(7)}$$

$$Z(\phi \to \phi_g) \to h\, Z(\phi_g, g^{-1} Q) \qquad h \in SU(8)$$

$$V(\phi, Q) = V(\phi_g, g^{-1} Q)$$

but if we compute V at a critical point

$$V\Big|_{\partial_i V = 0} = V(Q) = V(g^{-1} Q)$$

$$V \sim \sqrt{|J_4|} = V(e, m) = V\left(Z_{AB}\Big|_H\right)$$

Cartan quartic invariant of the 56 of E_7

$$Z_{AB} = e^{i\phi/4}\begin{pmatrix} \rho_1\epsilon & & & \\ & \rho_2\epsilon & & \\ & & \rho_3\epsilon & \\ & & & \rho_4\epsilon \end{pmatrix} \qquad \epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

SU(8) rotation ↗

$$J_4 = \left[(\rho_1+\rho_2)^2-(\rho_3+\rho_4)^2\right]\left[(\rho_1-\rho_2)^2-(\rho_3-\rho_4)^2\right] + 8\rho_1\rho_2\rho_3\rho_4(\cos\phi - 1)$$

Orbits with $J_4 \neq 0$ (large black-holes)
(Gunaydin, S.F.)

$J_4 > 0$ $\quad E_{7(7)}/E_{6(2)}$ $\quad$ BPS ($\frac{1}{8}$)

$J_4 < 0$ $\quad E_{7(7)}/E_{6(6)}$ $\quad$ non BPS

$E_{6(2)} \supset SU(2) \times SU(6)$ $\quad$ MCS

$E_{6(6)} \supset USp(8)$ $\quad$ MCS

It can be shown that at $\mathcal{H}$ $N=8$ attractor points (Kallosh, S.F.)

BPS $\quad \rho = \rho_1 \neq 0 \quad \rho_2 = \rho_3 = \rho_4 = 0$

non BPS $\quad \phi = \pi \quad \rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho$

Critical Points for the $N=8$ black-hole potential (Kallosh, S.F.)

$$Z_{[AB}Z_{CD]} + \frac{1}{4!}\epsilon_{ABCDEFGH}\bar{Z}^{EF}\bar{Z}^{GH} = 0$$

$$Z_{AB} = \begin{pmatrix} z_1\epsilon & & & 0 \\ & z_2\epsilon & & \\ & & z_3\epsilon & \\ 0 & & & z_4\epsilon \end{pmatrix} \qquad \epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$z_i z_j + z_k^* z_l^* = 0 \qquad i \neq j \neq k \neq l$$
$$(i = 1,2,3,4)$$

$$V\Big|_{\text{Attractor}} = \sum_{i=1}^{4} |z_i|^2$$

$$D_i Z_{AB} = \frac{1}{2} P_{i\,ABCD}\bar{Z}^{CD}$$

$Z_{12} \neq 0 \quad Z_{34} = Z_{56} = Z_{78} = 0$

solve $\quad D_i Z_{12} = 0 \quad$ 1/8 Susy flow.

Z_{AB} has (in normal form) a symmetry $SU(2)^4$, it gets enhanced at the critical points:

$$Z^{BPS}_{AB_H} \to \begin{pmatrix} Z_1\epsilon & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad SU(2)\times SU(6)$$

$$Z^{NBPS}_{AB_H} \to e^{i\frac{\pi}{4}}\rho \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & \epsilon \end{pmatrix} \qquad USp(8)$$

Massless Directions of the Black-Hole potential

BPS: $70 = (1,15) + (1,\overline{15}) + (2,20)$

(Andrianopoli, D'Auria, S.F.)

$m \neq 0$ (N=2 Vectormult) $\quad m = 0$ (N=2 hyper)

NBPS

(Marrani, S.F.)

$70 = 42 + 27 + 1$

$m=0 \quad m\neq 0 \quad m \neq 0$

Actually these massless directions are flat directions of the potential as it can be seen by the fact that the stabilizer of the Q orbit is non-compact so that

$$g_Q Q^{BPS} = Q^{BPS} \qquad g_Q \in E_{6(2)}$$

$$g_Q Q^{NBPS} = Q^{NBPS} \qquad g_Q \in E_{6(6)}$$

and then at the critical point

$$V(\phi_{g_Q}, g_Q^{-1} Q) = V(\phi_{g_Q}, Q) = V(\phi, Q)$$

So there is an associated moduli space of solutions for the $N=8$ attractors:

BPS $\qquad E_{6(2)}/SU(6)\times SU(2)$ dim 30
(Quaternionic Manifold)

NBPS $\qquad E_{6(6)}/USp(8)$ dim 42
(5D N=8 supergra) (Marrani, SF.)

The same reasoning will apply to all N-extended supergravities based on homogeneous spaces when the stabilizer of the orbit of the "charge vector Q" is non-compact (Mazzoni, S.F.)

For $N>2$ this will apply both to BPS and non-BPS critical points (as in $N=8$). However for $N=2$ the stabilizer of the BPS orbits is compact and no flat directions will occur (apart from hypermultiplets).

Indeed from special geometry we have the general result (S.F. Gibbons Kallosh)

$$\mathcal{D}_i \mathcal{D}_{\bar{j}} V \Big|_{\text{BPS Attractor}} = 2 g_{i\bar{j}} V \Big|_{\text{BPS attractor}}$$

$g_{i\bar{j}}$: metric in moduli space

Two general classes of non-BPS attractor solutions ($\mathcal{D}_i Z \neq 0$)

$$2\bar{Z}\mathcal{D}_i Z + i C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \mathcal{D}_{\bar{j}}\bar{Z}\mathcal{D}_{\bar{k}}\bar{Z} = 0$$

1 $\bar{Z} \neq 0$, $J_4 < 0$

2 $\bar{Z} = 0$ $J_4 > 0$

Violation of the BPS bound:

$$M^2_{ADS}\big|_H = 4|Z|^2_H > |Z|^2_H \qquad 1)$$

$$M^2_{ADS}\big|_H = |\mathcal{D}_i Z|^2 > 0 \qquad 2)$$

$$\left(C_{ijk} g^{j\bar{j}} g^{k\bar{k}} \mathcal{D}_{\bar{j}}\bar{Z}\mathcal{D}_{\bar{k}}\bar{Z} = 0 \right)$$

For symmetric spaces one has

$$\mathcal{D}_i C_{jkp} = 0$$

$$g^{k\bar{k}} g^{r\bar{j}} C_{r(pq} C_{i)jk} \bar{C}_{\bar{k}\bar{r}\bar{j}} = \frac{4}{3} g_{(q|\bar{r}} C_{i|jp)}$$

$$\left(E_{\bar{r}qijp} = 0 \right)$$

	$\frac{G_V}{H_V}$	r	$dim_{\mathbb{C}} \equiv n_V$
quadratic sequence $n \in \mathbb{N}$	$\frac{SU(1,n)}{U(1)\otimes SU(n)}$	1	n
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,n)}{SO(2)\otimes SO(n)}$	$2\ (n=1)$ $3\ (n \geqslant 2)$	$n+1$
$J_3^{\mathbb{O}}$	$\frac{E_{7(-25)}}{E_{6(-78)}\otimes U(1)}$	3	27
$J_3^{\mathbb{H}}$	$\frac{SO^*(12)}{U(6)}$	3	15
$J_3^{\mathbb{C}}$	$\frac{SU(3,3)}{S(U(3)\otimes U(3))} = \frac{SU(3,3)}{SU(3)\otimes SU(3)\otimes U(1)}$	3	9
$J_3^{\mathbb{R}}$	$\frac{Sp(6,\mathbb{R})}{U(3)}$	3	6

Table 1: $\mathcal{N}=2$, $d=4$ **homogeneous symmetric special Kähler manifolds**

Quartic Norm of the exceptional Freudenthal triple

$$q = \begin{pmatrix} \alpha & x \\ y & -\beta \end{pmatrix} \qquad x, y \in J_3$$

$$I_4(q) = -\{\alpha\beta + T(x,y)\}^2 + \frac{1}{4}\{\alpha I_3(y) - \beta I_3(x) - T(x^*, y^*)\}$$

$$x^{**} = I_3(x) x$$

$$x^A \to p^A \quad y_A \to q_A \qquad \alpha \to p^0 \quad \beta \to q_0$$

$$x \to x^* \qquad p^A \to \frac{1}{2} d^{ABC} q_B q_C$$

$$y \to y^* \qquad q_A \to \frac{1}{2} d_{ABC} p^B p^C$$

$$x^{**} \to I_3(x) x \qquad p^A \to I_3(p)\, p^A$$

$$I_3(p) = \frac{1}{3!} d_{ABC} p^A p^B p^C$$

$$I_4 = -(p^0 q_0)^2 + \text{----}$$

Special Geometry

$$R_{i\bar{j}\ell\bar{k}} = -g_{i\bar{j}}\,g_{\ell\bar{k}} - g_{i\bar{k}}\,g_{\ell\bar{j}} + C_{i\ell p}\,C_{\bar{j}\bar{k}\bar{p}}\,g^{p\bar{p}}$$

$$\left(\bar{D}_{\bar{i}}\, C_{i\ell p} = 0 \qquad D_{[k}\, C_{i]\ell p} = 0 \right)$$

Symmetric Spaces

$$D_k\, C_{i\ell p} = 0$$

$$C_{i\ell p} = e^{K}\,\partial_i \partial_\ell \partial_p\, f(t^i)$$

$$f(t^i) = \frac{1}{3!}\, d_{ijk}\, t^i t^j t^k \qquad ijk \to ABC$$

$$d_{ABC}\, d^{B(PQ}\, d^{LM)C} = \frac{4}{3}\,\delta_A^{(P}\, d^{QLM)}$$

(Cremmer, van Proeyen ; Gunaydin, Sierra, Townsend)

The associated moduli spaces for non BPS attractors are given in the tables.

Hessian of V ($2n \times 2n$ symmetric)
Eigenvalues

1) $Z \neq 0$ $n-1$ massless (Tripathy, Trivedi)

2) $Z = 0$ massless according to tables

J_3^A ($A = 1, 2, 4, 8$ for R, C, H, O)

flat directions
$Z \neq 0$ $3A + 2$
$Z = 0$ $2A_c$

($D = 5$) ~~$Z \neq 0$~~ $2A$

	H_0	$\widehat{H}$	$\widetilde{H}$	$\widehat{h} \equiv m.c.s.\left(\widehat{H}\right)$	$\widetilde{h}' \equiv \frac{m.c.s.\left(\widetilde{H}\right)}{U(1)}$
I	$SU(n+1)$	$-$	$SU(1,n)$	$-$	$SU(n)$
II	$SO(2) \otimes SO(2+n)$	$SO(1,1) \otimes SO(1,1+n)$	$SO(2) \otimes SO(2,n)$	$SO(1+n)$	$SO(2) \otimes SO(n)$
III	$E_6 \equiv E_{6(-78)}$	$E_{6(-26)}$	$E_{6(-14)}$	$F_4 \equiv F_{4(-52)}$	$SO(10)$
IV	$SU(6)$	$SU^*(6)$	$SU(4,2)$	$USp(6)$	$SU(4) \otimes SU(2)$
V	$SU(3) \otimes SU(3)$	$SL(3,\mathbb{C})$	$SU(2,1) \otimes SU(1,2)$	$SU(3)$	$SU(2) \otimes SU(2) \otimes U(1)$
VI	$SU(3)$	$SL(3,\mathbb{R})$	$SU(2,1)$	$SO(3)$	$SU(2)$

Table 8: **Stabilizers and corresponding m.c.s.s of the non-degenerate classes of orbits of $N=2$, $d=4$ symmetric MESGTs.** $\widehat{H}$ and $\widetilde{H}$ are real (non-compact) forms of H_0, the stabilizer of $\frac{1}{2}$-BPS orbits.

The J_3^{O} (Octonion) example

Special geometry ($D=4$) $E_{7(-25)}/E_6 \times U(1)$

$Q = 56$ of $E_{7(-25)}$ 1 quasiphoton 27 matter vector multiplets

BPS orbit $E_{7(-25)}/E_{6(-78)}$

no flat directions

NBPS orbits: $Z \neq 0$ $E_{7(-25)}/E_{6(-26)}$

flat directions: $E_{6(-26)}/F_{4(-52)}$
26

WBPS orbit: $Z = 0$ $E_{7(-25)}/E_{6(-14)}$

flat directions: $E_{6(-14)}/SO(10) \times U(1)$
$32 = 16_c$

$D=5$ Attractors of real special geometry

non BPS orbit: $E_{6(-26)}/F_{4(-20)}$ → dim 26

Flat directions: $F_{4(-20)}/SO(9)$: 16

	$\frac{\widetilde{H}}{\widetilde{h}} = \frac{\widetilde{H}}{\widetilde{h}' \otimes U(1)}$	r	$dim_{\mathbb{C}}$
quadratic sequence $n \in \mathbb{N}$	$\frac{SU(1,n-1)}{U(1) \otimes SU(n-1)}$	1	$n-1$
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$\frac{SO(2,n-2)}{SO(2) \otimes SO(n-2) \otimes U(1)},\ n \geqslant 3$	$1\ (n=3)$ $2\ (n \geqslant 4)$	$n-2$
$J_3^{\mathbb{O}}$	$\frac{E_{6(-14)}}{SO(10) \otimes U(1)}$	2	16
$J_3^{\mathbb{H}}$	$\frac{SU(4,2)}{SU(4) \otimes SU(2) \otimes U(1)}$	2	8
$J_3^{\mathbb{C}}$	$\frac{SU(2,1)}{SU(2) \otimes U(1)} \otimes \frac{SU(1,2)}{SU(2) \otimes U(1)}$	2	4
$J_3^{\mathbb{R}}$	$\frac{SU(2,1)}{SU(2) \otimes U(1)}$	1	2

Table 1: **Moduli spaces of non-BPS $Z = 0$ critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}= 2$, $d = 4$ homogeneous symmetric supergravities. They are (non-special) homogeneous symmetric Kähler manifolds.**

	$\frac{\hat{H}}{\hat{h}}$	r	$dim_{\mathbb{R}}$
$\mathbb{R} \oplus \mathbf{\Gamma}_n,\ n \in \mathbb{N}$	$SO(1,1) \otimes \frac{SO(1,n-1)}{SO(n-1)}$	$1\ (n=1)$ $2\ (n \geqslant 2)$	n
$J_3^{\mathbb{O}}$	$\frac{E_{6(-26)}}{F_{4(-52)}}$	2	26
$J_3^{\mathbb{H}}$	$\frac{SU^*(6)}{USp(6)}$	2	14
$J_3^{\mathbb{C}}$	$\frac{SL(3,\mathbb{C})}{SU(3)}$	2	8
$J_3^{\mathbb{R}}$	$\frac{SL(3,\mathbb{R})}{SO(3)}$	2	5

Table 1: **Moduli spaces of non-BPS $Z \neq 0$ critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}=2$, $d=4$ homogeneous symmetric supergravities. They are the $\mathcal{N}=2$, $d=5$ homogeneous symmetric real special manifolds.**

	$\frac{\tilde{H}_5}{K_5}$	r	$dim_{\mathbb{R}}$
$\mathbb{R}\oplus\mathbf{\Gamma}_n,\ n\in\mathbb{N}$	$\frac{SO(1,n-2)}{SO(n-2)},\ n\geqslant 3$	$1\ (n\geqslant 3)$	$n-2$
$J_3^{\mathbb{O}}$	$\frac{F_{4(-20)}}{SO(9)}$	1	16
$J_3^{\mathbb{H}}$	$\frac{USp(4,2)}{USp(4)\otimes USp(2)}$	1	8
$J_3^{\mathbb{C}}$	$\frac{SU(2,1)}{SU(2)\otimes U(1)}$	1	4
$J_3^{\mathbb{R}}$	$\frac{SL(2,\mathbb{R})}{SO(2)}$	1	2

Table 1: **Moduli spaces of non-BPS critical points of $V_{BH,\mathcal{N}=2}$ in $\mathcal{N}=2$, $d=5$ homogeneous symmetric supergravities.**