OPE coefficients, string field theory vertex and integrability

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Outline

The AdS/CFT correspondence – Key questions

$\boxed{\mathcal{N} = 4 \text{ SYM theory}} \equiv \boxed{\text{type IIB superstring theory on } AdS_5 \times S^5}$

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

- Find the OPE coefficients C_{ijk} defined through

$$\langle O_i(x_1)O_j(x_2)O_k(x_3)\rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_i+\Delta_j-\Delta_k}|x_1 - x_3|^{\Delta_i+\Delta_k-\Delta_j}|x_2 - x_3|^{\Delta_j+\Delta_k-\Delta_i}}$$

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Main current problem:

Find a framework for determining the OPE coefficients of $\mathcal{N} = 4$ SYM at any coupling

Why (light-cone) string field theory is interesting?

- It may serve as the appropriate framework...
- The use of integrability for string **interactions** would be fascinating!

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The AdS/CFT dictionary

Local operators in $\mathcal{N}=4$ SYM	$\equiv$	string states in $AdS_5 \times S^5$
Δ	$\equiv$	E
$\mathrm{tr}\, ZZXZXZ$		$J \equiv J_1 = 4 \; J_2 = 2$
		angular momentum on S^5
many scalar fields		spinning strings ($J_i \propto \sqrt{\lambda}$)

The spectral problem in $\mathcal{N}=4$ SYM is

- equivalent to finding the quantized energy levels of a string in $AdS_5 \times S^5$
- once we pass to e.g. uniform light cone gauge, this is equivalent to finding the energy levels of a specific integrable 2D QFT on a cylinder of size J

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Interesting classes of operators

many Z's and X's	$\longleftrightarrow$	large angular momenta $\supset$ classical string states **Heavy** operators ($\Delta \propto \lambda^{\frac{1}{2}}$)
few Z's and X's	$\longleftrightarrow$	supergravity modes ($\Delta \propto \lambda^{0}$) or lightest massive string modes ($\Delta \propto \lambda^{\frac{1}{4}}$) **Light** (or **Medium**) operators
many Z's and **few** X's with $p_i = \mathcal{O}(1)$	$\longleftrightarrow$	**Heavy** operators but not of *spinning string* type

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How to solve the spectral problem?

I) solve the theory on an infinite plane

symmetry + Yang-Baxter equation
+ crossing
+ unitarity
$\longrightarrow$ **S-matrix**

II) solve the theory on a (large!) cylinder

Bethe Ansatz Quantization

$$e^{ip_k L} \prod_{l \neq k} S(p_k, p_l) = 1$$

Get the energies from
$$E = \sum_k E(p_k) = \sum_k \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p_k}{2}}$$

This gives the spectrum up to wrapping corrections...

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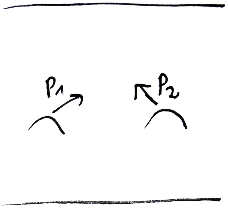

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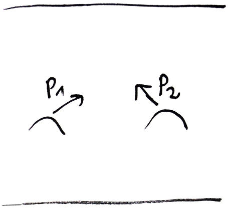

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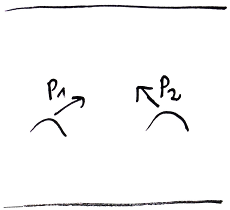

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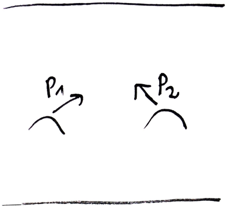

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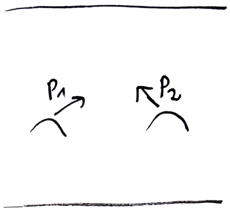

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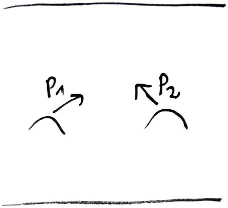

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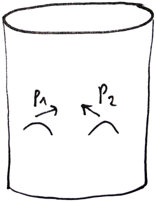

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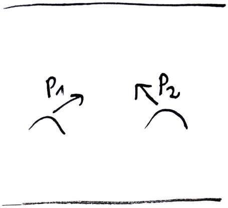

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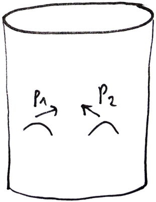

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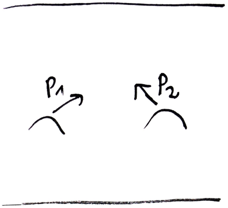

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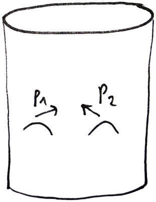

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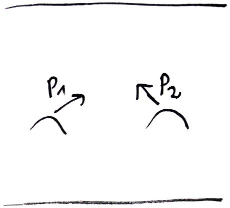

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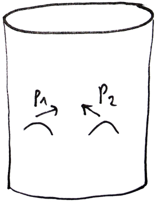

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III) Include leading wrapping corrections...

— generalized Lüscher formulas

IV) Resum all wrapping corrections

— Thermodynamic Bethe Ansatz
$\longrightarrow$ Quantum Spectral Curve

Comments

- The basic steps follow the strategy used for solving ordinary relativistic integrable quantum field theories...
  (despite numerous subtleties and novel features)
- **Key role** of the infinite plane $\longrightarrow$ only there do we have crossing+analyticity which allows for solving for the S-matrix
  (functional equations for the S-matrix)
- Up to wrapping corrections, the finite volume spectrum follows very easily

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Why are the OPE coefficients challenging?

We need to compute a quantum amplitude:

figure from Zarembo 1008.1059

- There is no analogous problem in relativistic integrable theories!
- This is a worldsheet 3-point function in conformal gauge of the string but we do not have any integrable (or other) formulation of this!!

Nevertheless a lot of progress has been made...

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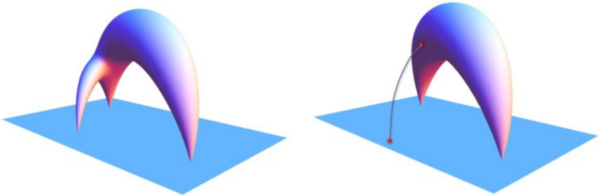

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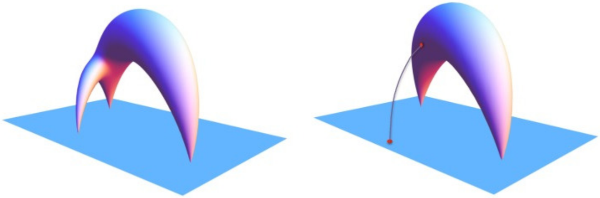

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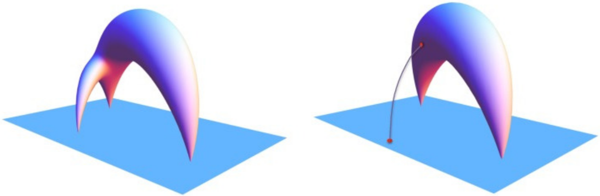

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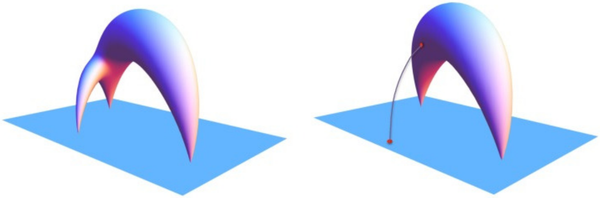

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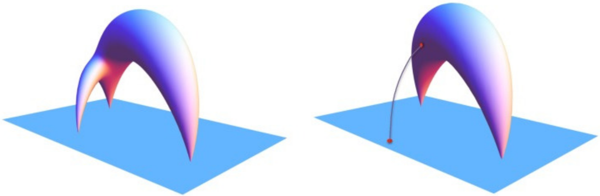

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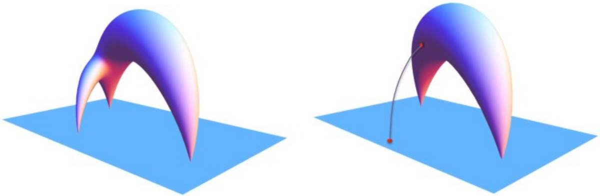

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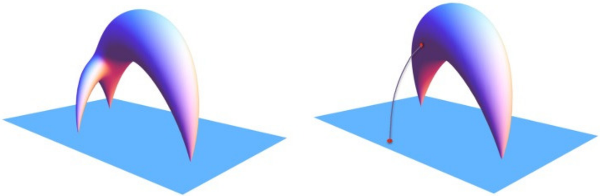

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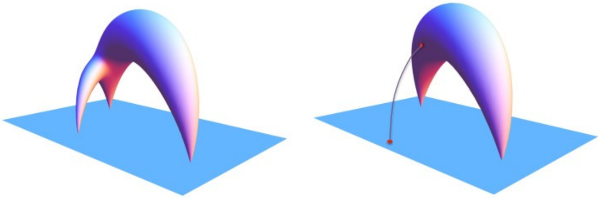

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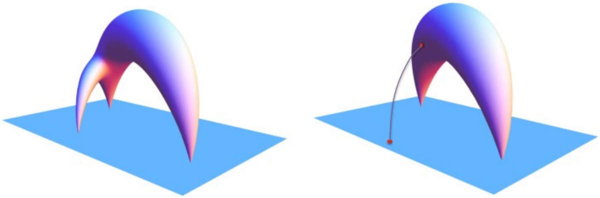

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- This **does not** mean that we are concentrating on the strong coupling side!
- An integrable approach should work at any coupling...
- We would like to develop an approach neglecting *wrapping corrections*

Recall the spectral problem...

- It was crucial to have **an infinite volume formulation** in order to derive functional equations
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$$ -i\, \mathrm{res}_{\theta'=\theta}\, f_{n+2}(\theta', \theta + i\pi, \theta_1, \ldots, \theta_n) = (1 - \prod_i S(\theta, \theta_i))\, f_n(\theta_1, \ldots, \theta_n) $$

- Solutions explicitly known for numerous relativistic integrable QFT's

Form factors

- Up to wrapping corrections ($\sim e^{-mL}$), very simple way to pass to finite volume (cylinder of circumference L): Pozsgay, Takacs

$$\langle \varnothing | \mathcal{O}(0) | \theta_1, \theta_2 \rangle_L = \frac{1}{\sqrt{\rho_2 \cdot S(\theta_1, \theta_2)}} \cdot f(\theta_1, \theta_2)$$

where θ_1, θ_2 satisfy Bethe ansatz quantization and ρ_2 is essentially the Gaudin norm

- Relation to **Heavy-Heavy-Light** correlators:

Bajnok, RJ, Wereszczyński

$$\longrightarrow \quad C_{HHL} \sim \int_{Moduli} \int d^2\sigma \; V_L(X^I(\sigma))$$

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- Definitely requires testing away from strong coupling...

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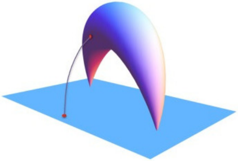

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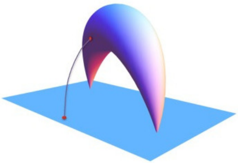

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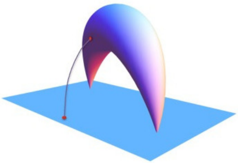

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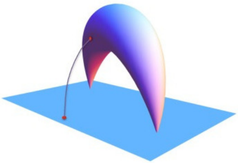

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Pros:

- In principle can work at any coupling!
- Natural ∞-volume setting and finite volume reduction
- Distinctive finite volume behaviour (in the relevant diagonal case)

Cons:

- For OPE coefficients applicable directly only when J charge (all R-charges?) of the initial and final state/operator coincide! (J charge defines the size of the cylinder)
- This is not a generic situation as typically we only have $J_1 + J_2 = J_3$ in a 3-point correlation function
- The formulation is very asymmetrical between the two operators corresponding to the initial and final state and the third 'local' worldsheet operator
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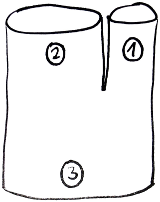

- String Field Theory vertex describes the splitting/joining of 3 strings with generic sizes $J_1 + J_2 = J_3$
- In the case of the pp-wave limit of $AdS_5 \times S^5$, SFT vertex was used to compute various OPE coefficients for a class of gauge theory operators (so-called BMN operators)
- However, in general, the relation between the SFT vertex and OPE coefficients has not been settled c.f. Dobashi, Yoneya but see also Zayakin, Schulgin

Our goal: Concentrate first on defining the string field theory vertex for a generic integrable worldsheet theory

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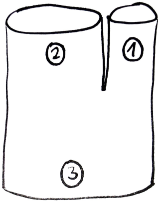

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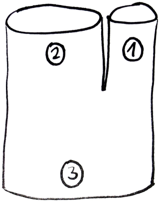

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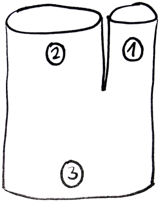

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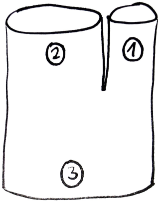

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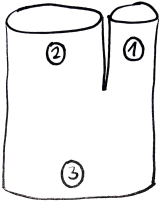

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- impose continuity conditions for ϕ and $\Pi \equiv \partial_t \phi$
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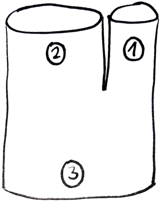

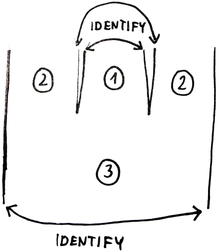

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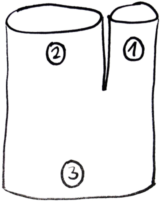

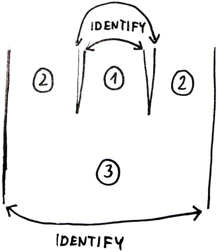

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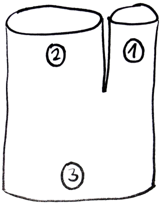

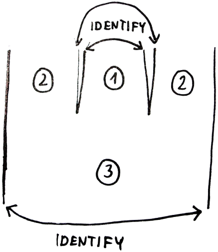

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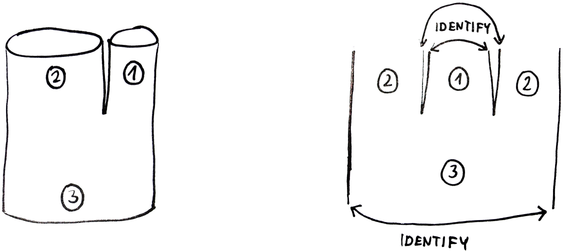

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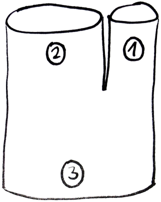

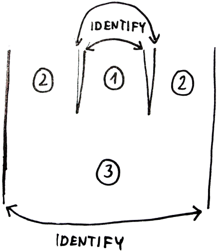

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- Implement these relations as operator equations acting on a state $|V\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$
- The state has the form up to a possible prefactor...

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- Obtaining the Neumann matrices is surprisingly nontrivial as it involves inverting an infinite-dimensional matrix

He, Schwarz, Spradlin, Volovich
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$$\sum_{r=1}^{3} \frac{X^r_{nm}}{\sqrt{\omega^r_m}} \left(a^{+(r)}_m - a^{(r)}_m \right) = 0 \quad \sum_{r=1}^{3} s_r X^r_{nm} \sqrt{\omega^r_m} \left(a^{+(r)}_m + a^{(r)}_m \right) = 0$$

- Implement these relations as operator equations acting on a state $|V\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$
- The state has the form up to a possible prefactor...

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- Obtaining the Neumann matrices is surprisingly nontrivial as it involves inverting an infinite-dimensional matrix

He, Schwarz, Spradlin, Volovich
$\rightarrow$ Lucietti, Schafer-Nameki, Sinha

- Involves some novel special functions $\Gamma_\mu(m)$

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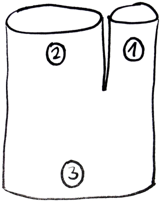

Or equivalently...

String #1 still remains of finite size... ($L = L_1$)

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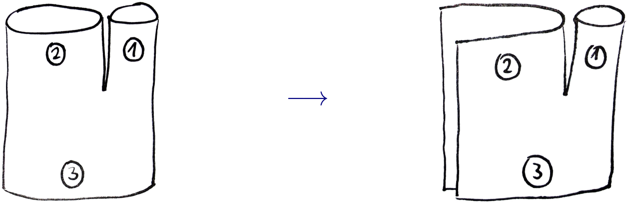

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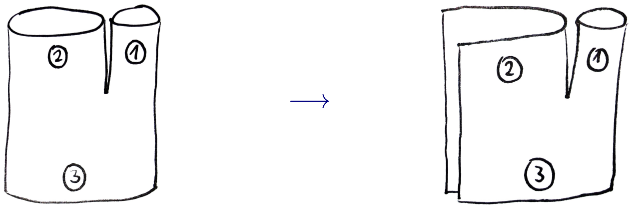

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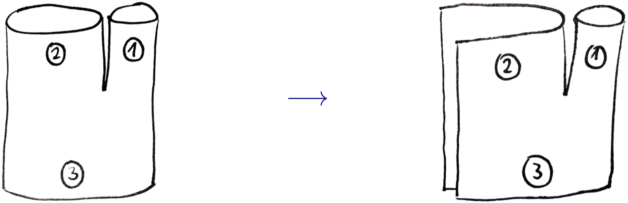

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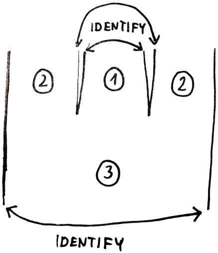

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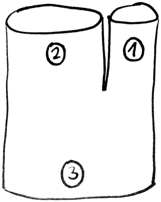

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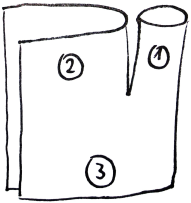

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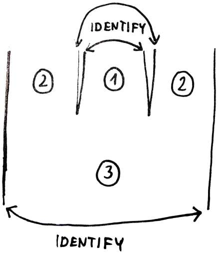

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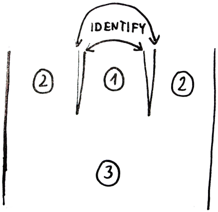

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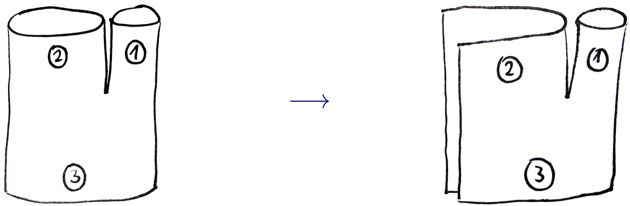

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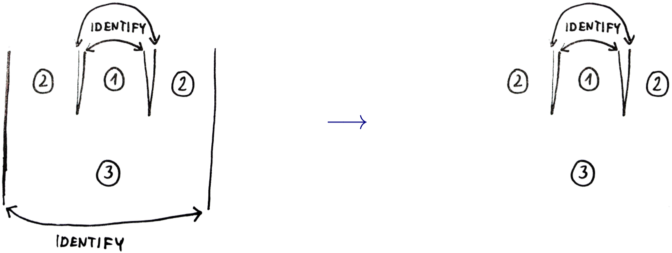

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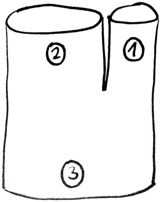

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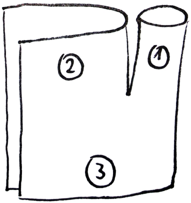

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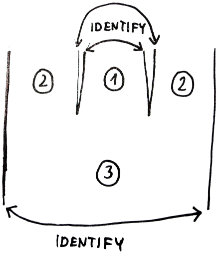

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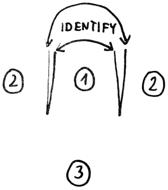

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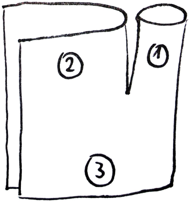

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Formulate functional equations...

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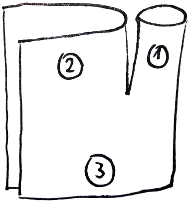

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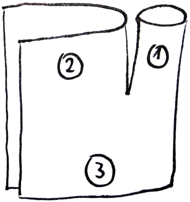

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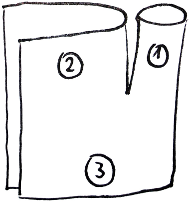

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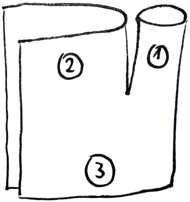

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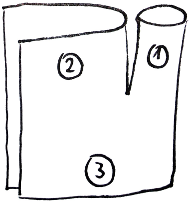

- The emission of string #1 can be understood as an insertion of some macroscopic (not completely local) operator...
- Looks like some kind of generalized form factor with ingoing particles in string #3 and outgoing ones in string #2
- **Key difference:** string #1 'eats up volume' $\longrightarrow$ the operator should have a e^{ipL} branch cut defect...

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Functional equations for the (decompactified) string vertex

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N^{33}(\theta+i\pi+\varepsilon,\theta) &= \frac{1}{\varepsilon}(1-e^{ipL})F_0+reg
\end{aligned}
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In addition, we have phase factors for crossing

$$N^{32}(\theta_1,\theta_2)=e^{i\frac{p_1L}{2}}N^{33}(\theta_1,\theta_2-i\pi)$$

- The exact pp-wave solution (for $S(\theta_1,\theta_2)=1$), involving the $\Gamma_\mu(m)$ special function solves these equations (and can be reconstructed from them...)
- This includes all wrapping corrections for the #1 string
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Comments:

- The solution of the above equations involves all e^{-mL} corrections..
- The asymptotic solution is much simpler:

$$N^{33}_{asympt}(\theta_1,\theta_2) = \frac{-1}{\cosh\frac{\theta_1-\theta_2}{2}} \cdot \sin\frac{p_1 L_1}{2} \sin\frac{p_2 L_1}{2}$$

- Functional equations for $N^{rs}_{asympt}(\theta_1,\theta_2)$???

Surprise: Large L limit does not commute with $\theta \to \theta + i\pi$!!

- In order to have a chance for a unique solution we need to understand the analyticity properties of the solutions
- By examining the explicit case of the pp-wave we see that the knowledge about the location of zeroes is crucial...

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Analyticity properties – some heuristics

- Recall the expression

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- The puzzling $\sin \frac{p_1 L_1}{2}$ appear also in the exact expression
- In contrast $N^{22}(\theta_1, \theta_2)$ does not have these factors:

$$N^{22}_{asympt}(\theta_1, \theta_2) = \frac{1}{4 \cosh \frac{\theta_1 - \theta_2}{2}}$$

- What is the difference?

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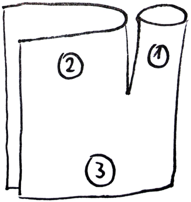
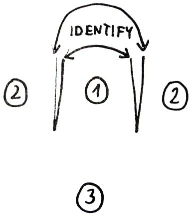


The decompactified string vertex – analyticity properties



- The condition $\sin \frac{pL_1}{2}$ means that the phase factor

$$e^{ipL_1} = 1$$

  so a plane wave with such a momentum does not *feel* string #1...
- Such a plane wave incoming from string #3 is a perfectly smooth plane wave on string #2...
- So it should be 'orthogonal' to the vacuum $\longrightarrow$ the Neumann coefficient should vanish
- On the other hand, such a plane wave on string #2 continued back in time to string #3 will always have some junk below string #1
- So there should be nonzero overlap with everything on string #3, hence nonzero Neumann coefficient

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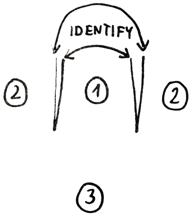

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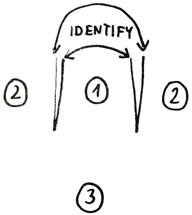

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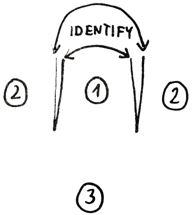

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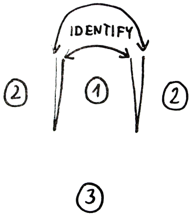

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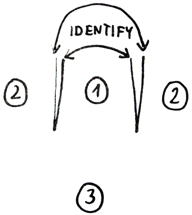

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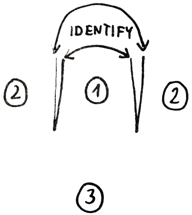

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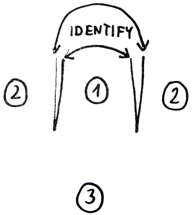

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$$e^{ipL_1} = 1$$

  so a plane wave with such a momentum does not *feel* string #1...
- Such a plane wave incoming from string #3 is a perfectly smooth plane wave on string #2...
- So it should be 'orthogonal' to the vacuum $\longrightarrow$ the Neumann coefficient should vanish
- On the other hand, such a plane wave on string #2 continued back in time to string #3 will always have some junk below string #1
- So there should be nonzero overlap with everything on string #3, hence nonzero Neumann coefficient

The program — back to finite volume

We considered so far the 'decompactified string vertex'...

but ultimately we are interested in the finite volume one...

Main idea:

- Look at the vertex from two points of view
  1. Keep strings #2 and #3 decompactified
  2. Keep strings #1 and #3 decompactified
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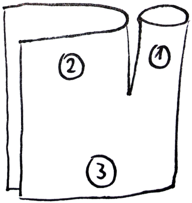

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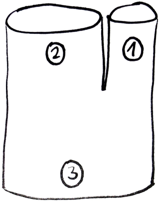

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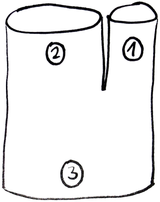

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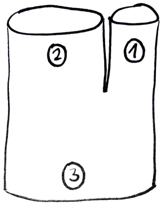

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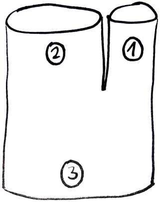

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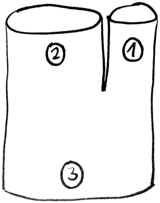

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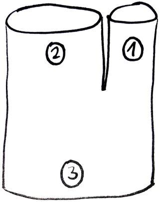

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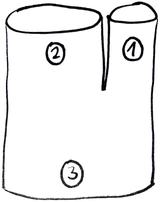

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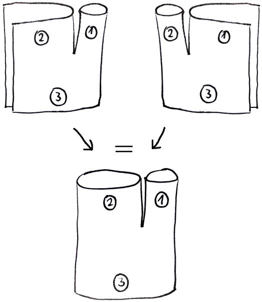

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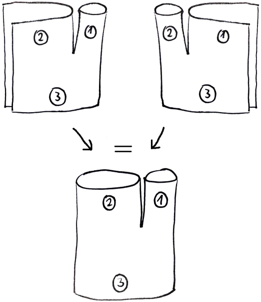

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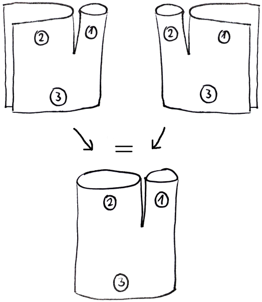

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- Ideally, these approaches should work at any coupling (possibly up to wrapping corrections)
- A key step is the existence of an infinite volume setup, which allows for formulating functional equations incorporating e.g. crossing
- Second step involves reduction to (large) finite size
- Form factors and string field theory vertex seem to be promising (complementary) candidates
- String field theory axioms are similar in flavour to form factor ones..
- We reproduced pp-wave string field theory formulas for the Neumann coefficients
- Kinematical singularity can be observed also in some weak coupling results
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